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EFFECT OF TEMPERATURE AND AIR IN RF CAVITIES

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The presence of air inside an RF cavity lowers its resonant frequency due to the change in ε. The CRC Handbook of Chemistry and Physics lists ε of dry air as 1.00054. Using this value the frequency in air would be

$$f_{air} = \frac{f_{vac}}{\sqrt{\varepsilon}} = 0.99973\ f_{vac}$$

But we know that the air surrounding an RF cavity is not absolutely dry (if nothing else, from the sweat of the person tuning it). The humidity increases the value of ε. John Power has used the value 1.00059 for ε, giving

$$f_{air} = \frac{f_{vac}}{\sqrt{\varepsilon}} = 0.99971\ f_{vac}$$

and he also found a rule of thumb in a book that gives the above factor as 0.9997. We decided to use this last number. Thus, our vacuum resonant frequency of 1300 MHz becomes 1299.610 when the cavity contains air.

The effect of the temperature comes from the thermal expansion of the copper. Every linear dimension of the cavity will expand (or contract) according to the coefficient of thermal expansion of the copper. Thus, the wavelength of the resonant frequency of the cavity will scale together with the cavity dimensions:

$$\lambda \propto \frac{1}{f} \propto L$$

where L is any linear dimension of the cavity. Let's call the operating temperature of the cavity T_{hot}, as opposed to the room temperature T_{room}. Correspondingly, the linear dimension L will be called L_{hot} and L_{room}, and the resonant frequencies f_{hot} and f_{room}. Thus:

$$f_{room} \propto \frac{1}{L_{room}} \qquad \text{and} \qquad f_{hot} \propto \frac{1}{L_{hot}}$$

$$\Rightarrow \quad \frac{f_{room}}{f_{hot}} = \frac{L_{hot}}{L_{room}}$$

From this equality we can write

$$\frac{f_{hot} - f_{room}}{f_{hot}} = \frac{L_{room} - L_{hot}}{L_{room}}$$

and now using the definition of the thermal expansion coefficient α:

$$\frac{L_{hot} - L_{room}}{L_{room}} = \alpha \left(T_{hot} - T_{room}\right)$$

we can write

$$f_{room} = f_{hot} \left(1 + \alpha \left(T_{hot} - T_{room}\right)\right)$$

The CRC Handbook of Chemistry and Physics lists the thermal expansion coefficient α for copper as $16.5\times10^{-6}\ {}^{\circ}C^{-1}$.

Finally,

$$f_{room,\ air} = 0.9997\ f_{room} = 0.9997\ f_{hot}\left(1+\alpha \left(T_{hot} - T_{room}\right)\right)$$

Thus, for $T_{hot} = 35^{\circ}C = 95^{\circ}F$ we get

$T_{room}\left({}^{\circ}C\right)$	$T_{room}\left({}^{\circ}F\right)$	$f_{room,\ air}\ (MHz)$
20	68	1299.932
25	77	1299.824
30	86	1299.717
35	95	1299.610

For $T_{hot} = 50^{\circ}C = 122^{\circ}F$ we get

$T_{room}\left({}^{\circ}C\right)$	$T_{room}\left({}^{\circ}F\right)$	$f_{room,\ air}\ (MHz)$
20	68	1300.253
25	77	1300.146
30	86	1300.039
35	95	1299.932

Remark: The dimensions of the coupling iris also scale with the thermal expansion of the copper, therefore the coupling parameter β for f_{room} at T_{room} will be the same as β for f_{hot} at T_{hot}.