Phase Space Mapping and Emittance Measurement Of Intense Particle Beams Using A Tomography Technique

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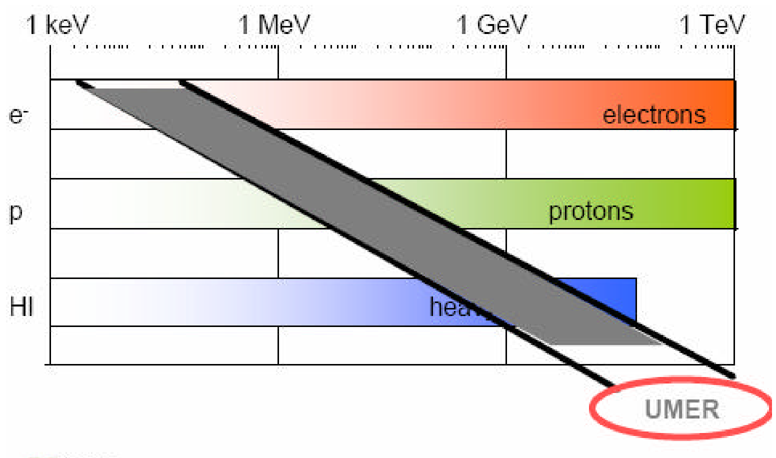
AAC 2006, Lake Geneva, WI

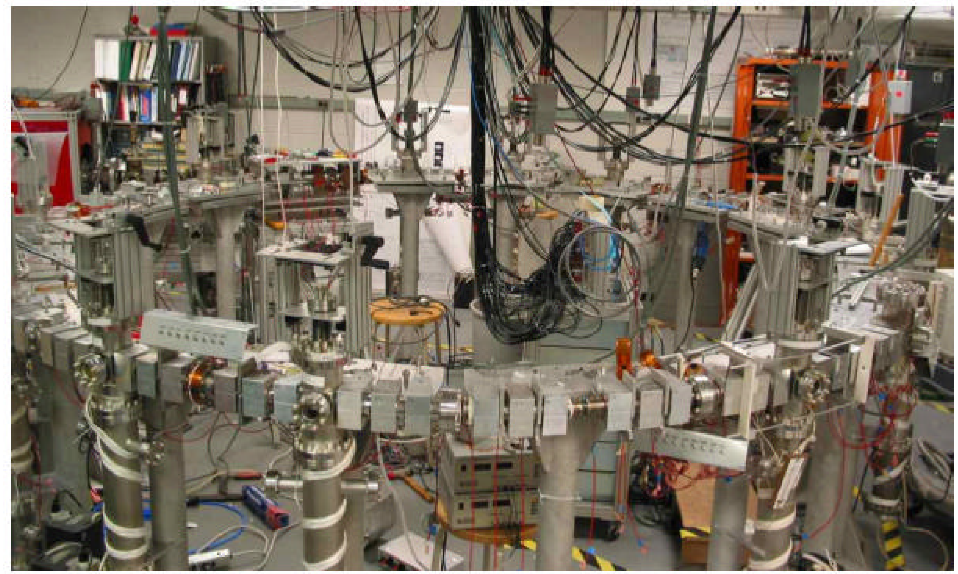
Research supported by the US Dept. of Energy

The University of Maryland Electron Ring (UMER)

- UMER will serve as a low-cost model of high intensity accelerators

  Scaling laws: GeV, kA heavy ions = keV, mA electrons





$$K = \frac{qI}{2\pi\epsilon_0 m(c\beta\gamma)^3}$$

$$\chi \equiv \frac{K}{k_o^2 a^2}$$

Energy	10-50 keV
Energy Spread	20 eV
rms Emittance, nor	0.2-3 µm
Current range	0.6-100 mA

Circulation time	200 ns
Pulse length	20-100 ns
Zero-Current Tune	7.6
Depressed Tune	1.5 – 6.5

Motivation

- Test a new technique for mapping phase spaces based on tomography
- Extend it for beams with space charge
- Simulate it:
  - Error sensitivities
  - Different distributions
  - Accuracy of space charge modeling
- Use it on experiments

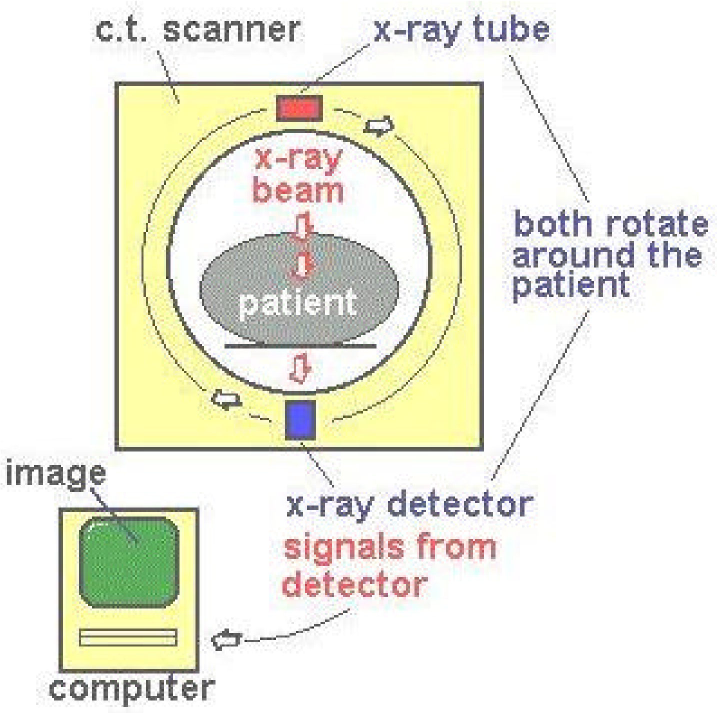
Literature Review

Article	Beam/Facility	Energy / Current	G. Perveance
McKee et al. 1995	Duke Mark III FEL	44MeV / 0.2A	$3.5\ 10^{-11}$
Connolly et al. 2000	RHIC	100 Gev/u / 1.76A	$3.5\ 10^{-10}$
Hancock et al. 2000	CERN PSB	50 MeV / 180 mA 1 GeV	$2.1\ 10^{-11}$
Montag et al. 2002	RHIC	100 Gev/u / 1.76A	$3.5\ 10^{-10}$
Yakimenko et al. 2003	ATF at BNL	50 MeV/ 100A	$1.2\ 10^{-8}$
Loos et al.2004	DUV-FEL	38 MeV/ 148A	$4.0\ 10^{-6}$
Li H. PhD Dis. 2004	UMER	10 keV/ 7mA	$1.0\ 10^{-4}$
Chalut et al. 2005	OK-4 Duke FEL	0.8 GeV/ 20mA	$6.1\ 10^{-16}$

For UMER: G. Perveance 10^{-6} to 10^{-3}

Computed Tomography (CAT Scan)

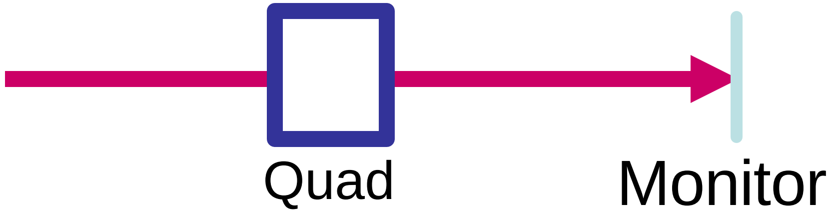
- Tomography is the technique of reconstructing an image from its projections (1826 Abel, 1917 Radon)



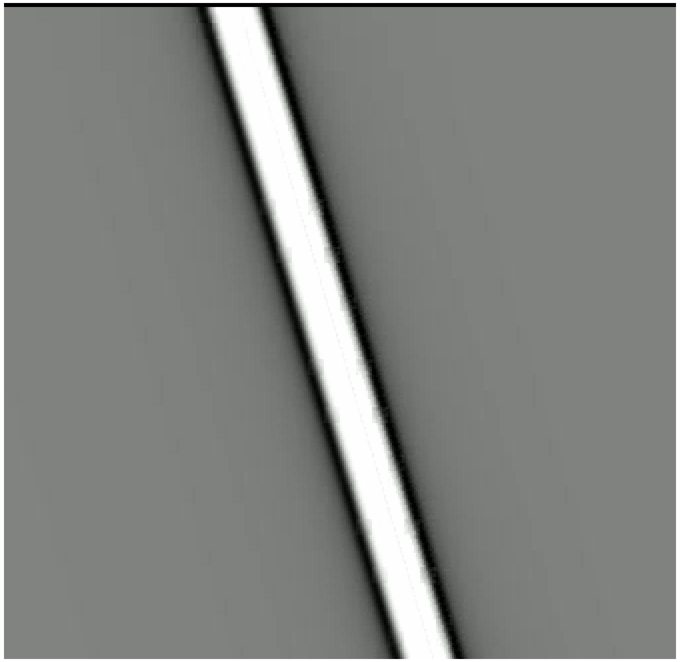
http://www.dbh.nhs.uk

http://universe-review.ca

Beam Phase Space Tomography

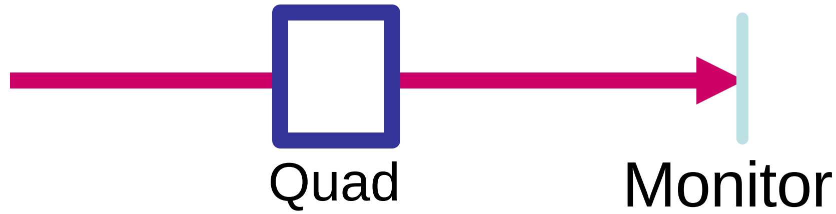


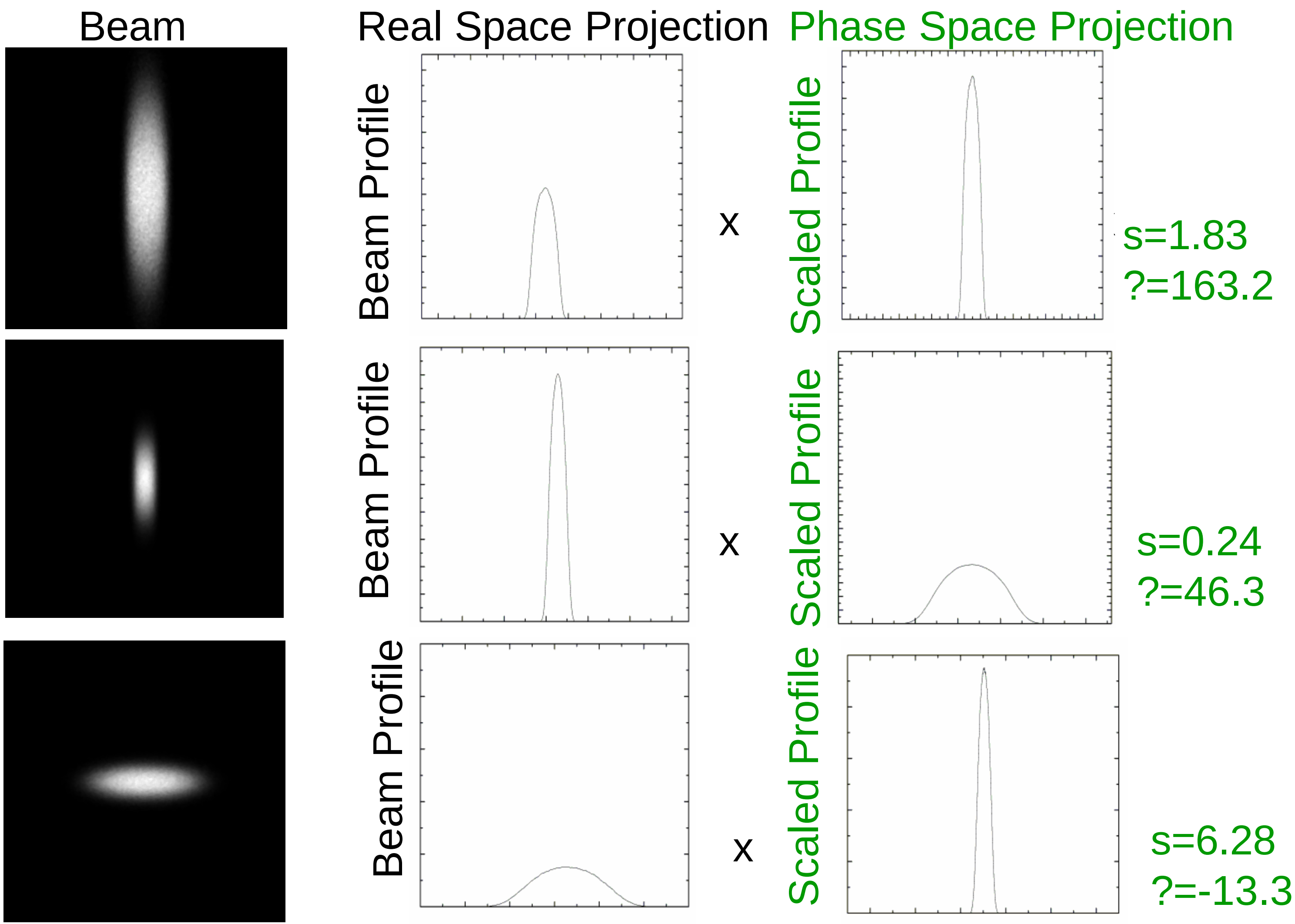
- We measure the beam phase space by combining a simple quadrupole-scan with tomography



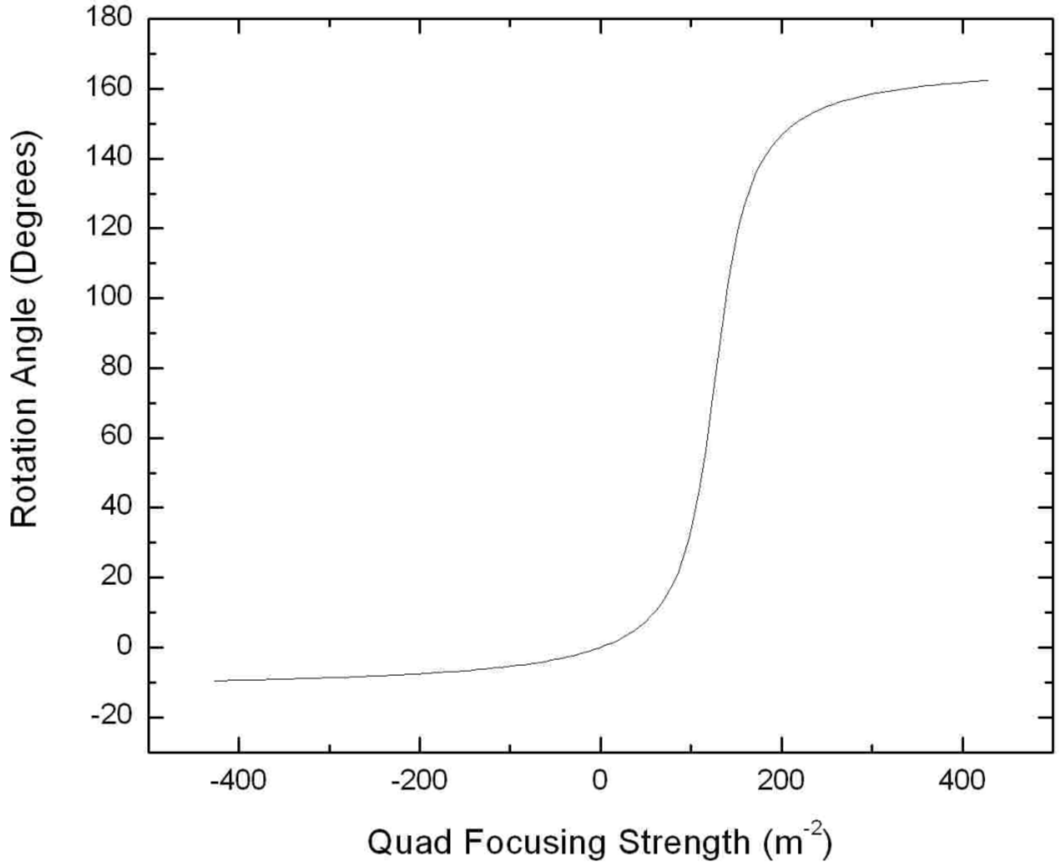
McKee et al. 1995

Beam Phase Space Tomography





Beam Phase Space Tomography



Equations of Motion

- Single particle equation

$$x'' = -k_{x,0}x + F_{linear} + F_{Nonlinear}$$

- No space charge:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos\sqrt{k_x}z & \frac{1}{\sqrt{k_x}}\sin\sqrt{k_x}z \\ -\sqrt{k_x}\sin\sqrt{k_x}z & \cos\sqrt{k_x}z \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \end{pmatrix}$$ where $k_x = k_{x,0}$

- Space charge: Calculations are very complicated and approximations need to be made in order to generate the transfer matrices.

Space Charge Dominated Beams - Assumptions

- <u>Linear space charge</u>: Calculation of the transfer matrix is easy.

$$x'' = -k_{x,0}x + F_{linear} + \cancelto{0}{F_{Nonlinear}} \Rightarrow x'' = -\left(k_{x,0} - \frac{2K}{X(X+Y)}\right)x$$

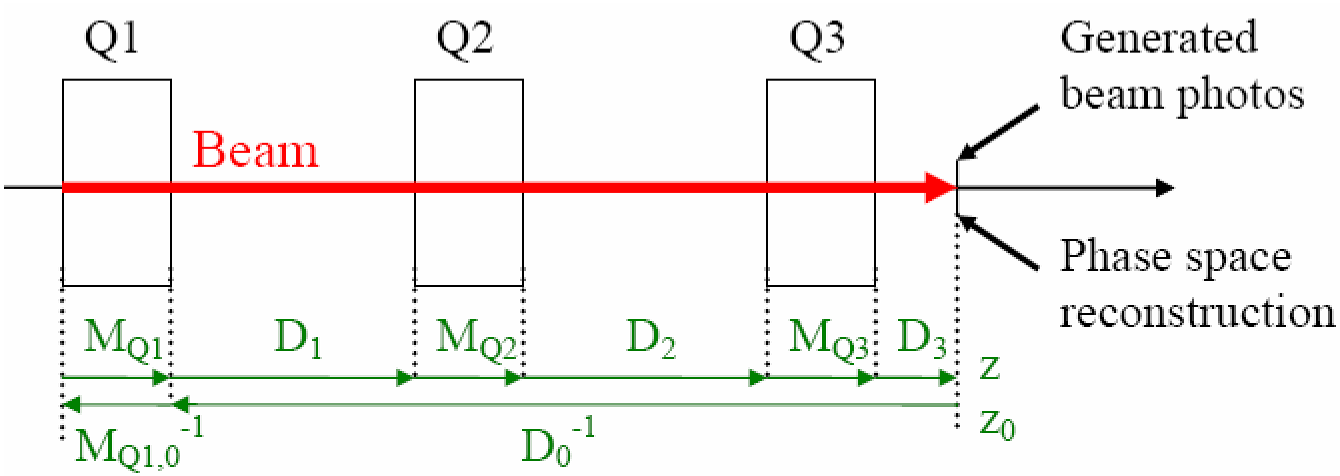
Problem: X and Y vary with z

- <u>No emittance growth</u>. The beam size will be calculated using the beam envelope equations.

$$X'' + k_x X - \frac{2K}{X+Y} - \frac{e_x^{\,2}}{X^3} = 0 \qquad Y'' + k_y Y - \frac{2K}{X+Y} - \frac{e_y^{\,2}}{Y^3} = 0$$

Tomography Set - Up

- We simulate the tomography process using the particle-in-cell code WARP.

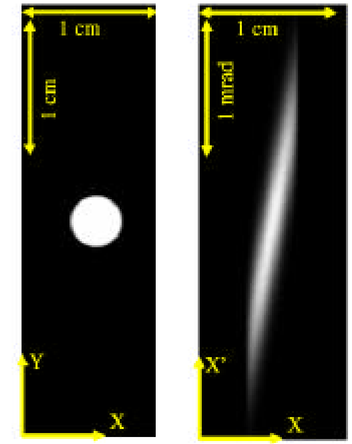


- Reconstructed phase space by Tomography will be compared to that generated by WARP.

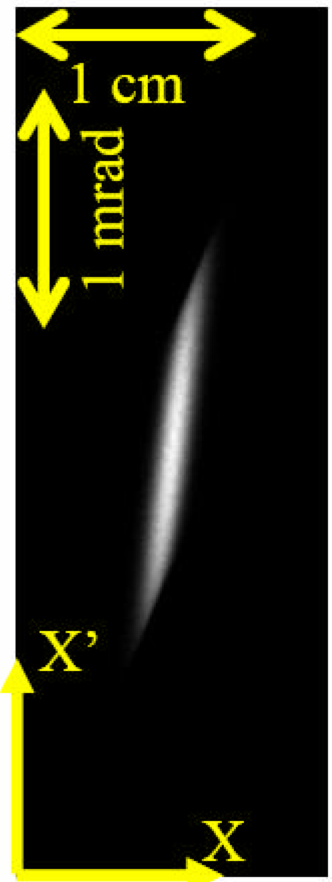
WARP	Tomography
Nonlinear space charge	Linear space charge
Emittance growth	Constant emittance
Bends included	Bends ignored
Image charge forces	Image charge force ignored

Phase Space Tomography - No Space Charge

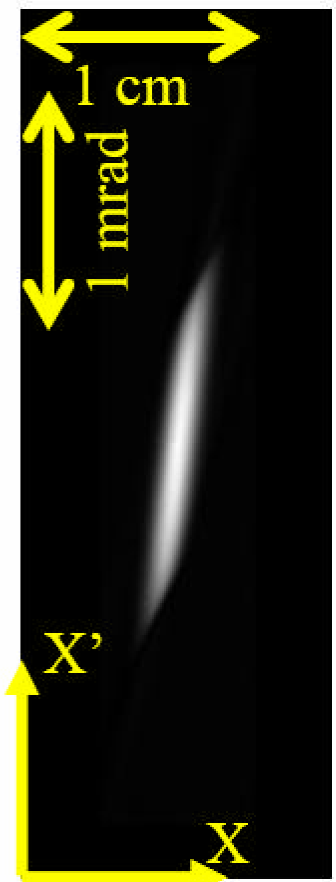
- ?=0.3, I=0.6mA
- Initial beam distribution: Semi-Gaussian



Uniform Gaussian



WARP



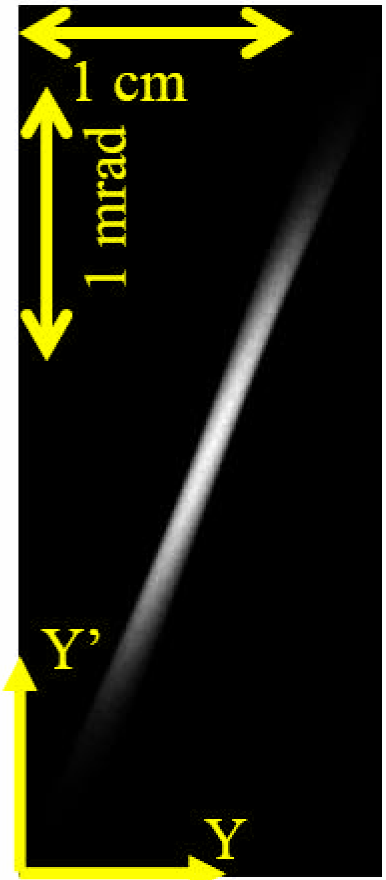
Tomography

	Direct WARP	Tomo	Error (%)
$\varepsilon_x (4\times rms)\mu m$	5.46	5.37	1.6
$X(2\times rms)mm$	2.28	2.23	2.2

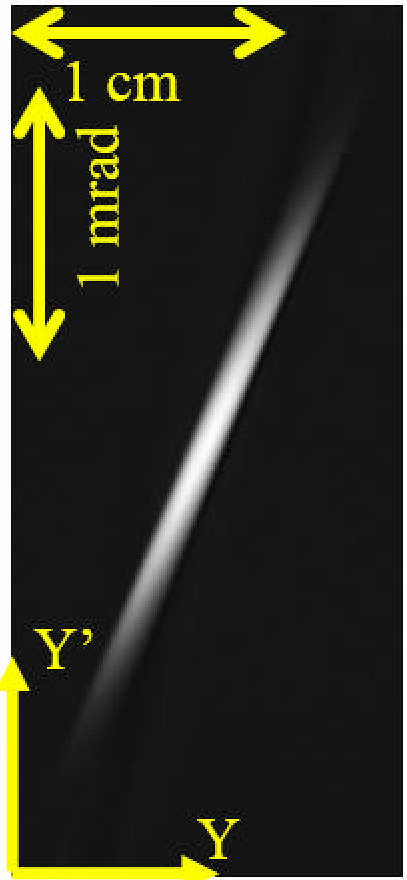
Stratakis et al. Physical Review ST - AB (submitted 2006)

Phase Space Tomography - No Space Charge

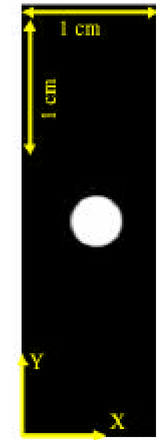
- ?=0.3, I=0.6mA
- Initial beam distribution: Semi-Gaussian



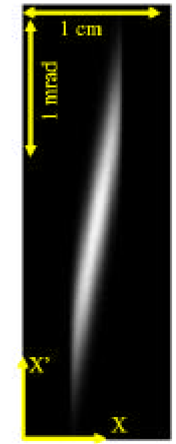
WARP



Tomography



Uniform



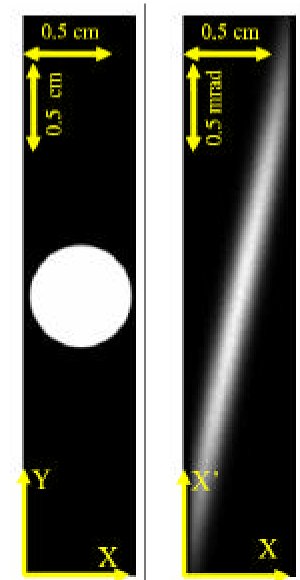
Gaussian

	Direct WARP	Tomo	Error (%)
$\varepsilon_y(4\times rms)\mu m$	5.49	5.23	4.9
$Y(2\times rms)mm$	3.68	3.46	6.0

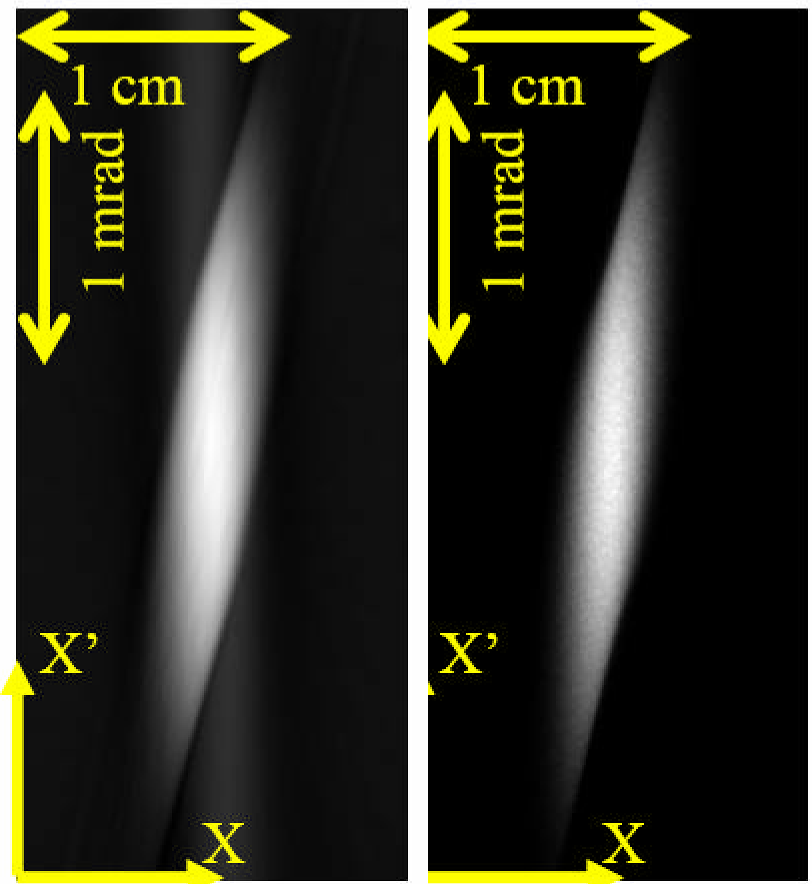
Stratakis et al. Physical Review ST - AB (submitted 2006)

Phase Space Tomography - With Space Charge

- ?=0.72, I=7mA
- Initial beam distribution: Semi-Gaussian



Uniform Gaussian



Tomography WARP

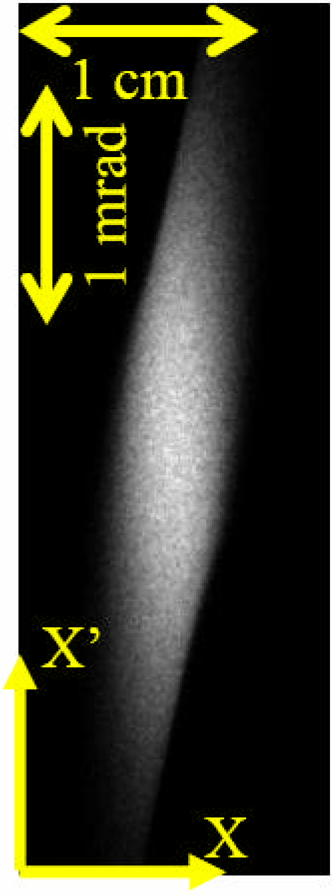
	Direct WARP	Tomo	Error (%)
$\varepsilon_x (4\times rms)\mu m$	14.5	13.4	7.6
$X(2\times rms)mm$	1.88	1.72	8.5

Stratakis et al. Physical Review ST - AB (submitted 2006)

Phase Space Tomography - With Extreme Space Charge

- ?=0.90, I=24 mA
- Initial beam distribution: Semi-Gaussian

Tomography

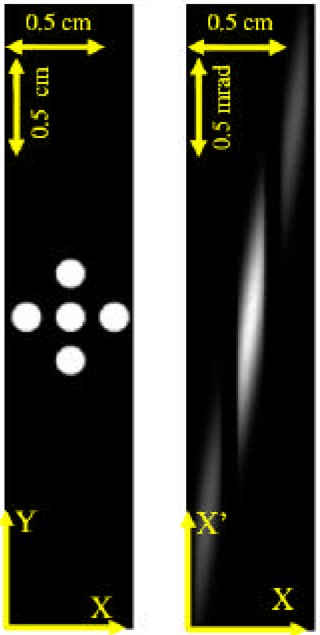


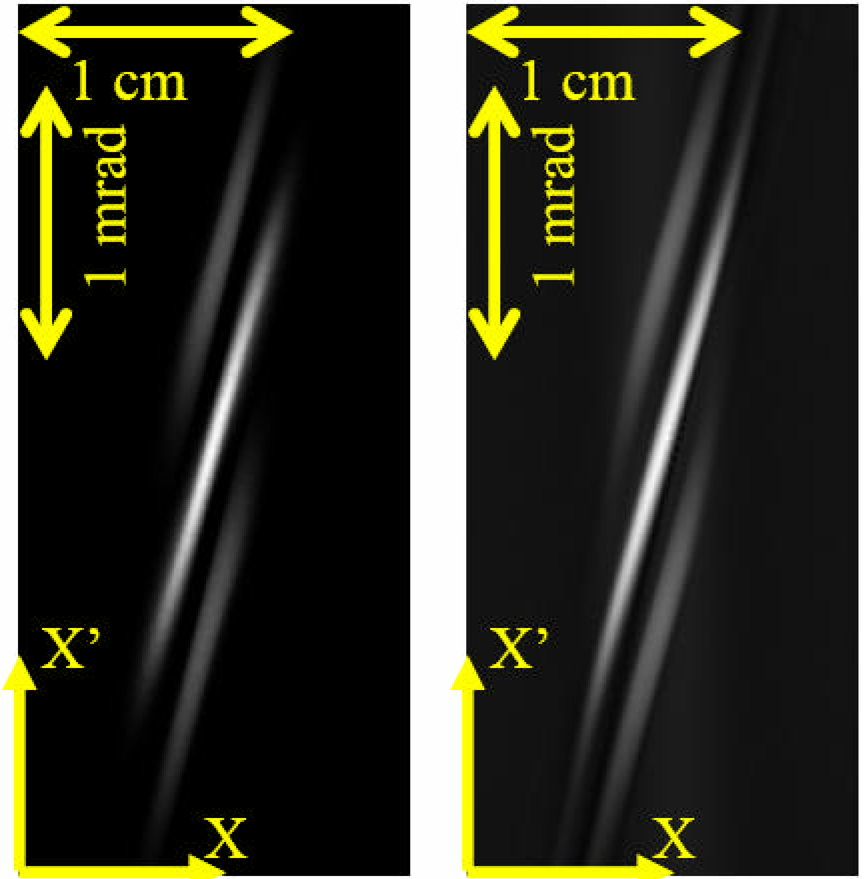
WARP

	Direct WARP	Tomo	Error (%)
$\varepsilon_x (4\times rms)\mu m$	28.8	24.4	15.2
$X(2\times rms)mm$	2.38	2.62	9.2

Phase Space Tomography - Different Distributions

- ?=0.72 , I=7mA (space charge)
- Initial Distribution: Five Beamlet
- Highly non-uniform distribution





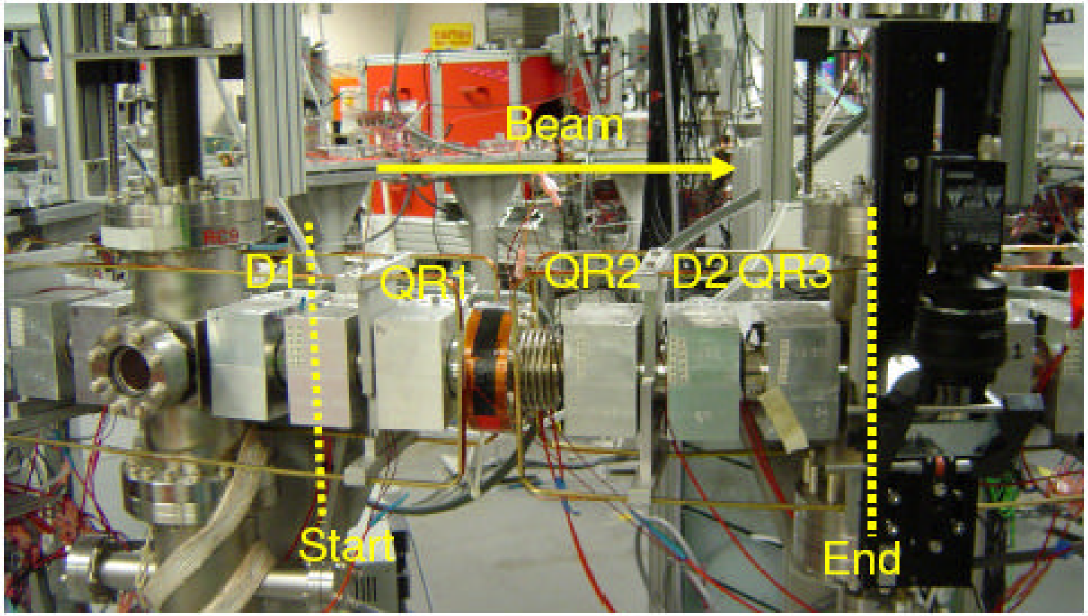
WARP Tomography

	Direct WARP	Tomo	Error (%)
$\varepsilon_x(4\times rms)\mu m$	19.0	17.6	7.5
$X(2\times rms)mm$	2.39	2.26	5.4

Tomography can be used to map the phase space of complex multi-beamlet distributions

Phase Space Tomography - Experiments in UMER

Set-up

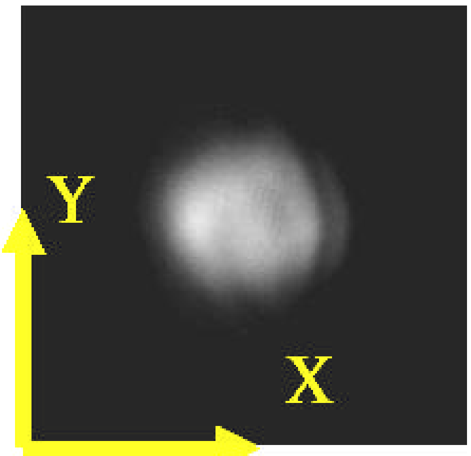


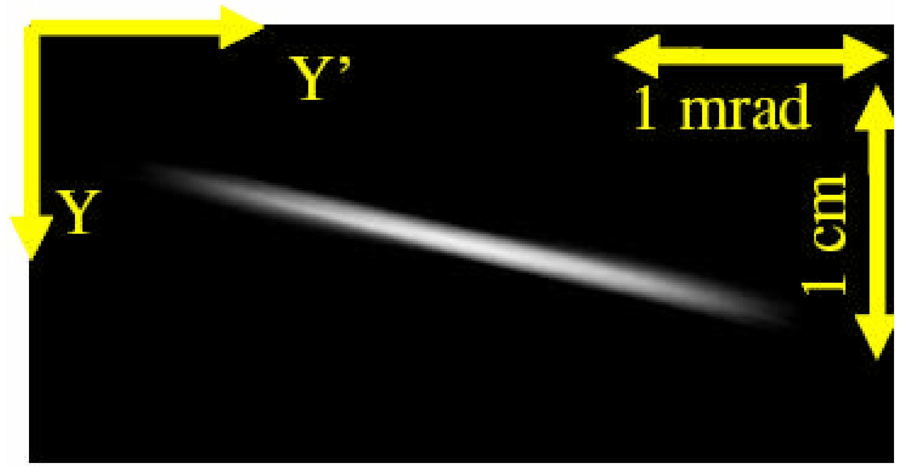
Beam photo capture

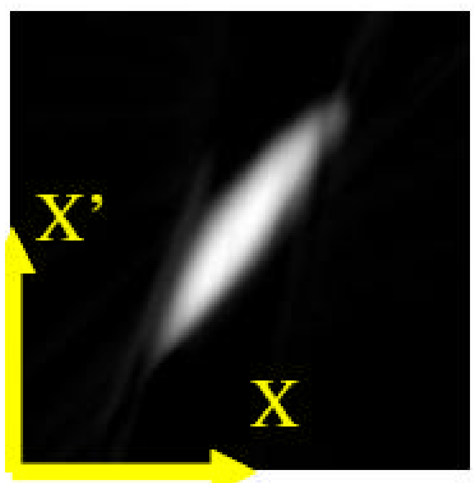
Beam photos were collected with the aid of phosphor screen which intercept the beam

Phase Space Tomography - Experiments in UMER

- Phase space reconstruction of a low current beam (?=0.30) along the injector line







4xRMS emittance:

$$e_x = 5.8 \pm 0.1 mm$$

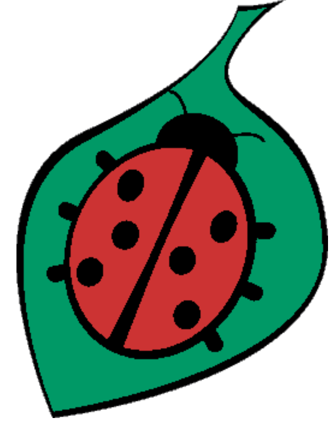
$$e_y = 6.8 \pm 0.1 mm$$

Stratakis et al. Physical Review ST - AB (submitted 2006)

Conclusions

- We designed a simple, portable technique to map the beam phase space based on tomography.
- Tomography accurately reproduces the beam phase space predicted by WARP simulations for both emittance and space charge dominated beams
- Tomography can be used to map the phase space of more complex, non-equilibrium distributions
- First experiments with tomography in UMER have been completed

Thanks to



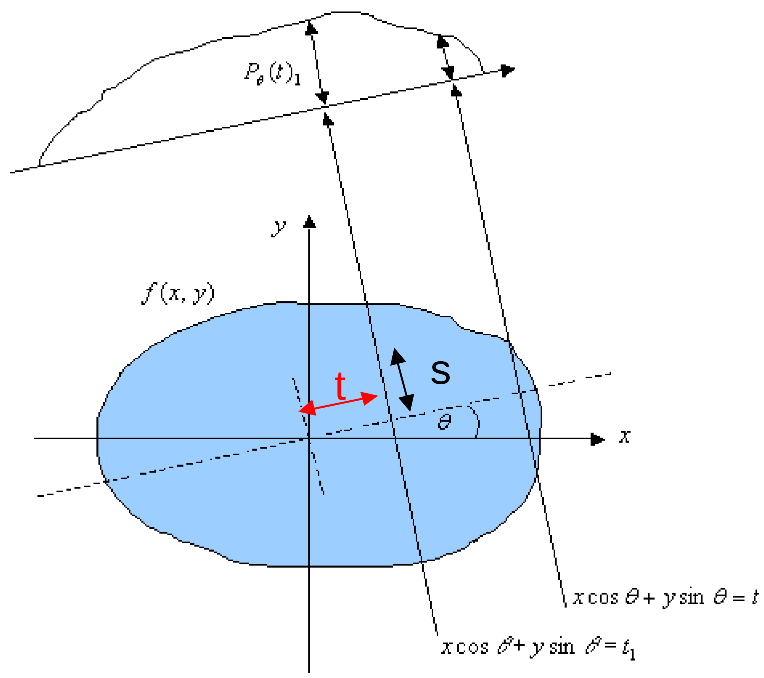
- Prof. Patrick O'Shea
- Prof. Rami Kishek
- Dr. Ralph Fiorito
- Dr. Irving Haber
- Dr. David Sutter
- Dr. Santiago Bernal
- Dr. Mark Walter
- Bryan Quinn
- External: Dr. H. Li (Microsoft), Dr. V. Yakimenko (Brookhaven)
- UMER graduate students: Brian, Chao, Charles, Christos, Gang, and Kai

Backup Slides

Computed Tomography (CAT scan)

- <u>Radon Transform</u>

  We can recover an object in n-dimensional space from projections onto (n-1)-dimensional space.



$$P_?(t) = \int_{(?,t)_{\text{line}}} f(x,y)ds$$ or

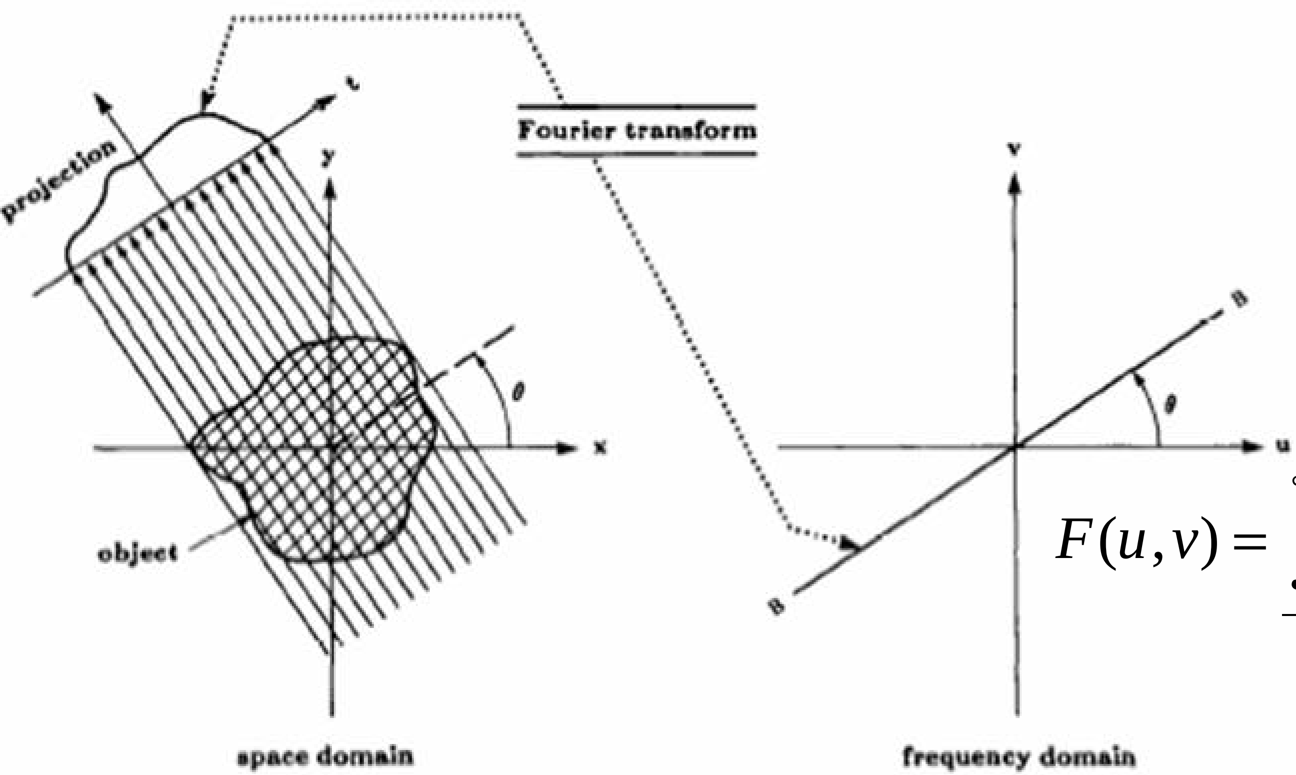
$$P_q(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dxdyf(x,y)d(t-\vec{x}\hat{x})$$

where $\hat{x} = \cos q\,\hat{i} + \sin q\,\hat{j}$ and $\vec{x} = x\,\hat{i} + y\,\hat{j}$

Tomography Algorithm

- <u>Fourier Slice Theorem</u>

  Fourier transform of a parallel projection is equal to a slice of the two-dimensional Fourier transform of the original object.



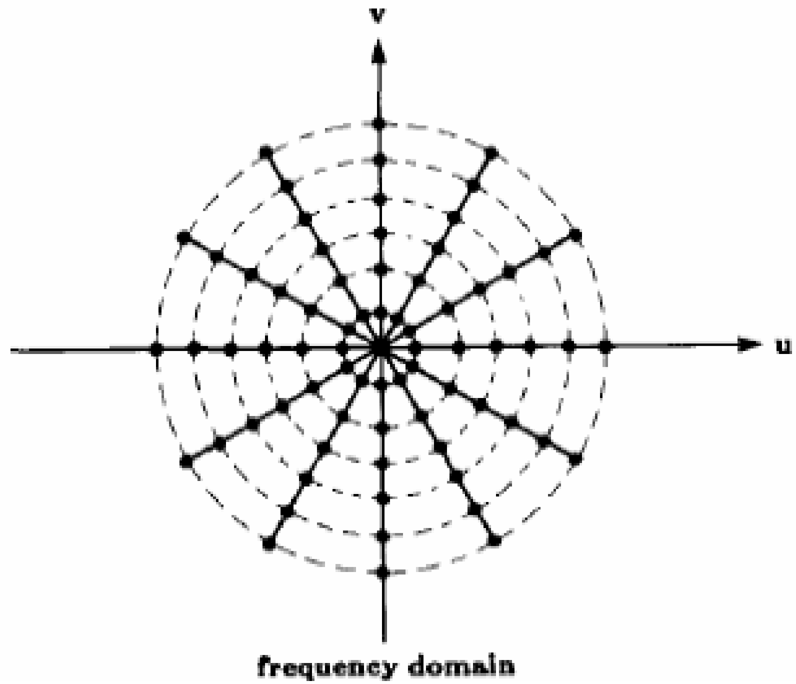
$$S_\theta(\omega) = \int e^{-j2\pi\omega t} P_\theta(t)\,dt$$

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dx\,dy\,f(x,y)\,e^{-j2\pi(xu+yv)}$$

Kak and Slaney, Principles of Computerized Tomographic Imaging

Tomography Algorithm

- Knowledge of the F(u,v) the object function f(x,y) can be recovered by using the inverse Fourier Transform



$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} du\,dv\,F(u,v)\,e^{j2p(xu+yv)}$$

Kak and Slaney, Principles of Computerized Tomographic Imaging

Filtered Backprojection Algorithm (FBA)

- A simple weighting in the frequency domain is used to take is projection and estimate a pie-shaped wedge of the object's Fourier transform.
- We multiply the value of the Fourier transform of the projection of the projection and multiply it by the width of the wedge at that frequency
- Apply inverse Fourier Transform of the filtered projections

Backprojection Algorithm

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} \, du \, dv$$

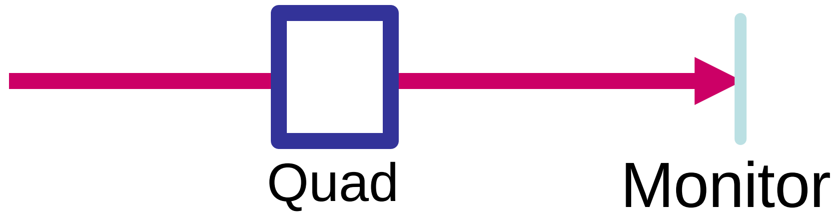
$$f(x, y) = \int_{0}^{2\pi} \int_{0}^{\infty} F(w, \theta) e^{j2\pi w(x\cos\theta + y\sin\theta)} w \, dw \, d\theta.$$

$$f(x, y) = \int_{0}^{\pi} \left[\int_{-\infty}^{\infty} S_\theta(w) |w| e^{j2\pi wt} \, dw \right] d\theta.$$

$$f(x, y) = \int_{0}^{\pi} Q_\theta(x\cos\theta + y\sin\theta) \, d\theta$$

$$Q_\theta(t) = \int_{-\infty}^{\infty} S_\theta(w) |w| e^{j2\pi wt} \, dw.$$

Beam Phase Space Tomography



- We can reconstruct the beam phase space distribution using its projections in real space.

$$C(x) = \int A(x,y)dy = \int m(x, x^{'})dx^{'}$$

- Variation of the quadrupole lens strength rotates the distribution in phase space generating a number of independent projections on the screen.
- There is a simple scaling equation that relates these profiles to the radon transform of the phase space.

$$\begin{pmatrix} x \\ x^{'} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ x_0^{'} \end{pmatrix}$$

$$\tan q = \frac{T_{12}}{T_{11}}$$

$$s = \sqrt{T_{11}{}^2 + T_{12}{}^2}$$

C. B. McKee, P. G. O'Shea, J.M. Madey, NIMPR, 1995