Left-Handed Structures for Accelerator Applications

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Abstract. Metamaterials are artificial periodic structures made of small elements and designed to obtain specific electromagnetic properties. As long as the periodicity and the size of the elements are much smaller than the wavelength of interest, an artificial structure can be described by a permittivity and permeability, just like natural materials. When the permittivity and permeability are simultaneously negative in some frequency range, the metamaterial is called double negative or left-handed and has some unusual properties. Left-handed metamaterials (LHM) have potential applications in active and passive devices at millimeter waves and at much higher frequencies. Waveguides loaded with metamaterials are of interest because the metamaterials can change the dispersion relation of the waveguide significantly. Slow backward waves can be produced in a LHM-loaded waveguide without corrugations. The dispersion relation of a LHM-loaded waveguide has several interesting frequency bands which are described. Left-handed structures can be employed at X-band accelerators to suppress wakefields. In this paper we present theoretical studies and computer modeling of waveguides loaded with 2D anisotropic metamaterials, dispersion relations for LHM-loaded waveguides, and describe current efforts toward designing a left-handed accelerator.

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INTRODUCTION

For most of materials ε and μ are positive for propagating frequencies of electromagnetic waves. Therefore phase vector (k) of the wave forms a right-handed system with the field vectors E and B. The Poynting vector is co-directed with k.

It has been shown in [1], that propagation is also possible when ε and μ are simultaneously negative. Propagating waves in this double-negative media will exhibit several unusual properties. First of all phase vector forms a left-handed system with the field vectors in this case. That's why materials with simultaneously negative ε and μ are called left-handed (LHM). In such media the Poynting vector, which is collinear with the group velocity, is counter-directed to the phase vector. This gives rise to several unusual effects like the reversed Doppler Effect, reversed Cherenkov radiation [1, 12, 13 and 14] and negative refraction [1, 10]. Cherenkov radiation is widely used in accelerator physics. It has particle detector application and it may be that reverse Cherenkov radiation is uniquely useful for beam detection [13, 14 and 19].

In this paper we discuss a possible accelerating application of left-handed media. LHM can create a narrow accelerating band and effectively suppress wakefields.

Left-handed materials do not exist naturally. The first LHM was artificially constructed in 2001 [9, 10] using wire array (which provided $\varepsilon<0$ [3]) and array of split rings (which provided $\mu<0$ [2]). It has been shown [3] that a wire array exhibits plasma-like behavior (1) in the GHz frequency range (figure 1).

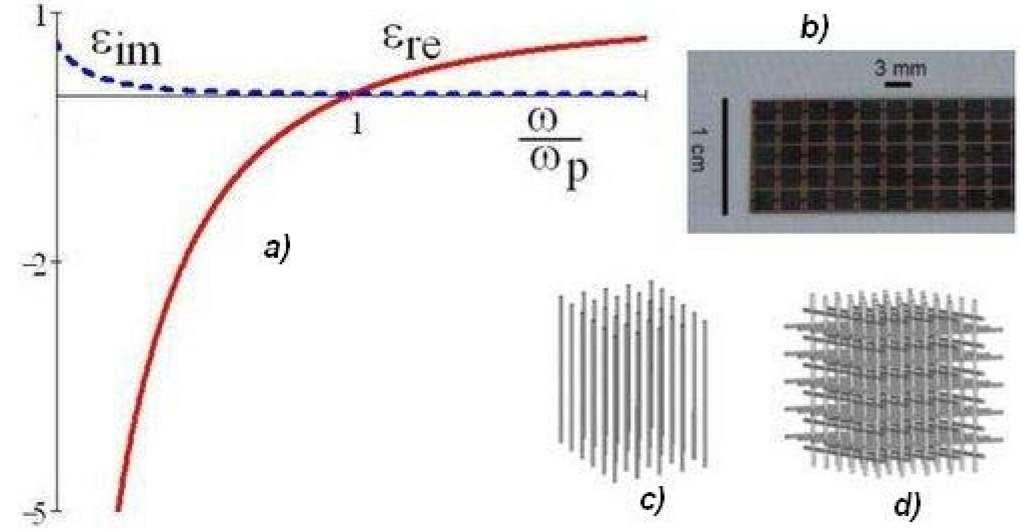

FIGURE 1. Artificial dielectrics. A) plasma-like permittivity of the wire arrays c) and d). B) capacitively loaded wire array studied at AWA [8, 20].

$$\varepsilon(\omega)=1-\frac{\omega_p^2}{\omega^2+i\gamma\omega}\quad \omega_p^2=\frac{2\pi c^2}{a^2\ln(a/r)}\ \text{and}\ \gamma_e=\frac{c^2}{2\sigma S\ln(a/r)} \qquad (1)$$

Where $\boldsymbol{a}$ is the periodic spacing between wires, $\boldsymbol{r}$ is the wire radius, $\boldsymbol{c}$ is the speed of light and $\boldsymbol{S}$ is the wire cross section. A simple antenna analysis gives the same result [5, 6]. The wire array shown in figure 1, c) can produce plasma-like behavior (negative permittivity) only for electric fields that are parallel to the direction of the wires. This structure is anisotropic. In order to have an isotropic tensor for $\varepsilon=\varepsilon\cdot\hat{I}$, where $\hat{I}$ is the identity matrix, the wires should be configured in a 3D grid, figure 1, d.

In order to realize an artificial μ we turn to magnetic dipoles. A loop of current creates a magnetic dipole. An assembly of small loop structures behaves like a continuous media, provided the radiation wavelength is significantly greater than the geometric scale of the loops. This assembly produces a response to magnetic field, when it penetrates the rings. These rings are usually made thin, so other polarization of magnetic field does not produce any effect on the rings. This makes metamaterials strongly anisotropic. One has to make an additional effort to make the structure isotropic in terms of $\mu=\mu\cdot\hat{I}$ by having all 3 possible orientations of SRR $\perp$ to x, y and z. To create a resonant response one needs to cut the ring (figure 2). The resulting split ring resonator (SRR) [2] has a distributed self inductance of the loop and a small capacitance in the cut. Therefore the system behaves similarly to RLC circuit and has a resonance. Our rings have a square form for more efficient space usage. Typical response of such structure is almost Lorenz-like [7] (figure 1).

$$\mu_{eff}=1-\frac{F\omega^2}{\omega^2-\omega_{res}^2+i\gamma\omega} \qquad (2)$$

Here F is geometrical factor. Constants ω_{res} and γ are determined by geometry.

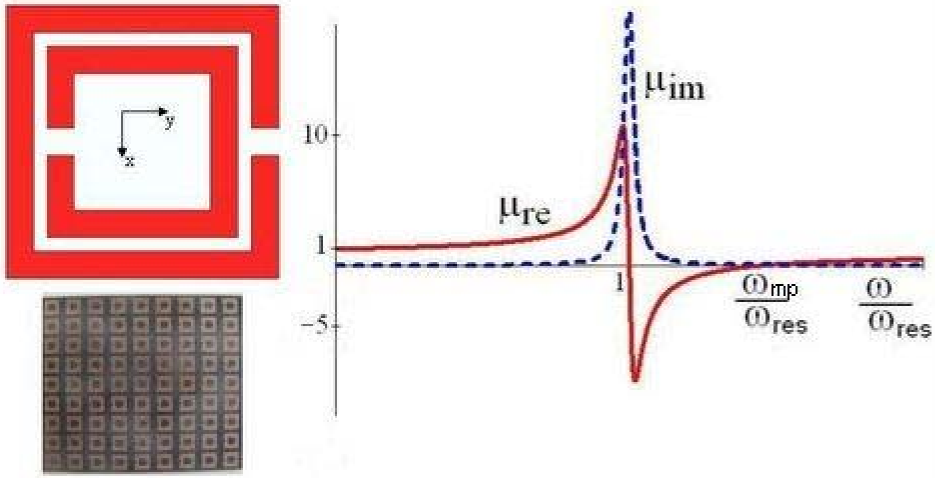

FIGURE 2. The split ring resonator [2]. Picture of manufactured layer of split ring resonators. Typical permeability behavior of the SRR as a function of frequency.

Different geometrical variations are also possible for particular purposes. We use two concentric split rings to increase the capacitive region and lower the resonance frequency. In the design we studied at Argonne [19, 20] we had a *2.54mm* overall size of the ring. The resonance frequency was designed and measured at *11.4GHz* ($\lambda >> d$).

We follow with a brief discussion of a proof of principle experiment that was done using our first metamaterial design, continued with a more detailed discussion of the calculation and simulation of the TM_{11} accelerating mode in a rectangular LHM loaded waveguide. Than we explain the mechanism of wakefield suppression in a metamaterial-loaded structure. Finally we conclude with a preliminary calculation of the figures of merit for such an accelerating structure.

EXPERIMENTAL STUDIES OF METAMATERIALS

We have designed and manufactured a double-negative metamaterial. We have experimentally verified that the refraction on the left-handed metamaterial is negative.

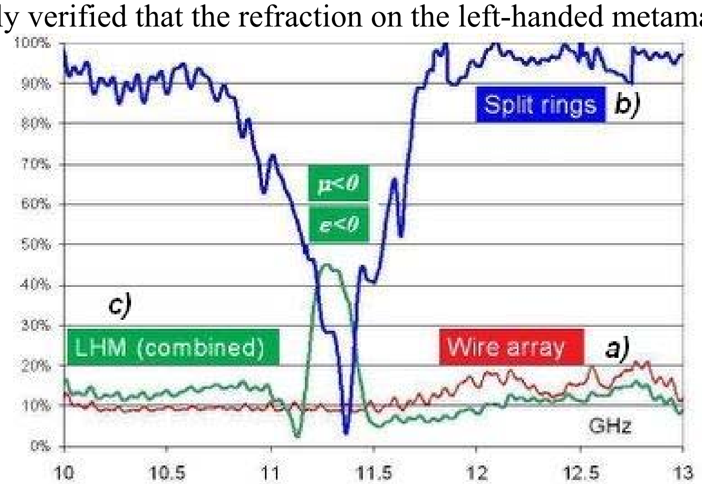

FIGURE 3. Metamaterial-loaded waveguide experiment. TE_{10} mode propagates through the waveguide, loaded with a) wire array (red), b) split rings (blue), c) combined wire array + split rings (green). Transmission parameter as a function of frequency on linear scale (*100%* - full transmission).

This experiment was similar to [10]. In addition, using a metamaterial-loaded waveguide, we have demonstrated the existence of a narrow double-negative band of propagation of the fundamental mode. These results are shown in figure 3. Our experiment was similar to the one where a propagation of radiation through LHMs was first demonstrated [9]. When a waveguide is loaded with the wire array we observe no propagation (fig 3, a) because the effective media inside the waveguide has negative permittivity of the wide range of frequencies (1). Split ring resonators have negative values of μ only in proximity of the resonance (2), so when the waveguide was loaded with only the split rings, there was a narrow band transmission drop (b). However, when the wire array and split rings were both inserted, we observed narrow band transmission (c) in the region where ε and μ are simultaneously negative.

WAVEGUIDES LOADED WITH ANISOTROPIC MEDIA

Let's consider a waveguide (along z) with an anisotropic media inside. We analyze a rectangular waveguide because it is easier to load a rectangular waveguide with metamaterial than a cylindrical one. The cylindrical waveguide analysis is similar. We assume that the media inside the waveguide has the following tensors for ε and μ.

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix}, \quad \hat{\mu} = \begin{pmatrix} \mu_\perp & 0 & 0 \\ 0 & \mu_\perp & 0 \\ 0 & 0 & \mu_\parallel \end{pmatrix} \tag{3}$$

Here $\varepsilon_\perp$ has the form (1) and $\mu_\perp$ - (2) other components are positive constants. A metamaterial can be made to realize tensors (3). The dispersion relation for the TM-modes of such structure can be derived using standard methods [11, 16, and 22]:

$$k_z = k_0 \sqrt{\varepsilon_\perp \mu_\perp \left(1 - \frac{\chi_x^2 + \chi_y^2}{\varepsilon_\parallel \mu_\perp k_0^2}\right)} \tag{4}$$

Here $k_0=\omega/c$, $\chi_x=\pi m/a$ and $\chi_y=\pi n/b$, m, n – mode indices and a, b – dimensions of the waveguide. The analysis is done for TM_{11} mode and is similar for any other mode. Our system now has several characteristic frequencies: 1) cutoff frequency of the empty waveguide ω_{cutoff} (for mode TM_{11}), 2) plasma frequency for transverse permittivity ω_p, 3) resonance frequency for permeability ω_{res} and 4) magnetic plasma frequency for permeability ω_{mp}. The dispersion relation is of particular interest when $\omega_{cutoff} < \omega_{res} < \omega_{mp} < \omega_p$ (figure 4). There are five characteristic frequency bands in this case.

I. This is the non-magnetic band similar to the one discussed in [16, 17 and 18]. We see (region I, figure 4) the propagation of the mode (real k_z) below the cutoff frequency, similar to [21(non-ε)].

II. This region is characterized by negative ε and μ in the waveguide above the cutoff frequency. There is no propagation in this region according to (4).

III. In this region we observe classical left-handed behavior. The resonant behavior of negative μ (2) causes the dispersion curve of the loaded waveguide to intersect with the dispersion of highly relativistic electrons ($\omega=kc$) (next paragraph and figure 5).

Therefore an interaction between the electrons and the backward mode is possible, allowing acceleration.

IV. In this region $\varepsilon<0$ and $1>\mu>0$. Nevertheless, propagation is possible, because low values of μ make an effective cutoff frequency (ω^2_{cutoff}/μ) higher than the frequency in this band. This band does not require negative values of μ to create a backward propagating mode. This quasi-non-magnetic band requires the condition $1>\mu>0$ which can be realized by natural diamagnetics.

V. At the frequencies in this band $\varepsilon>0$ and $\mu>0$. The behavior of the system resembles the behavior of an empty waveguide. There is no interaction between the relativistic electrons and the mode.

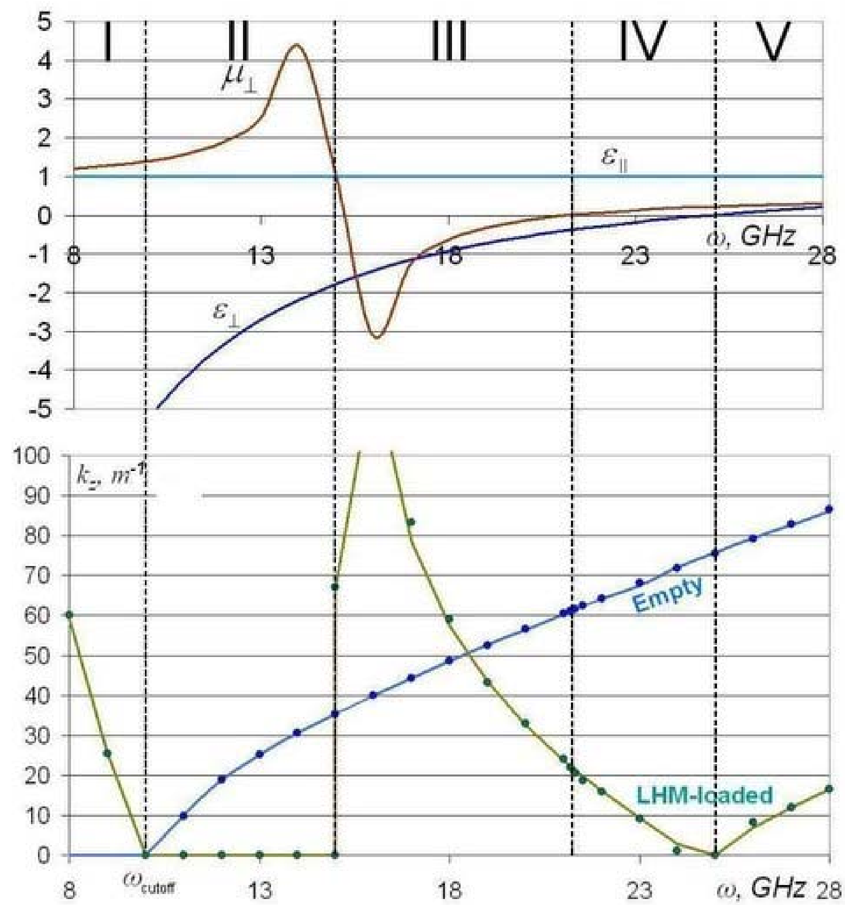

FIGURE 4. Effective permittivity and permeability of the metamaterial (top). Dispersion for TM_{11} mode of empty (blue) and metamaterial-loaded waveguide (green). Solid lines – theory, dots – simulation, using representation of ε and μ (1, 2). It is practically impossible to simulate full-scale metamaterial loaded waveguide due to mesh requirements. We observe five different frequency bands between ω_{cutoff} (*10GHz*)< ω_{res} (*15GHz*)< ω_{mp} (*21GHz*)< ω_p (*25GHz*).

Overcritical Propagation in Metamaterial-Loaded Waveguide

It has been shown that the dispersion relation (4) supports a non-magnetic band (I). In this band a backward propagation is realized due to the negative permittivity of the media and the fact that we operate below cutoff frequency. The waveguide below cutoff frequency presents a continuous media of effective negative permeability.

If the waveguide is loaded with a continuous anisotropic dielectric satisfying (1, 3) than a non-magnetic band exists. A THz metamaterial design using liquid helium cooled bismuth [23] was proposed [17]. However, our metamaterial design will contain a wire array to produce negative ε. It has been shown in [15] that there is no propagation through a wire array embedded into a continuous medium with negative μ. Thus, a wire array in the waveguide below cutoff frequency is not a left-handed material (LHM) and there will be no propagation in the non-magnetic band. Microfields in the space between the wires are vacuum-like. The overcritical waveguide suppresses them accordingly.

Metamaterials we use do not support the overcritical propagation in the waveguide.

METAMATERIAL-BASED ACCELERATOR

Loading a waveguide with a metamaterial changes the dispersion significantly, in some cases causing several slow-backward wave bands to appear.

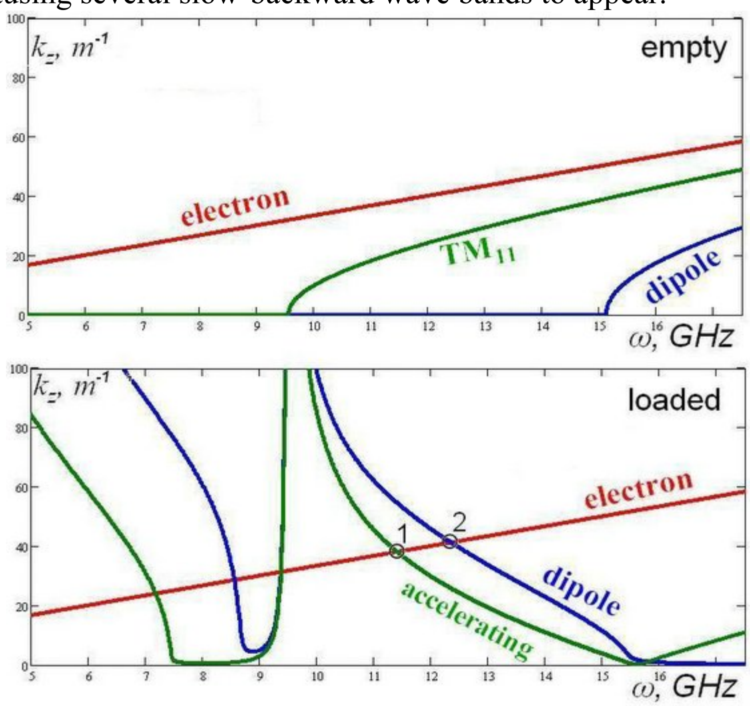

FIGURE 5. Dispersion for TM_{11} and first dipole mode of empty (top) and metamaterial-loaded rectangular waveguide (bottom). There is no possibility for acceleration in the empty waveguide. Metamaterial-loaded waveguide presents synchronization points 1 and 2. Synchronization for the dipole mode occurs below the cutoff frequency of this mode in a non-magnetic region (point 2). Therefore in a real structure this mode will be suppressed. Three low frequency points are below the cutoff frequency and will be suppressed too. The only mode left will be a TM_{11} (accelerating mode) in rectangular waveguide (or TM_{01} in similar design for cylindrical waveguide) at *11.424GHz* (point 1).

The dispersion of a waveguide loaded with an anisotropic media (1, 2 and 3) is shown in figure 5. A metamaterial can be designed to have electromagnetic parameters (1, 2 and 3). Slow TM_{11} mode supports a particle – wave interaction needed for acceleration (point 1 on fig. 5). The metamaterial of our design (wire array and split rings) does not support overcritical modes, so there will be no synchronization between particles and the dipole mode in the waveguide (at point 2 on fig. 5).

Therefore, the resulting structure presents the possibility for particle acceleration with a wakefield suppression mechanism.

SUMMARY

A metamaterial-loaded waveguide as an accelerator was presented. We have not discussed several practical issues, like breakdown limits and the manufacturing problems. In the table below we present calculated values for the accelerating structure parameters.

TABLE 1. Left-Handed Accelerator Parameters

Parameters	**Values**
Frequency, GHz	11.424
Quality factor (Q)	150
Shunt Impedance, MOhm/m	7
R_s/Q, kOhm/m	35
Group Velocity, c	0.208
Accelerating Gradient, MV/m	$2.5\sqrt{P[MW]}$

Due to resonant losses in the metamaterial (figure 1) the quality factor Q is rather low. Other parameters are typical. Taking care of losses in a metamaterial is a major direction for improvement in left-handed accelerator. One possibility is to operate in the quasi-non-magnetic regime to reduce the losses in the accelerator. The condition $1>\mu>0$ can be realized by natural diamagnetic materials.

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