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WZW INTERACTIONS IN DIMENSIONAL DECONSTRUCTION AND ATAVISTIC LIE ALGEBRAS

Dimensional deconstruction reduces higher-dimensional pure gauge theories into 4D matter-coupled gauge theories, such as current models for electroweak interactions.

• Anomaly and topological structure of such pure (mesonless) gauge theories: 5D Chern-Simons terms,

$$\mathcal{L} = \frac{N_c}{48\pi^2}\epsilon^{ABCDE}\,\mathrm{Tr}\Big(A_A\partial_B A_C\partial_D A_E - \frac{3i}{2}A_A A_B A_C \partial_D A_E - \frac{3}{5}A_A A_B A_C A_D A_E\Big)$$

$\rightsquigarrow$ Wess-Zumino-Witten terms in 4D $SU(N)\times SU(N)$ chiral models,

$$\frac{2N_c}{15\pi^2 f_\pi^5}\epsilon_{\mu\nu\rho\sigma}\,\mathrm{Tr}(\tilde\pi\partial^\mu\tilde\pi\partial^\nu\tilde\pi\partial^\rho\tilde\pi\partial^\sigma\tilde\pi) + O(\tilde\pi^7),$$

required in effective QCD, but, so far, enigmatically, inaccessible to deconstruction methods.

$\rightsquigarrow$ Pegs parity and allows for odd-point-functions of pseudoscalar mesons (e.g., $2K3\pi$), or K-K modes—the future!

Development of a new deconstruction technique: compactify the theory on an orbifold, by structuring special delocalized brane assignments dictated by underlying anomalies. The pseudoscalar mesons (goldstons) arise from lattice Wilson links, $U = P\exp\left(-i\int_0^a dx^4 A_4\right) = \exp(2i\tilde{\pi}/f_\pi)$. Chiral interactions due to delocalization—vanish upon branes merging.

These new correspondence rules were discovered by examining the consistency of covariant derivatives involving the lowest K–K modes with the anomaly and topological structure of these theories (Bianchi-consistent).

• Revisit, gingerly, $SU(3)$ group invariant volume ,"Euler angles" (cf Holland, Marinov, Byrd).
Technical obstacle: since its inception, the 4D WZW term has **not been written down explicitly to all oders** for chiral models. (It has

for 2D; and for 4D hyperspherical σ-models.) So all-orders comparison not at hand. Intermediate implicit form found with 4 extra parametric "dimensions",

$$\propto \epsilon_{\mu\nu\rho\sigma} \int_0^1 dt_1 dt_2 dt_3 dt_4 \, \Big(\theta(t_1 - t_2)\theta(t_2 - t_3)\theta(t_3 - t_4) + \text{perms}\Big)$$

$$\times \quad \mathrm{Tr}\Big(U^{t_4-t_1}\partial^\mu \tilde{\pi} \; U^{t_1-t_2}\partial^\nu \tilde{\pi} \; U^{t_2-t_3}\partial^\rho \tilde{\pi} \; U^{t_3-t_4}\partial^\sigma \tilde{\pi}\Big).$$

• Transition of groups along a link from site to site (brane to brane), leads to the consideration/introduction of an entire class of infinite-dimensional "Atavistic Lie Algebras". Effectively, these encompass **most Lie algebras utilized in physics**, ($GL(N)$), Classical Lie, Moyal, Poisson, Virasoro, Vertex):

$$\boxed{[J^a_{m_1,m_2}, J^b_{n_1,n_2}] =}$$

$$e^{is(m_1 e^{-a} n_2 - m_2 e^{a} n_1)} J^{a+b}_{m_1+e^a n_1, m_2+e^{-a} n_2} - e^{is(n_1 e^{-b} m_2 - n_2 e^{b} m_1)} J^{a+b}_{n_1+e^b m_1, n_2+e^{-b} m_2}.$$

• Corresponds to a delocalized "twist" of the star product,

$$\star\ D(a) \equiv \exp\left(-is(\overleftarrow{\partial}_x \overrightarrow{\partial}_p - \overleftarrow{\partial}_p \overrightarrow{\partial}_x)\right)\ \exp\left(a(x\ \overrightarrow{\partial}_x - p\ \overrightarrow{\partial}_p)\right),$$

so it acts differently on the left and right branes.

• Coherent state realization,

$$\boxed{J^a_{m_1,m_2} = e^{im_1\alpha^\dagger + sm_2\alpha}\ e^{a\ \alpha^\dagger\alpha}},$$

underlain by (dim 4, rank 2) oscillator group; $\rightsquigarrow$ vertex operator, finite matrix, and tensor product realizations,

• Applications in deconstruction, noncommutative QFT, and possibly twisted CFT.