Wakefields in a DLA structure and Dielectric based Wakefield Power Extractor

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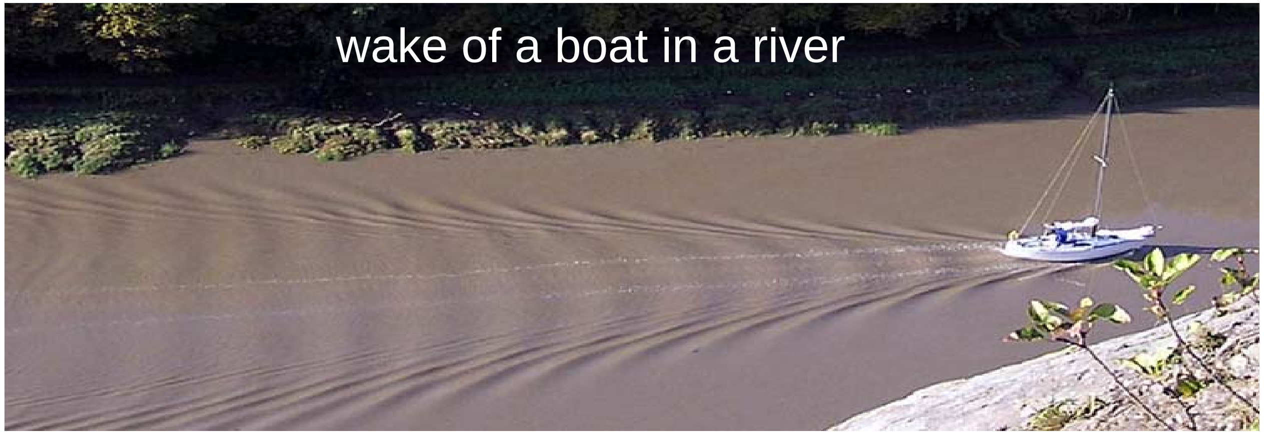
Contents

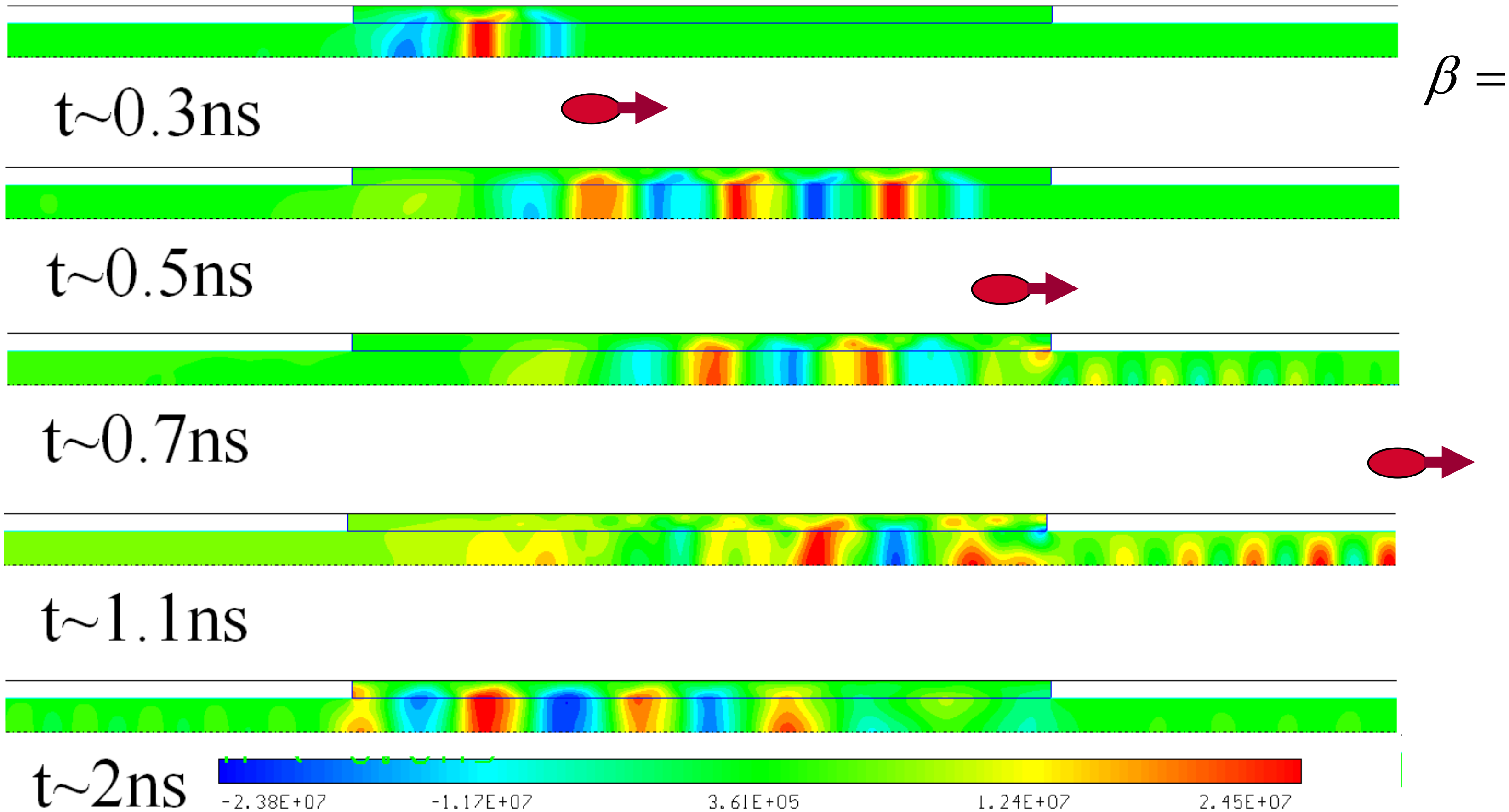
- Wakefield Calculation
  - Direct calculation
  - Indirect calculation
  - CST PS simulation
- Wakefield Acceleration
  - Collinear scheme
  - Two Beam scheme
- Special Topic: dielectric based wakefield power extraction
  - Principle
  - How to design a DWPE using CST
  - An example: 7.8GHz DWPE

Part I:
Wakefield in DLA structures



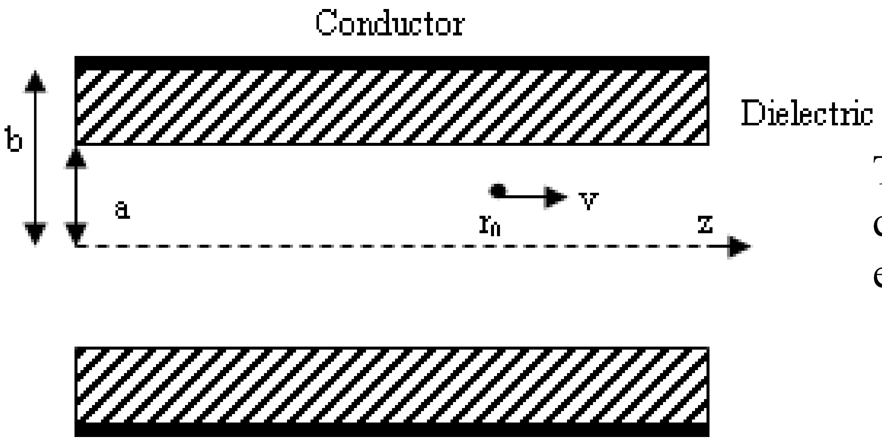
Condition for the Cherenkov radiation

$$\beta = v / c > \varepsilon^{-0.5}$$



1.Direct calculation of the wakefields in a DLA structure

[ref1] Ng, K.-Y., "Wake Fields in a Dielectric-lined Waveguide." Physical Review D (1990):1891-28.
[ref2] Rosing, M., and W. Gai. "Longitudinal- and Transverse-wake-field Effects in Dielectric Structures." Physical Review D (1990):1829-34.



The wakefields produced by the motion of a charged particle are given by Maxwell's equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{D} = \rho, \quad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{j}$$

As a standard way of solving electrodynamics problems we introduced a scalar and a vector potential ϕ and $\vec{A}$, under the Lorentz gauge they satisfy uncoupled inhomogeneous wave equations respectively as

$$\nabla^2 \phi + \omega^2 \mu\varepsilon \frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\varepsilon}, \quad \nabla^2 \vec{A} + \omega^2 \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu\vec{j}$$

$$\rho = \frac{e\delta(r - r_0)}{r}\delta(\theta)\delta(z - vt), \quad \vec{j} = \vec{v}\rho$$

Then we solve for the scalar and vector potentials and bring them into equations

$$\vec{E} = -\nabla \phi - \frac{\partial \vec{A}}{\partial t}, \quad \vec{H} = \frac{1}{\mu}\nabla \times \vec{A}$$

so that we can obtain electric and magnetic field components.

For a given n^{th} mode (n=0 is monopole, n=1 dipole, n=2 quadrupole), the longitudinal EM fields are

$$E_z^n(r,\theta,z) = -8e\cos(n\theta)\cos\left(\frac{k_0}{\beta}(z-vt)\right)\cdot I_n(k_1 r)I_n(k_1 r_0)\cdot\frac{K_n(k_1 a)}{I_n(k_1 a)}\left.\frac{T(k_2)}{k_2\, dC(k_2)/dk_2}\right|_{k_2=k_{2\lambda}}$$

$$H_z^n(r,\theta,z) = -8e\sin(n\theta)\sin\left(\frac{k_0}{\beta}(z-vt)\right)\cdot I_n(k_1 r)I_n(k_1 r_0)\cdot\frac{K_n(k_1 a)}{I_n(k_1 a)}\left.\frac{T(k_2)}{k_2\, dC(k_2)/dk_2}\right|_{k_2=k_{2\lambda}}$$

Where,

$$T(k_2) = k_2^2 k_1^2\left[\frac{k_1 G_{nn}'(k_2 a)}{G_{nn}(k_2 a)} + \frac{k_2 I_n'(k_1 a)}{I_n(k_1 a)}\right]\cdot\left[\frac{k_2 I_n'(k_1 a)}{I_n(k_1 a)} + \frac{\varepsilon k_1 F_{nn}'(k_2 a)}{F_{nn}(k_2 a)}\right] - \frac{n^2}{a^2}k_1^4\beta^2\left(\frac{\varepsilon-1}{1-\beta^2}\right)^2$$

$$C(k_2) = \frac{n^2 k_2}{a^2}k_1^2\beta^2\left(\frac{\varepsilon-1}{1-\beta^2}\right)^2 - k_2^3\left[\frac{k_1 G_{nn}'(k_2 a)}{G_{nn}(k_2 a)} + \frac{k_2 I_n'(k_1 a)}{I_n(k_1 a)}\right]\cdot\left[\frac{k_2 I_n'(k_1 a)}{I_n(k_1 a)} + \frac{\varepsilon k_1 F_{nn}'(k_2 a)}{F_{nn}(k_2 a)}\right]$$

In the case of the particle travels at the center of the DLA structure:

When particle moving at center of circular waveguide ultrarelativisticly, that is, r_0 is zero, n=0 and β=1, then

$$E_z(r,z_0)=\frac{4e}{\varepsilon a}\sum_{\lambda}\left(\frac{F_{00}(k_2a)Y_0(k_2b)}{\frac{d}{dk_2}\left(F_{00}{}'(k_2a)Y_0(k_2b)-\frac{k_2a}{2\varepsilon}F_{00}(k_2a)Y_0(k_2b)\right)}\right)_{k_2=k_{2\lambda}}\cos\left(\frac{\omega_\lambda}{c}z_0\right)$$

$$H_z(r,z_0)=0 \longrightarrow \text{TM0n modes}$$

$$z_0=z-vt$$

In general, with assumption of a Gaussian line charge with rms bunch length σ, the wakefields can be expressed as the integration

$$E_z^m(r,\theta,z_0)=\frac{N}{\sigma\sqrt{2\pi}}\int_{-\infty}^{z_0}E_z^m(r,\theta,z)e^{-[(z_0-z)/\sigma]^2}dz$$

Here N is the total number of charges in the driving bunch and z_0 is the distance behind the center of the driving bunch where the field is measured.

The classical example:

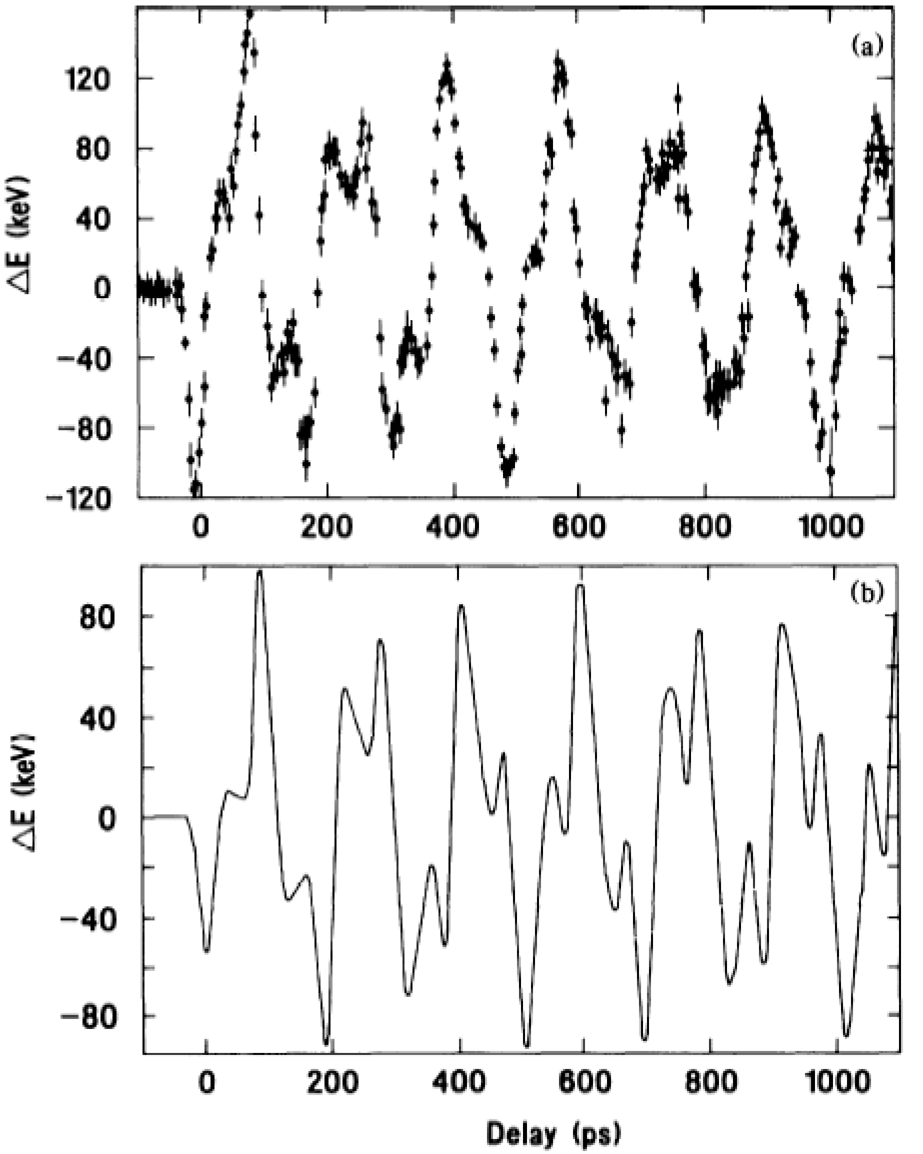


FIG. 3. Steatite scan: (a) measured and (b) calculated wake potential.

[ref3] W. Gai, et al, Phys. Rev. Lett. 61, 34(1988):2756-2758.

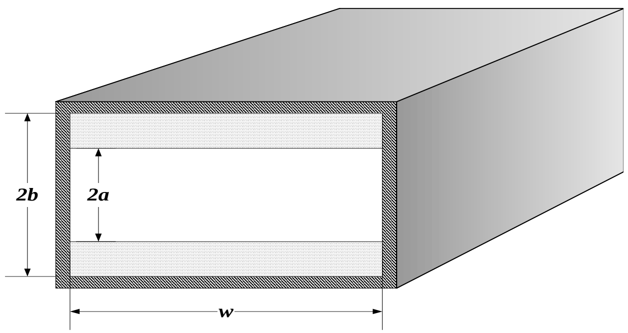
TABLE I. Experimental cavities and beam conditions.

Material	Length (cm)	a (cm)	b (cm)	Driver charge (nC)	Driver length FWHM (ps)
Polystyrene	51	1.27	0.63	~2.6	~23
Steatite	51	1.27	0.63	~2.0	~23
Nylon	51	1.27	0.63	~2.6	~30

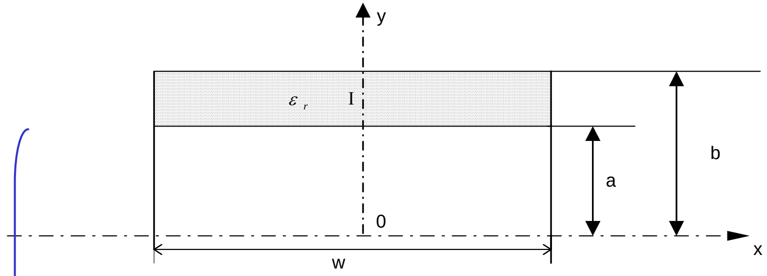
2.Indirect calculation of the wakefields in a DLA structure

[ref4] C. Jing, et al. PHYSICAL REVIEW E 68, 016502 (2003)

Classify modes based on guided modal decomposition in waveguide



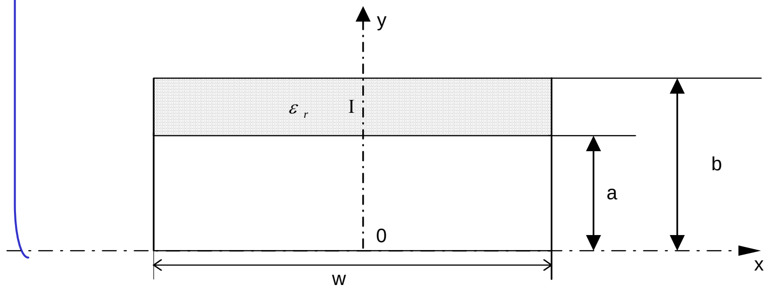
Equivalent boundary conditions are applied by geometrical symmetry of the structure



Open plane

$LSE^{(open)}_{mn}$ & $LSM^{(open)}_{mn}$

(electrical symmetrical modes)



Short plane

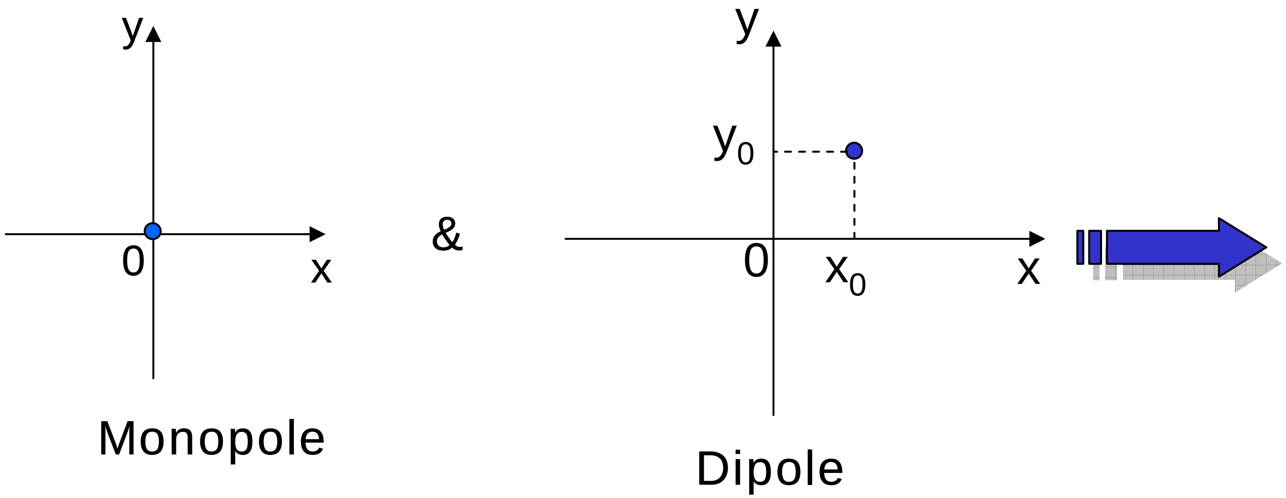
$LSE^{(short)}_{mn}$ & $LSM^{(short)}_{mn}$

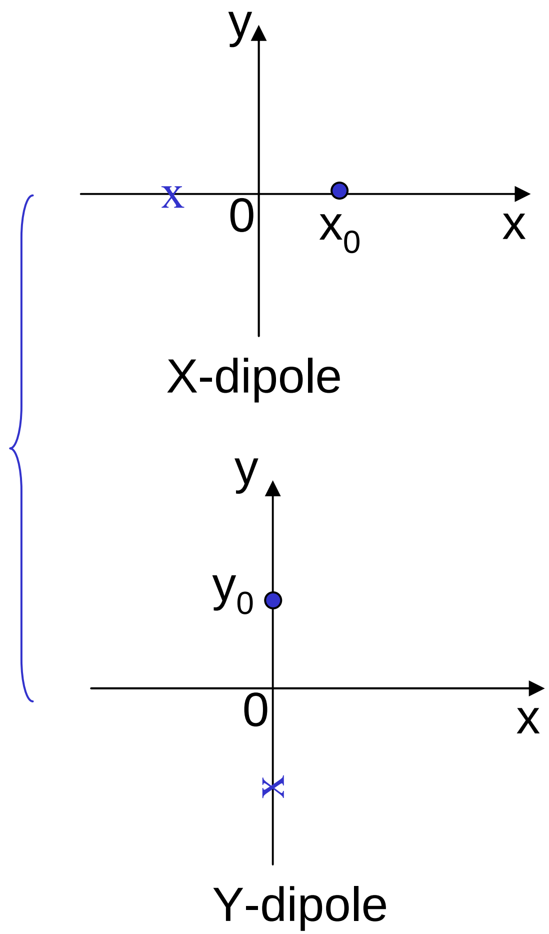
(electrical asymmetrical modes)

LSM_{mn} $(H_y = 0)$ — longitudinal section magnetic mode

LSE_{mn} $(E_y = 0)$ — longitudinal section electric mode

Classify modes based on their leading-order azimuthal harmonic





Electromagnetic dipole modes depicted according to the symmetry of the characteristic voltage witnessed by a relativistic beam, traveling into the page

Mode matching

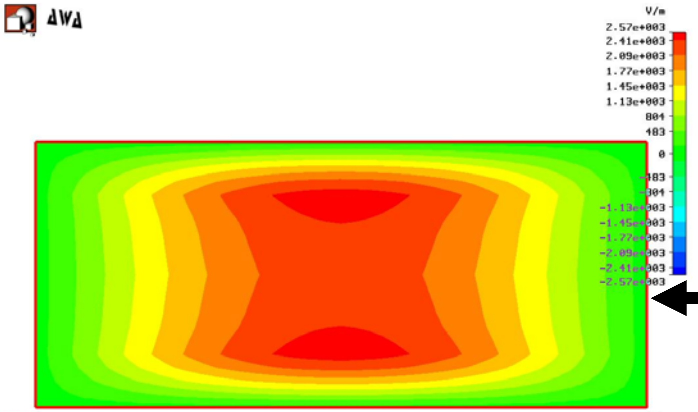
By mode pattern we have

Monopole modes----- Σ $\{LSM^{(open)}_{kn}$ & $LSE^{(open)}_{kn}\}$

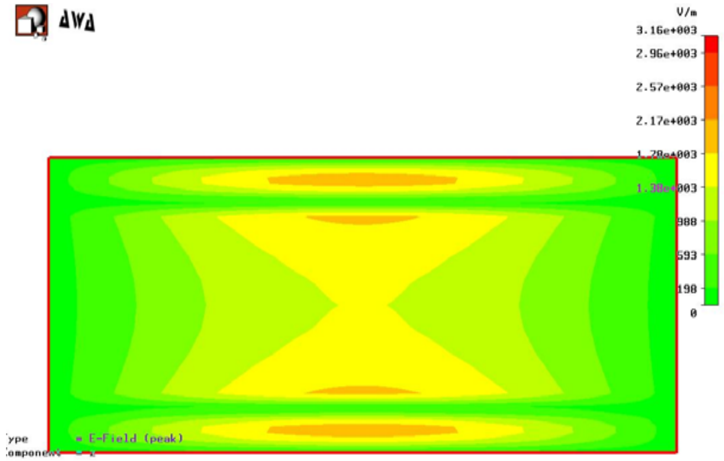
X-dipole modes------- Σ $\{LSM^{(open)}_{(2k)n}$ & $LSE^{(open)}_{(2k)n}\}$

Y-dipole modes------- Σ $\{LSM^{(short)}_{(2k-1)n}$ & $LSE^{(short)}_{(2k-1)n}\}$
$(k = 1, 2, 3\ldots..)$

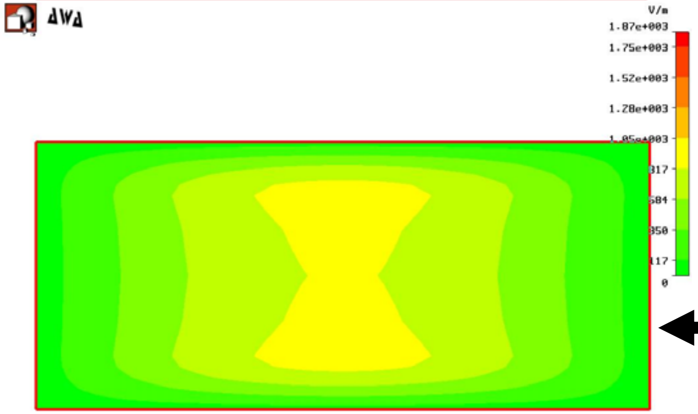
Mode matching ---- monopole



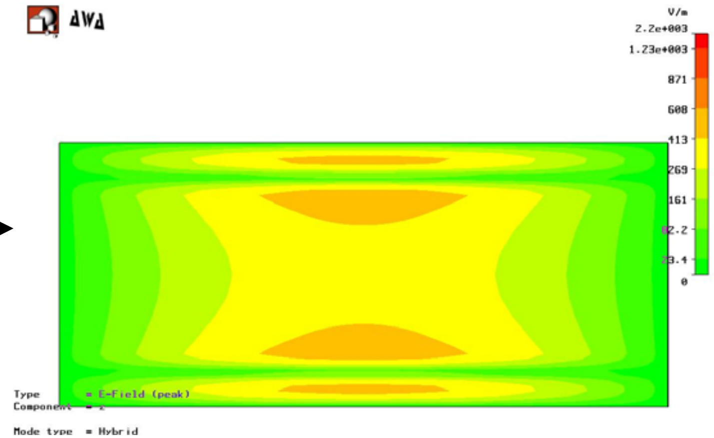
$LSM^{(open)}_{11}$



$LSM^{(open)}_{12}$



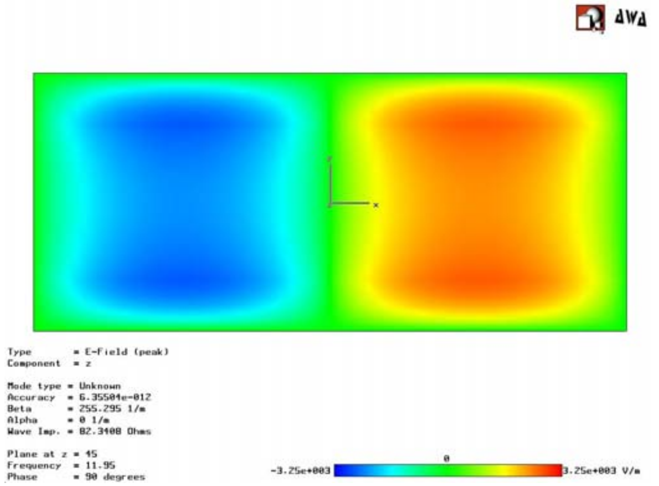
$LSE^{(open)}_{11}$

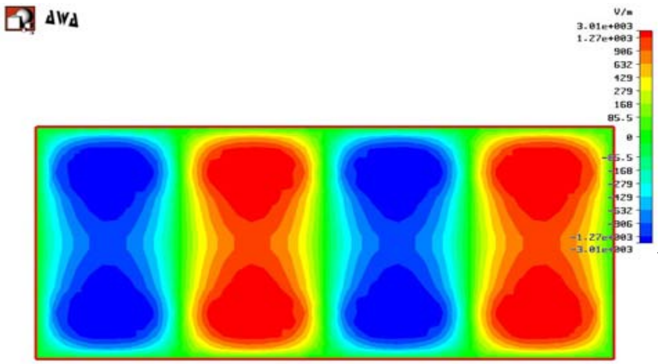


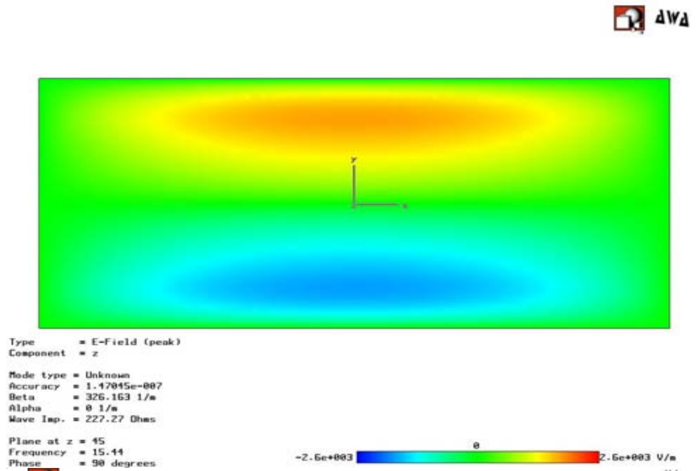
$LSE^{(open)}_{12}$

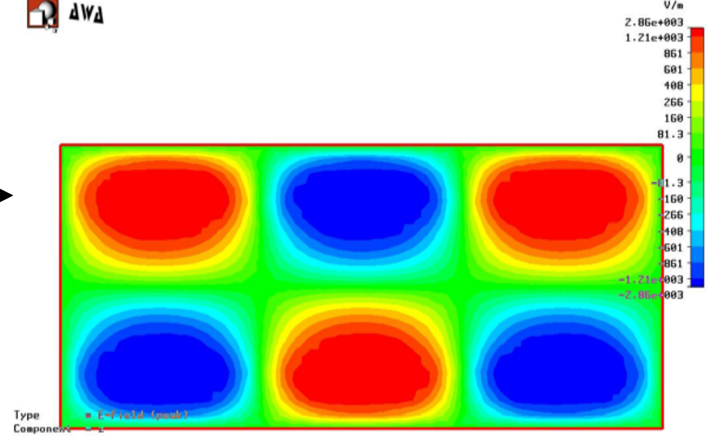
All figs show Ez Amplitude

Mode matching ---- dipole









$\mathbf{LSM}^{(short)}_{11}$

$\mathbf{LSM}^{(open)}_{21}$

$\mathbf{LSM}^{(short)}_{31}$

$\mathbf{LSM}^{(open)}_{41}$

All figs show Ez Amplitude

Method Overview

Wake fields solution

$$E_z = \sum_n A_{1n} \cos\frac{\pi}{w} \cosh\left|k_y^{(0)}\right| y e^{-j\beta z} \qquad monopole$$

$$E_z = \sum_{k,n} \begin{cases} B_{(2k)n} \sin\frac{(2k)\pi}{w} x \cosh\left|k_y^{(0)}\right| y e^{-j\beta z} & x-dipole \\ C_{(2k-1)n} \cos\frac{(2k-1)\pi}{w} x \sinh\left|k_y^{(0)}\right| y e^{-j\beta z} & y-dipole \end{cases}$$

From field analysis

Next: how to determine amplitude for each mode?

$$\left|E_{z_i}\right| = V = 2k_{l_i} q \text{ (normalized)} \qquad k_{l_i} = \frac{1}{4}\omega_i \left(\frac{R_{x_0,y_0}}{Q}\right)_i$$

General WF theory

@(x_0,y_0)

$$\begin{cases} A_{(2k-1)n} = 2k_{l_{1n}} q = \frac{q\omega_{1n}}{2}\left(\frac{R_{0,0}}{Q}\right)_{1n} & monopole \\ B_{(2k)n} \sin\frac{(2k)\pi}{w} x_0 \cosh\left|k_y^{(0)}\right| y_0 = 2k_{l_{(2k)n}} q = \frac{q\omega_{(2k)n}}{2}\left(\frac{R_{x_0,y_0}}{Q}\right)_{(2k)n} & x-dipole \\ C_{(2k-1)n} \cos\frac{(2k-1)\pi}{w} x_0 \sinh\left|k_y^{(0)}\right| y_0 = 2k_{l_{(2k-1)n}} q = \frac{q\omega_{(2k-1)n}}{2}\left(\frac{R_{x_0,y_0}}{Q}\right)_{(2k-1)n} & y-dipole \end{cases}$$

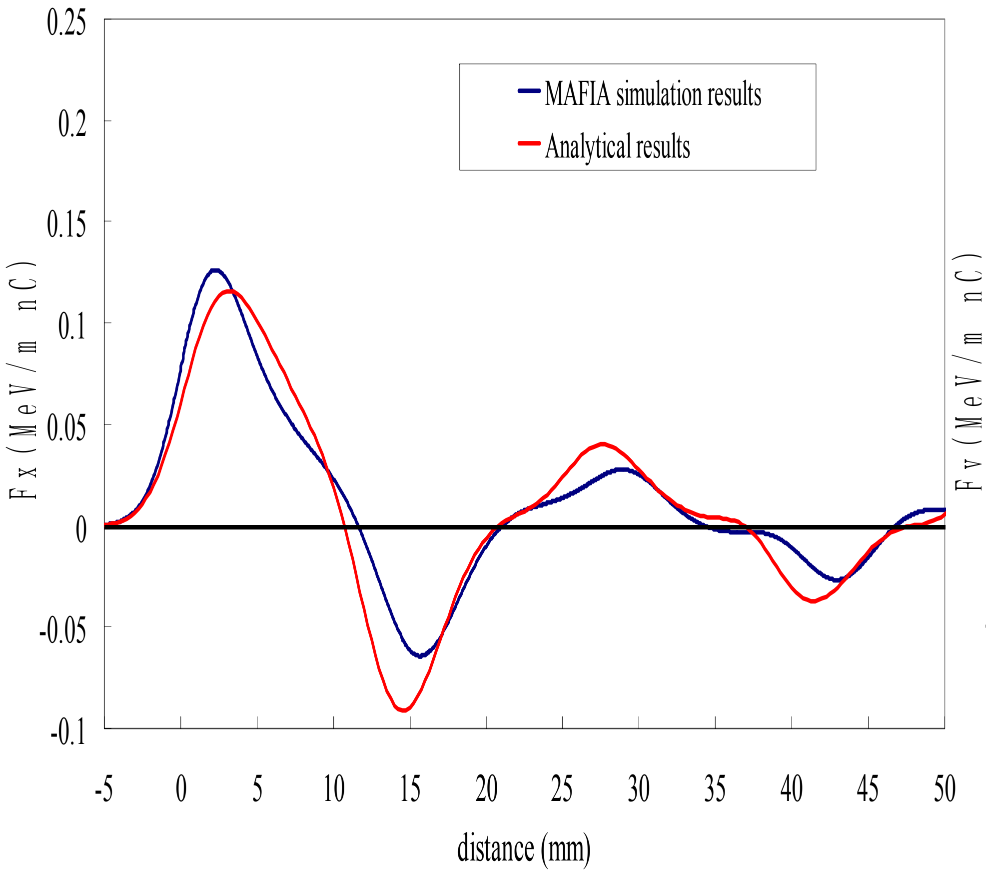
Next: how to find transverse wake fields?

$$\frac{\partial \vec{F}_\perp}{\partial z} = e\nabla_\perp E_z$$

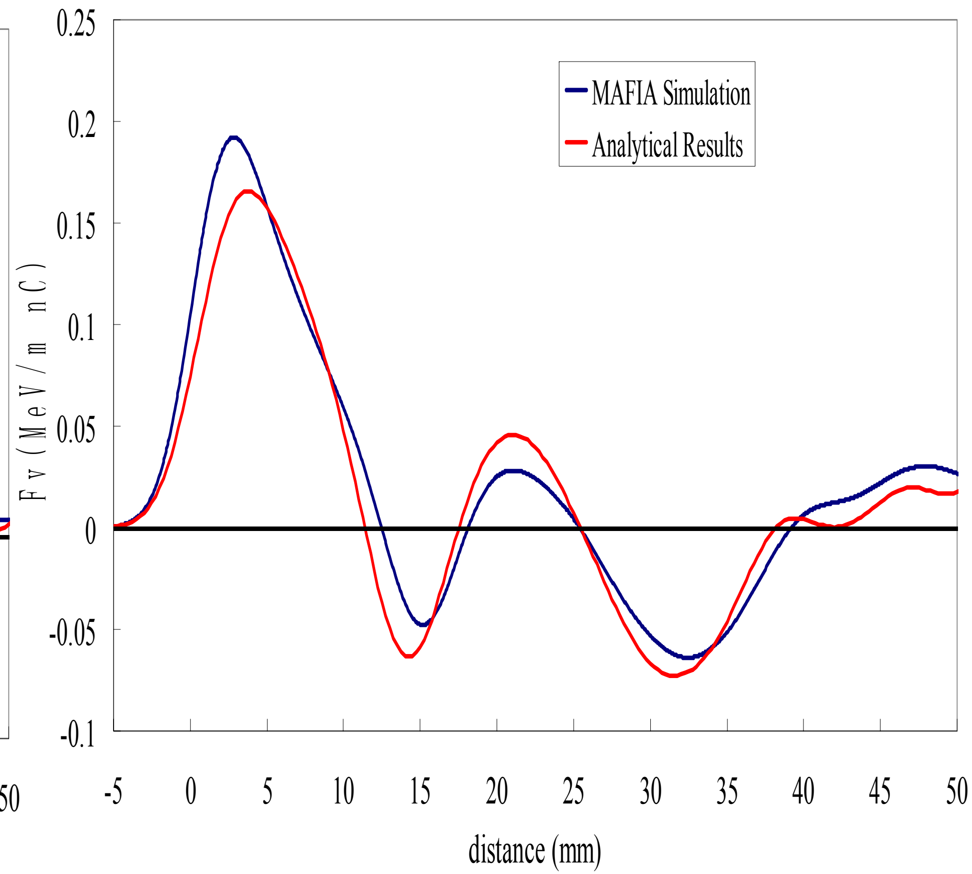
Panofsky-Wenzel theorem

Numerical Example

Transverse wake fields for dipole modes



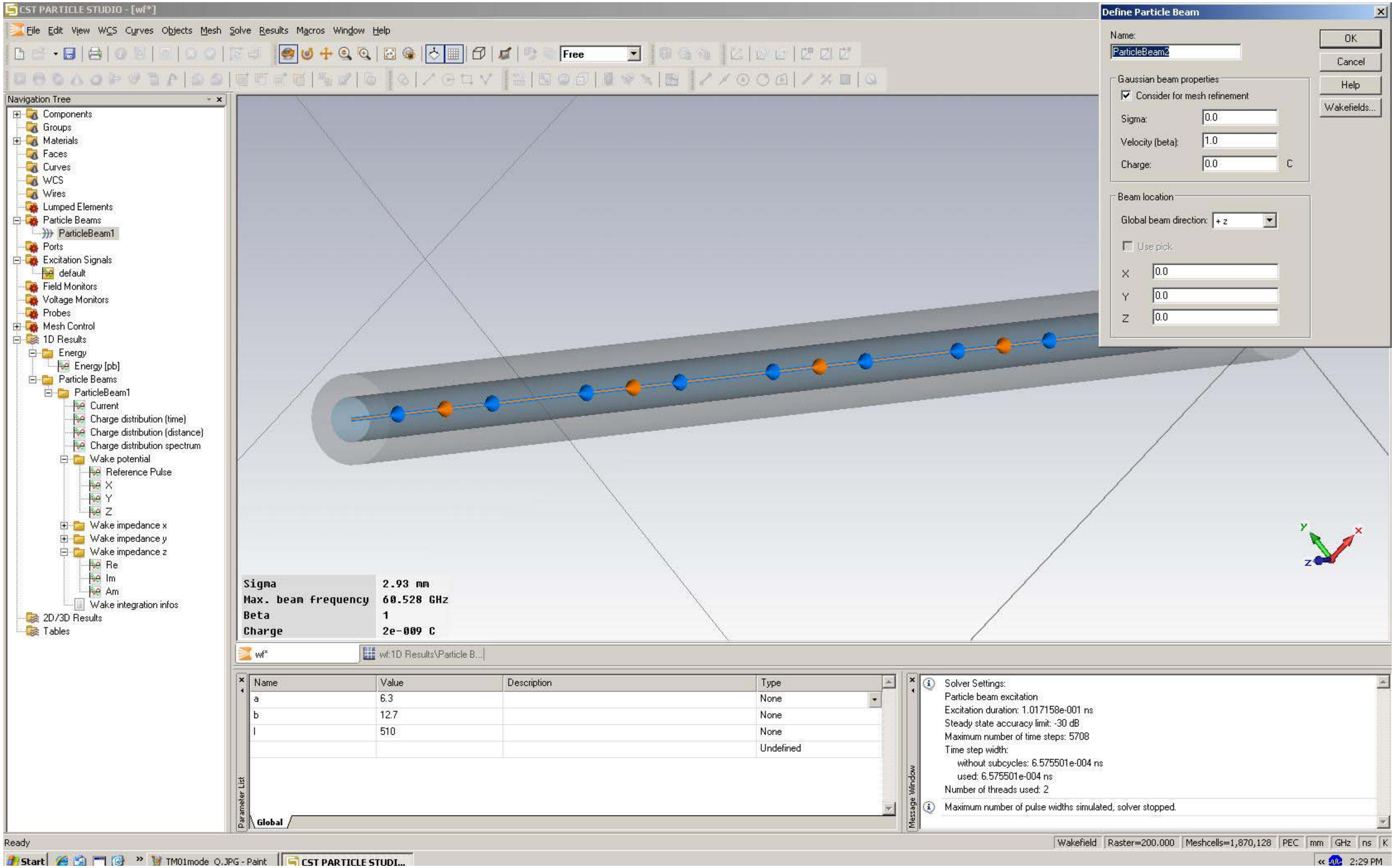
Fx



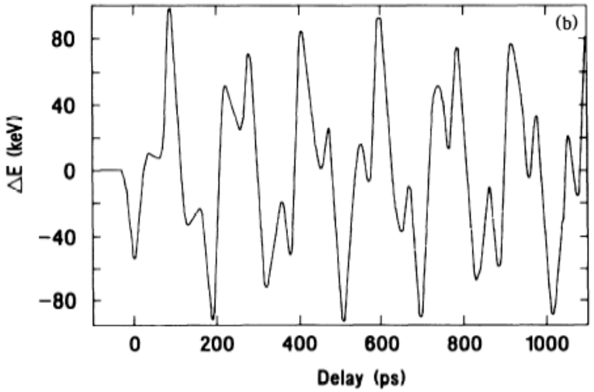
Fy

3.Wakefield Simulation using CST Particle Studio

Wakefield Solver



Example:



Analytical cal

TABLE I. Experimental cavities and beam conditions.

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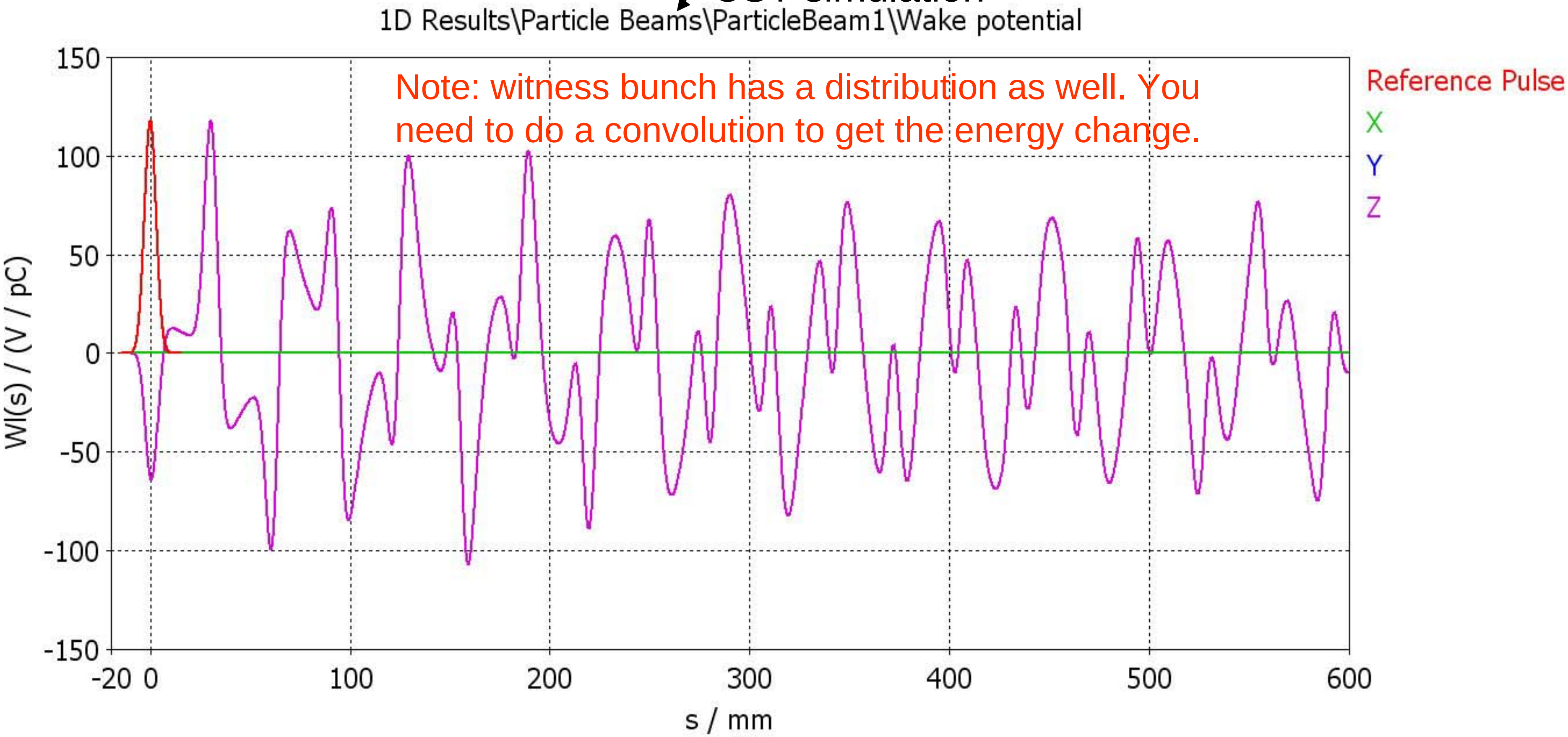
Computer lab HW

CST simulation

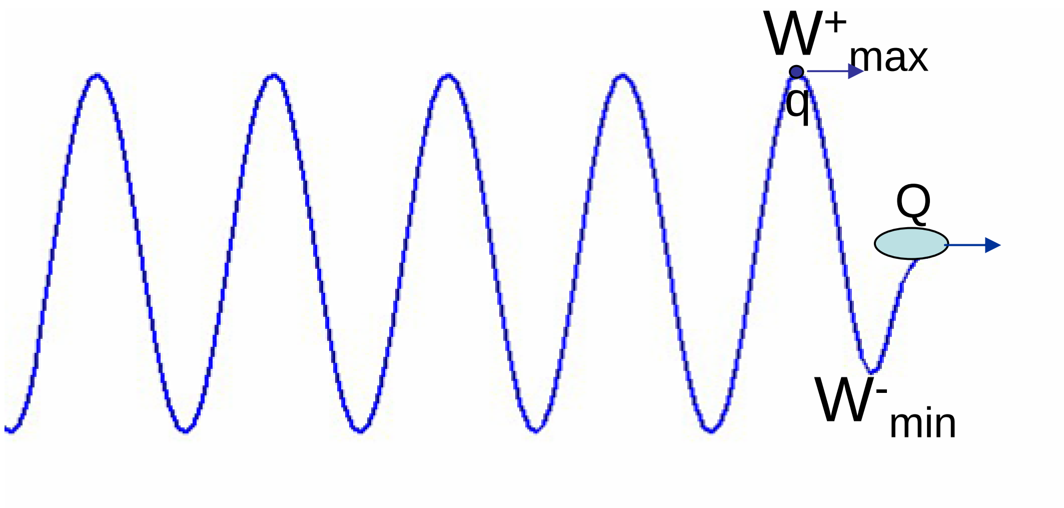
Note: witness bunch has a distribution as well. You need to do a convolution to get the energy change.

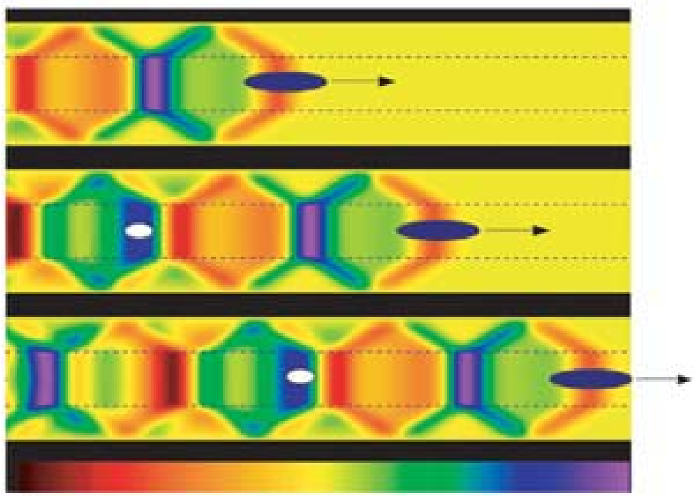


Part II:
Wakefield Acceleration

1.Collinear Wakefield Acceleration

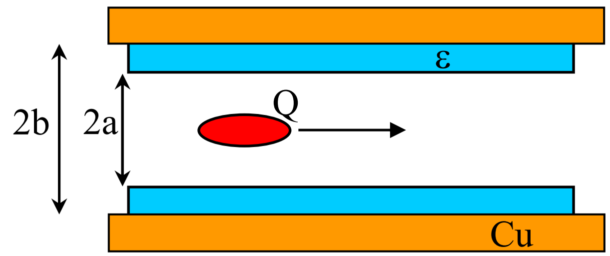
$$W_z = -e\int_0^L E_z dz$$





Electron Beam Driven High Gradient Wakefields

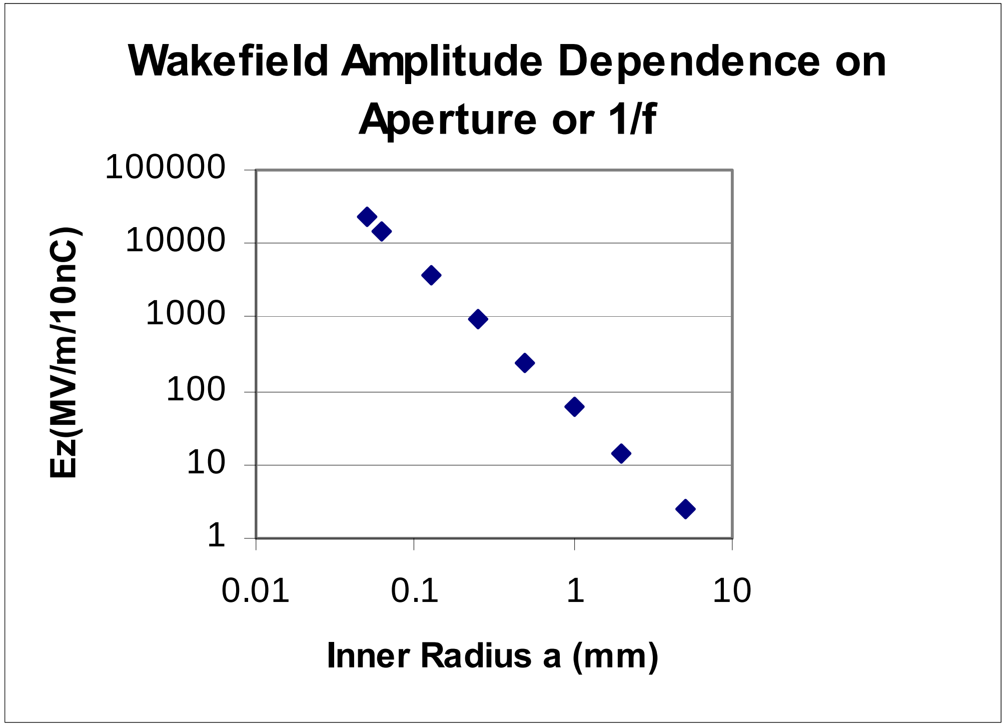
- A new way to power the accelerating structure by transporting the power in the electron beam.
- Applications
  - ***Collinear wakefield acceleration***
  - ***Two-beam acceleration***
- Keys to the success: Drive beam, drive beam and drive beam!
- Energy ↑, Charge ↑ Bunch length ↓ Emittance ↓



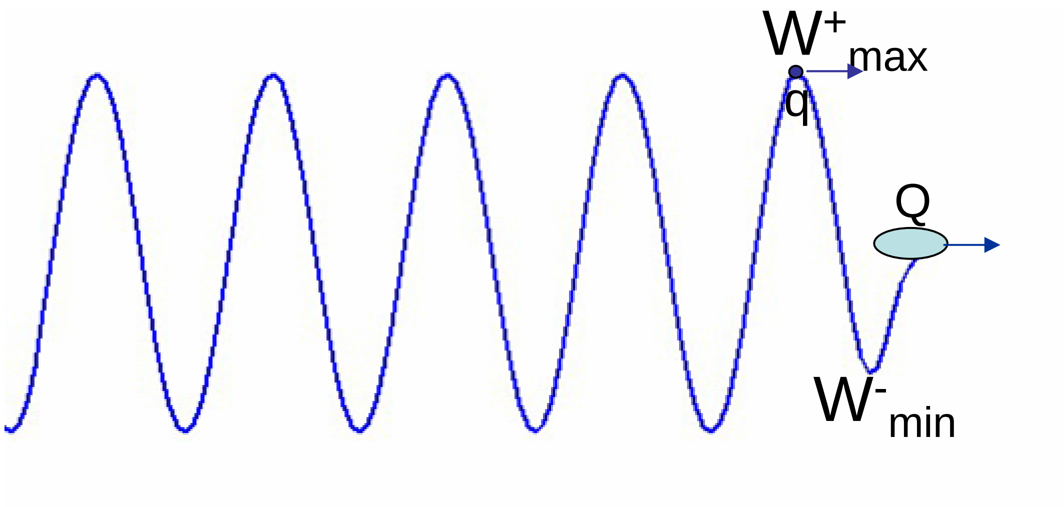
$$W_Z(z) \approx \frac{Q}{a^2} \exp\left[-2\left(\frac{\pi\,\sigma_z}{\lambda_n}\right)^2\right]\cos(kz)$$

$$\sigma_r = \left(\frac{\varepsilon_N}{\gamma}\beta\right)^{1/2}$$

10 nC a=0.1 mm → 5 GV/m.



Wakefield Transformer Ratio

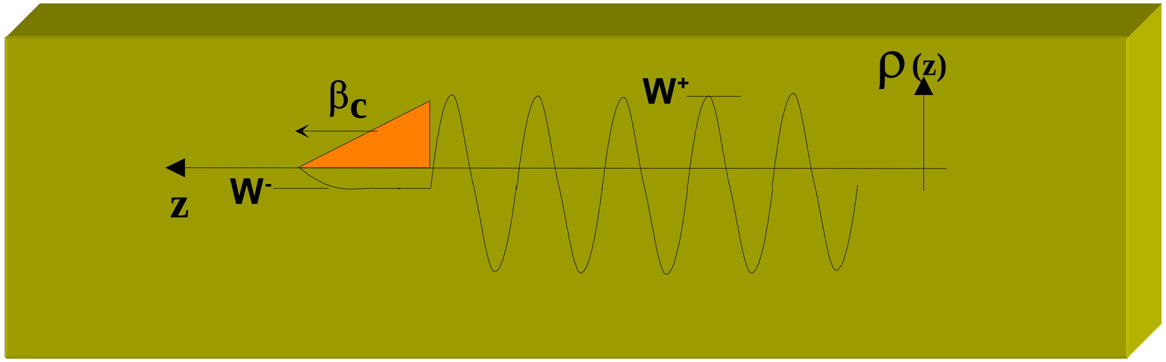


$$\text{Transformer ratio } R = \frac{\text{Max energy gain of the witness bunch}}{\text{Max energy loss of the drive bunch}}$$

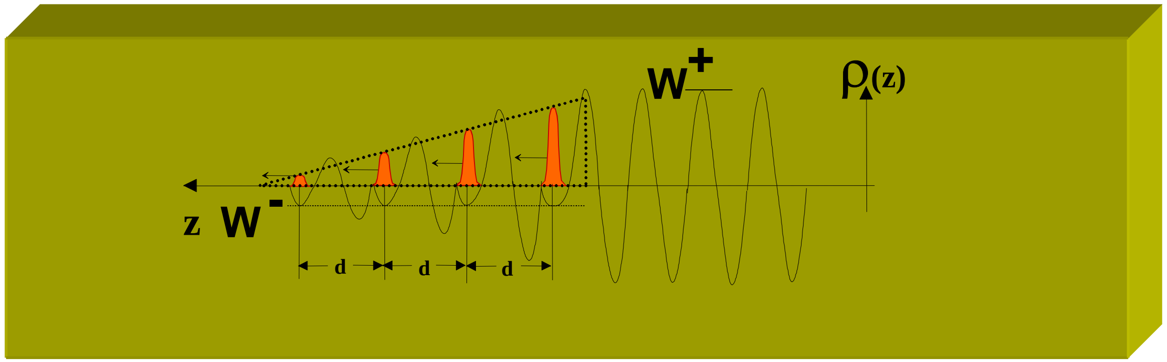
Transformer ratio limited: R≤2 @ a longitudinally symmetric drive bunch, but it can be enhanced greater than 2 using asymmetric bunch.

To Enhance the WTR

Scheme I---Single
Triangular Bunch



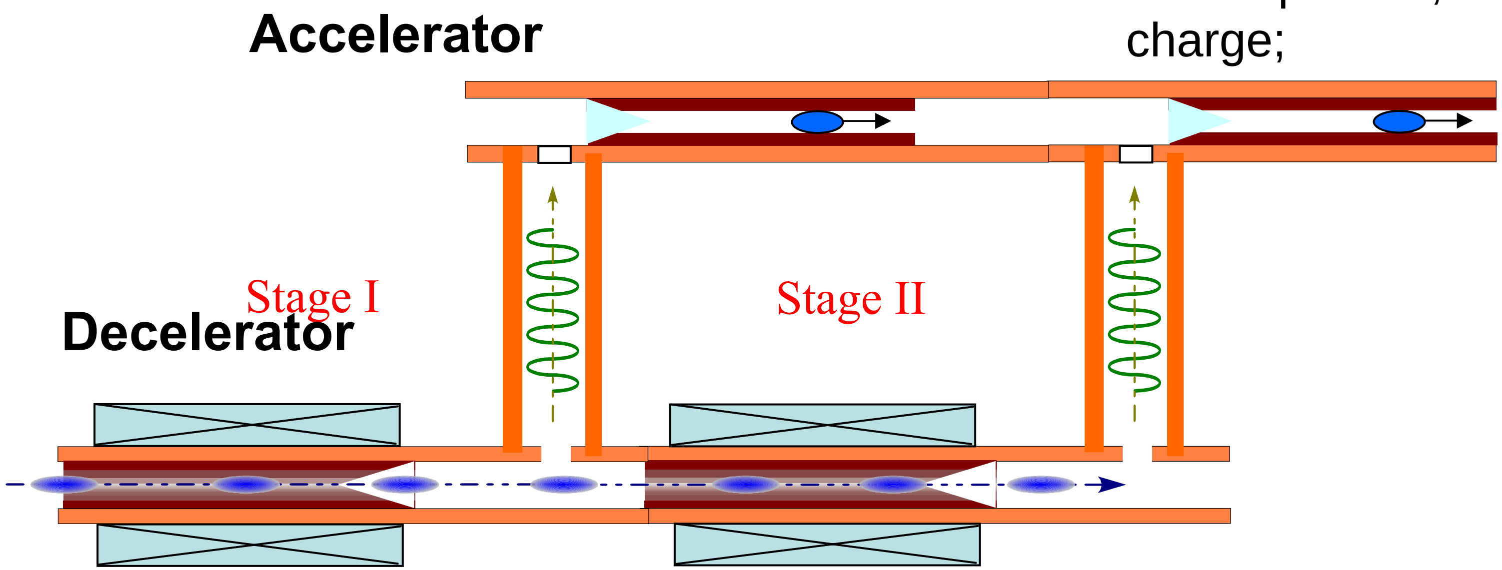
Scheme II---Ramped
Bunch Train



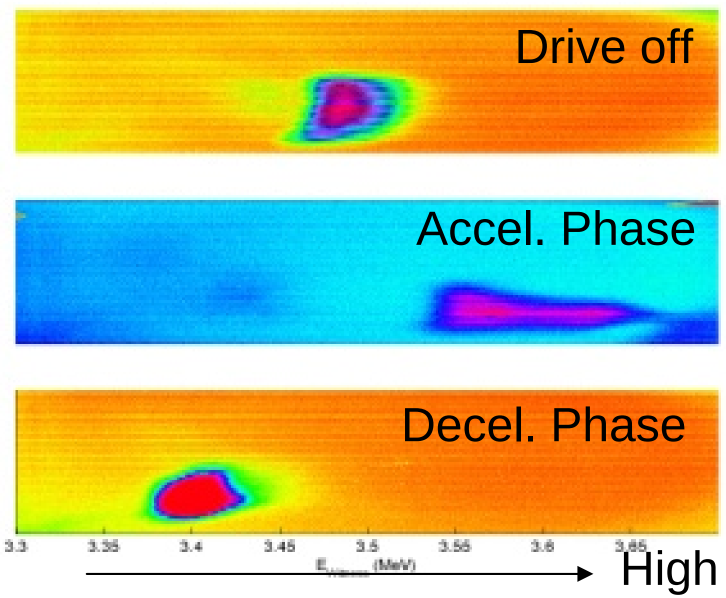
RBT: d=(1+1/2) λ , acceleration for the second bunch, $Q_1=3Q_0$,
$W^+=(3-1)W_0^+=2W_0^+$, $W_0^-=(3-2)W_0^-=W_0^-$, $R=2R^0$
R_n= nR_0~2*n for the large number of bunches

2.Two Beam Wakefield Acceleration

High dielectric const; small aperture; low charge;



Low dielectric const; big aperture; high charge;



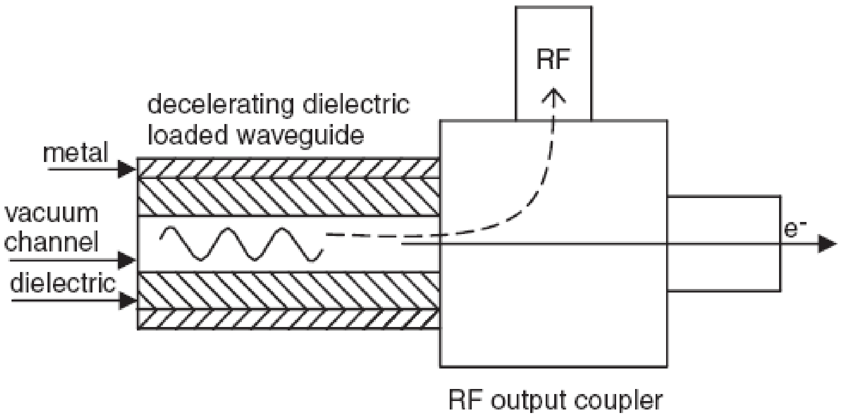
High Energy

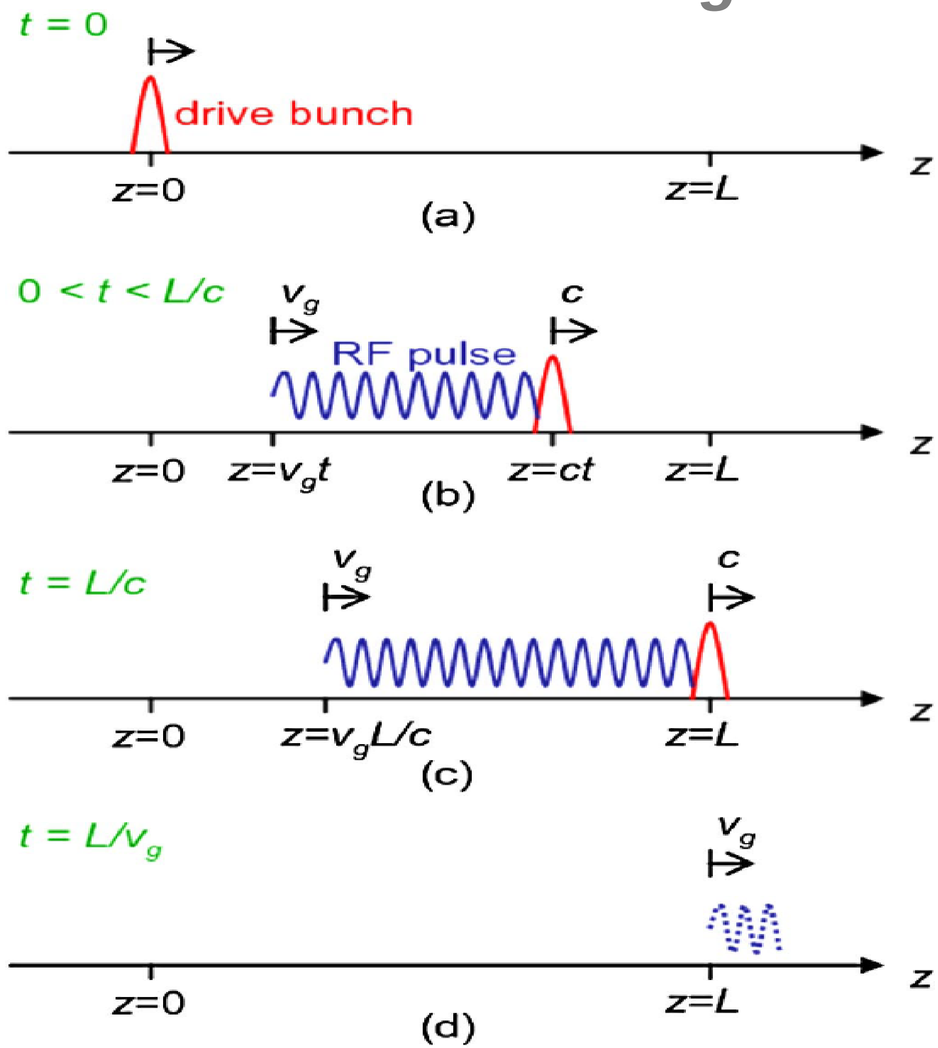
Part III:
Special Topic: dielectric based wakefield power extraction

1. Principle

[ref5] F. Gao, et al. Phys. Rev. S.T. Accel. Beams, 11, 041301 (2008).

A Single Bunch Excites an RF Pulse in the Waveguide



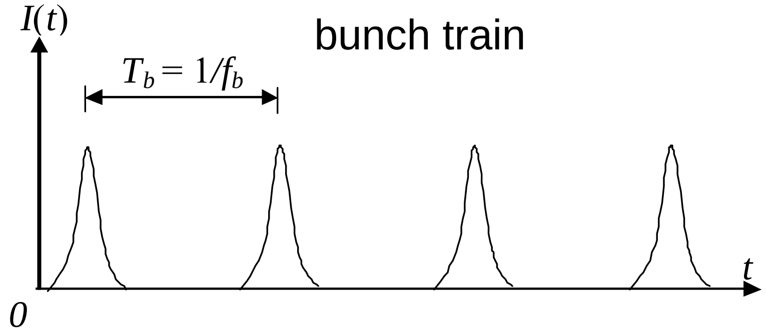


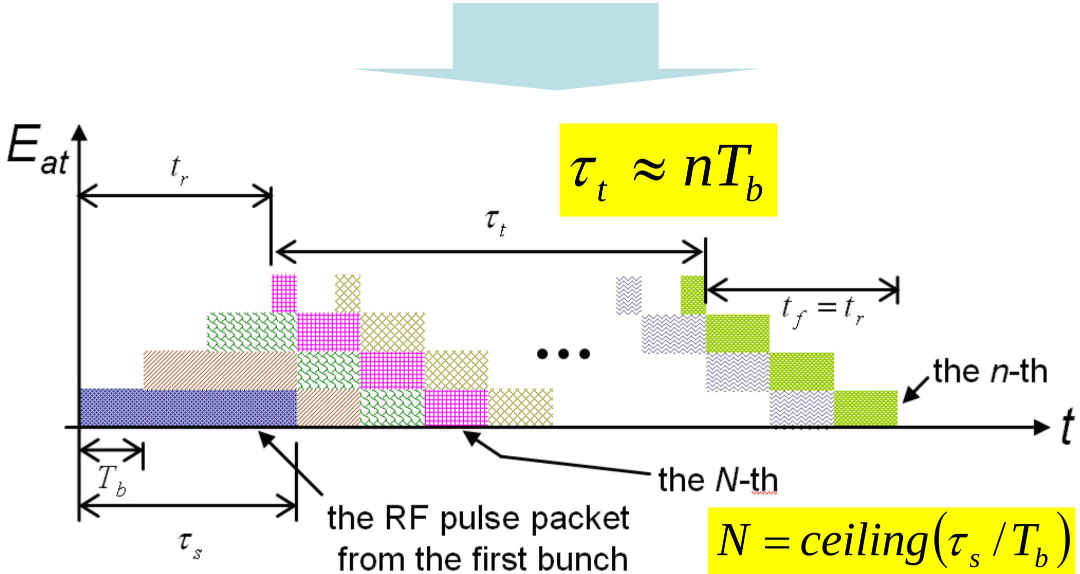
Pulse length: $\tau_s = L / v_g - L / c$

Probed RF Pulse at the Downstream End

single bunch

$$\tau_s = L / v_g - L / c$$





$$\tau_t \approx nT_b$$

$$N = ceiling\left(\tau_s / T_b\right)$$

* **Feng Gao *et al*, *Phys. Rev. ST-AB*, vol. 11, 041301, 2008.**

Calculation of the Generated Power

Relation between power and gradient

$$P_w = \frac{E_a^{\ 2}}{2\alpha_0 r_L}$$

Energy conservation with a drive beam

$$\frac{dP_w}{dz} = \frac{qE_a}{T_b}\Phi - 2\alpha_0 P_w$$

beam loading

attenuation due to finite Q

	lossless	lossy
bunch train	$P_t = q^2 \frac{k_z}{4\beta_g}\left[\frac{r}{Q}\right]\left(\frac{L}{T_b}\right)^2 \Phi^2$	$P_t = q^2 \frac{k_z}{4\beta_g}\left[\frac{r}{Q}\right]\left(\frac{1-e^{-\alpha_0 L}}{\alpha_0 T_b}\right)^2 \Phi^2$
single bunch	$P_s = q^2 \frac{k_z\beta_g}{4}\left[\frac{r}{Q}\right]\left(\frac{c}{1-\beta_g}\right)^2 \Phi^2$	$P_s(t) = q^2 \frac{k_z\beta_g}{4}\left[\frac{r}{Q}\right]\left(\frac{c}{1-\beta_g}\right)^2 \Phi^2 e^{-2\alpha_0 v_g t}$

$$\Phi = \left|\frac{1}{q}\int_{-\infty}^{+\infty} f(z)e^{-jk_z z}dz\right|$$

is the bunch form factor which represents the spectrum of the drive beam.

2.Design a dielectric-based wakefield power extractor with help from CST

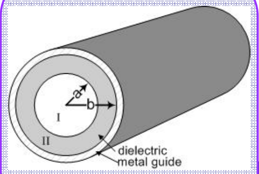
Analytical Solutions

CST MW Studio

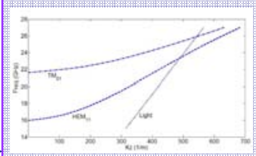
CST PS Wakefield

CST PS PIC

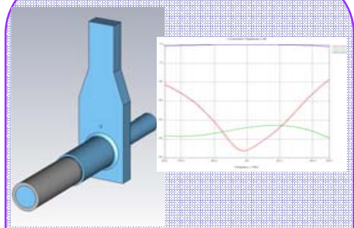
DWPE



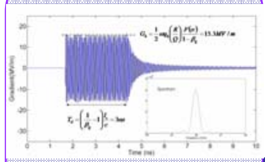
Calculate f, a, b, L, Q, R/Q, Vg, G, P, etc.



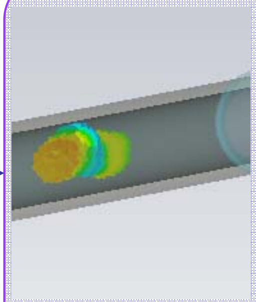
Verify, modes dispersion curves, EM fields, Q, etc.



Coupler Design

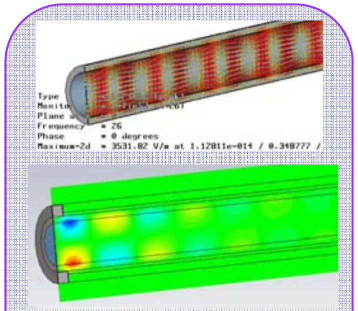


Verify Grad. & output rf power.

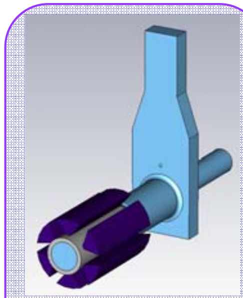


Study BBU effect.

TDDWPE



Verify, modes dispersion curves, EM fields, Q, etc.

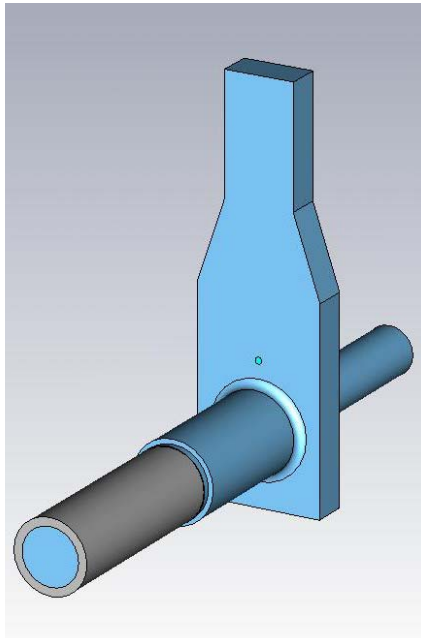


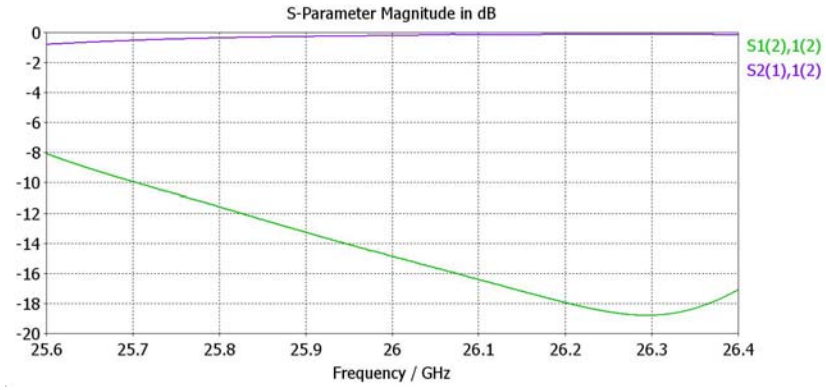
Coupler Design

Study BBU effect.

26GHz DWPE

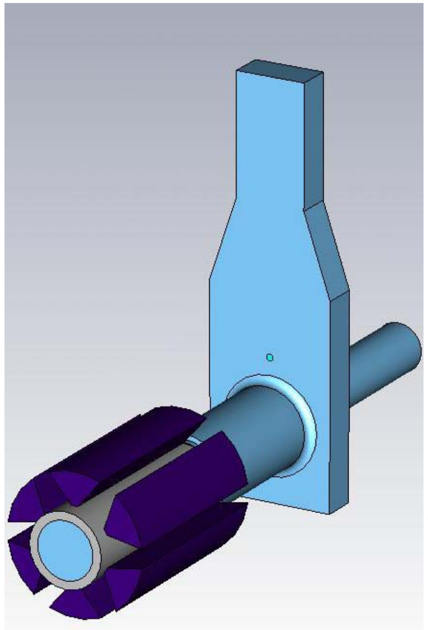
a=3.5mm
b=4.53mm
L=300mm
eps=6.64
Q=2950 (TM01)
R/Q=9788ohm/m

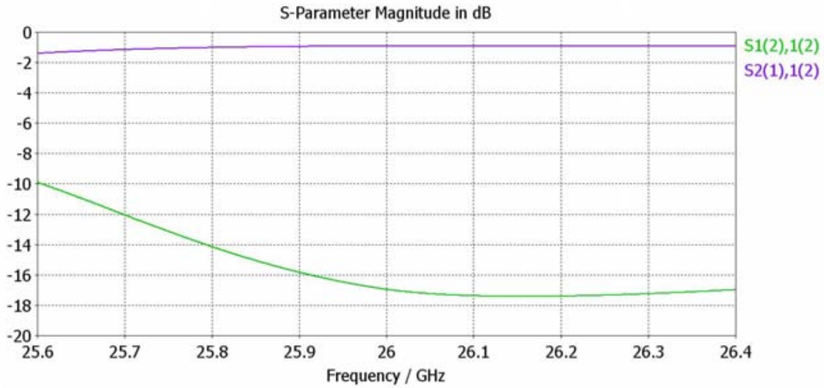


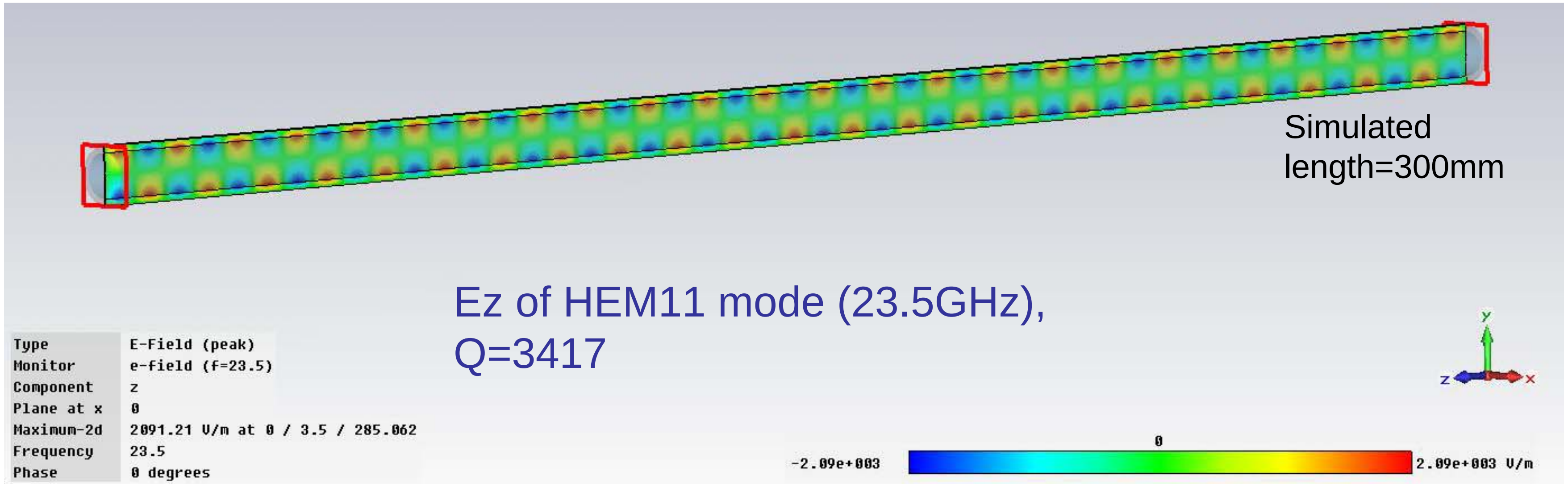


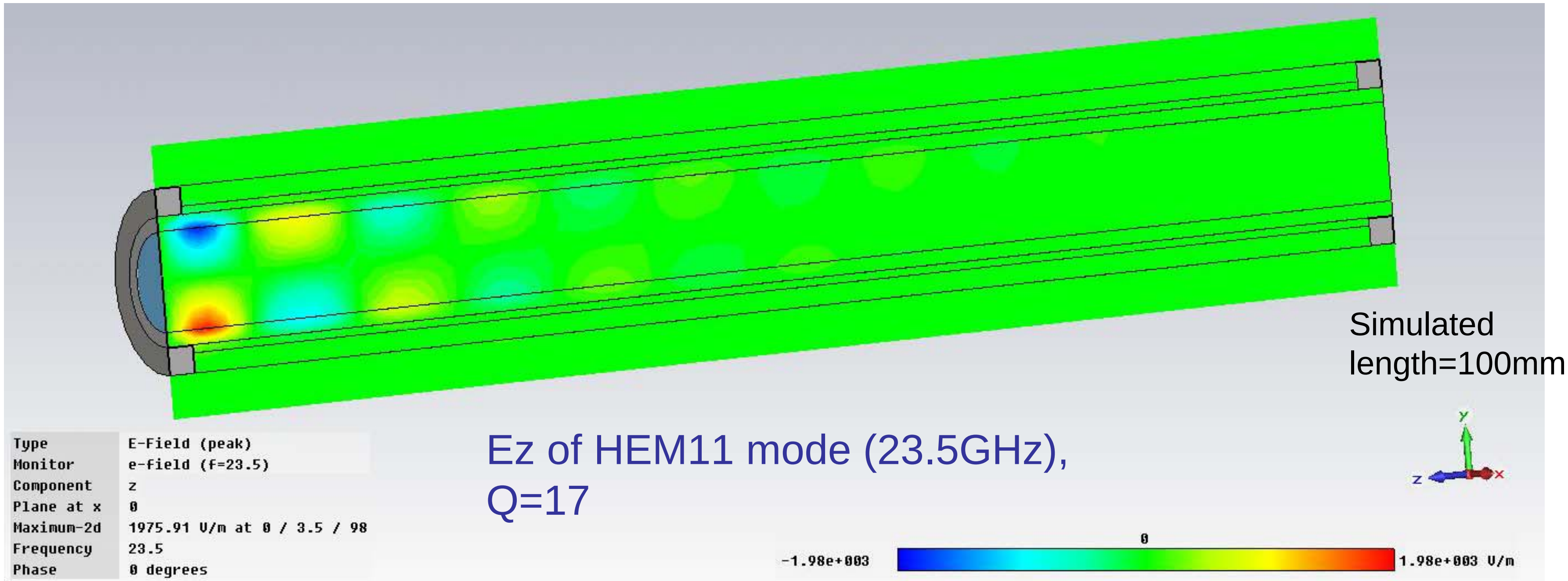
26GHz TDDWPE

6 damping slots fully filled with rf absorption material.





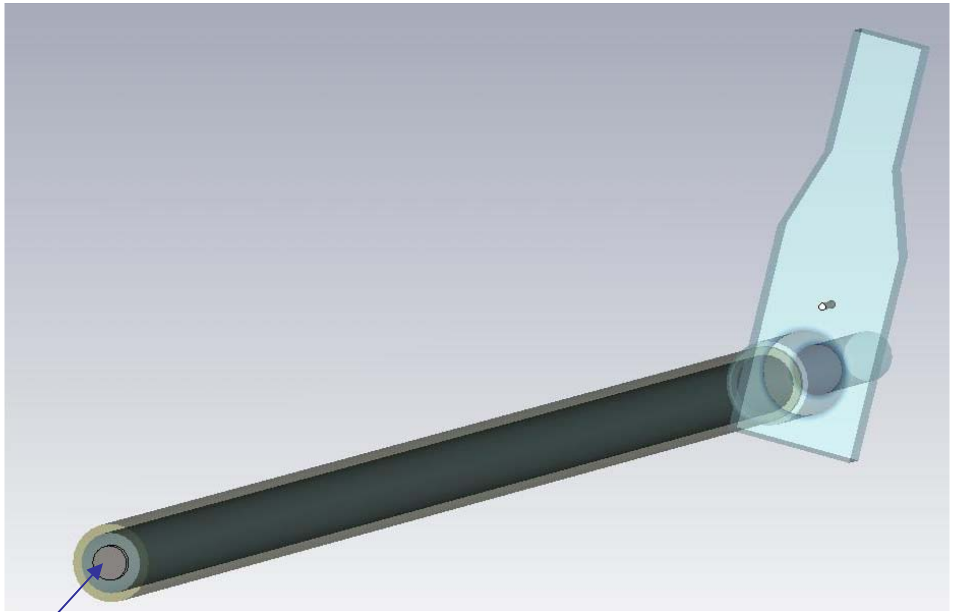

Simulated
length=300mm
Ez of HEM11 mode (23.5GHz),
Q=3417
Type E-Field (peak)
Monitor e-field (f=23.5)
Component z
Plane at x 0
Maximum-2d 2091.21 V/m at 0 / 3.5 / 285.062
Frequency 23.5
Phase 0 degrees
-2.09e+003
0
2.09e+003 V/m
y
z
x


Simulated
length=100mm
Ez of HEM11 mode (23.5GHz),
Q=17
Type E-Field (peak)
Monitor e-field (f=23.5)
Component z
Plane at x 0
Maximum-2d 1975.91 V/m at 0 / 3.5 / 98
Frequency 23.5
Phase 0 degrees
-1.98e+003
0
1.98e+003 V/m
y
z
x

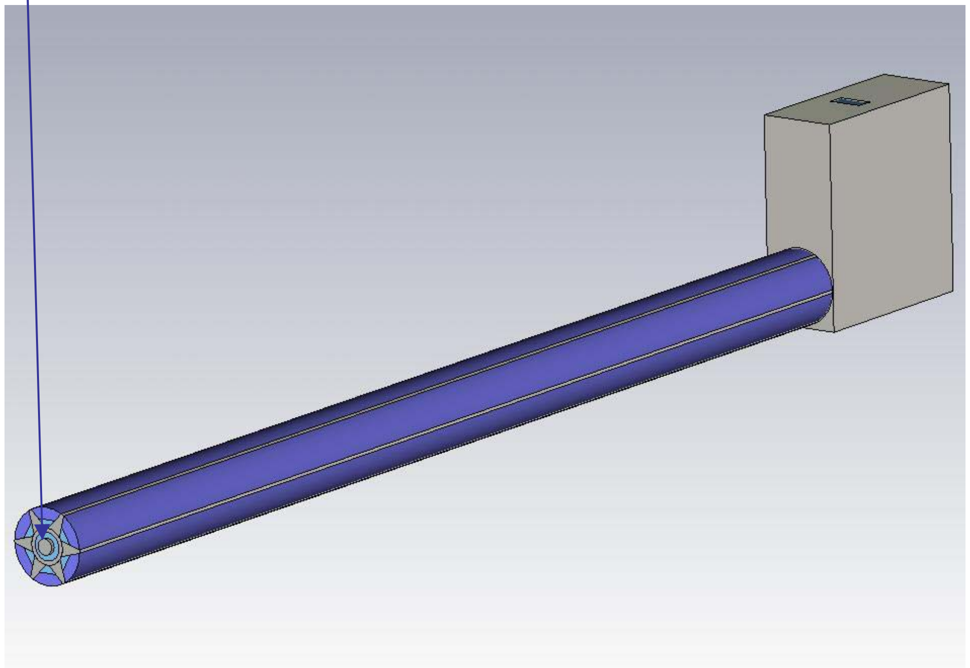
Set up simulation in CST PS PIC module

Beam parameters at the entrance

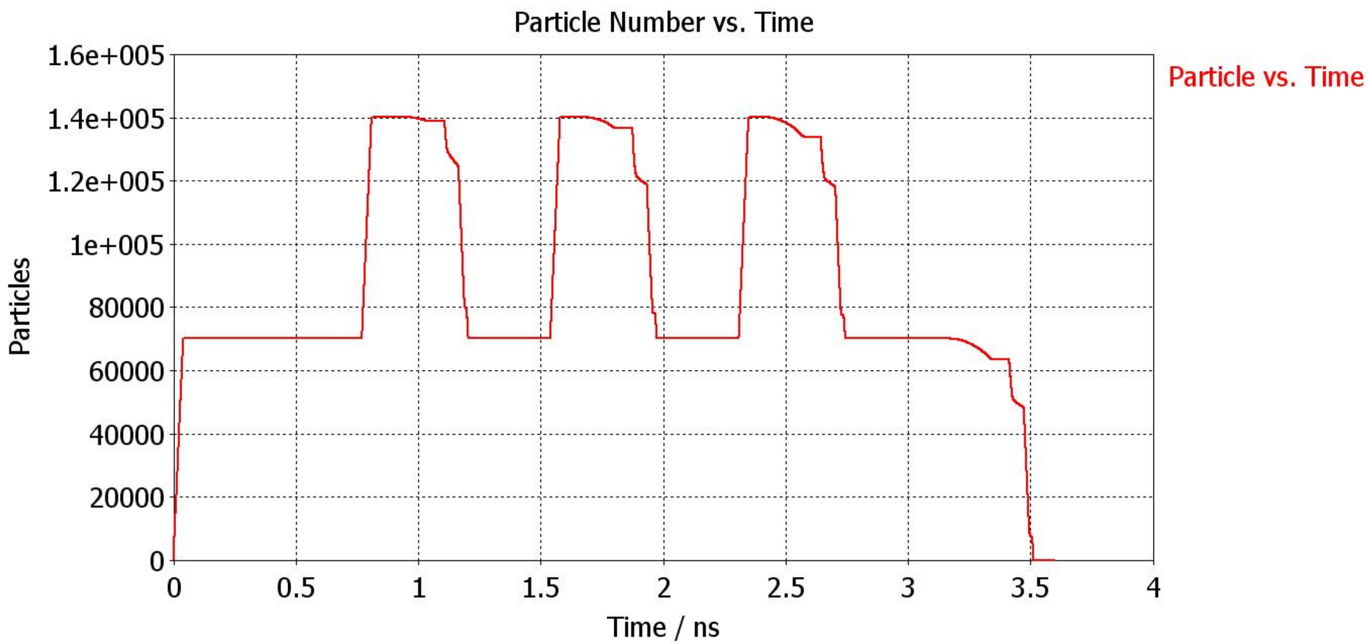
Source type	Face
Particle type	electron
Charge	-1.602177e-019 C
Mass	9.109390e-031 kg
Emission Model	Gauss
Kinetic type	Energy
Value	1.500e+007 eV
Kinetic spread	1.000e+000 %
Angle spread	5.700e-002 °
Bunch settings	
Charge (abs)	2.000e-008 C
Sigma	2.000e+000 mm
Cutoff	6.000e+000 mm
Offset	6.000e+000 mm
Bunches	4
Distance	2.306e+002 mm



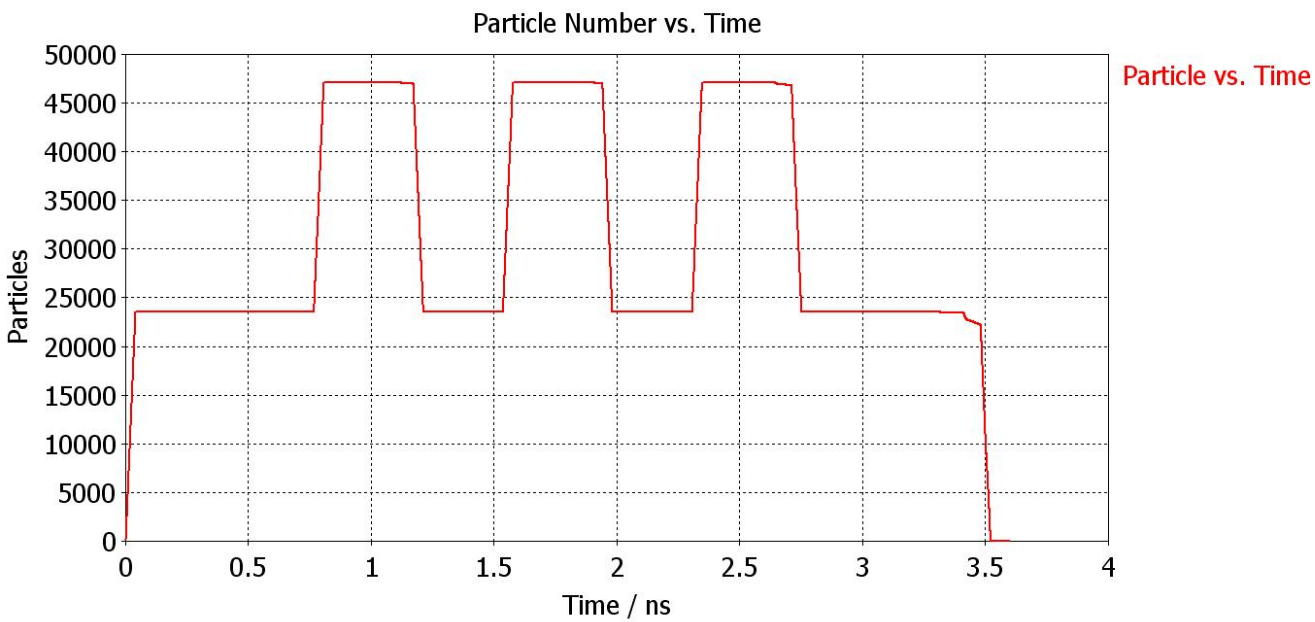
Beam launching surface



26GHz DWPE



26GHz TDDWPE

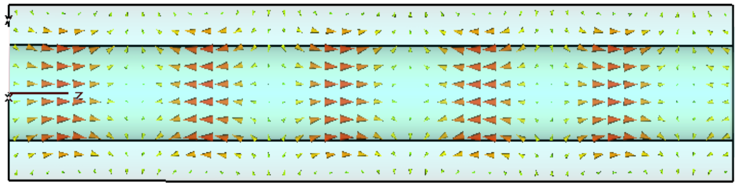


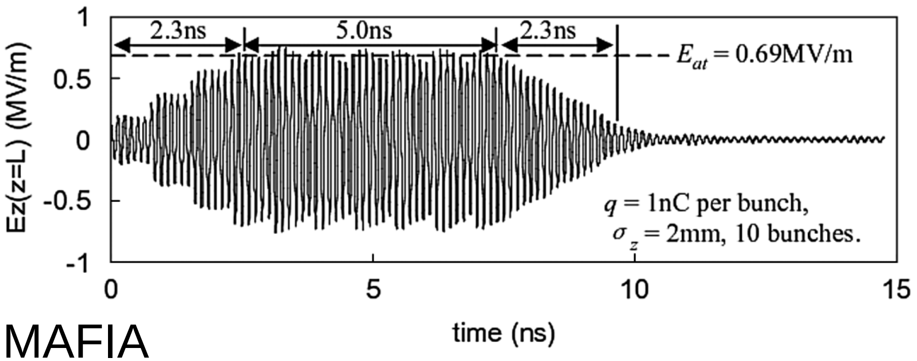
Movie of 4-20nC- bunch train go through the 26GHz DWPE

Movie of 4-20nC- bunch train go through the 26GHz TDDWPE

3. An Example: 7.8GHz DWPE

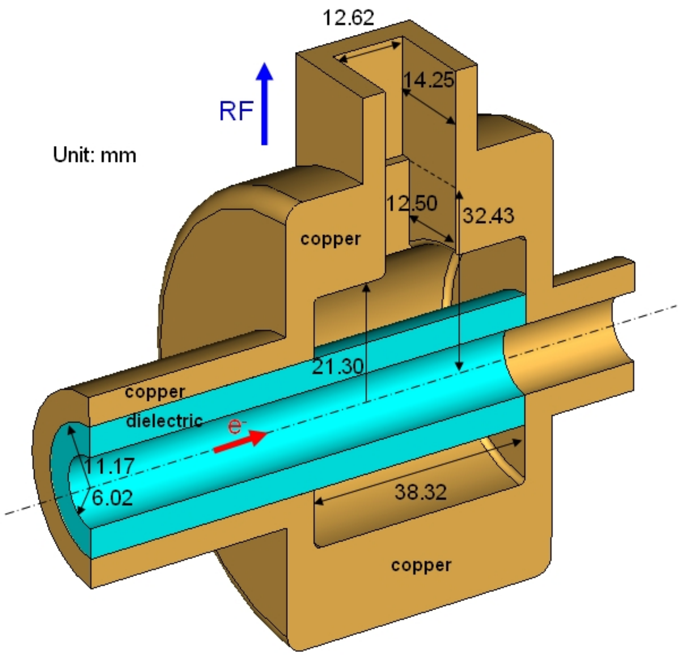
7.8GHz Decelerating waveguide + Output coupler

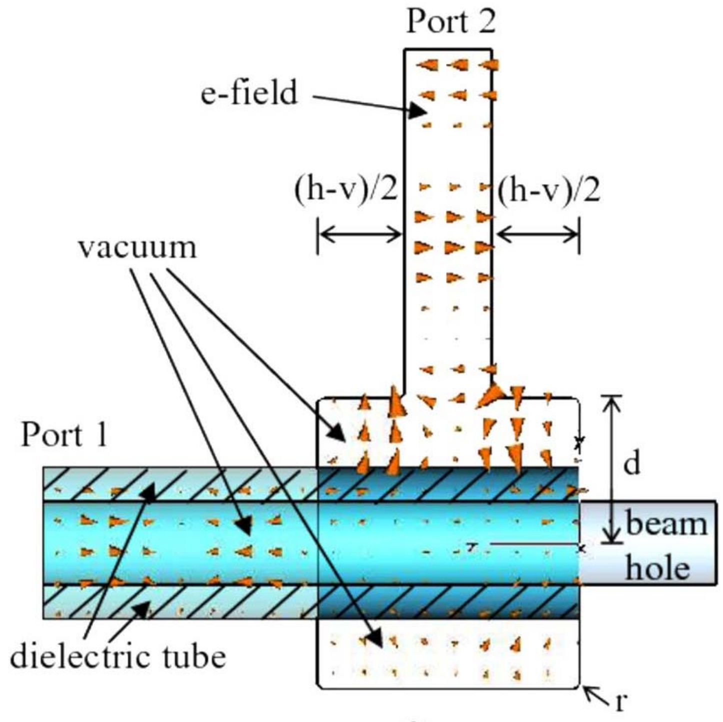




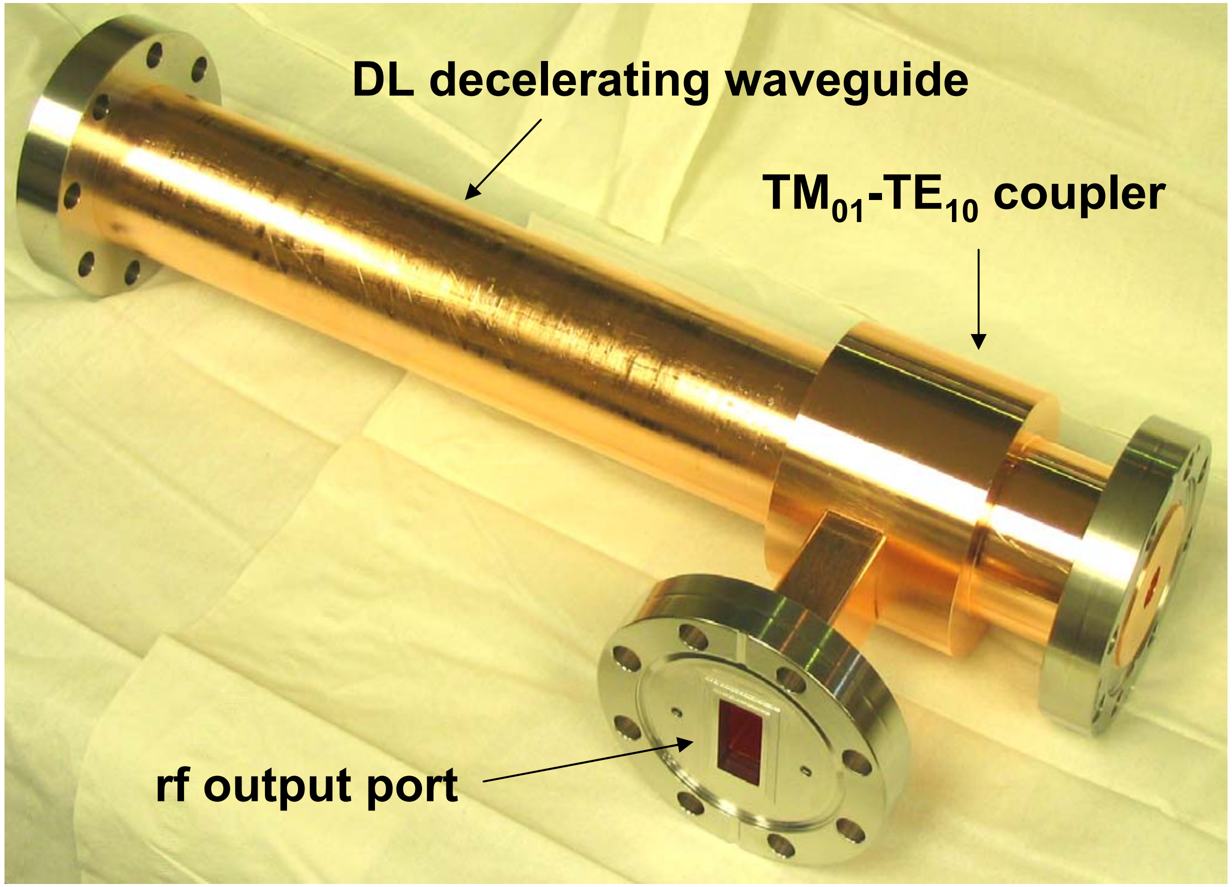
MAFIA

Microwave Studio





The 7.8GHz Power Extractor

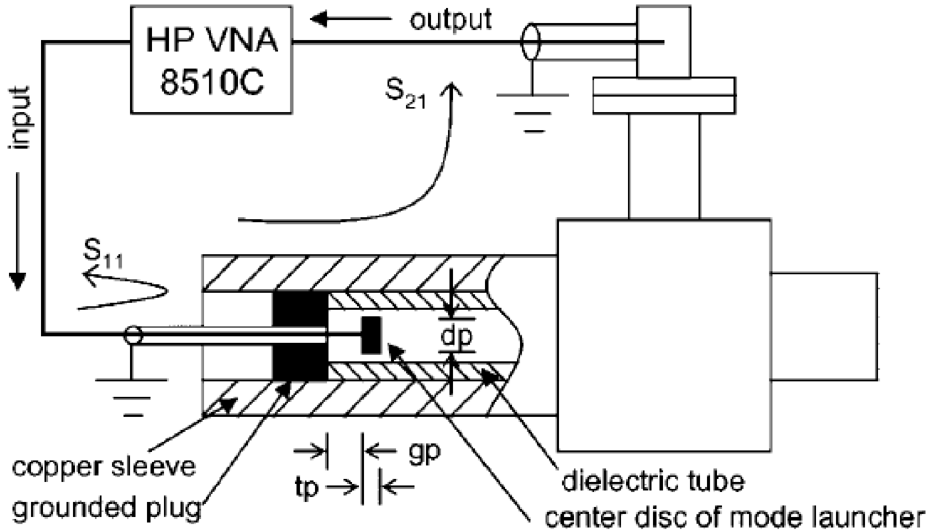


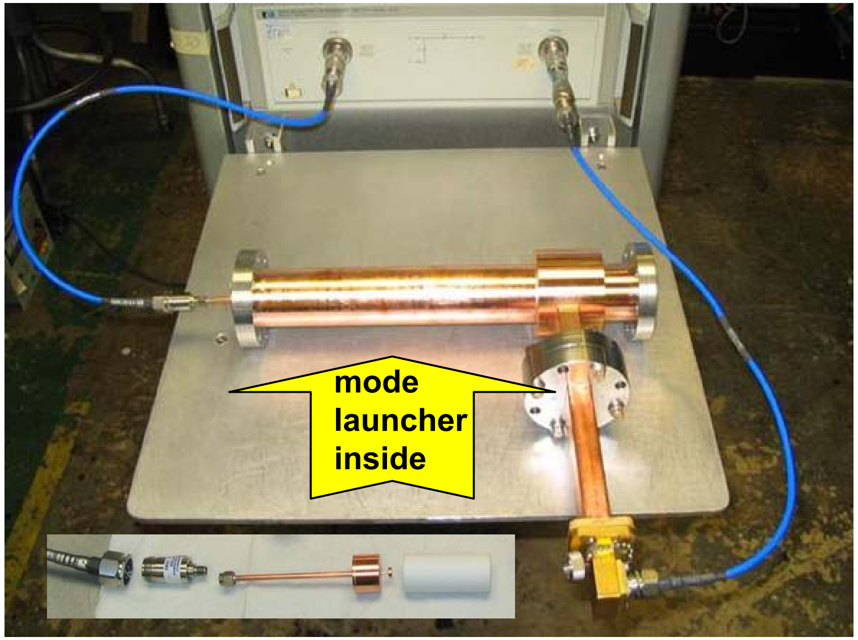
Frequency:	7.8GHz
Inner diameter:	12.04mm
Outer diameter:	22.34mm
Dielectric constant:	4.6
Waveguide length:	266mm
Group velocity:	0.23c

$$P_t = \left(\frac{q}{T_b}\right)^2 L^2 \frac{k_z}{4\beta_g}\left[\frac{r}{Q}\right]\Phi^2$$

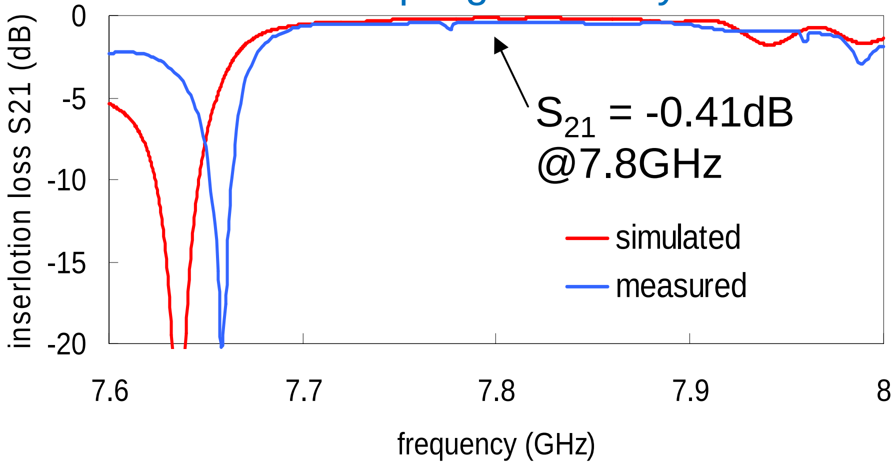
Generated power (Gaussian electron bunch σ_z = 2mm):
Single bunch: 79MW @100nC per bunch
Bunch train (T_b = 769ps): 280MW @ 50nC per bunch
1.1GW @ 100nC per bunch

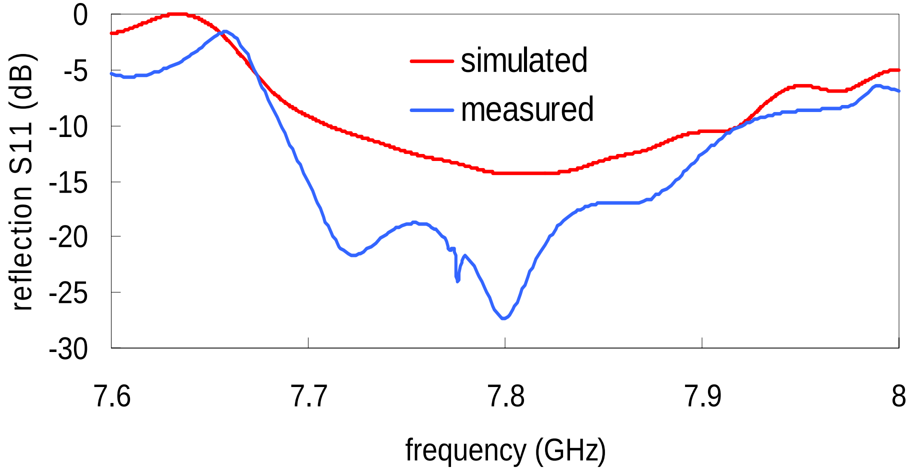
Test of the 7.8GHz output coupler



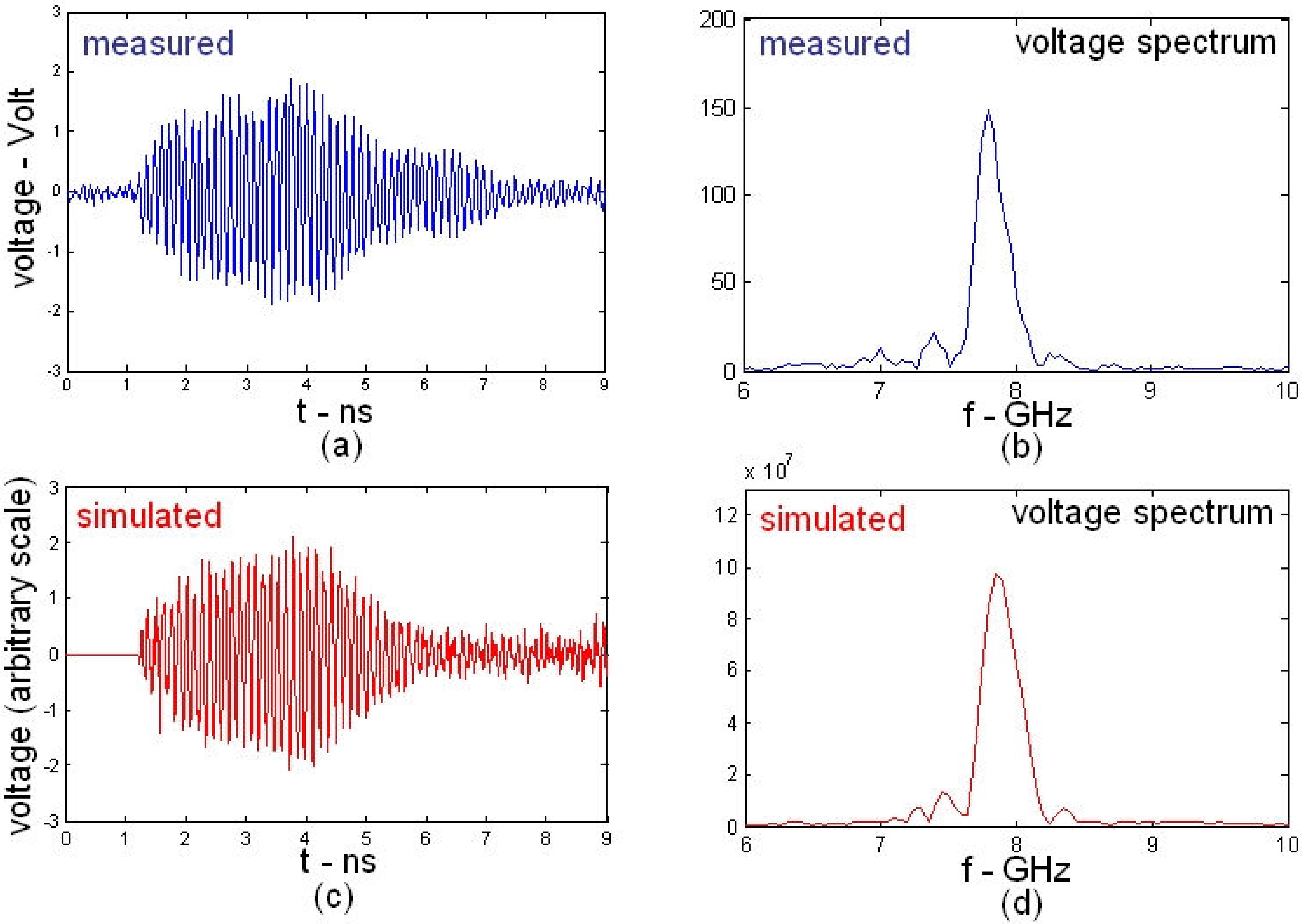


Power coupling efficiency: 91%.

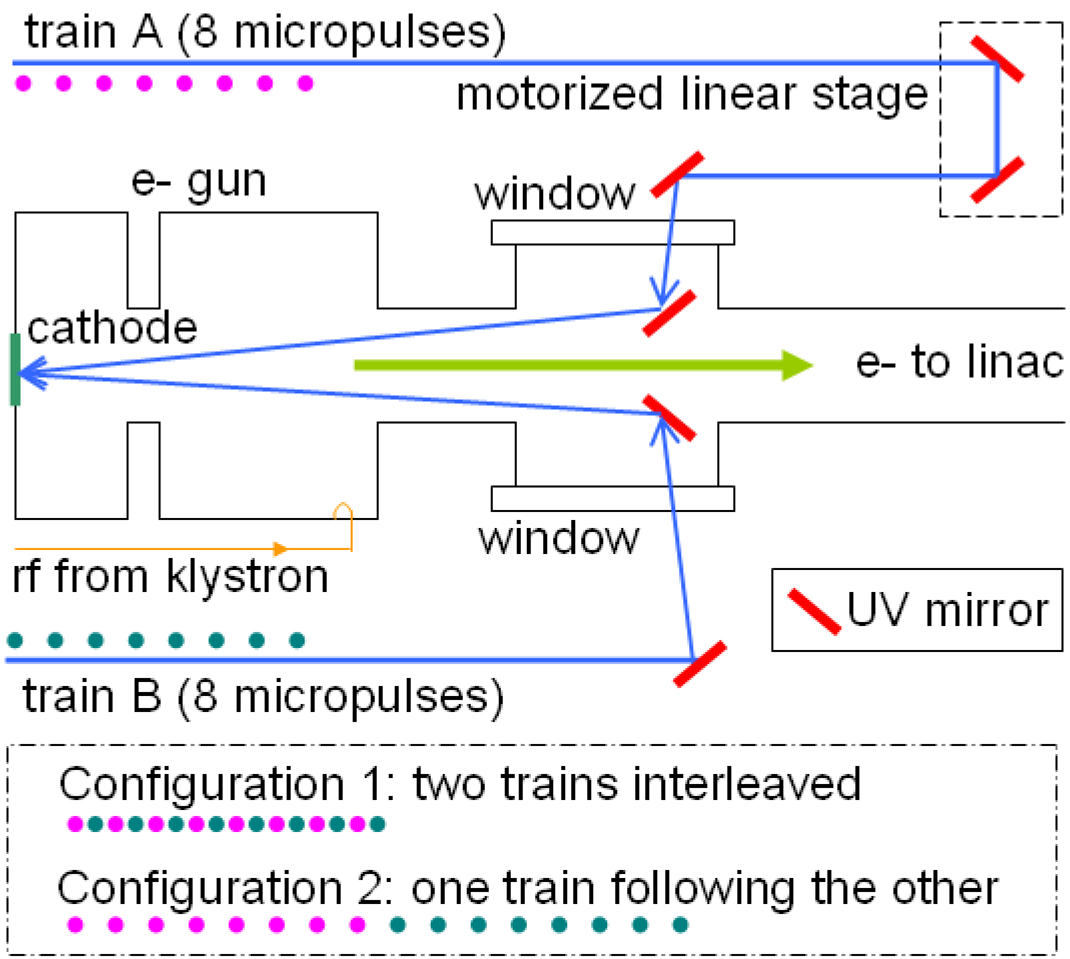


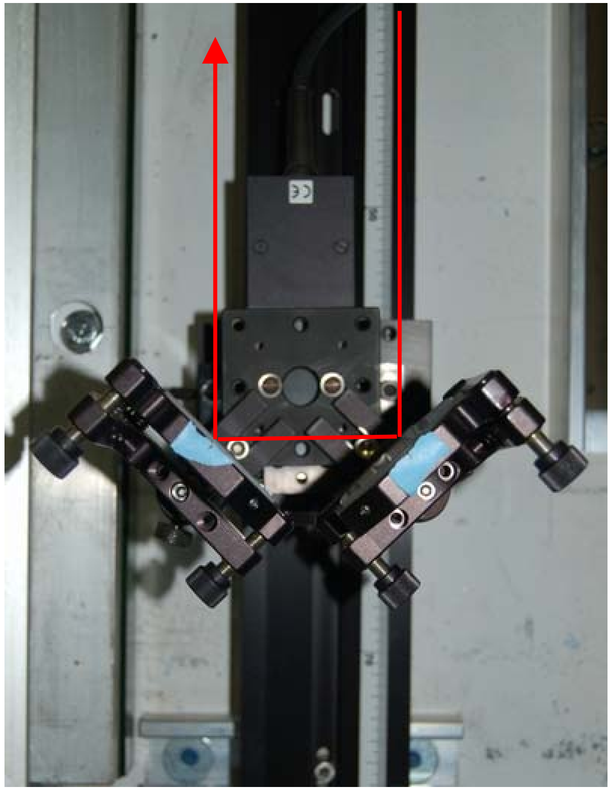


Single bunch test – measurement v.s. simulation



Bunch train test – electron bunch train generation

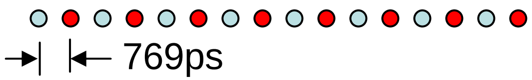


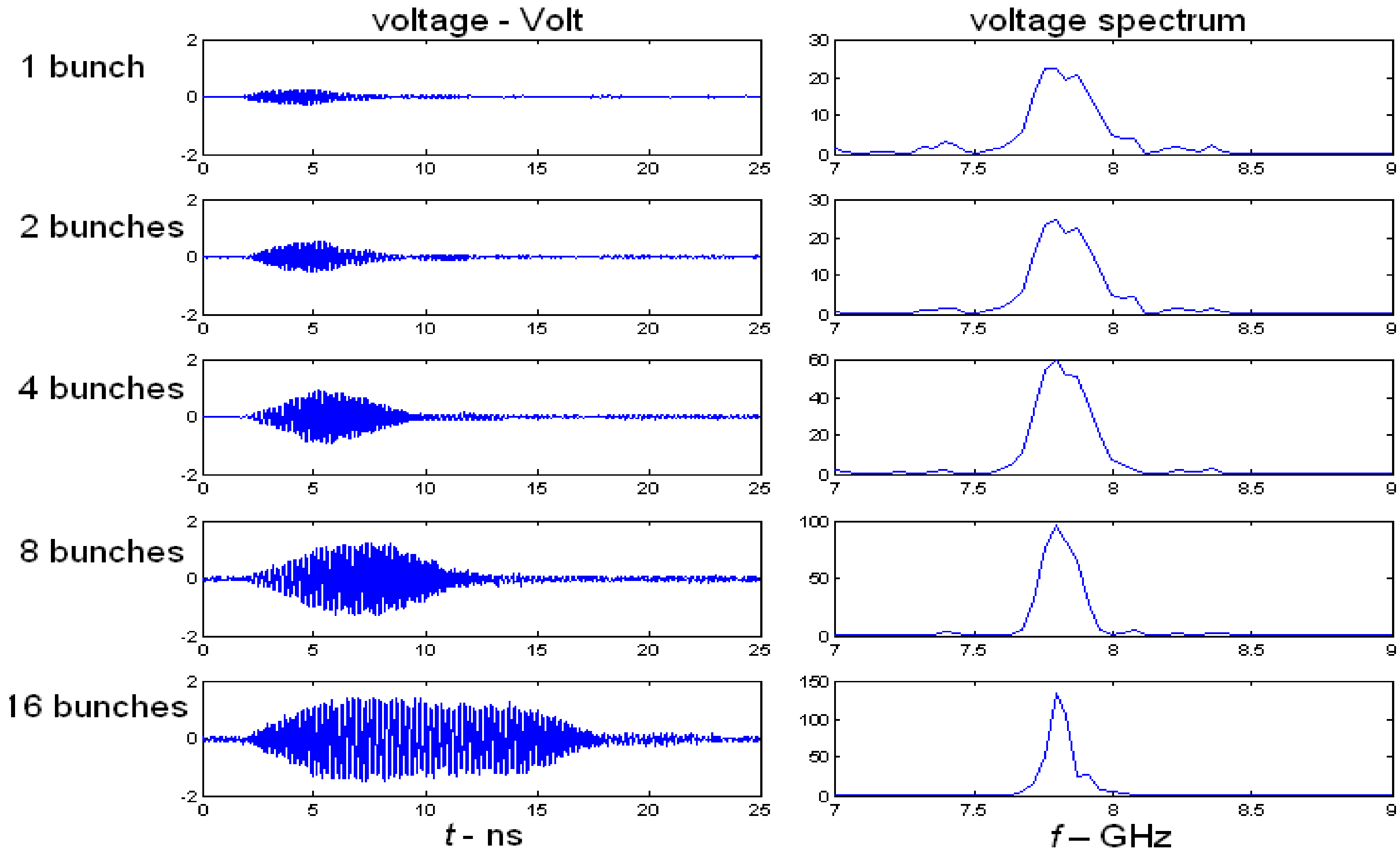


10ns long RF pulse generation experiment

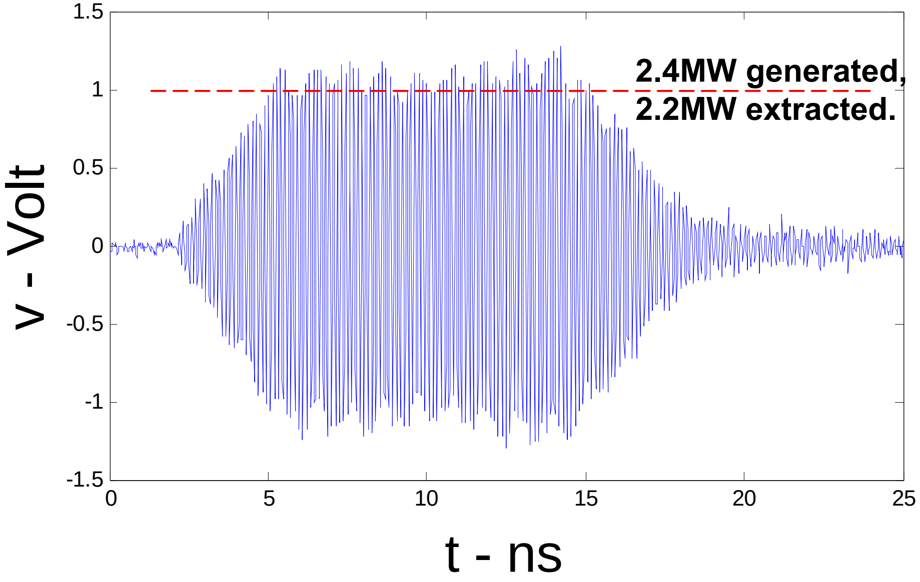
$$P_t = \left(\frac{q}{T_b}\right)^2 L^2 \frac{k_z}{4\beta_g}\left[\frac{r}{Q}\right]\Phi^2$$

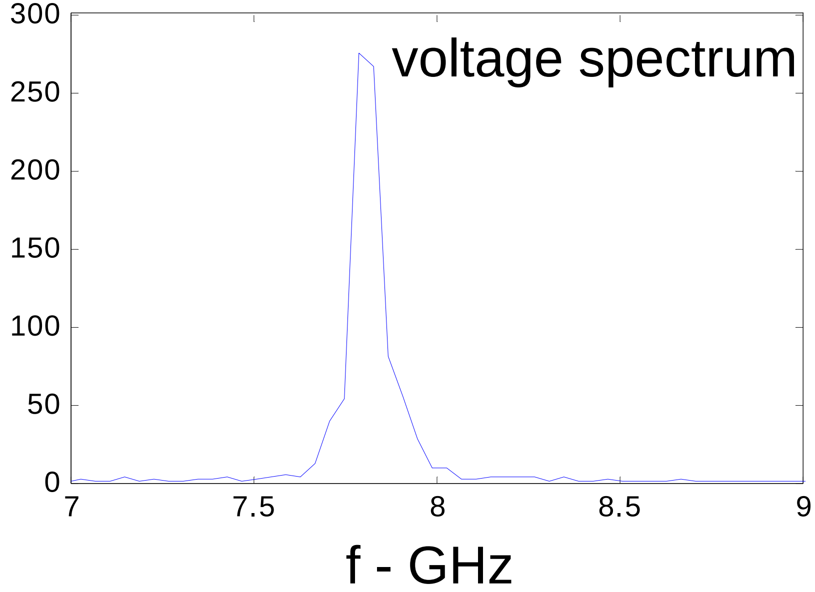
Beam for ~10ns RF pulse generation



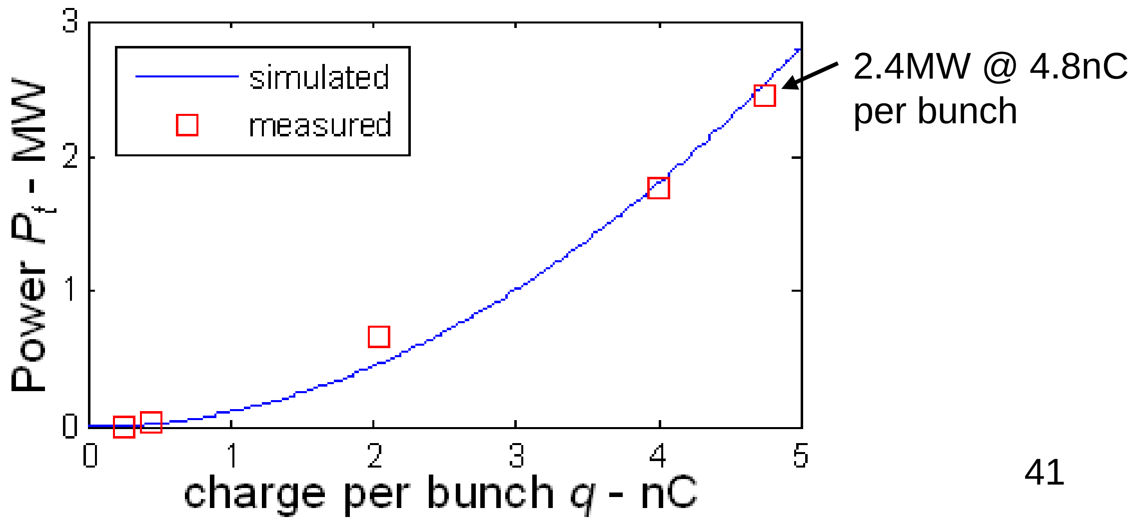


2.4MW, 10ns long RF pulse generation





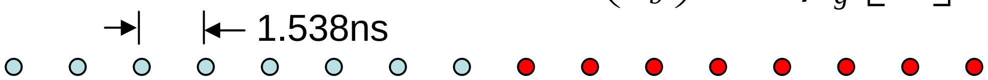
$$P_t = \left(\frac{q}{T_b}\right)^2 L^2 \frac{k_z}{4\beta_g}\left[\frac{r}{Q}\right]\Phi^2$$

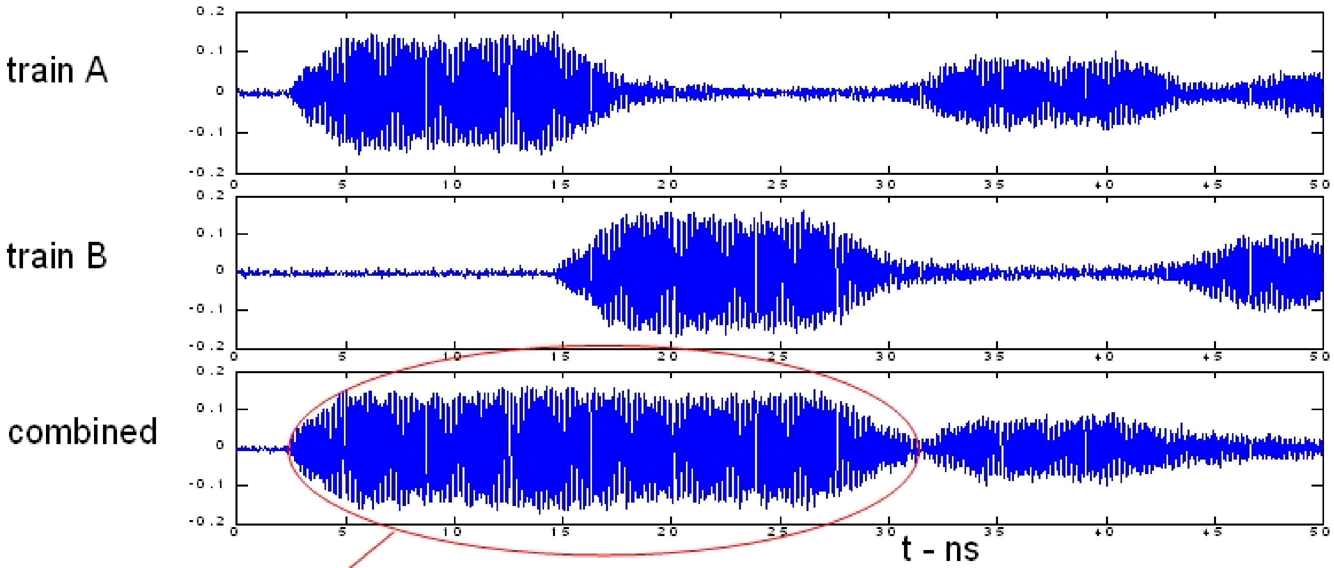


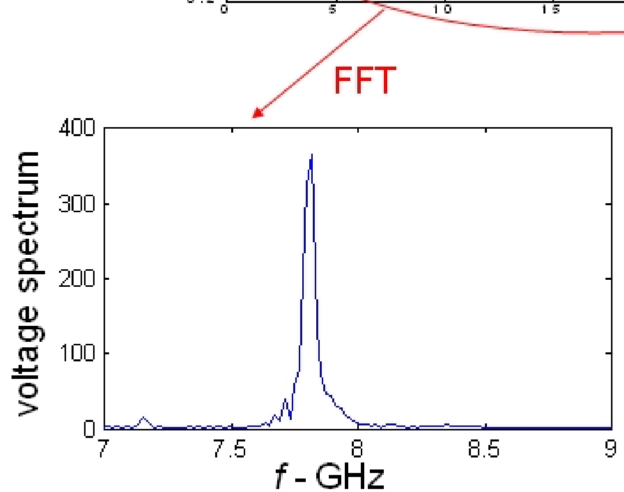
22ns long RF pulse generation

$$P_t = \left(\frac{q}{T_b}\right)^2 L^2 \frac{k_z}{4\beta_g}\left[\frac{r}{Q}\right]\Phi^2$$

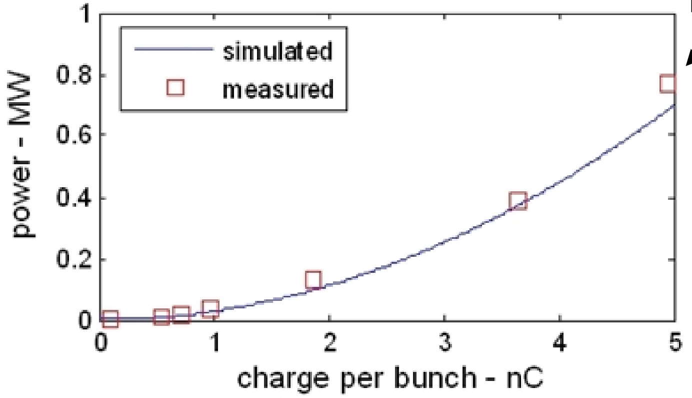
Beam for 22ns RF pulse generation



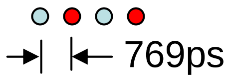




0.77MW @ 4.9nC per bunch

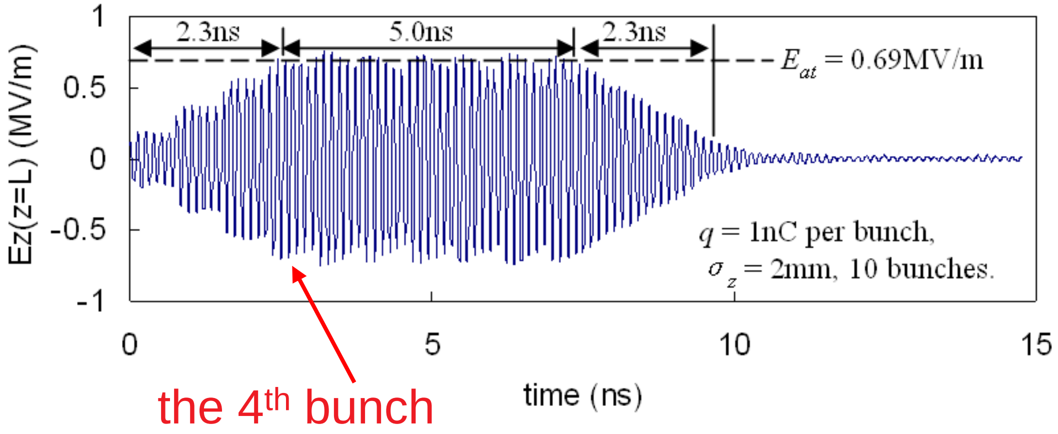


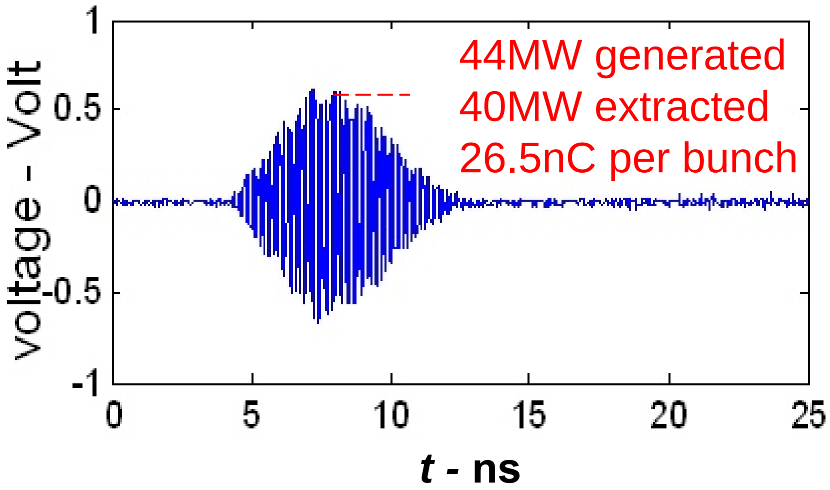
4-bunch test for high power generation

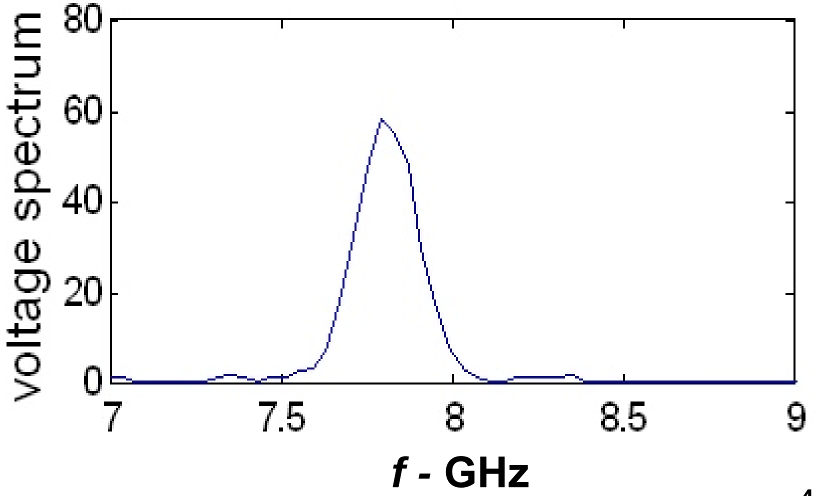


$$P_t = \left(\frac{q}{T_b}\right)^2 L^2 \frac{k_z}{4\beta_g}\left[\frac{r}{Q}\right]\Phi^2$$

- Simulation shows the power reach “flat-top” saturation level when the drive bunch contains 4 or more consecutive bunches spaced by 769ps.
- To maximize this power level the UV laser bunch was only split into 4 bunches.







* Feng Gao et al, *IEEE Tran. Nucl. Sci.* 56, pp. 1492-1497, 2009.

The CLIC Two Beam Scheme

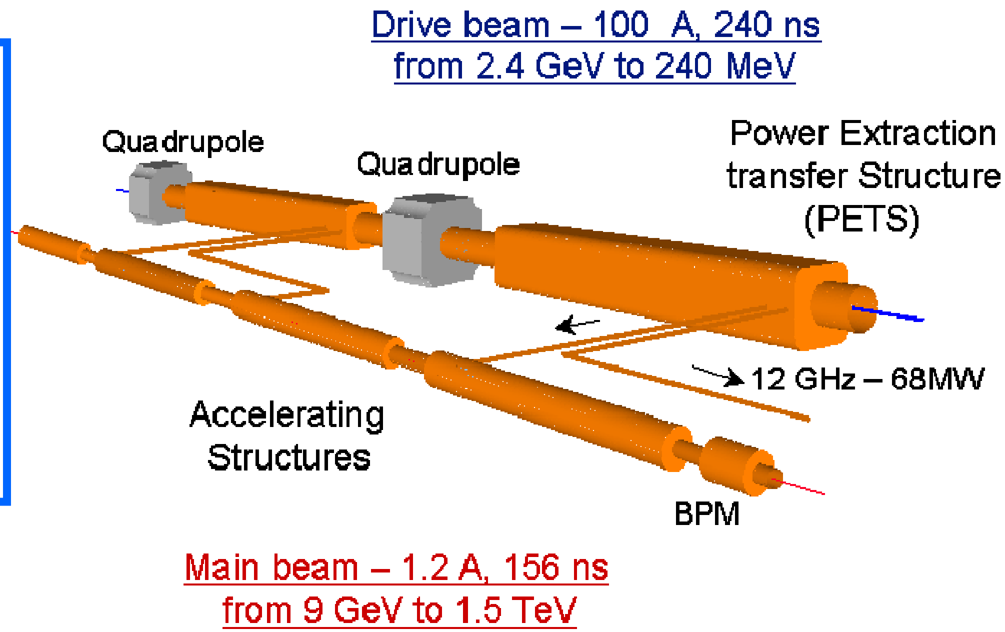
Individual RF power sources ?

➔ Not for the 1.5 TeV linacs

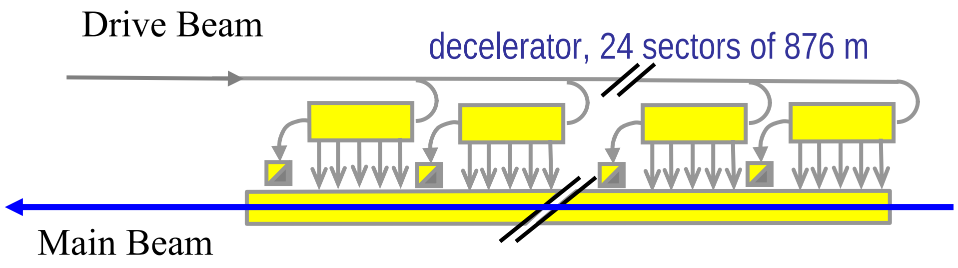
Two Beam Scheme:

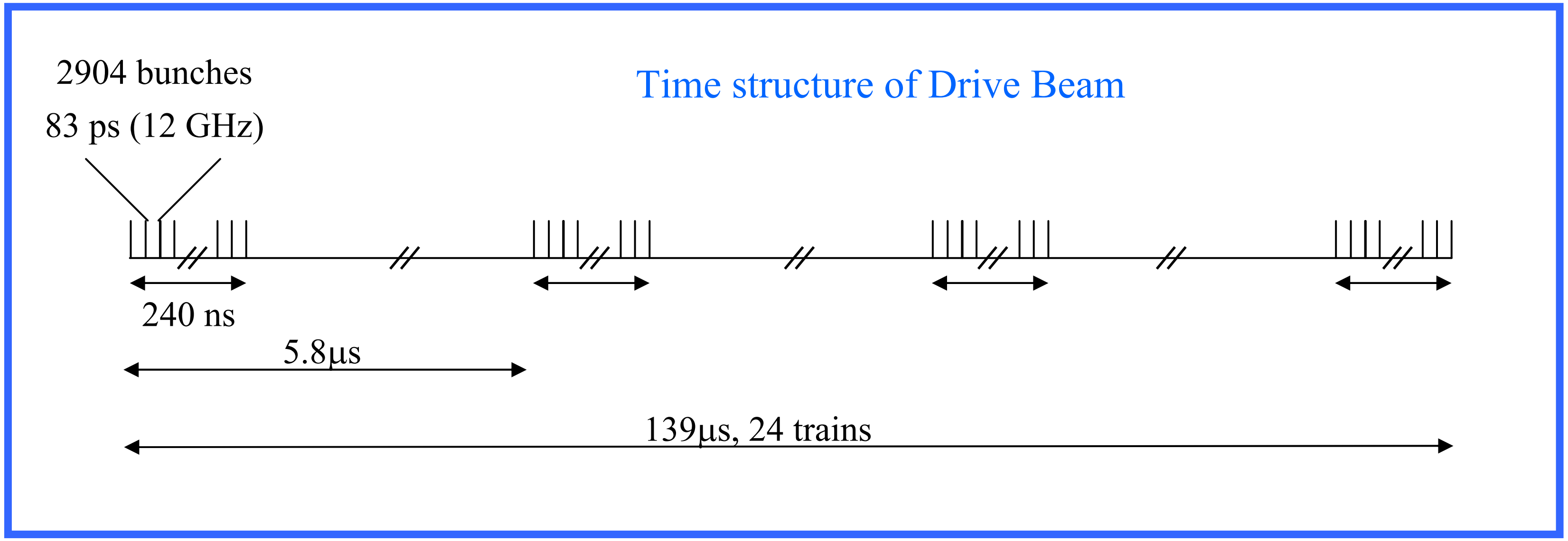
Drive Beam supplies RF power

- 12 GHz bunch structure
- low energy (2.4 GeV - 240 MeV)
- high current (100A)



The CLIC Two Beam scheme

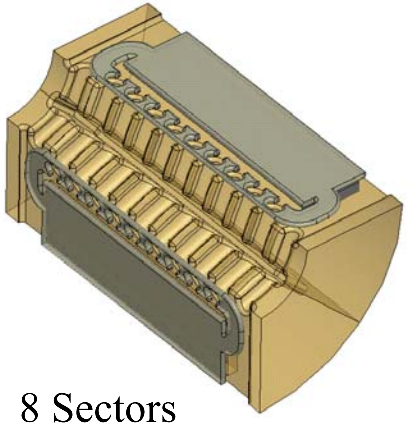




Bunch charge: 8.4 nC, Current in train: 100 A

PETS

Special development for CLIC



8 Sectors

damped

on-off possibility

• Travelling wave structures	136 MW RF @ 240 ns per PETS
• Small R/Q : 2.2 kΩ/m	(2 accelerating structures)
(accelerating structure: 15-18 kΩ/m)	0.21 m active length
• 100 A beam current	total number : 35’703 per linac

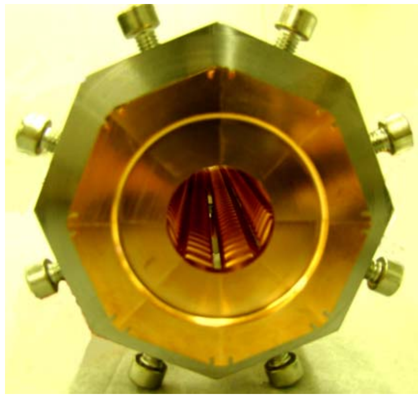
Status:

Advanced design,

RF power testing at SLAC planned July 08

with beam in CTF3 in autumn 2008

ref: Igor Syratchev



	CLIC PETS
Freq	12GHz
Effective Length	21cm
Beam channel	23mm
Dielectric const.	N/A
Q	7200
R/Q	2.29kΩ/m
Vg	0.453c
Gradient	6.5MV/m
Steady Power	135MW

Homework: fill the blanks in the table

12GHz Quartz-Based Power Extractor Using CLIC Parameters:

σz=1mm, Q=8.4nC, Tb=83ps

Freq	11.994GHz
Effective Length	23cm
Beam channel	23mm
Thickness of the dielectric tube	
Dielectric const.	3.75(Quartz)
Loss tangent (@10GHz)	6*10^-5
Q	
R/Q	
Vg	
Peak surface Gradient	
Steady Power	