Lattice Gauge Theory

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Lattice field theories beyond the standard model

- Lattice Born-Infeld QED

Thermodynamics of Lattice QCD

- The universality class of the finite temperature transition with staggered quarks.
- Lattice QCD at finite temperature and density

Lattice field theories beyond the standard model

We are interested in applying lattice gauge theory simulation methods to beyond the standard model physics. Born-Infeld QED is a simple demonstration project.

Born-Infeld non-linear QED

Born-Infeld action arises naturally as a field theory for strings and branes.

$$\boldsymbol{S = b^2 \int d^{n+1}x[1 - \sqrt{-\det(g_{\mu\nu} + b^{-1}F_{\mu\nu})}]}$$

Typically $n = 9$. Dimensionally reduced to $n = p$, this describes a p-brane with $n - p$ transverse degrees of freedom.

We will consider the original $n = 3$ theory. Defining $\boldsymbol{D} = \partial\mathcal{L}/\partial\boldsymbol{E}$ and $\boldsymbol{H} = \partial\mathcal{L}/\partial\boldsymbol{B}$, the equations of motion have the normal Maxwell form. The classical fields for a point charge are

$$\boldsymbol{D} = \frac{e}{4\pi r^3}\boldsymbol{r} \qquad \boldsymbol{E} = \frac{e}{4\pi r}\frac{\boldsymbol{r}}{\sqrt{r^4 + r_0^4}}$$

with $r_0 = \sqrt{|e|/4\pi b}$ – short distance screening of $\boldsymbol{E}$.

Quantize by performing lattice Monte-Carlo simulations in Euclidean space-time. Point charge is introduced by including a Wilson line (Polyakov loop) in the functional integral. Sign problem associated with this Wilson line can be circumvented.

$\boldsymbol{D}$ field is identical to Maxwell case as expected.

$\boldsymbol{E}$ field is enhanced by quantum fluctuations. It exhibits short-distance screening as in the classical case.

The $b \to 0$ (or $a \to 0$) limit is a conformal field theory with $\mathcal{L}_E = |\boldsymbol{E} \cdot \boldsymbol{B}|$ and $\mathcal{H} = |\boldsymbol{D} \times \boldsymbol{B}|$ which describes the small $\beta = b^2 a^4$ behaviour of its electrostatics.

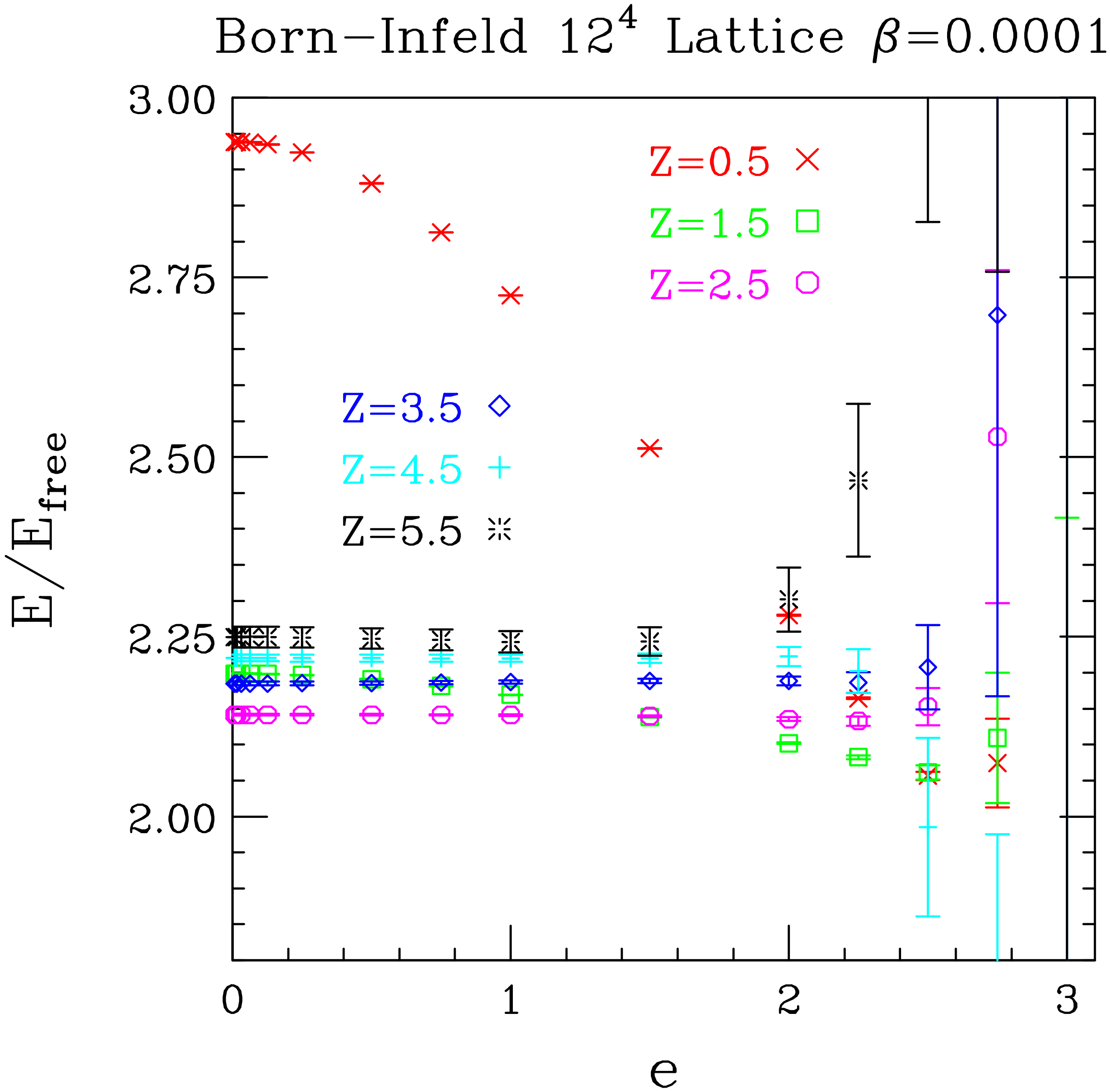

Figure 1: Screening of the electric field at small β (non-linear limit).

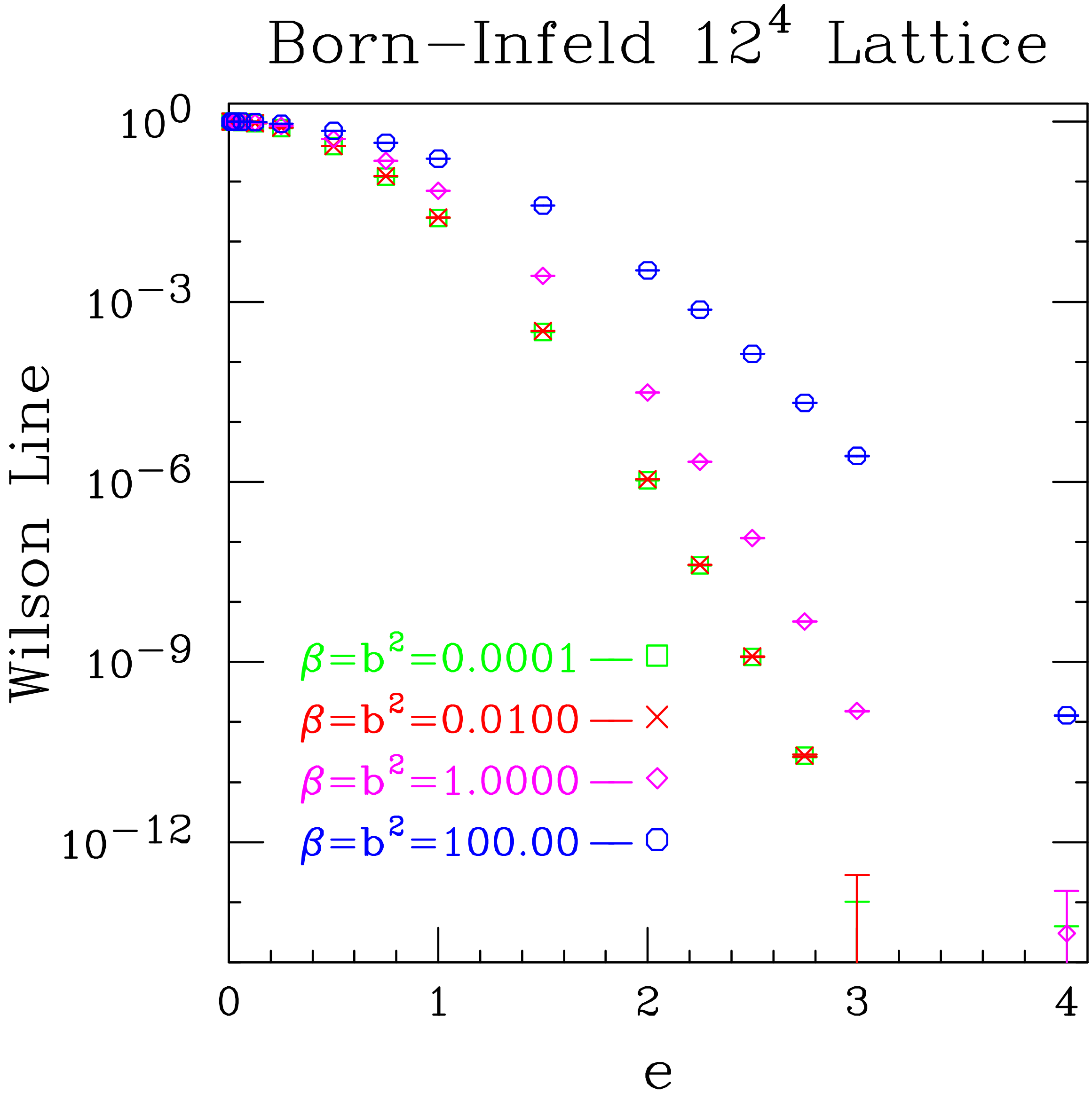

Figure 2: Wilson lines (Polyakov loops) as functions of e. Essentially exp[−(free energy of charge e)× time].

Thermodynamics of Lattice QCD

QCD at finite temperature and/or densities.

Relevant to physics of the early universe, relativistic heavy-ion collisions – RHIC, CERN, GSI-SIS... –, neutron stars.

A very useful probe of QCD dynamics – chiral symmetry breaking, confinement, alternate symmetry breaking patterns in extreme environments (colour superconductivity...).

Sensitivity of lattice QCD thermodynamics to details of fermion dynamics allows crucial tests of lattice fermion transcriptions and simulation methods. Hadron masses and matrix elements are much less sensitive to these details.

Lattice QCD with 2 massless staggered quark flavours: The universality class of the finite temperature transition

For 2 massless quark flavours, the universality class of the finite temperature transition from hadronic matter to a quark-gluon plasma is argued to be that of the $O(4)$ spin-model (non-linear sigma model) in 3 dimensions.

The reduced symmetry of the staggered quark action could be expected to reduce this to that of the $O(2)$ spin model.

Previous simulations have had difficulty in making either of these identifications. The problems are 2-fold. First it is difficult to get the quark masses small enough to see the chiral limit. Second, finite volume effects are large on the lattice sizes used.

We have introduced a new lattice action incorporating an irrelevant chiral 4-fermion interaction of the Gross-Neveu type "χQCD" which allows zero-mass simulations.

We circumvent the finite volume problem by fitting the behaviour of the chiral condensate to the magnetizations of the $O(2)$ spin model also on finite volumes. These fits indicate that the critical behaviour of lattice QCD with staggered quarks is consistent with that of the 3-dimensional $O(2)$ spin model.

Further studies of the $O(2)$ spin model on finite volumes indicate why previous attempts to establish $O(2)$ or $O(4)$ universality failed.

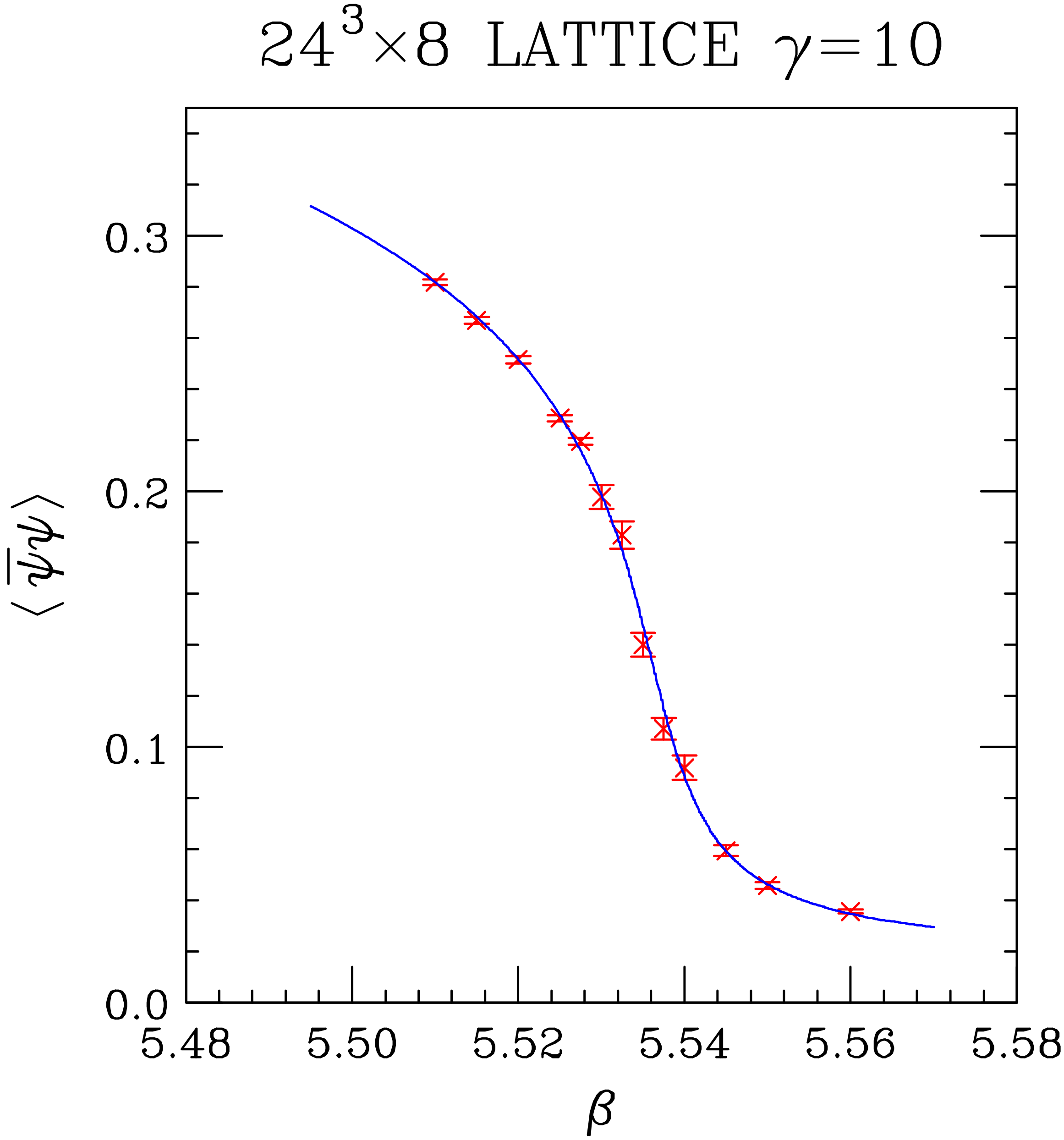

Figure 3: Fit of the lattice QCD chiral condensate on a $24^3 \times 8$ lattice to the magnetization of the $O(2)$ spin model on an 8^3 lattice.

Lattice QCD at non-zero temperature and density

We study lattice QCD at small isospin density, close to the finite temperature transition from hadronic matter to a quark-gluon plasma. Here it can be argued that the phase structure and transition temperature should be the same as that for QCD at non-zero quark/baryon-number density.

Finite isospin density is achieved by incorporating an isospin chemical potential μ_I in the action. This preserves the positivity of the fermion determinant allowing simulations to be performed. (In contrast, a quark-number chemical potential makes the fermion determinant complex).

We are searching for a critical endpoint where the transition changes from being a crossover to being a first-order transition. We are using 3 quark flavours where the existence of a critical mass where the zero chemical potential transition also changes from a crossover to a first-order transition suggested that one could tune the critical endpoint to be as close to zero chemical potential as one desired.

Binder cumulants B_4 are used to search for the critical endpoint where $B_4 = 1.604(1)$.

$$B_4(\bar{\psi}\psi) = \left[\frac{\langle\delta(\bar{\psi}\psi)^4\rangle}{\langle\delta(\bar{\psi}\psi)^2\rangle^2}\right]$$

where $\delta(\bar{\psi}\psi) = \bar{\psi}\psi - \langle\bar{\psi}\psi\rangle$.

We discovered that unfortunately this quantity is very sensitive to the updating increment dt in the hybrid molecular-dynamics simulation algorithm that we and everyone else were using.

We have now switched to a new simulation method, RHMC which has no such finite dt systematic errors.

Our preliminary results indicate that there is probably no critical endpoint for small chemical potentials, and that claims to the contrary probably suffer from uncontrolled finite dt systematics.

The critical mass (in lattice units) on this size lattice is ≈ 0.025 compared with $0.033(1)$ from the earlier predictions from 3 separate groups.

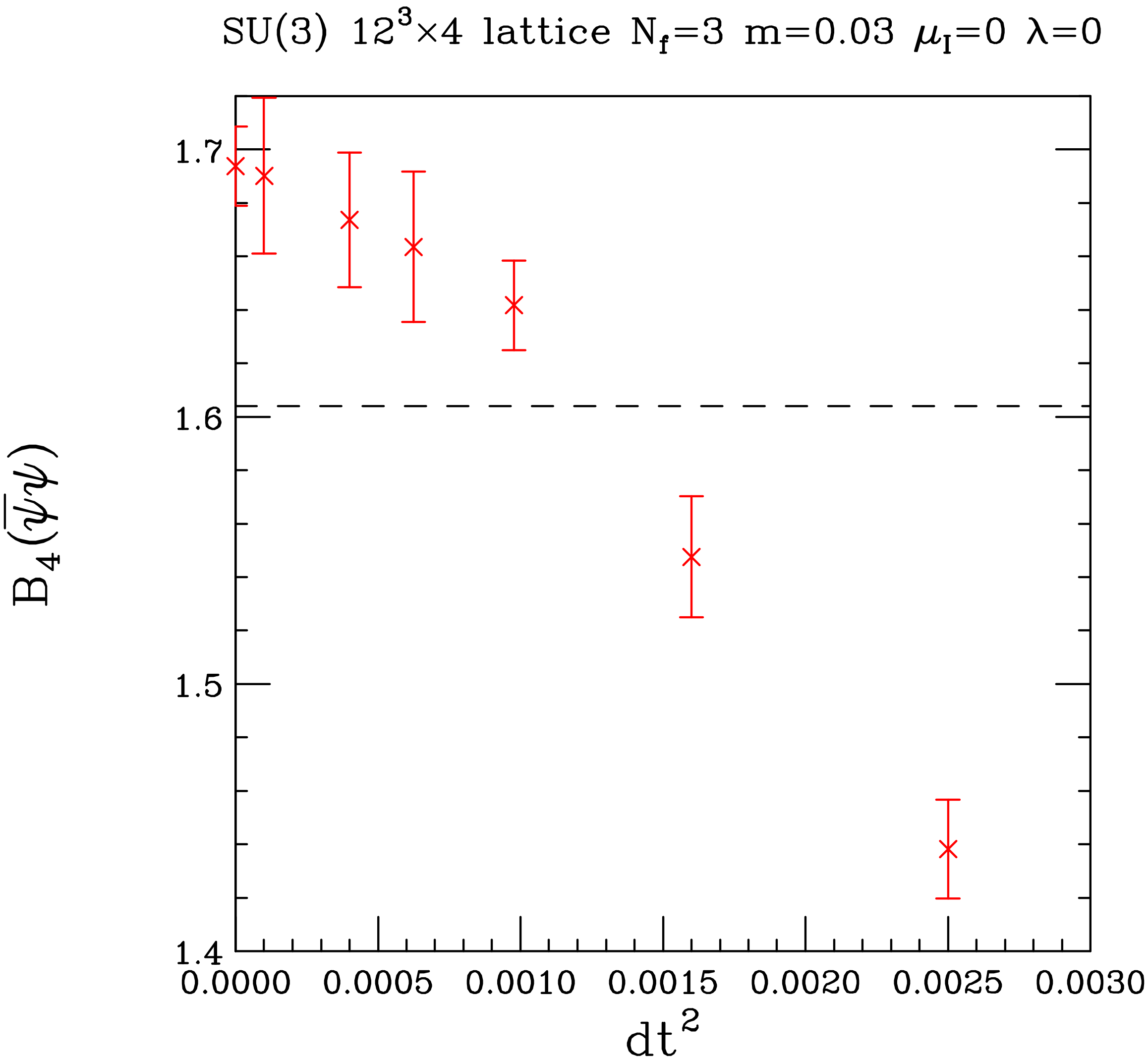

Figure 4: dt dependence of the Binder cumulant at $m = 0.03$, $\mu_I = 0$.

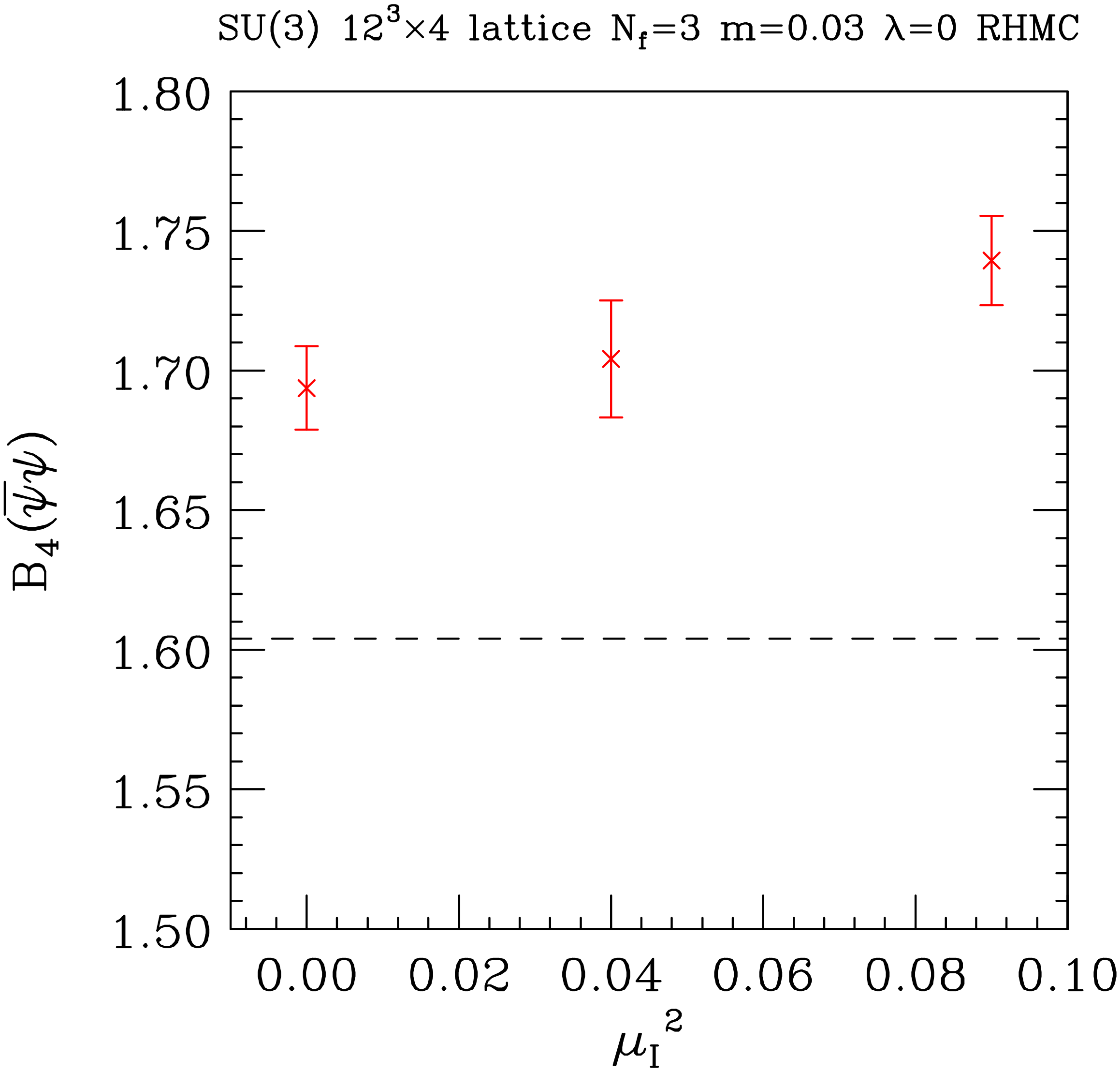

Figure 5: μ_I dependence of the Binder cumulant at $m = 0.03$.

Computers used for simulations

- ANL HEP — pcl8, theory
- ANL LCRC — Jazz
- ANL MCS — BlueGene/L
- NERSC — Seaborg, Jacquard, Bassi
- NCSA — Tungsten, Cobalt
- SDSC/NPACI — BlueHorizon, DataStar
- PSC — Rachel

Discussion

- Born-Infeld electrodynamics was successfully quantized using Lattice Gauge Theory simulations. The similarities and differences between classical and quantum theories were elucidated.
- We will now add transverse dimensions and quantize strings/branes.
- By simulating 2-flavour lattice QCD at zero quark masses and comparing with $O(2)$ spin model simulations we have seen evidence of the expected $O(2)$ universality of the chiral phase transition.
- We will compare the critical scaling with other spin models to check that other universality classes can be ruled out. We will move to finite quark mass to study other properties of the critical point.
- We have searched for the critical end-point for QCD at finite densities. Preliminary results appear to exclude

such a critical end point for small chemical potentials. Previous positive results appear to be due to finite dt systematics. New simulation algorithms avoid such systematic errors.

- We will finish these finite density simulations. These will be used as a basis for equation-of-state calculations.
- We will use finite isospin density as a starting-point for finite quark/baryon-number simulations.
- We have generated 1000 zero-temperature $16^3 \times 32$ configurations with our χQCD lattice action and zero quark mass.
- We will calculate hadron spectrum in the chiral limit on these (and larger) lattices. The χQCD action allows this even though $1/m_\pi$ is larger (infinite) than the spatial extent of the lattice.
- More quarkonium studies?