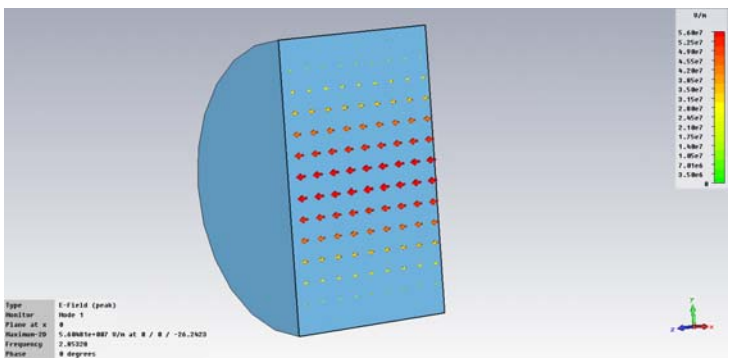
Benchmark studies of CST post processing template for calculation of accelerator parameters using pillbox cavity.

Sergey Antipov, Chunguang Jing and John Power.

The goal of this study was to benchmark CST MWS calculations in template based postprocessing. TM010 mode was simulated (eigensolver). Accelerating parameters were calculated using "template based postprocessing". File: "tm_benchmark.cst"
These results were compared to John Power's mcd script. In this script he writes out the exact solution for the TM modes in pillbox cavity. He obtains all accelerating parameters by definition via integration. File name "TM_benchmark.xmcd"

In both cases CST and MCD one can change the resonant frequency along with the dimensions of the cavity. See description in MCD file.



Note: The exact solution is built the following way. Ez field amplitude is chosen and all other field are scaled accordingly. All accelerating parameters are calculated from the fields. CST normalizes eigenmodes to have 1J stored energy. So for exact solution we pick Ez amplitude to have 1 Joule stored. This would be ~ 56 MV/m.
Note: When CST integrates (template integration) *int(E_z,dz)* it would measure dz in mm, while E_z in MV/m!
Note: CST calculates #D Eigenmode result\Voltage in Volts.
Note: CST calculates #D Eigenmode result\Total Energy in Joule. Normalized to 1 Joule.
Note: CST calculates #D Eigenmode result\Total Loss in Watts.
Note: CST calculates #D Eigenmode result\Shunt Impedance = EFFECTIVE SHUNT IMPEDANCE calculates as Voltage^2 / Power loss in Ohms. Not per unit length!
Note: CST calculates #D Eigenmode result\R over Q as shunt impedance (effective – see above) divided by Q in Ohms. Not per unit length!

In the following table we summarize the results:

Parameter	Parameter	Exact MCD calculations	CST original
Frequency	TM_{010} mode eigenfrequency	2.856 GHz	λ_0=104.97mm
a and d	Cavity size: d=52.4847mm	a=40.1789mm	
Electric field amplitude	E_0	56 MV/m	56 MV/m
Conductivity	σ_{copper}	59.6e6 S / m	5.8e7
Stored Energy	$U = \frac{\epsilon}{2}\int_V \vec{E}^2 dV$	0.996 Joule	1 Joule
Power dissipation	$P = \frac{R_s}{2}\int_S H^2 dS$	958 kWatt	968 kW
Quality factor	$Q = \frac{\omega U}{P}$	1.865e4	1.85e4
Voltage (Panofsky)	$V_0 = T \cdot E_0 \cdot d$	1.87 MV	1.87e6 V
Transit factor	$T = \frac{\int_{-L/2}^{L/2} E(0,z) cos(2\pi z/\beta\lambda) dz}{\int_{-L/2}^{L/2} E(0,z) dz}$	0.636	0.6378*
Effective gradient	$T \cdot E_0$	35.65 MV/m	35.7 MV/m
Energy Gain	$T \cdot E_0 \cdot d$	1.87 MeV	1.875 MeV
Effective Shunt Impedance	$R_{sh}=(T \cdot E_0 \cdot d)^2/P$, per structure!	3.65 MOhm	3.6 MOhm
Effective Per length Shunt impedance	$r_{sh} = V_0^2/P/d$ Voltage gain per power loss per meter	69 MOhm/m	68.6 MOhm/m
R effective over Q	R_{sh} / Q, per structure! Called Z/Q	196 Ohm	196 Ohms
R effective over Q per length	$R_{sh} / Q / d$, per unit length	3.73 kOhm/m	3.73 kOhm/m

*transit factor can be computed in two ways: 1. Integrate equation for transit factor. or 2. from Voltage and Ez amplitude and cavity size: $T=V_0/E_0/d$;
Directly calculated in postprocessing