Solution of the Puzzle of $e^+e^- \to J/\psi + \eta_c$

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The Conflict Between Theory and Experiment (April 2006)

- Experiment
  Belle: $\sigma(e^+e^- \to J/\psi + \eta_c) \times B_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb.
  BABAR: $\sigma(e^+e^- \to J/\psi + \eta_c) \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb.

- NRQCD at LO in α_s and v
  Braaten, Lee: $\sigma(e^+e^- \to J/\psi + \eta_c) = 3.78 \pm 1.26$ fb.
  Liu, He, Chao: $\sigma(e^+e^- \to J/\psi + \eta_c) = 5.5$ fb.
  The two calculations employ different choices of m_c, NRQCD matrix elements, and α_s.
  Braaten and Lee include QED effects.

- Trends
  - The Belle cross section has moved down from $33^{+7}_{-6} \pm 9$ fb.
  - The BABAR cross section is even lower.
  - Braaten and Lee found a sign error in the QED interference term that raised the prediction from 2.31 ± 1.09 fb.
  - Zhang, Gao, Chao: A calculation of corrections at NLO in α_s shows that the K factor is approximately 1.96.
  - The trends are in the right direction, but a discrepancy remains.

Relativistic Corrections to $e^+e^- \to J/\psi + \eta_c$

- Braaten, Lee: It has been known for some time that the order-v^2 corrections to $\sigma(e^+e^- \to J/\psi + \eta_c)$ are potentially large: $\sigma_0 \times 2.0^{+2.9}_{-1.1}$.
- The obstacles to a precise result are the large uncertainties in the NRQCD matrix elements of higher order in the heavy-quark-antiquark relative velocity v.
- A large relativistic correction would cast doubt on the reliability of the v expansion.
- We addressed these problems by
  - calculating the matrix elements in a potential model,
  - resumming a class of relativistic corrections.

Potential-Model Calculation of Higher-Order Matrix Element

- Typical Nonrelativistic QCD (NRQCD) matrix element at LO in v^2:

$$\psi(0) \equiv \int \frac{d^3p}{(2\pi)^3}\widetilde{\psi}(\boldsymbol{p}) = \frac{1}{\sqrt{2N_c}}\langle 0|\chi^\dagger\psi|H(^1S_0)\rangle,$$

  - ψ annihilates a heavy quark;
    $\chi^\dagger$ annihilates a heavy antiquark.
  - $\psi(\mathbf{x})$ and $\tilde{\psi}(\mathbf{p})$ are coordinate-space and momentum-space Schrödinger wave functions.

- Typical NRQCD matrix element at NLO in v^2:

$$\psi^{(2)}(0) = \int \frac{d^3p}{(2\pi)^3}\boldsymbol{p}^2\widetilde{\psi}(\boldsymbol{p}) = \frac{1}{\sqrt{2N_c}}\langle 0|\chi^\dagger\left(-\boldsymbol{\nabla}^2\right)\psi|H(^1S_0)\rangle.$$

  This and related NLO matrix elements appear in the relativistic (relative-order-v^2) corrections to many quarkonium decay and production processes.

- Notation: $\langle \boldsymbol{p}^2\rangle \equiv \psi^{(2)}(0)/\psi(0)$, $\langle v^2\rangle = \langle \boldsymbol{p}^2\rangle/m_c^2$.

- Attempts to determine $\psi^{(2)}(0)$ have largely been stymied.
  - Phenomenological attempts: large uncertainties from uncalculated higher orders in α_s, m_c.
  - Lattice attempts: large uncertainties because of large cancellations in converting from lattice regularization to dimensional regularization.
  - Gremm-Kapustin relation (from the NRQCD equations of motion):

$$\psi^{(2)}(0)/[m_c^2\psi(0)] = \langle p^2\rangle/m_c^2 \approx (M_H - 2m_c)/m_c.$$

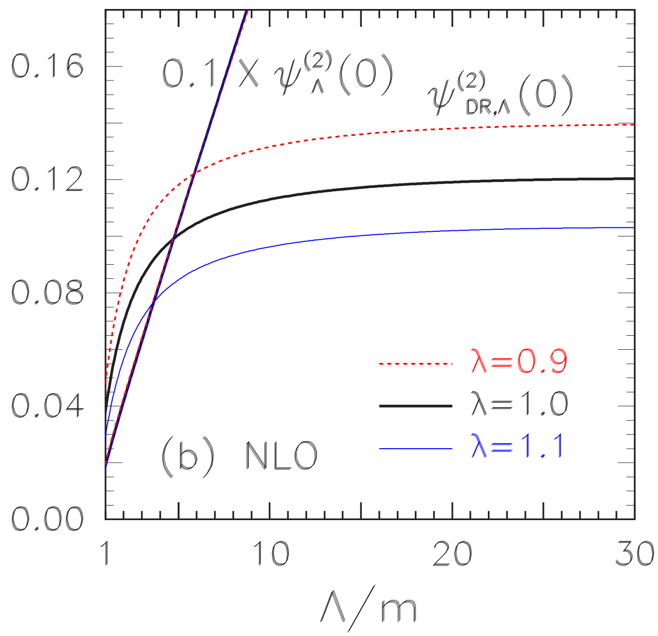
    Large uncertainties in m_c make the method unreliable.
- Even the sign of $\psi^{(2)}(0)$ was not known with great confidence.

Strategy

- Use a potential-model calculation (Cornell potential) to determine $\psi^{(2)}(0)$.
- If we know the potential exactly (lattice), then the potential model is accurate up to errors of order v^2.
- Complication: $\psi^{(2)}(0)$ contains a linear UV divergence and must be regulated.
  Want a dimensionally regulated matrix element to use in phenomenology.

Method 1

- First regulate using a simple, analytic momentum-space hard cutoff $\Lambda^2/(\boldsymbol{p}^2+\Lambda^2)$.
- Compute the difference between the hard cutoff and dimensional regularization $\Delta\psi^{(2)}(0)$ in perturbation theory.
- Subtract $\Delta\psi^{(2)}(0)$ from the hard-cutoff result to obtain the dimensionally regulated result.
- Extrapolate to $\Lambda=\infty$ to eliminate uncompensated $1/\Lambda$ power corrections.



- $\psi^{(2)}_{\Lambda} = -\nabla^2\psi(0)$ for a hard cutoff.
  $\psi^{(2)}_{\mathrm{DR},\Lambda} = -\nabla^2\psi(0)$ for dim. reg.
- Large cancellations occur in the conversion.
- Requires high numerical precision.
- Final result ($\Lambda\to\infty$) is well behaved.
- For the potential-model parameter $\lambda=1.0$, 1.1, the Cornell potential brackets the lattice string tension.

Method 2

- Use the Bethe-Salpeter equation to expose an explicit loop in the wave function.
- Regulate the loop dimensionally.
- The result from Method 2 is in good agreement with the $\Lambda \to \infty$ result from Method 1.

Results

- $\psi^{(2)}(0) = 0.118 \pm 0.024 \pm 0.035$ GeV$^{7/2}$.
- $\langle v^2 \rangle \approx 0.25 \pm 0.05 \pm 0.08$,
  in agreement with the v-scaling rules of NRQCD.
- First error: uncertainty in the input potential-model parameters and the wave function at the origin.
  Second error: neglected relative-order-v^2 corrections.
- This is the first reliable determination of $\psi^{(2)}(0)$.

Resummation

- NRQCD matrix elements of higher order in v are

$$\psi^{(2n)}(0) \equiv \int \frac{d^3p}{(2\pi)^3}\,\boldsymbol{p}^{2n}\widetilde{\psi}(\boldsymbol{p}) = \frac{1}{\sqrt{2N_c}}\langle 0|\chi^\dagger(-\boldsymbol{\nabla}^2)^n\psi|H\rangle.$$

$$\langle \boldsymbol{p}^{2n}\rangle \equiv \psi^{(2n)}(0)/\psi^{(0)}(0).$$

- We can extend Method 2 to compute these higher-order matrix elements.
- Use the equation of motion, dimensional regularization, and the scalelessness of the individual terms in the Cornell potential to obtain

$$\langle \boldsymbol{p}^{2n}\rangle = \langle \boldsymbol{p}^2\rangle^n.$$

- This formula allows us to resum a class of the relativistic corrections to the S-wave quarkonium amplitude to all orders in v.
- It is applicable to many quarkonium production and decay processes.

K Factors

- Relativistic corrections to the short-distance coefficient for $e^+e^- \to J/\psi + \eta_c$:
  1.42 without resummation (order v^2),
  1.34 with resummation.

- Relativistic corrections to the phenomenological determination of $|\psi(0)|^2$ from $J/\psi \to e^+e^-$ (one factor each for J/ψ, η_c):
  $(1.45)^2$ without resummation (order v^2),
  $(1.32)^2$ with resummation $\Rightarrow (1.34)^2$ in the presence of α_s corrections.

- Order-α_s corrections (Liu, He, Chao):
  1.96.

- Complete K factor:

$$[1 + \underbrace{0.96}_{\alpha_s} + \underbrace{0.34}_{\text{s.d.}}] \times (\underbrace{1.34}_{|\psi(0)|^2})^2 = 4.15 \pm 1.37.$$

This large factor comes from a cocktail of moderately sized effects.

Cross Section

- Preliminary result:

$$\sigma(e^+e^- \to J/\psi + \eta_c) = 17.5 \pm 5.7 \text{ fb}.$$

  - Includes a 6% increase in the PDG value for $J/\psi \to e^+e^-$.
  - Only uncertainties from $m_c = 1.4 \pm 0.2$ GeV and $\langle \boldsymbol{p}^2 \rangle$ are shown.
  - Uncertainties from higher orders in α_s, scale dependence, order-$\alpha_s v^2$ corrections, power corrections to factorization will be taken into account as well.

- Experiment

  Belle: $\sigma(e^+e^- \to J/\psi + \eta_c) \times B_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb.

  BABAR: $\sigma(e^+e^- \to J/\psi + \eta_c) \times B_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb.

- Experiment and theory agree within uncertainties.

  A long-standing puzzle has now been resolved.

- Scale dependence of the order-α_s K factor 1.96 is large $\approx {}^{+75}_{-29}\%$.

- A further refinement:

  In the potential model, $\langle \boldsymbol{p}^2 \rangle$ depends on $\psi(0)$.

  $\psi(0)$ also depends on $\langle \boldsymbol{p}^2 \rangle$ through relativistic corrections to $J/\psi \to e^+e^-$.

  Work is in progress to solve these coupled nonlinear equations.

Exclusive Two-Vector-Meson Production in e^+e^- Annihilation

G.T. Bodwin, E. Braaten, J. Lee, C. Yu

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- Motivated by recent measurements by BABAR of exclusive production of two vector mesons.
- Calculated the production cross sections for all combinations of ρ^0, ω, ϕ, J/ψ, $\psi(2S)$.
- Calculated the fragmentation amplitudes using VMD.
  - VMD incorporates some corrections of higher order in α_s and v.
  - Minimizes theoretical uncertainties.
- For light mesons, the nonfragmentation amplitudes are negligible (order $\Lambda^2_{\rm QCD}/E^2_{\rm beam}$).
- Calculated the nonfragmentation amplitudes for the J/ψ and $\psi(2S)$ using NRQCD.
- Included effects from the finite width of the ρ^0 by using the experimentally-measured line shape. Included effects from the finite width of the ϕ by using a Breit-Wigner form.
- Also computed the rates using the BABAR cuts on the production angle and the ρ^0 mass.
- Our results for $\rho^0 + \rho^0$ and $\rho^0 + \phi$ are in good agreement with the measurements of BABAR.

$$\sigma[e^+e^- \to \rho^0 + \rho^0]^{\mathrm{theory}} = 17.71 \pm 0.61 \text{ fb.}$$
$$\sigma[e^+e^- \to \rho^0 + \rho^0]^{\mathrm{expt}} = 20.7 \pm 0.7_{\mathrm{stat}} \pm 2.7_{\mathrm{syst}} \text{ fb.}$$

$$\sigma[e^+e^- \to \rho^0 + \phi]^{\mathrm{theory}} = 5.61 \pm 0.20 \text{ fb.}$$
$$\sigma[e^+e^- \to \rho^0 + \phi]^{\mathrm{expt}} = 5.7 \pm 0.5_{\mathrm{stat}} \pm 0.8_{\mathrm{syst}} \text{ fb.}$$

Inclusive Charm Production in χ_b Decays

G.T. Bodwin, E. Braaten, D. Kang, J. Lee

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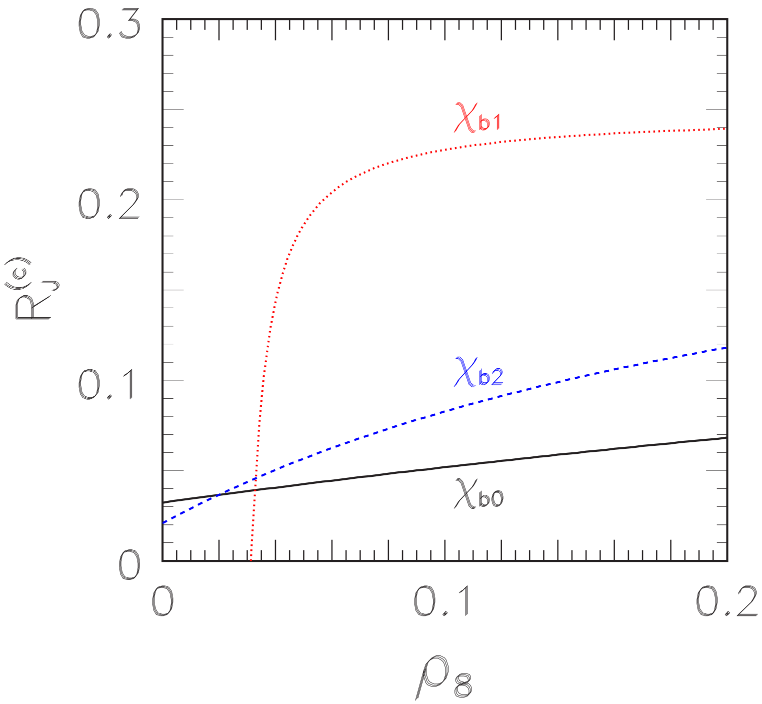
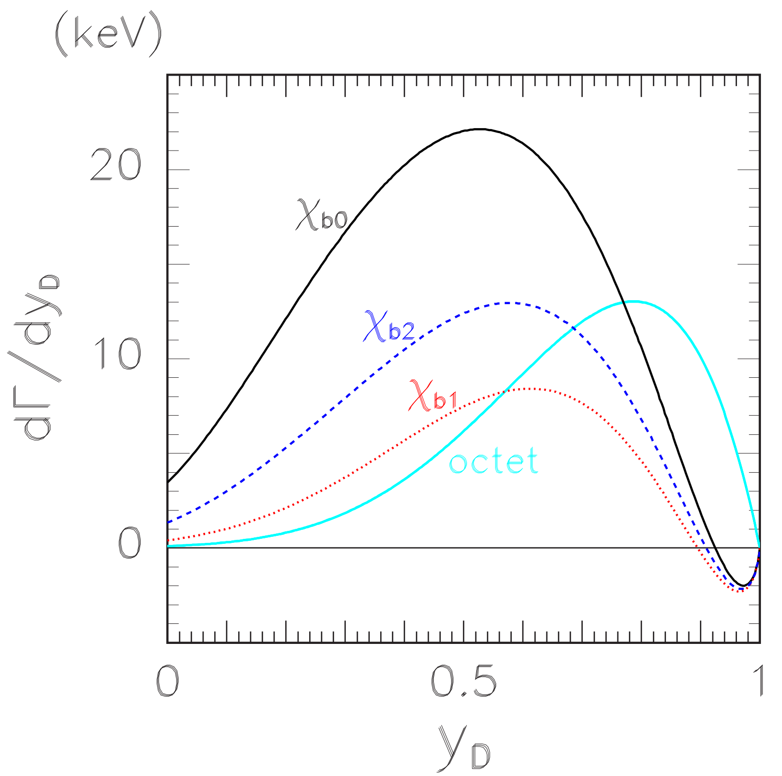
- Calculation motivated by a request from CLEO.
- Calculated the decay rates for $\chi_{bJ} \to c\bar{c} + X$ for $J = 0, 1, 2$ at order α_s^3 using the NRQCD formalism.
- Calculated the total charm rate and the charm rate differential in the momentum of the c-quark.
- The rate in the color-singlet channel is IR divergent, but, in the NRQCD formalism, the divergence is absorbed into a color-octet matrix element.
- All of the 3-body phase space integrals were calculated analytically for a finite c-quark mass.



- The ratio of rates $R_J^{(c)} = \Gamma^{\text{charm}}/\Gamma^{\text{total}}$ depends on the ratio of matrix elements $\rho_8 = m_b^2 \langle \mathcal{O}_8 \rangle_{\chi_b} / \langle \mathcal{O}_1 \rangle_{\chi_b}$.
- Measurements of $R_J^{(c)}$ can be used to determine the color-octet matrix element.



- Convolutions of our results with a quark fragmentation function give the D meson spectrum.
- $y = p_D/p_D^{\max}$.
- Unphysical negative cross sections near the kinematic endpoint signal the breakdown of the α_s expansion.
  Can be cured by resummation of logs.
  Work in progress.

The Quarkonium Working Group (QWG)

- An international organization consisting of more than 150 theorists and experimenters.
- Aims:
  - To promote discussions on theory and experiment in heavy-quarkonium production, decay, and spectroscopy.
  - To foster interactions between theorists and experimenters working in these fields.
- GTB is a convener of the QWG and a convener of the QWG Production subgroup.
- Yellow Report on Quarkonium Physics
  - Detailed (521 page) review of the status of theory and experiment in quarkonium physics.
  - Published as CERN-2005-05.
  - GTB was a principal author of the Production section.
- The QWG has organized
  - 4 international workshops in 2002–2006 at CERN, Fermilab, Beijing (IHEP), BNL
  - The Topical Seminar School on Heavy Quarkonia at Accelerators (September 2004, Beijing)
- Next QWG international workshop: DESY (Hamburg), October 17–20, 2007.