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Design, fabrication and testing of left-handed metamaterials

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ABSTRACT

Recently, there has been a growing interest in applying artificial materials, known as Left-Handed Metamaterials (LHM), to accelerator physics applications. These materials have both negative permittivity and permeability and therefore possess several unusual properties: the index of refraction is negative, and when electromagnetic radiation propagates through the material, the direction of the group velocity is antiparallel to the direction of the phase velocity (along k). This results in many interesting effects, such as reverse Doppler Effect, Negative Refraction and the reverse Cherenkov Effect, where the emitted light will propagate in direction opposite to the particle velocity [1].The magnetic and electric coupling of a material to radiation can be controlled through its design and construction [2]. A material can be made to have both negative permittivity and permeability for a certain frequency range [2, 3, 4, 5, and 6].

Several LHM devices with different configurations were designed. The permittivity and permeability retrieval techniques that were developed [9, 10, 11] and applied to these metamaterials. The mechanism of negative response is discussed.

INTRODUCTION

When studying the interaction of an EM wave with a material it is impossible to take the response of each atom or electron into account to the radiation. We rely on electromagnetic parameters such as the index of refraction, the permittivity (ε) and the permeability (μ), to replace the complex and irrelevant electromagnetic details of structures much smaller than the wavelength.

Permittivity (ε) is an index describing the response of a medium to an electric field. Electric field in the medium is ε times smaller. The same way permeability (μ) was defined in respect to magnetic field.

Usually, over broad range of frequencies of applied field, materials have positive permeability and permittivity. The term for materials with positive ε and μ is double positive. Negative values of ε and μ are quite rare. Double negative material has not been found in nature.

There are some materials, which exhibit negative permittivity, such as a plasma below the plasma frequency, some metals like gold, silver, aluminum at optical frequencies and silicon carbide at 10μm wavelength. Materials with negative μ include resonant anisotropic ferromagnetic or antiferromagnetic systems. Materials having only one of the two indexes ε or μ negative do not allow propagating solutions of Maxwell system of equations (see the General Theory chapter). If a wave enters such media its amplitude decays exponentially.

How do negative values of ε and μ occur in materials? The Drude–Lorentz model of a material is a good starting point, because it introduces a physical mechanism for the material response. It conceptually replaces the atoms and molecules of a real material with a set of harmonically bound electron oscillators resonant at some frequency ω_0. At frequencies far below ω_0, an applied electric field displaces the electrons from the positive cores and induces a polarization in the same direction as the applied field. At frequencies near resonance, the induced polarization becomes very large, as is typical in resonance phenomena; the large response represents accumulation of energy over many cycles, such that a considerable amount of energy is stored in the resonator (in this case, the medium) relative to the driving field. So large is this stored energy that even changing the sign of the applied electric field has little effect on the polarization near resonance! That is, as the frequency of the driving electric field is swept through the resonance, the polarization flips from in–phase to out–of–phase with the driving field, and the material exhibits a negative response. If instead of electrons the material response was due to harmonically bound magnetic moments, then a negative magnetic response would exist.

In 1968 V.Veselago [1] studied hypothetical material, which has both ε and μ negative. Naturally this material does not occur. Frequency regions in which some of the materials exhibit negative ε do not overlap with the regions where other materials exhibit negative μ. He pointed out several unusual properties of the hypothetic material, which will be discussed further. But, for a long time his work was just a curious exercise until the technology caught up.

Recently, it began possible to construct a material with negative ε and μ. Artificially constructed material or metamaterial is a structure, built specifically to obtain the desired electromagnetic properties. Such metamaterials are composites of resonant structures which couple to magnetic fields and conducting elements which couple to electric fields to achieve certain effects at the desired frequency ranges. The coupling frequency of the magnetic and dielectric elements is controlled by their geometry [2]. Usually, metamaterial is a periodic arrangement of basic elements. As long as the size and spacing between the elements are much smaller than the electromagnetic wavelengths of interest, incident radiation cannot distinguish the collection of elements from a homogeneous material. Since frequencies on the order of 10GHz can be studied at Argonne Wakefield Facility, the double negative metamaterial designed for this research project exhibits negative ε and μ at around 11.5GHz. Microwave range of frequencies gives not too much freedom when it comes to the cell size (size of the repeatable basic element). Wavelength of 3*cm* corresponding to a frequency of 10GHz dictates the size of 3*mm* for the metamaterial cell. Since each element has to have some features within it, it took some time until the material with negative ε and μ were designed and made.

In 1999 John Pendry, a London scientist, introduced the first design for a negative ε and μ material [2]. Later, the next year the first such material was constructed by David Smith's group in San Diego [3]. In past few years there were hundreds of publications devoted to such materials. Most of the predicted properties were experimentally proven and several applications were introduced.

This field of physics is very young. The idea of making a Double-Negative Metamaterials came up in 1999 [2]. The first realization was just in 2000 [3, 5]. Negative refraction was shown in 2001 [3] and 2003 [4]. Interesting applications such as flat lens [1] made out of the LHM and left-handed photonic band gap accelerator [15] were proposed.

My research is focused on the reverse Cherenkov Effect in double negative media.

There has been little work done on this effect [7, 8]. To perform the experiment it is necessary to have a beamline with relatively narrow beam spot and low emittance. That is why Argonne Wakefield Accelerator facility got interested and supports this research. This facility is a good place to test different wakefield structures because it has appropriate beam characteristics. A wakefield structure is based on the inverse Cherenkov Effect (when an external field couples into the beam).

Experimental verification of reverse Cherenkov Effect in left-handed metamaterials is planned for the near future at AWA. This will be the first attempt to observe a reversed field generated by a beam in LHM [16].

Moreover it is not only a proof of principal experiment, but there is an interest to use this effect for detection purposes. The main advantage is that the emitted signal is behind the beam, since the radiation propagates in direction opposite to the beam. We expect it to be easier to get a cleaner signal then in a conventional Cherenkov detector.

Mechanism of negative response.

Let's consider a model of harmonic oscillator with losses (β) under the external force. According to Drude-Lorentz model atoms and molecules in material can be replaced by a system of such oscillators.

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = fe^{i\omega t} \qquad (1)$$

Looking for a solution on the external force frequency - ae^{iwt}, we obtain for complex amplitude a:

$$a = \frac{f}{(\omega_0^2 - \omega^2) + 2\beta i\omega} \text{ or} \qquad (2)$$

$$|a| = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}} \quad \text{and} \quad tg(\varphi) = \frac{2\beta\omega}{\omega_0^2 - \omega^2} \qquad (3)$$

Resonance frequency is easy to calculate: $\omega_{res} = \sqrt{\omega_0^2 - 2\beta^2}$ which is close to ω_0 when losses are relatively low. Maximum amplitude will be equal to:

$$|a| = \frac{f}{2\beta\sqrt{\omega_0^2 - \beta^2}} \approx \frac{f}{2\beta\omega_0} \qquad (4)$$

On the plot, note, that the amplitude on the resonance and close to it can be bigger then the amplitude of the external force, moreover the phase of the oscillation can be opposite (π) to the force, which cause the displacement – *negative response*.

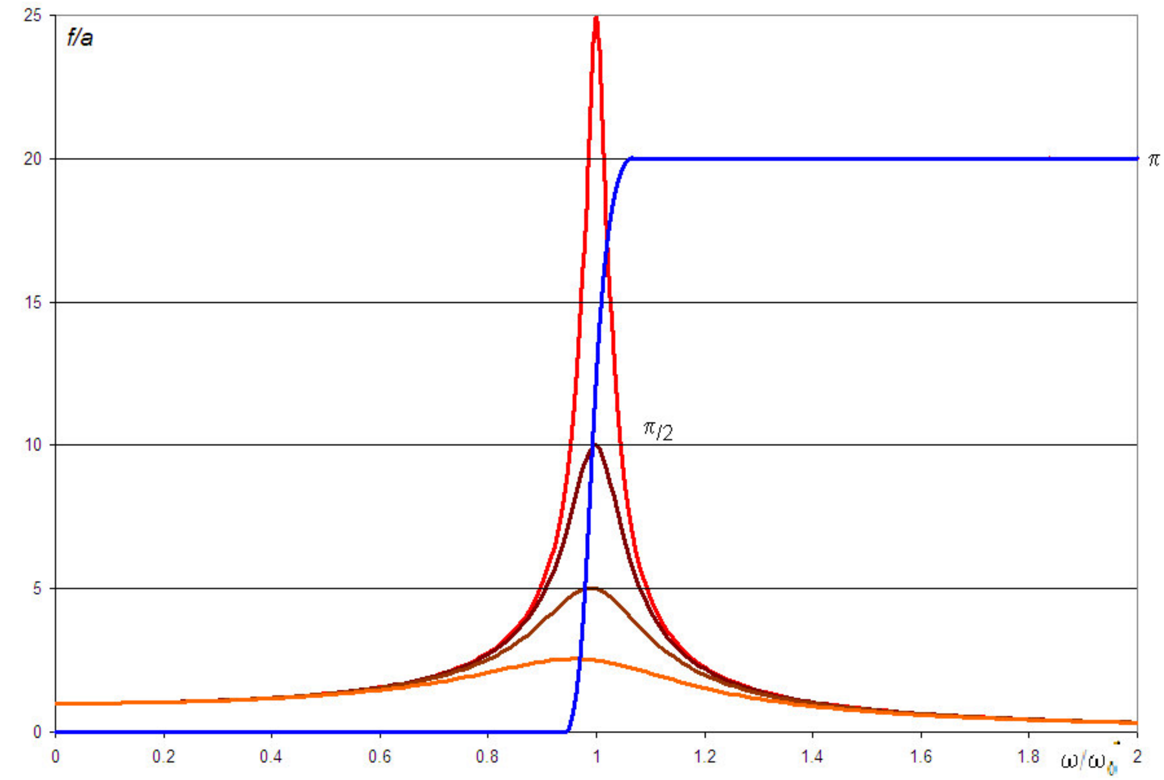


Figure 1. Amplitude and phase dependence on external field frequency (8). The amplitude of response can be a lot bigger then the amplitude of the external force. Blue line – phase of a response dependence on external field frequency. The response flips out-of-phase when frequency of the external force passes the value of systems eigenfrequency – *negative response*

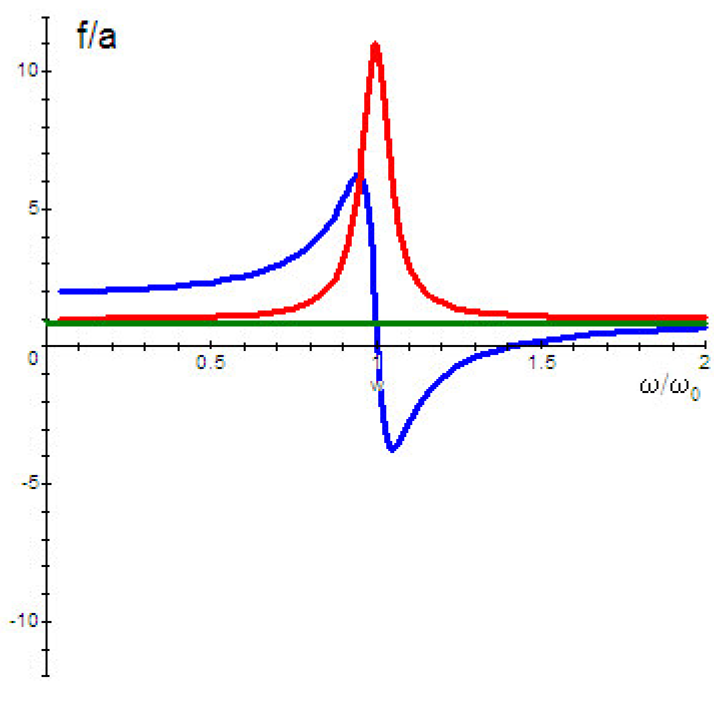


Figure 2. Response to the field. Green line is the driving force amplitude (electric or magnetic field) Blue line is the real part of the complex amplitude (real part of ε or μ) and the red line is imaginary part (imaginary part of ε or μ - losses).

Properties of materials with negative ε and μ.

Index of Refraction.

The index of refraction of such material can be calculated:

$$n = n_r + in_i = \sqrt{(\varepsilon_r + i\varepsilon_i)(\mu_r + i\mu_i)} \quad (5)$$

The square root of a complex number has two values. One of the values is non-physical. It was shown that it is the one where real part of ***index of refraction is negative***. The main idea on how to pick which root is physical is to look at imaginary part – it should not allow amplification of wave in the media (should be positive) [1, 11].

$$n = -\sqrt{|\varepsilon_r \mu_r|}\left[1 - \frac{i}{2}\left(\frac{\varepsilon_i}{|\varepsilon_r|} + \frac{\mu_i}{|\mu_r|}\right)\right] \quad (6)$$

Using the property of negative refraction one can testify that the media is double negative. It was predicted by Veselago [1] in 1967 and experimentally verified by D. Smith in 2001 [3] and A. Houck in 2003 [4].

$$\frac{n_1}{n_2} = \frac{\sin(\varphi_{inc1})}{\sin(\varphi_{inc2})} \quad (7)$$

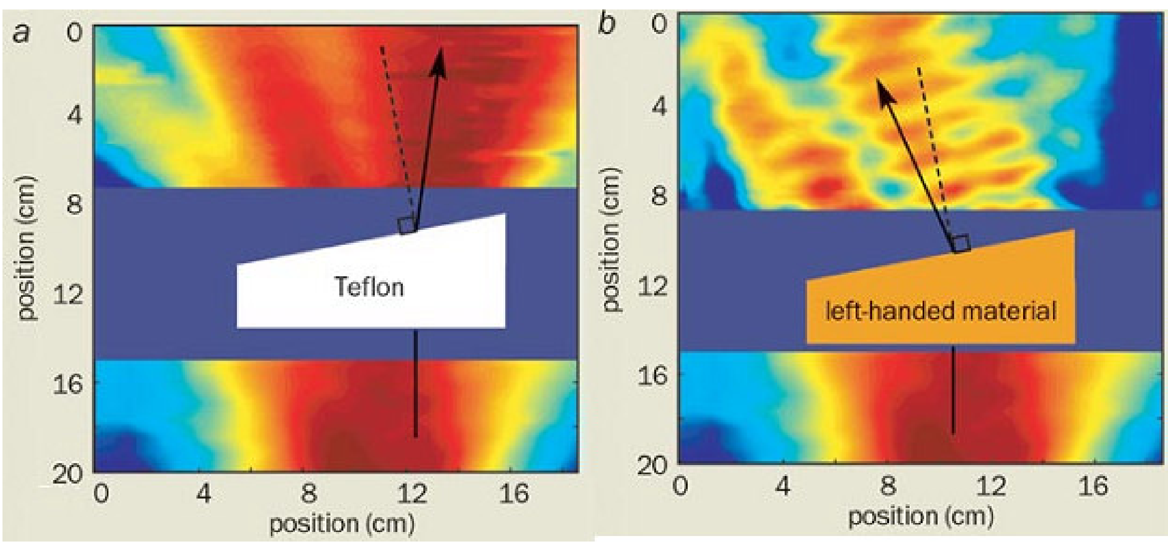


Figure 3. Experiment by A. Houck [4]. We can see the difference between refraction on Teflon and Left-Handed Metamaterial. We can also see that Left-Handed Media is very lossy.

Left-Handness.

Let's propagate plane monochromatic wave $e^{-i\omega t+ikz}$ through a media with negative permittivity and permeability. Then the rotor equations from Maxwell system can be rewritten:

$$\left[\vec{k},\vec{E}\right]=\frac{\omega}{c}\mu\vec{H} \quad \text{and} \quad \left[\vec{k},\vec{H}\right]=-\frac{\omega}{c}\varepsilon\vec{E} \qquad (8)$$

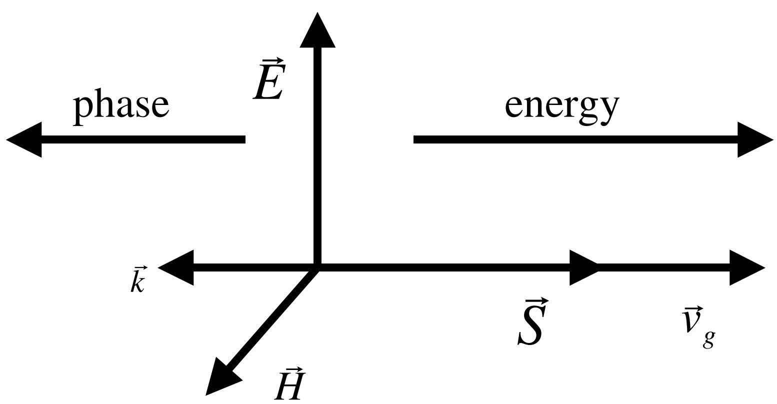
Now we can see that the wavevector *k*, electric field *E* vector and magnetic field vector *B* form a left-handed system instead of usual right-handed.

That is why these materials were called by V. Veselago [1] ***"Left-Handed"*** Pendry and Smith [2, 3] prefer to call them ***"Negative-Index Metamaterials"***, referring to the negative index of refraction (see above) and motivating, that left-handness can be confused with chirality. R. Ziolkowski [11] uses electrical engineering terminology ***"Double Negative"***. In literature you can see several abbreviations which all refer to materials with negative permittivity and permeability.

DNG MTM	double negative metamaterial (media)
LHM	left-handed metamaterial (media)
NIM, NRI	negative (refractive) index metamaterial (media)

Poynting vector is defined through field vectors:

$\vec{S} \overset{def}{=} \frac{c}{4\pi}\left[\vec{E},\vec{H}\right]$, and always form a right-handed system with them. Therefore in double-negative media ***group velocity (Poynting vector)*** is ***counterdirected*** with the ***wavevector (the direction of phase velocity).***



Energy propagates in different direction then a phase front advances.

The unusual index of refraction also causes an unusual phase velocity in the media. This leads to two more properties of LHMs – ***reverse Cherenkov Effect*** and ***reverse Doppler Effect*** in such media [12, 14].

Reverse Doppler Effect.

The Doppler shift formula is:

$$\Delta\omega = -\omega_0 \frac{v}{V} \tag{9}$$

ω_0 is the radiation source; v is the source-receiver relative speed, V – is the phase velocity of radiation in the media and $\Delta\omega$ is the difference between the source frequency and the frequency, which is picked up by the receiver.

Rewriting the phase velocity within the refractive index we get:

$$\Delta\omega = -\omega_0 \frac{v}{(c/n)} \tag{10}$$

This formula is valid for both cases of negative and positive refractive indexes n. Doppler shift in LHM is positive, which means that if the receiver is moving towards the source in double negative media it will register a frequency, smaller than the original one, because the phase front advances from the receiver in LHM.

Reverse Cherenkov Effect.

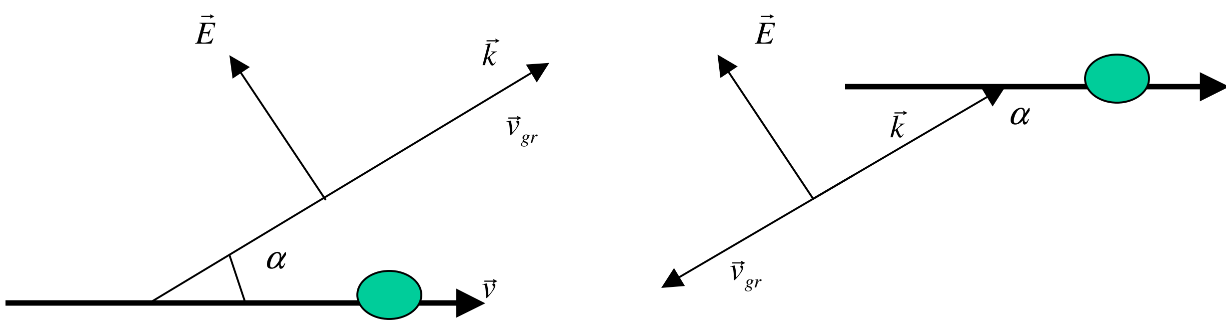


Figure 4. Cherenkov Effect in normal media (left) and LHM (right). In negative permittivity and permeability material fast particle will radiate backward, rather then normal forward direction.

The radiation coupling condition for Cherenkov Effect is [13]

$$\omega = \vec{k}\vec{v} = \frac{\omega n v}{c}\cos\alpha \tag{11}$$

Here ω is the radiated frequency, v is the particle velocity, n – material index of refraction and α is the angle between the particle trajectory and radiated photon. There are two possibilities for Cherenkov Effect in LHM, but the one on the Figure 9 (right) is the real one, because of Sommerfield radiation condition [8, 13]. Energy has to go from the source not toward it. Reverse Cherenkov Effect is a core of the research. There is an interest to observe reverse Cherenkov effect. It was not yet observed. We have an opportunity to do that at Wakefield Facility in Argonne. Besides, this effect clearly has potential beam diagnostics application.

Measurement and simulation systems

All the measurements so far have been done on network analyzer. With the aid of an antenna and a receiver, a network analyzer can measure transmission through and reflection from the material (or as they are called S-parameters) as functions of frequency. $\hat{S}\vec{a} = \vec{b}$, where *a* are incoming signals and *b* are outgoing signals. In case of antenna – receiver scheme, we have:

$$S_{11} = \frac{a_{reflected}}{a_{incident}} \quad \text{and} \quad S_{21} = \frac{a_{transmitted}}{a_{incident}}$$

The simulations, which I did to design the material, were done using a commercial software package – Microwave Studio. I defined the geometry of the metamaterial and signal ports for my simulations. The outcome of the simulation is also an S-parameters dependence on frequency.

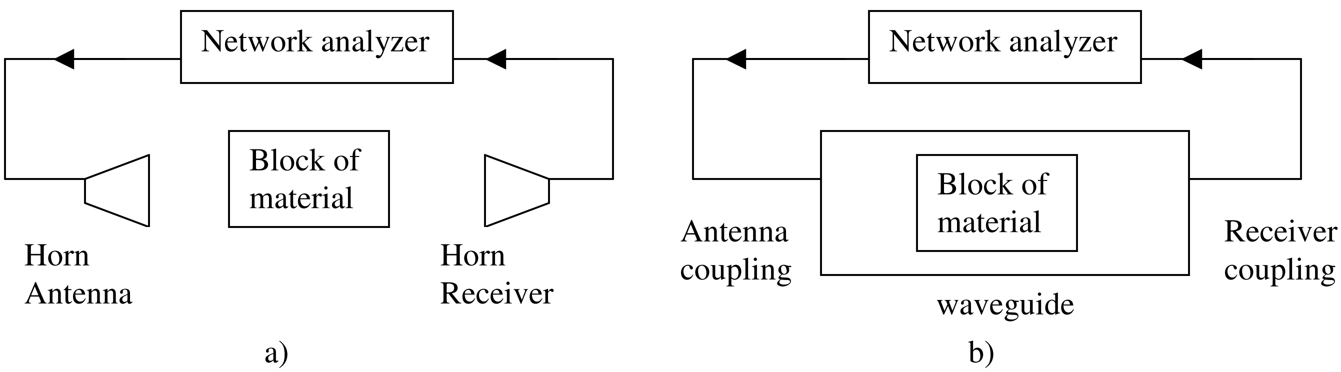
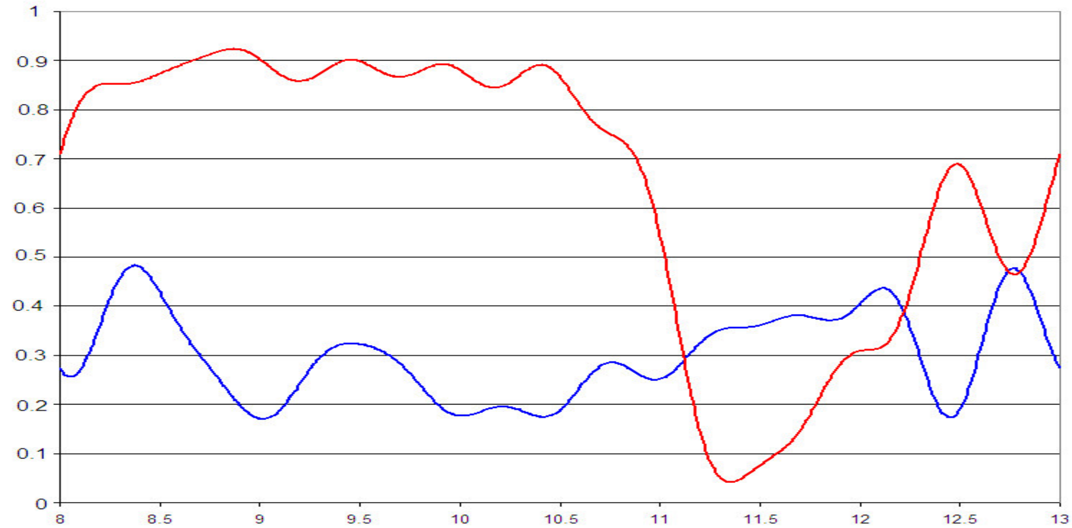


Figure 5. Experimental and simulation setup. a) Horn-Antenna testing. b) Loaded waveguide system. Experiment or simulation outcome is transmission/reflection dependence on frequency.

The main interest for metamaterial studies is the transmission parameter. It can be viewed as a function of frequency on linear or logarithmic (dB) scale. The linear scale is defined by the ratio of the transmitted power to the incident power and obviously occupies the [0, 1] region, where 1 – is a full transmission. The dB scale can be obtained from the linear scale: dB=10*log(linear). It ranges from minus infinity to 0, where 0 is a full transmission. Useful numbers 50% percent power transmission – linear=0.5 or -3dB, 10% - linear=0.1 or -10dB.

Different elements of the metamaterial should exhibit their own patterns of transmission parameter (S_{21}). For simplicity of design and measurement, metamaterials are designed so that one set of elements is responsible for the dielectric effect (response to electric field) and another set is responsible for the magnetic effect. There are some unavoidable interaction effects between the elements of these two sets which results in slight overall effect frequency shift.

Let say, that we got the following pattern of transmission (red) and reflection (blue) from measurement or simulation of the elements, responsible for magnetic effect:

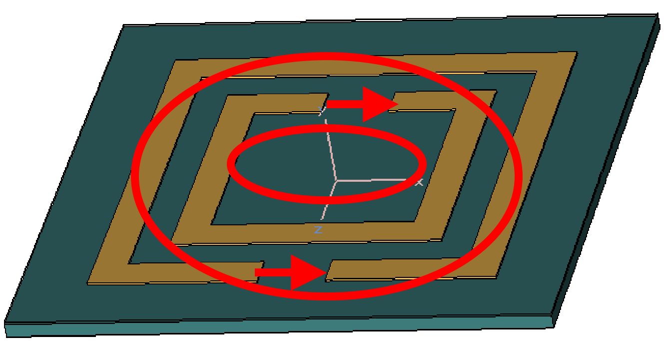


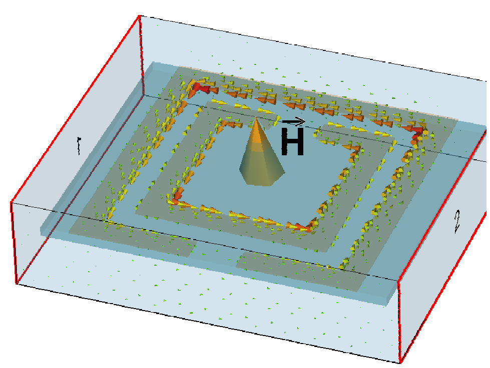
Since these elements do not give magnetic effect ε is approximately equal to 1. Transmission in general will be defined by epsilon then. On the picture above we see that there is a transmission drop between 11.2 and 11.7 GHz. Thus, the permeability is negative in this frequency range and propagation cannot occur in this region. The same argument applies for the magnetic elements.

METAMATERIAL DESIGN

Negative permeability.

Negative permeability material can be made up of the elements, called split ring resonators (SRR).





The element is manufactured like a piece of a circuit board. Its size is ~ 3 *mm*.

When a wave propagates through a media, composed of such structures it induces currents in the rings, creating a magnetic response (see figure above). The dielectric response is negligible (ε=1). The design provides two loops of current, which form a resonant system. In terms of effective circuits, the SRR can be considered a LC resonant circuit [2]. If transmission through SRRs experiences drops it can be only due to negative μ, since there is no dielectric effect [2, 12, and 14].

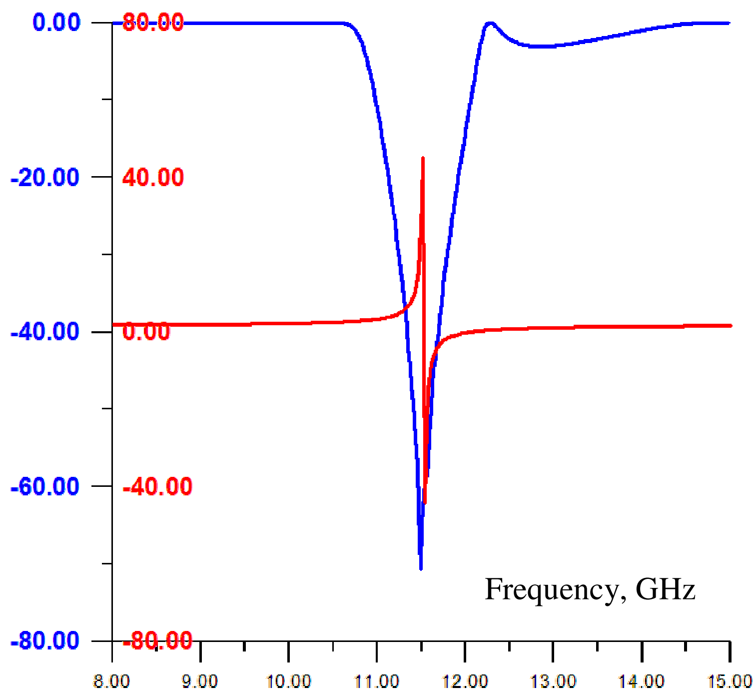


Figure 6. Transmission through the SRRs – dB scale. One can see the drop of transmission at designed frequency of 11.5 GHz. Red line is retrieved (my postprocessor) from the simulation permeability. Simulations were done on Microwave Studio. See Figure2. Permeability behavior is exactly the same as resonant oscillator.

Negative permittivity

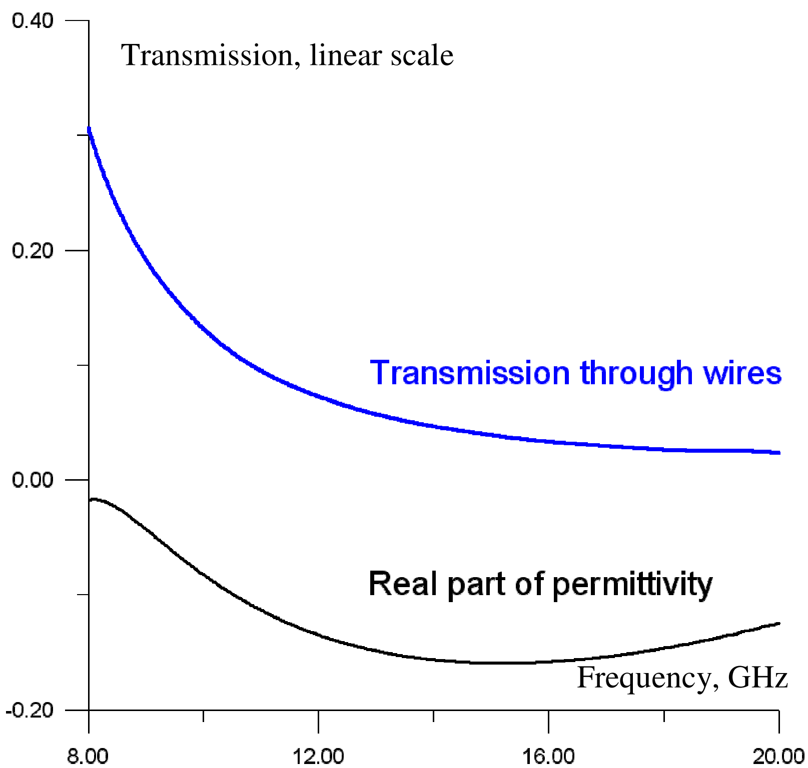
Negative permittivity can be realized with plasma-mimicking structures. As well known [18], plasma exhibits negative permittivity below plasma (Eigen) frequency. If plasma elements are displaced and then released they will start oscillating at ω_p frequency. It can be shown, that:

$$\varepsilon = 1 - \frac{\omega_p}{\omega} \qquad \omega_p^2 = \frac{4\pi e^2 N}{m} \tag{12}$$

Here e is an electron charge, m is a particle mass and N is a particle concentration in plasma [18].

A wire array is a plasma-mimicking structure. Such an assembly effectively acts as plasma, where charged particles can move only along the wires. The plasma frequency is determined by parameters of the array: length a and radius r of the wires[2, 6].

$$\varepsilon_{eff} = 1 - \frac{\nu_p^2}{\nu^2 + i\nu\gamma} \qquad \nu_p^2 = \frac{c^2}{2\pi a^2 \ln(a/r)} \tag{13}$$



The system does not have a way to give a magnetic response and $\mu \approx 1$, therefore transmission drop can be only due to negative ε [2, 12, 14, 18].

Figure 7. Transmission through the wire array – linear scale. One can see the drop of transmission at designed frequency of 11.5 GHz. Red line is retrieved (my postprocessor) from the simulation permeability. Simulations were done on Microwave Studio [17].

Negative permittivity can be achieved with a modified wire array [11].

These elements are called capacitively loaded strings (CLS) [11].

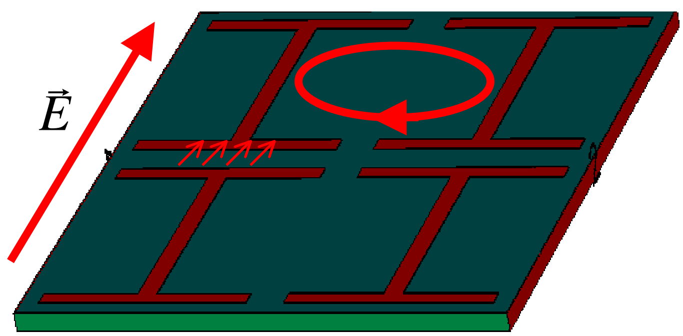
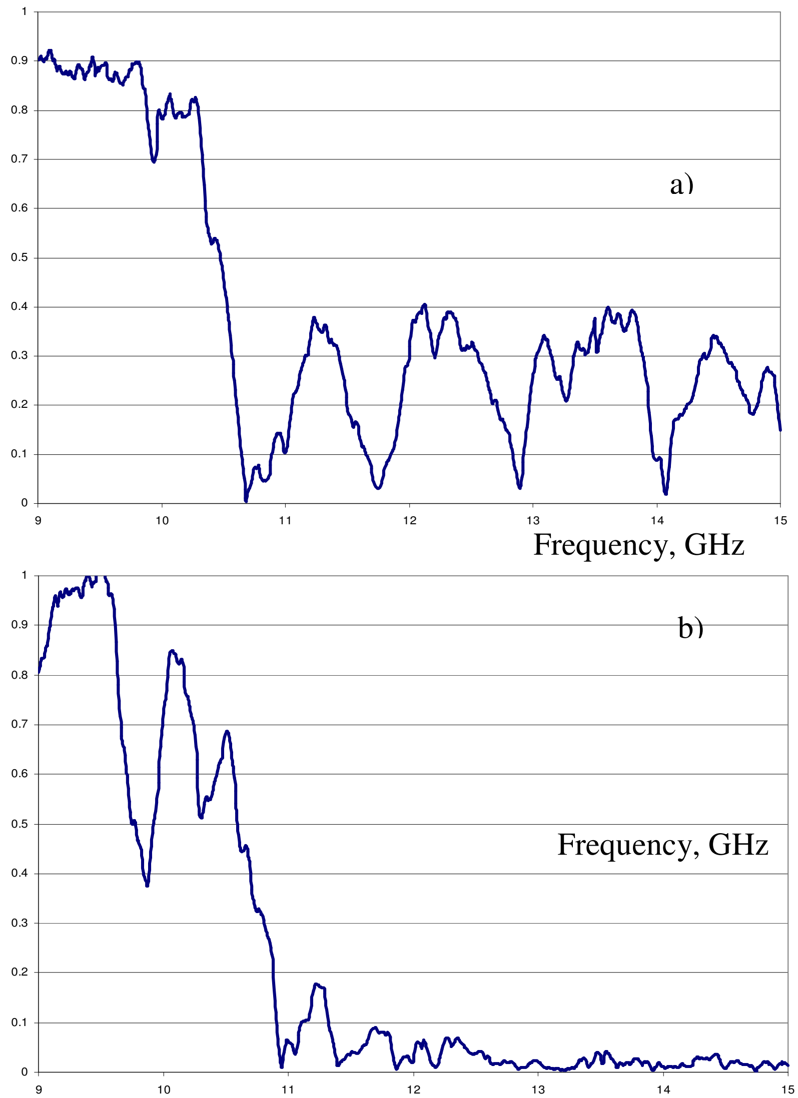


Figure 8. CLS elements. Transverse to the field elements do not give any response but work as a bank of electrons to enhance plasma effects.



There is a resonance effect due to effectively formed LC circuits (capacities – visible, loops of current are formed with geometry and displacement current), but this effect is very broad in frequency, because the bond between the elements is much stronger, then the one in the case of SRR elements [11]

Figure 9. Transmission linear scale. CLS media a) low density of elements one can see several resonant peaks. b) high density of elements broad transmission drop – like in a plasma [16].

Double-Negative Metamaterials.

CLS and SRR elements combined together and then assembled in metamaterial will exhibit negative ε and μ at designed frequency [2, 3, 4, 5, 11].

One of my simulations: several SRR elements and a wire array in the box with magnetic and electric walls. Two walls are determined to be ports. TE_{01} mode on the port excites the structure. Second port acts as a receiver. The outcome of simulation is transmission coefficients as a function of frequency.

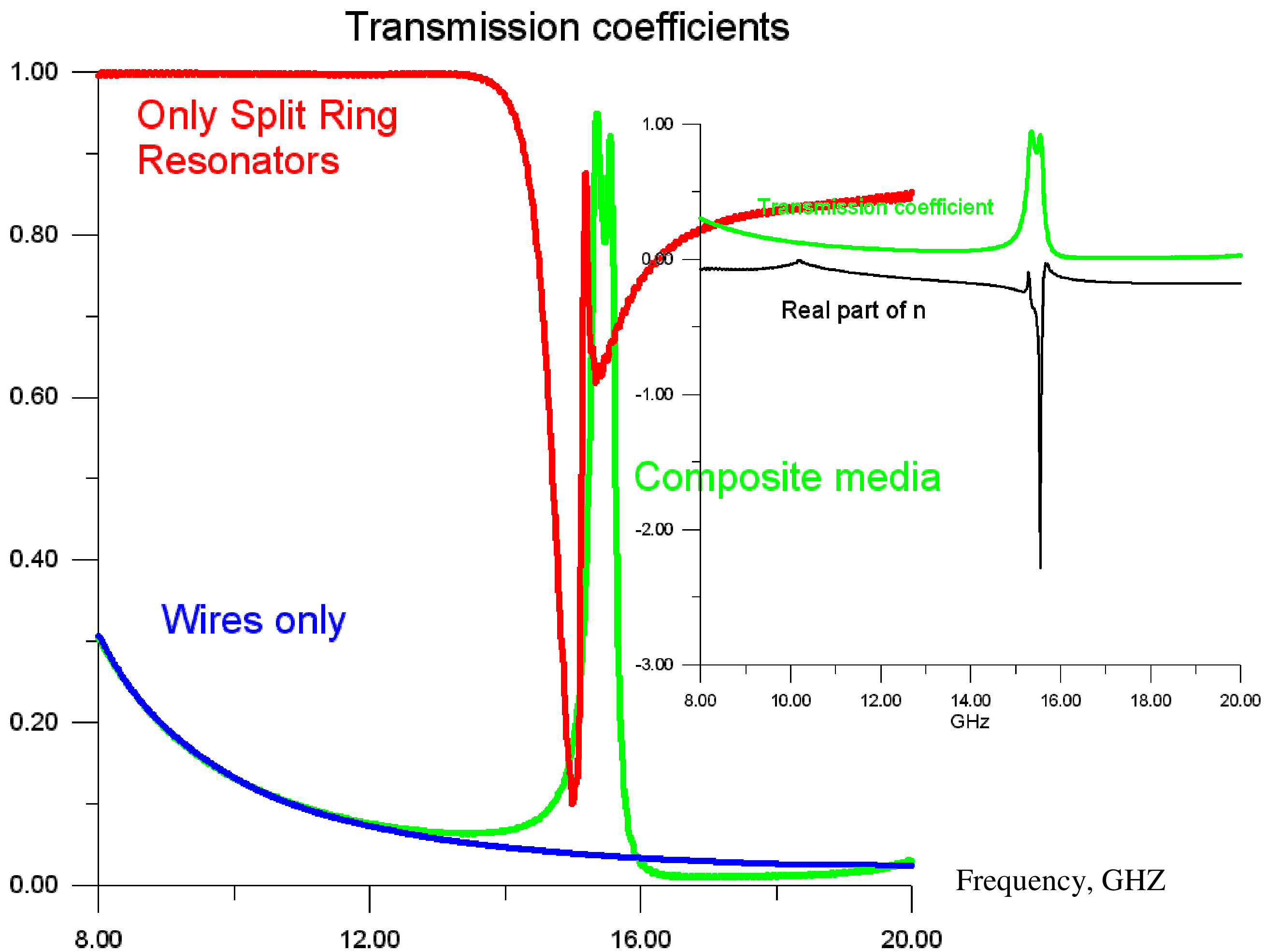


Figure 10.a) Transmission through SRR material, Wire Array and composite SRR-Wire Array media – linear scale.. We can see, that at the place where wire array does not allow propagation as well as SRRs composite media exhibits transmission peak (ε and μ are negative). (my simulations on Microwave Studio). b) transmission through the composite media and index of refraction (my postprocessor. At the left-handed transmission peak we observe negative refraction.

The actual design of my metamaterials.

Argonne Wakefield Facility has a good base for analysis of GHz structures. That is why the goal was to design a double negative material at around 11.5 GHz frequency range. I did a cycle of simulations aimed to determine the geometry of SRR and CLS units which will exhibit negative resonance response at 11.5 GHz. Then this elements were printed at American Standard Circuit facility. Rogers 5880 laminate was chosen as a substrate to achieve the lowest possible dielectric background constant, which is 2.2 at X-band frequency range.

The metamaterial itself is a stack of PCB pieces with the elements, printed on them.

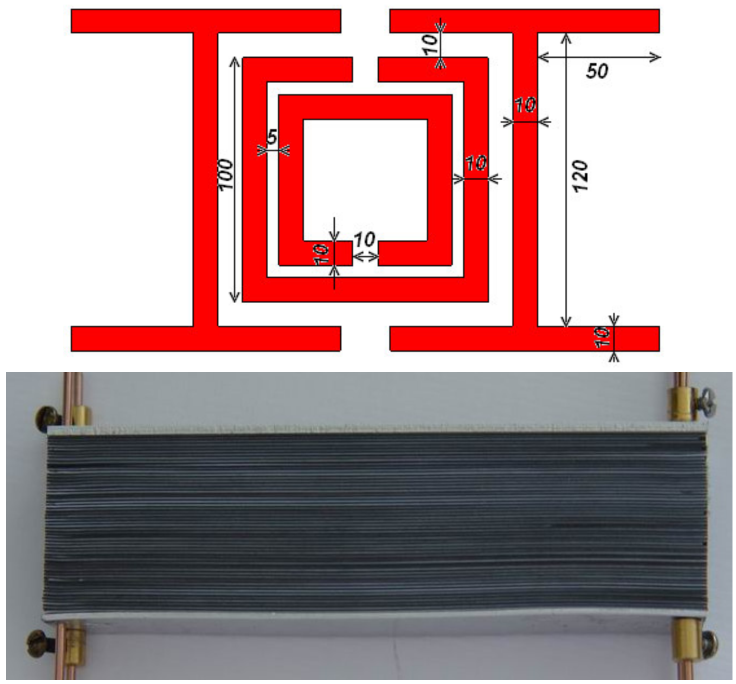


Figure 11. a) Sizes and spacing of basic elements of metamaterial. All dimensions are in mils (100mils=2.54mm). b) Rigid metamaterial - the horizontal stack of circuit boards with CLS, SRR elements.

There are two main material configurations planned for experimental studies: loaded waveguide and open structure with horn antennas (both X-band). Each configuration is designed with three different thicknesses to study losses and effect enhancement.

Several configurations are being tested: rigid structures with Teflon spacers of different thicknesses, structures in holders without spacers, loaded X- band waveguide, SRRs with CLSs, SRRs with wire arrays and SRRs with printed stripes of different space period.

FIRST MEASUREMENTS

The very first test was done with a comb-type holder. This holder allowed us to hold up to 60 PCBs parallel to each other. The main advantage of such holder is that one can load the PCBs of different types and sizes without taking the whole structure apart. More importantly, this holder does not have a big dielectric background: there is an air in between the PCB elements.

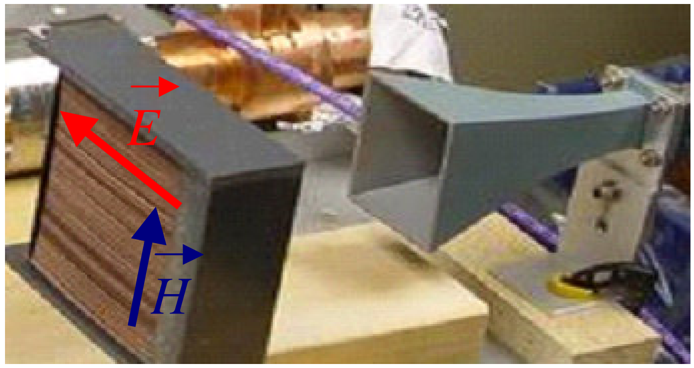


Figure 12. Comb-type holder. Horizontal stack of pieces of circuit boards with elements printed on them. Experiments with open structure. I am using X-Band horn antennas, polarization is shown.

The main disadvantage of this holder is that PCBs are not precisely aligned in it. The substrate is very soft and thin, so the boards sag in the holder. This makes the orientation with respect to the fields not identical for each element. Consequently, there are bunch of resonant elements with slightly different resonant frequencies. That is why the results differ from the original simulation: The bandwidth of the effects broadens, while the quality of negative responses gets lower. To eliminate this problem, future studies will be performed on rigid structures with Teflon spacers to maintain precise alignment (see figure 11 above).

CLS units measurement.

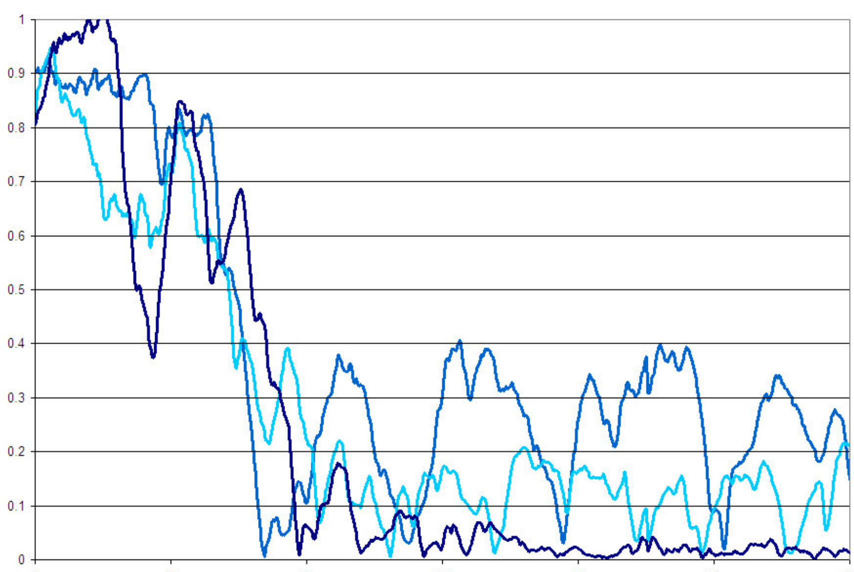


Figure 13. CLS units test. Transmission (1 is 100%) vs. frequency (9-15 GHz). Medium blue – 20 PCBs in the holder, light blue – 40 PCBs in the holder, dark blue – 55 PCBs in the holder. Starting at 11 GHz there is no transmission which means that ε (permittivity) is negative at least in the region of [11,15] GHz.

That means, that starting at around 11GHz permittivity is negative over a broad bandwidth. I tried to overlap this region with the region, where split ring resonators exhibit negative permittivity.

SRR units test.

The effect, which provides negative μ in split rings, is highly resonant. Therefore, even just by putting one PCB with SRRs between the source and receiver, one can observe a transmission drop at the design frequency by 50%.

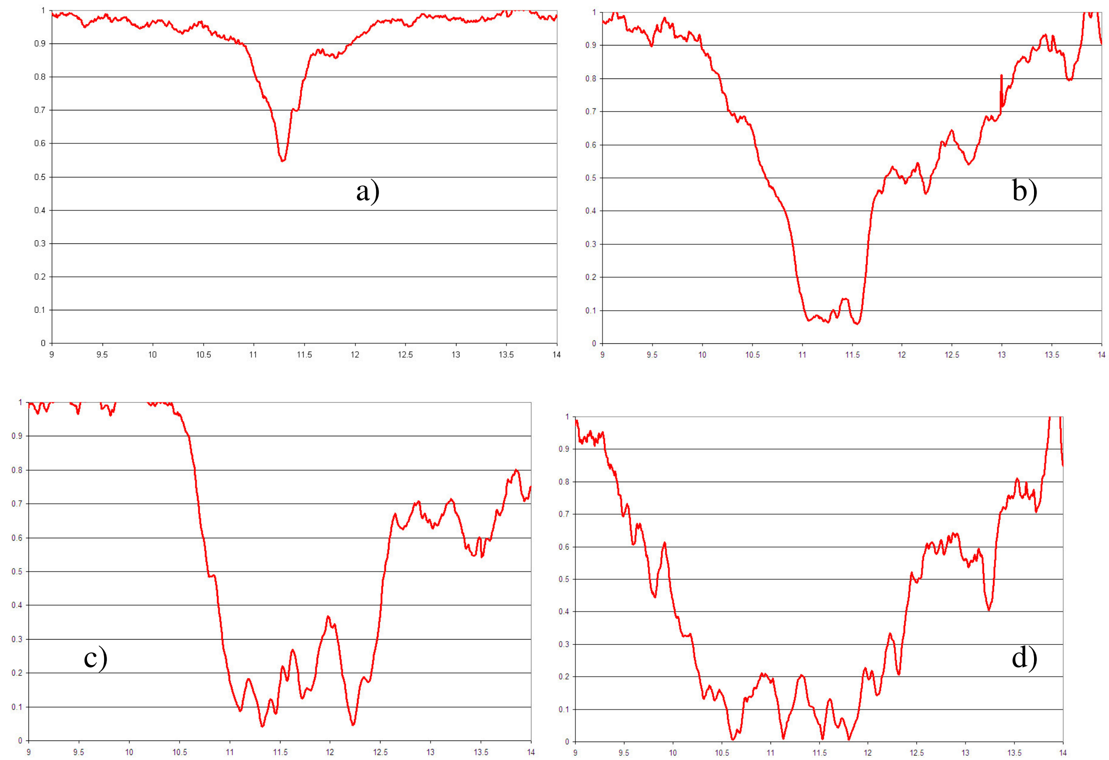


Figure 14. Transmission through the SRR systems. a) one PCB in the holder, b) 15 PCBs in the holder, c) 25 PCBs in the holder and d) 50 PCBs in the holder. Due to holder imperfection SRRs are inclined at different angles towards magnetic field. Therefore they have slightly different resonant frequencies in the range 9.5-13 GHz. Therefore instead of having a sharp transmission drop I have a wide broadband region, where effective permittivity is negative.

LHM (SRR + CLS) units test.

Assembly of SRR and CLS units in the holder as expected gave several transmission enhancement peaks, where being tested separately SRRs and CLSs did not give a good transmission.

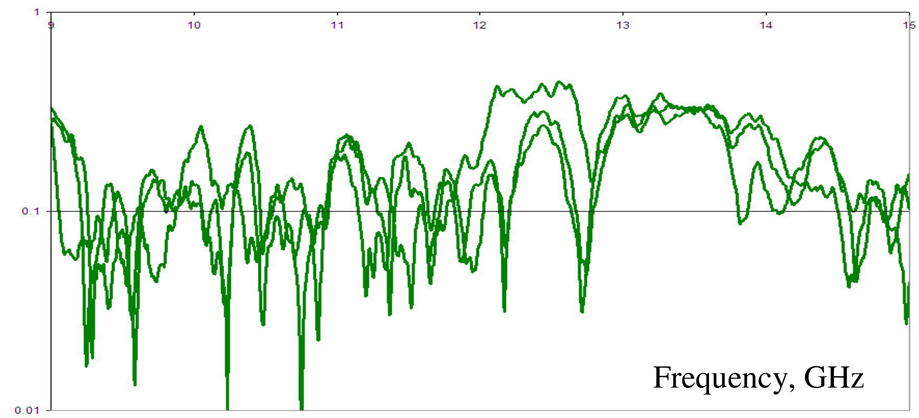


Figure 15. Logarithmic scale. Transmission through SRR+CLS units at different positions between the source and receiver. Shows imperfection of the holder. Due to not precise alignment of PCB pieces in the holder transmission slightly depends on position. System is very sensitive to the outside trembling.

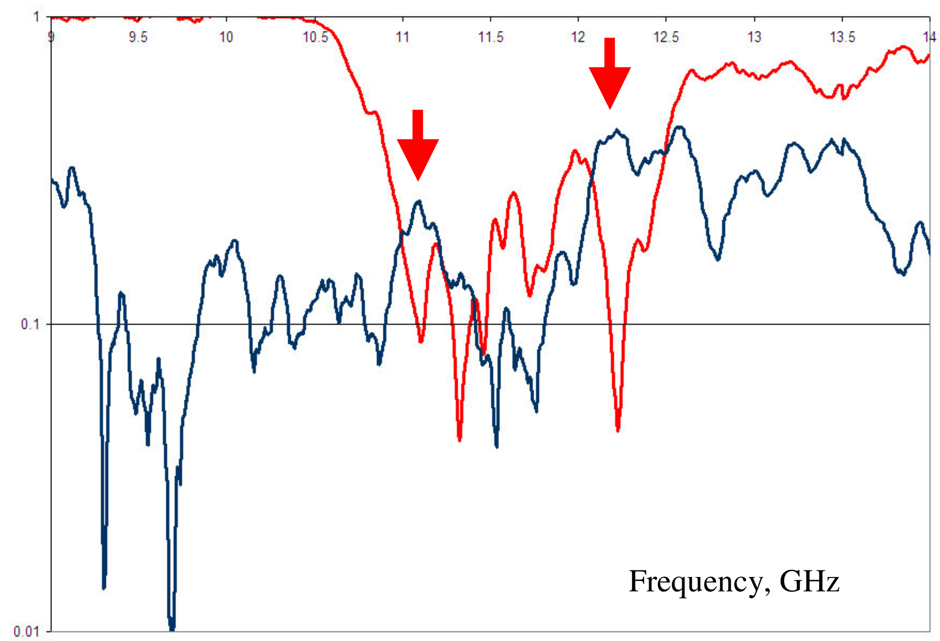


Figure 16. Logarithmic scale. Red line – transmission through the SRR elements. Blue line – transmission through the composite (SRR+CLS media). At 11.1 and 12.2 There is transmission enhancement in respect to SRR only system. Prediction: this is left-handed transmission peaks.

To eliminate the problem with alignment I will create a rigid structure with spacers. This way PCB boards will be separated by Teflon sheets and the stack will be pressed to keep elements aligned. Loaded waveguide structures will be tested soon too.

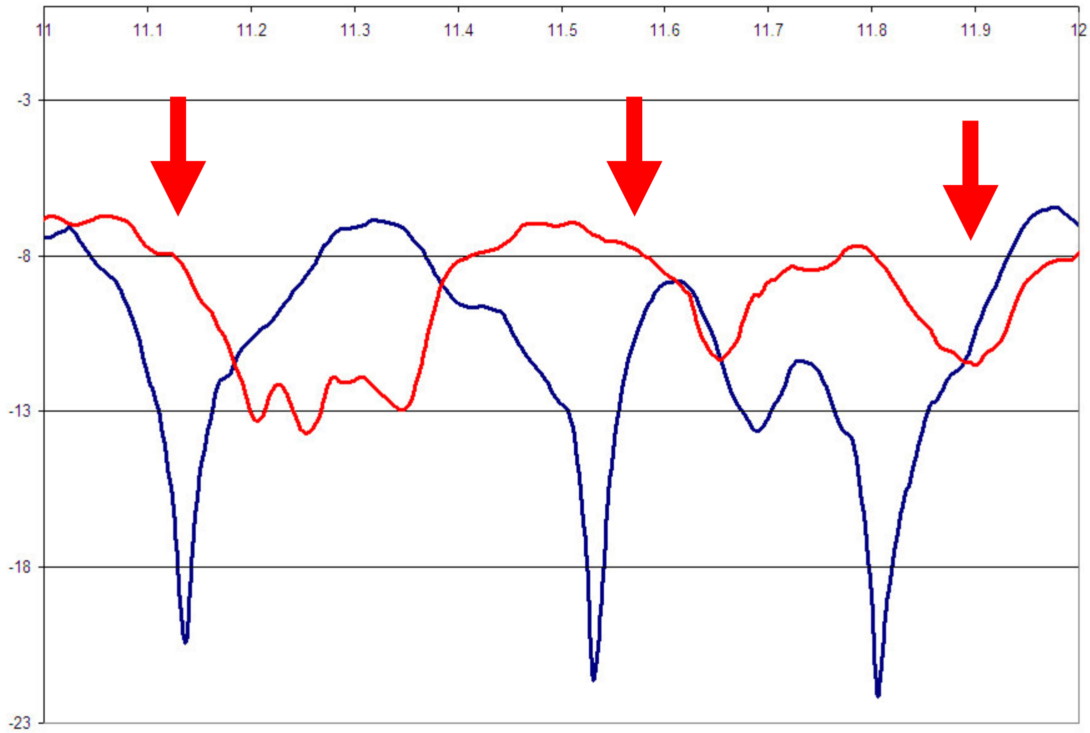


Figure 17. Blue line – transmission through the SSRs. Red line – transmission through the composite material SRR+CLS. Zoom on frequency region 11-12 GHz. One can clearly see, that at the transmission drops when only SRR exposed to radiation become transparent (transmission enhances), when CLS units are added. Clearly, this is a transmission through double-negative media. The design frequency is 11.5GHz. Peaks near this location are due to imperfection of the folder which allows sagging of the boards and thus creates not sharply peaked at 11.5GHz resonant frequency distribution of the elements.

POSTPROCESSING THEORY

Transmission and reflection parameters should give enough information to retrieve material parameters ε and μ. Methods of retrieval for open and loaded-waveguide structures are slightly different.

The Modified Ross-Weir [9, 10, and 11] approach for permittivity and permeability retrieval procedure begins by introducing the composite terms:

$$V_2 = S_{21} - S_{11} \text{ and } V_1 = S_{21} + S_{11} \tag{14}$$

Then, we derive:

$$X = \frac{1 + V_1 V_2}{V_1 + V_2} = \frac{1 + Z^2}{2Z} \tag{15}$$

$$Y = \frac{1 - V_1 V_2}{V_1 - V_2} = \frac{1 + \Gamma^2}{2\Gamma} \tag{16}$$

Therefore:

$$Z = X \pm \sqrt{X^2 - 1} \tag{17}$$

$$\Gamma = Y \pm \sqrt{Y^2 - 1} \tag{18}$$

Where $|Z| \leq 1$ is the phase advance $Z = e^{ikd}$ and $|\Gamma| \leq 1$ is the interface reflection coefficient. Choice of the sign has to match properties, listed above.

Usually, the values of S11 and S21 are highly frequency dependent and achieved values near zero and unity. The standard extraction expressions are unsatisfactory, particularly in the frequency regions where the permittivity and permeability resonances are expected, i.e., where those values would transit quickly between positive and negative values [11]. The presence of the square root values is particularly difficult for those regions. One can not anticipate what branches the square root values should lie on without potentially biasing the end results.

Using the same process, however, one can derive many other expressions for Γ and Z [11]. Ones that could handle typical MTM. For instance, one can obtain the transmission term Z and reflection Γ as:

$$Z = \frac{V_1 - \Gamma}{1 - \Gamma V_1} \text{ and } \Gamma = \frac{Z - V_2}{1 - Z V_2} \tag{19}$$

From previous equations we get $\left(\eta = \sqrt{\varepsilon/\mu}\right)$:

$$1 - Z = \frac{(1 - V_1)(1 + \Gamma)}{1 - \Gamma V_1} \text{ and } \eta = \frac{1 + \Gamma}{1 - \Gamma} = \frac{1 + Z}{1 - Z}\frac{1 - V_2}{1 + V_2} \tag{20}$$

Assuming, that the thickness of MTM slab is not too large $k_{real}d \approx 1$, and knowing that the complex wave number $k = \omega\sqrt{\varepsilon\mu}/c$, one can write $Z \approx 1 - jkd$ to obtain approximate results for permittivity and permeability:

$$\varepsilon \approx \frac{2}{ik_0 d}\frac{1-V_1}{1+V_1} \text{ and } \mu \approx \frac{2}{ik_0 d}\frac{1-V_2}{1+V_2} \tag{21}$$

For index of refraction:

$$n \approx -\sqrt{|\varepsilon\mu|}\left[1 - j\frac{1}{2}\left(\frac{\varepsilon_i}{|\varepsilon_r|} + \frac{\mu_i}{|\mu_r|}\right)\right] \tag{22}$$

The postprocessor was written in FORTRAN by me and has shown consistent results (see figures 4, 5 and 8). The problem for using the retrieval method with experimental data is a difficulty of calibrating reflection coefficient for horn-antenna measurements [11]. Unfortunately single transmission curve does not give enough information about the media to retrieve its parameters. Usually retrieval procedure is performed with the data obtained from a simulation. Prior to that one have to show that his simulation data agrees with the measurement.

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