ORBIFOLD GRAND UNIFIED THEORY WITH DYNAMICAL ELECTROWEAK SYMMETRY BREAKING

Tianjun Li

Institute for Advanced Study

Tianjun Li and Hitoshi Murayama, in preparation.

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I. INTRODUCTION

4-dimensional Grand Unified Theory (GUT) with $\mathcal{N} = 1$ Supersymmetry (SUSY) is very interesting:

- Unification of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge interactions in the Standard Model (SM).
- Gauge coupling unification in Minimal Supersymmetric Standard Model (MSSM).
- Gauge hierarchy problem.
- Charge quantization.
- Radiative electroweak symmetry breaking due to large top quark Yukawa coupling.
- Weak mixing angle at weak scale M_Z.
- Neutrino masses by see-saw mechanism.
- ....

Existing problems in the 4D SUSY GUT with gauge symmetries broken by the Higgs mechanism:

- Proton decay problem
- Higgs doublet-triplet splitting problem
- How to explain the fermion masses and mixings? For example in $SU(5)$, $m_e/m_\mu = m_d/m_s$.
- How to break the GUT gauge symmetry down to the SM gauge symmetry?
- ....

These problems in 4D GUT can be solved neatly in the high dimensional orbifold GUT models [a].

[a] Y. Kawamura; G. Altarelli and F. Feruglio; L. Hall and Y. Nomura; A. Hebecker and J. March-Russell; T. Li; R. Dermisek and A. Mafi; I. Gogoladze, Y. Mimura and S. Nandi.

Electroweak symmetry breaking (EWSB) is still a mystery in particle physics.

Technicolor theory explains the EWSB elegantly due to the condensations of one or several pairs of fermions [a].

Can we construct the orbifold GUTs with dynamical EWSB?

Embedding non-SUSY models with dynamical EWSB [b] into orbifold non-SUSY GUT, we have to give up the naturalness in SUSY.

The Minimal Supersymmetric fat Higgs Model (MSFHM).

[a] S. Weinberg; L. Susskind; C. T. Hill and E. H. Simmons.

[b] T. Appelquist and R. Shrock; C. T. Hill and E. H. Simmons.

II. Brief Review of the MSFHM

- A new gauge interaction $SU(2)_H$ that becomes strong at Λ_H
- New fields T^i, P^1, P^2, Q^1, Q^2, S and S'

The superpotential is

$$W = W_1 + W_2 + W_3$$

$$\begin{aligned} W_1 &= y_1 S T^1 T^2 + y_2 S' T^3 T^4 \\ W_2 &= -m T^5 T^6 + m' T^7 T^8 \\ W_3 &= y_3 P^1 (T^1, T^2) T^6 + y_4 P^2 (T^1, T^2) T^5 \\ &\quad + y_5 T^5 (T^3, T^4) Q^2 + y_6 T^6 (T^3, T^4) Q^1 \end{aligned}$$

The scale of electroweak symmetry breaking v_0 and the spectator masses are

$$v_0^2 \sim \frac{mm'}{(4\pi)^2}, \quad m_{\text{spect}} \sim \frac{m'}{4\pi}$$

$$m' \sim \Lambda_H$$

The quantum numbers of T^i, P^1, P^2, Q^1, Q^2, S and S' under $SU(2)_L \times SU(2)_H$ and $SU(2)_y \times U(1)_{g1} \times U(1)_{g2} \times U(1)_R$
The $U(1)_Y$ subgroup of $SU(2)_y$ is gauged.

Superfields	$SU(2)_L$	$SU(2)_H$	$SU(2)_y$	$U(1)_{g1}$	$U(1)_{g2}$	$U(1)_R$
$(T^1, T^2) \equiv T$	**2**	**2**	**1**	**0**	**0**	**0**
(T^3, T^4)	**1**	**2**	**2**	**0**	**0**	**0**
T^5	**1**	**2**	**1**	**1**	**0**	**1**
T^6	**1**	**2**	**1**	**−1**	**0**	**1**
T^7	**1**	**2**	**1**	**0**	**1**	**1**
T^8	**1**	**2**	**1**	**0**	**−1**	**1**
P^1	**2**	**1**	**1**	**1**	**0**	**1**
P^2	**2**	**1**	**1**	**−1**	**0**	**1**
Q^1	**1**	**1**	**2**	**1**	**0**	**1**
Q^2	**1**	**1**	**2**	**−1**	**0**	**1**
S	**1**	**1**	**1**	**0**	**0**	**2**
S'	**1**	**1**	**1**	**0**	**0**	**2**

Fermion masses:

- Four additional chiral multiplets are introduced

$$\varphi_u,\ \bar{\varphi}_d(\mathbf{1},\mathbf{2},+\frac{1}{2})\ ;\qquad \varphi_d,\ \bar{\varphi}_u(\mathbf{1},\mathbf{2},-\frac{1}{2})$$

- Superpotential is

$$\begin{aligned} W_f &= M_f(\varphi_u\bar{\varphi}_u + \bar{\varphi}_d\varphi_d) + y_7\bar{\varphi}_d(TT^4) + y_8\bar{\varphi}_u(TT^3) \\ &\quad + h_u^{ij}Q_iu_j\varphi_u + h_d^{ij}Q_id_j\varphi_d + h_e^{ij}L_ie_j\varphi_d \end{aligned}$$

$$(\Lambda_4/\Lambda_H)^{1/2} \sim 4\pi \quad M_f \sim m' \sim \Lambda_H$$

- Yukawa superpotential below compositeness scale is

$$W_f = h_u^{ij}Q_iu_jH_u + h_d^{ij}Q_id_jH_d + h_e^{ij}L_ie_jH_d$$

Gauge Coupling Unification.

β functions:

$$b_i\left(\mathrm{T}^{1,2,3,4} + \mathrm{P}^{1,2} + \mathrm{Q}^{1,2} + \varphi_{\mathrm{u,d}} + \bar{\varphi}_{\mathrm{u,d}}\right) = b_i(4\,\mathrm{H_u} + 4\,\mathrm{H_d})$$

The condition for gauge coupling unification is

$$b_i\,(\text{chiral fields}) = b_i(\mathrm{H_u} + \mathrm{H_d}) \bmod b_i(\mathrm{SU}(5)\ \text{Multiplets})$$

We introduce D_i and $\overline{D}_i$ with $(\mathbf{3}, \mathbf{1}, \mathbf{1/3})$ and $(\bar{\mathbf{3}}, \mathbf{1}, -\mathbf{1/3})$.

The superpotential for D_i and $\overline{D}_i$ is

$$W_{D_i} = \sum_{i=1}^{3} m_{D_i} D_i \overline{D}_i \, ,$$

with $m_{D_1} \sim m_{D_2} \sim M_f \sim m'$, and $m_{D_3} \sim m'/4\pi$.

Questions:

- $T^{1,2,3,4}$, $P^{1,2}$ and $Q^{1,2}$ can not be obtained from a complete GUT representation, how to embed it into GUTs?

- New μ problem: why are m, m' and M_f, or m_{D_i} not at the Plank scale or GUT scale? And why

$$M_f \sim m' \sim m_{D_1} \sim m_{D_2} \,,\; m_{D_3} \sim m'/4\pi$$

$$v_0^2 \sim \frac{mm'}{(4\pi)^2}$$

- How to explain the fermion masses and mixings?

III. 6-D $N = 2$ SUSY ORBIFOLD $SU(5)$ MODEL

General setup:

- 6D space-time $M^4 \times S^1 \times S^1$, x^μ, $(\mu = 0, 1, 2, 3)$, $y \equiv x^5$ and $z \equiv x^6$.

- Radii for the circles along y direction and z direction are R_1 and R_2.

- The orbifold $S^1/Z_2 \times S^1/(Z_2 \times Z_2')$ is obtained by $S^1 \times S^1$ moduloing the equivalent classes

  $$y \sim -y \ , \ z \sim -z \ , \ z' \sim -z'$$

  where $z' \equiv z - \pi R_2/2$.

- 4 fixed points: $(y = 0, z = 0)$, $(y = 0, z = \pi R_2/2)$, $(y = \pi R_1, z = 0)$ and $(y = \pi R_1, z = \pi R_2/2)$
  4 fixed lines: $y = 0$, $y = \pi R_1$, $z = 0$ and $z = \pi R_2/2$.

- In order to make sure that the 5D masses for the hypermultiplets on the 4-brane at $z = 0$ are smaller than the unification scale or cut-off scale, we consider a local Z_2' discrete symmetry [a].
- We introduce a 4-brane at

$$y = s \equiv \frac{p\pi R_1}{p+q}$$

and assume a local Z_2' symmetry in the neighborhood of this 4-brane

$$y' \sim -y' \ , \ \text{where } y' \equiv y - s$$

- The effective radius r_1

$$r_1 = \frac{2R_1}{p+q}$$

- The masses of KK modes depend on the effective radius r_1 instead of R_1, and then, the KK modes can be relatively heavier comparing to the $1/R_1$ if $p + q$ is relatively larger than 2.
- The relatively large extra dimension and arbitrarily heavy KK modes [a] can be arranged.

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[a] T. Li.

- The 6D $N = 2$ vector multiplet can be decomposed to be one vector superfield, V, and three chiral superfields, Σ_5, Σ_6 and Φ in the 4D $N = 1$ SUSY language.

- 3 global Z_2 parity operators, P_y, P_z, and P'_z, and 1 local Z_2 parity operator P'_y

$$
\begin{aligned}
P_y &: y \sim -y, \quad P'_y : y' \sim -y' \\
P_z &: z \sim -z, \quad P'_z : z' \sim -z'
\end{aligned}
$$

Under P_y, the vector multiplet transforms as

$$
\begin{aligned}
V(x^\mu, -y, z) &= P_y V(x^\mu, y, z)(P_y)^{-1} \\
\Sigma_5(x^\mu, -y, z) &= -P_y \Sigma_5(x^\mu, y, z)(P_y)^{-1} \\
\Sigma_6(x^\mu, -y, z) &= P_y \Sigma_6(x^\mu, y, z)(P_y)^{-1} \\
\Phi(x^\mu, -y, z) &= -P_y \Phi(x^\mu, y, z)(P_y)^{-1}
\end{aligned}
$$

For P_z and P'_z, we just interchange the subscripts: $y \leftrightarrow z$ and $5 \leftrightarrow 6$.

- To break $SU(5)$, we choose the following representations for P_y, P'_y, P_z, and P'_z

$$P_y = (+1, +1, +1, +1, +1), \quad P'_y = (+1, +1, +1, -1, -1)$$
$$P_z = (+1, +1, +1, -1, -1), \quad P'_z = (+1, +1, +1, +1, +1)$$

- Under P'_y or P_z, the gauge generators for $SU(5)$ are separated into two sets: T^a and $T^{\hat{a}}$, the generators for SM and $SU(5)/G_{SM}$

$$P'_y\, T^a\, P'^{-1}_y = T^a\ , \ P'_y\, T^{\hat{a}}\, P'^{-1}_y = -T^{\hat{a}}\ ,$$
$$P_z\, T^a\, P_z^{-1} = T^a\ , \ P_z\, T^{\hat{a}}\, P_z^{-1} = -T^{\hat{a}}$$

- For all the KK modes:
  - 4D $N = 1$ SUSY is preserved on the 3-branes
  - 4D $N = 2$ SUSY is preserved on the 4-branes
- The surviving gauge group on each 3-brane or 4-brane is $SU(5)$ or $SU(3) \times SU(2) \times U(1)$.
- For the zero modes, the bulk 4D $N = 4$ supersymmetric $SU(5)$ gauge symmetry is broken down to the $N = 1$ supersymmetric $SU(3) \times SU(2) \times U(1)$.

Parity assignment and masses ($n \geq 0, m \geq 0$) for the vector multiplet in the supersymmetric orbifold $SU(5)$ model.

$(P^y, P^{y'}, P^z, P^{z'})$	Field	Mass
$(+,+,+,+)$	V_μ^a	$\sqrt{(2n)^2/r_1^2 + (2m)^2/R_2^2}$
$(+,-,-,+)$	$V_\mu^{\hat{a}}$	$\sqrt{(2n+1)^2/r_1^2 + (2m+1)^2/R_2^2}$
$(-,-,+,+)$	Σ_5^a	$\sqrt{(2n+2)^2/r_1^2 + (2m)^2/R_2^2}$
$(-,+,-,+)$	$\Sigma_5^{\hat{a}}$	$\sqrt{(2n+1)^2/r_1^2 + (2m+1)^2/R_2^2}$
$(+,+,-,-)$	Σ_6^a	$\sqrt{(2n)^2/r_1^2 + (2m+2)^2/R_2^2}$
$(+,-,+,-)$	$\Sigma_6^{\hat{a}}$	$\sqrt{(2n+1)^2/r_1^2 + (2m+1)^2/R_2^2}$
$(-,-,-,-)$	Φ^a	$\sqrt{(2n+2)^2/r_1^2 + (2m+2)^2/R_2^2}$
$(-,+,+,-)$	$\Phi^{\hat{a}}$	$\sqrt{(2n+1)^2/r_1^2 + (2m+1)^2/R_2^2}$

At the fixed point $(y=0, z=0)$, $(y=s, z=0)$, $(y=\pi R_1, z=0)$, $(y=0, z=\pi R_2/2)$, $(y=s, z=\pi R_2/2)$, or $(y=\pi R_1, z=\pi R_2/2)$.

3-Brane Position	Fields	SUSY	Gauge Symmetry
$(0,0)$ or $(\pi R_1, 0)$	V_μ^a, $\Sigma_6^{\hat{a}}$	N=1	G_{SM}
$(0, \pi R_2/2)$ or $(\pi R_1, \pi R_2/2)$	V_μ^A	N=1	$SU(5)$
$(s, 0)$	V_μ^a, $\Phi^{\hat{a}}$	N=1	G_{SM}
$(s, \pi R_2/2)$	V_μ^a, $\Sigma_5^{\hat{a}}$	N=1	G_{SM}

On the 4-brane which is located at the fixed line $y=0$, $z=0$, $y=s$, $y=\pi R_1$, or $z=\pi R_2/2$.

4-Brane Position	Fields	SUSY	Gauge Symmetry
$y=0$ or $y=\pi R_1$	V_μ^A, Σ_6^A	N=2	$SU(5)$
$z=0$	V_μ^a, Σ_5^a, $\Sigma_6^{\hat{a}}$, $\Phi^{\hat{a}}$	N=2	G_{SM}
$y=s$	V_μ^a, $\Sigma_5^{\hat{a}}$, Σ_6^a, $\Phi^{\hat{a}}$	N=2	G_{SM}
$z=\pi R_2/2$	V_μ^A, Σ_5^A	N=2	$SU(5)$

IV. EMBEDDING THE MSFHM

On the 4-brane at the fixed line $z = 0$, we only have the 4D $N = 2$ SUSY and the Standard Model gauge symmetry.

We can put the minimal fat Higgs model on the 4-brane at $z = 0$.

The $SU(2)_H$ gauge interaction can be either in the 6D space-time or on this 4-brane.

The 5D $N=1$ SUSY action

For the gauge group G and a hypermultiplet (X and X^c as 4D $N=1$ chiral multiplets), in terms of the 4D $N=1$ SUSY: [a]

$$
\begin{aligned}
S \;=\; & \int d^4x \int_0^{\pi R_1} dy \frac{1}{kg^2} \mathrm{Tr}\left[\frac{1}{4}\int d^2\theta\,(W^\alpha W_\alpha + \mathrm{H.C.})\right. \\
& \left. + \int d^4\theta \left((\sqrt{2}\partial_5 + \Sigma_5^\dagger)e^{-V}(-\sqrt{2}\partial_5 + \Sigma_5)e^V + \partial_5 e^{-V}\partial_5 e^V\right)\right] \\
& + \int d^4x \int_0^{\pi R_1} dy \left[\int d^4\theta\,(X^c e^V \bar{X}^c + \bar{X}e^{-V}X)\right. \\
& \left. + \int d^2\theta \left(X^c(\partial_5 + m_X^{bm}(\theta(s-y) - \theta(y-s)) - \frac{1}{\sqrt{2}}\Sigma_5)X + \mathrm{H.C.}\right)\right]
\end{aligned}
$$

where m_X^{bm} is the 5D mass term.

[a] N. Arkani-Hamed, T. Gregoire and J. Wacker.

- Under P_y and P'_y, the parities of X and X^c are $(+,+)$ and $(-,-)$, consequently X^c does not have zero mode
- The 5D wave function for the zero mode of X is [a]

$$f_0^X(y) = \begin{cases} a_X\, e^{-m_X^{bm} y} & \text{if } 0 \leq y \leq s \\ a_X\, e^{-m_X^{bm}(2s-y)} & \text{if } s \leq y \leq \pi R_1 \end{cases}$$

$$a_X = \sqrt{\frac{2\, m_X^{bm}}{1 + e^{2m_X^{bm}(\pi R_1 - 2s)} - 2e^{-2m_X^{bm} s}}}$$

m_X^{bm} is the 5D mass.

- The masses for the n-th KK modes of X and X^c are

$$M_n = \sqrt{(m_X^{bm})^2 + \frac{n^2}{r_1^2}}\ .$$

- We choose

$$p = 5\ ,\ q = 1\ .$$

So, we have

$$s = \frac{5\pi R_1}{6}\ ,\ r_1 = \frac{R_1}{3}\ .$$

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[a] N. Arkani-Hamed and M. Schmaltz; E. A. Mirabelli and M. Schmaltz; D. E. Kaplan and T. M. Tait; R. Kitano and T. Li; K. W. Choi, I. W. Kim and W. Y. Song; K. w. Choi.

Including the fields:

- Q_i, u_i, d_i, and T^j, P^k, Q^k, $\varphi_{u,d}$, $\bar{\varphi}_{u,d}$, D_i and $\overline{D}_i$ on the 4-brane at $z = 0$.
- The singlets S and S' on the 3-brane at $(y = 0, z = 0)$.
- One pair of vector-like particles S_1 and S_2 with quantum numbers $(\mathbf{1}, \mathbf{1}, +\mathbf{1})$ and $(\mathbf{1}, \mathbf{1}, -\mathbf{1})$ under the SM gauge symmetry on the 4-brane at $z = 0$, and their parities under (P_y, P'_y) are $(+, -)$ [a].

[a] T. Li and W. Liao.

The (P_y, P_y') parity assignment and the masses $(n \geq 0)$.

$(P^y, P^{y'})$	Field	Mass
$(+,+)$	Q_i, u_i, d_i, T^j, P^k, Q^k $\varphi_{u,d}$, $\bar{\varphi}_{u,d}$, D_i, $\overline{D}_i$	$\sqrt{(m_X^{bm})^2 + (2n)^2/r_1^2}$
$(+,-)$	S_1, S_2	$(2n+1)/r_1$
$(-,+)$	S_1^c, S_2^c	$(2n+1)/r_1$
$(+,+)$	Q_i^c, u_i^c, d_i^c, T^{jc}, P^{kc}, Q^{kc} $\varphi_{u,d}^c$, $\bar{\varphi}_{u,d}^c$, D_i^c, $\overline{D}_i^c$	$\sqrt{(m_X^{bm})^2 + (2n+2)^2/r_1^2}$

The 5D masses for the fields.

Field	m_X^{bm}	**Localization**
T^1, T^2, T^3, T^4	0	**No**
$T^7, \varphi_{u,d}, D_1, D_2$	$m_{T\varphi}^{bm}$	$(y = 0, z = 0)$
$T^8, \bar{\varphi}_{u,d}, \overline{D}_1, \overline{D}_2$	$-m_{T\varphi}^{bm}$	$(y = s, z = 0)$
T^5, P^2, Q^2, D_3	m_{PQ}^{bm}	$(y = 0, z = 0)$
$T^6, P^1, Q^1, \overline{D}_3$	$-m_{PQ}^{bm}$	$(y = s, z = 0)$

$$m \sim M_{D_3} \sim m'/4\pi$$

The 5D localized superpotential is

$$W_1^{(5)} = \left(y_1 S T^1 T^2 + y_2 S' T^3 T^4\right) \pi R_1 \delta(y)$$

$$\begin{aligned} W_2^{(5)} &= -T^5 T^6 \left[y_m^0 \delta(y) + y_m^s \delta(y-s) + y_m^1 \delta(y - \pi R_1)\right] \\ &\quad + T^7 T^8 \left[y_{m'}^0 \delta(y) + y_{m'}^s \delta(y-s) + y_{m'}^1 \delta(y - \pi R_1)\right] \end{aligned}$$

$$\begin{aligned} W_3^{(5)} &= \left[y_3 P^1(T^1, T^2) T^6 + y_6 T^6 (T^3, T^4) Q^1\right] \frac{\sqrt{\pi R_1}}{\sqrt{m_{T\varphi}^{bm} m_{PQ}^{bm}}} \delta(y-s) \\ &\quad + \left[y_4 P^2(T^1, T^2) T^5 + y_5 T^5 (T^3, T^4) Q^2\right] \frac{\sqrt{\pi R_1}}{2\sqrt{m_{T\varphi}^{bm} m_{PQ}^{bm}}} \delta(y) , \end{aligned}$$

$$\begin{aligned} W_f^{(5)} &= (\varphi_u \bar{\varphi}_u + \bar{\varphi}_d \varphi_d) \left[y_{M_f}^0 \delta(y) + y_{M_f}^s \delta(y-s) + y_{M_f}^1 \delta(y - \pi R_1)\right] \\ &\quad + \left[y_7 \bar{\varphi}_d (T T^4) + y_8 \bar{\varphi}_u (T T^3)\right] \frac{\pi R_1}{\sqrt{m_{T\varphi}^{bm}}} \delta(y-s) \\ &\quad + \left[h_u^{'ij} Q_i u_j \varphi_u + h_d^{'ij} Q_i d_j \varphi_d + h_e^{'ij} L_i e_j \varphi_d\right] \frac{\pi R_1}{\sqrt{2 m_{T\varphi}^{bm}}} \delta(y) \end{aligned}$$

$$W_{D_i}^{(5)} = \sum_{i=1}^{3} D_i \overline{D}_i \left[y_{m_{D_i}}^0 \delta(y) + y_{m_{D_i}}^s \delta(y-s) + y_{m_{D_i}}^1 \delta(y - \pi R_1)\right]$$

After compactification, we obtain 4D superpotential with

$$
\begin{aligned}
m &= \sqrt{2}\left(y_m^0 + y_m^s + y_m^1\right) m_{PQ}^{bm} e^{-m_{PQ}^{bm} s} \\
m' &= \sqrt{2}\left(y_{m'}^0 + y_{m'}^s + y_{m'}^1\right) m_{T\varphi}^{bm} e^{-m_{T\varphi}^{bm} s} \\
M_f &= \sqrt{2}\left(y_{M_f}^0 + y_{M_f}^s + y_{M_f}^1\right) m_{T\varphi}^{bm} e^{-m_{T\varphi}^{bm} s} \\
m_{D_1} &= \sqrt{2}\left(y_{m_{D_1}}^0 + y_{m_{D_1}}^s + y_{m_{D_1}}^1\right) m_{T\varphi}^{bm} e^{-m_{T\varphi}^{bm} s} \\
m_{D_2} &= \sqrt{2}\left(y_{m_{D_2}}^0 + y_{m_{D_2}}^s + y_{m_{D_2}}^1\right) m_{T\varphi}^{bm} e^{-m_{T\varphi}^{bm} s} \\
m_{D_3} &= \sqrt{2}\left(y_{m_{D_3}}^0 + y_{m_{D_3}}^s + y_{m_{D_3}}^1\right) m_{PQ}^{bm} e^{-m_{PQ}^{bm} s} \\
h_u^{ij} &= h_u^{'ij} f_0^{'Q_i}(0) f_0^{'u_j}(0) \\
h_d^{ij} &= h_d^{'ij} f_0^{'Q_i}(0) f_0^{'d_j}(0) \\
h_e^{ij} &= h_e^{'ij} f_0^{'L_i}(0) f_0^{'e_j}(0)
\end{aligned}
$$

where

$$
f_0^{'X}(0) \equiv \sqrt{\pi R_1} f_0^X(0) = \sqrt{\frac{2\pi R_1\, m_X^{bm}}{1 + e^{2m_X^{bm}(\pi R_1 - 2s)} - 2e^{-2m_X^{bm} s}}}
$$

Vector-like particle masses:

- We assume that

$$m' \sim M_f \sim m_{D_1} \sim m_{D_2} \sim m^{bm}_{T\varphi} e^{-m^{bm}_{T\varphi} s} \sim 10 \text{ TeV}$$

$$m \sim m_{D_3} \sim m^{bm}_{PQ} e^{-m^{bm}_{PQ} s} \sim 1 \text{ TeV}$$

- From the RGE running, $m^{bm}_{T\varphi} \sim m^{bm}_{PQ} \sim 10^{15}$ GeV.

$$e^{-m^{bm}_{T\varphi} s} \sim 10^{-11} \qquad e^{-m^{bm}_{PQ} s} \sim 10^{-12}$$

- Masses quantized in the unit of $1/\pi R_1$

$$m^{bm}_{T\varphi} = \frac{30}{\pi R_1} \qquad m^{bm}_{PQ} = \frac{33}{\pi R_1}$$

Fermion mass matrices:

For u, d, and l mass matrices, our goal is to produce the following textures

$$h_u^{ij} \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}, \quad h_d^{ij} \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}, \quad h_e^{ij} \sim \begin{pmatrix} \lambda^5 & \lambda^3 & \lambda \\ \lambda^4 & \lambda^2 & 1 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix},$$

with $\lambda \sim 0.22$. For the left-handed neutrinos, the Majorana mass matrix is

$$m_\nu \propto \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}.$$

Correct fermion mass hierarchies, CKM matrix and PMNS matrices can be generated from the above fermion mass matrices [a].

[a] H. K. Dreiner, H. Murayama and M. Thormeier.

Note:

$$
\begin{aligned}
h_u^{ij} &= h_u^{\prime ij} f_0^{\prime Q_i}(0) f_0^{\prime u_j}(0) \\
h_d^{ij} &= h_d^{\prime ij} f_0^{\prime Q_i}(0) f_0^{\prime d_j}(0) \\
h_e^{ij} &= h_e^{\prime ij} f_0^{\prime L_i}(0) f_0^{\prime e_j}(0)
\end{aligned}
$$

$$
f_0^{\prime X}(0) \equiv \sqrt{\pi R_1} f_0^X(0) = \sqrt{\frac{2\pi R_1\, m_X^{bm}}{1 + e^{2m_X^{bm}(\pi R_1 - 2s)} - 2e^{-2m_X^{bm} s}}}
$$

To generate the desired structure

$$
f_0^{\prime Q_i}(0) = [\lambda^3, \lambda^2, 1] \qquad f_0^{\prime u_i}(0) = [\lambda^5, \lambda^2, 1]
$$

$$
f_0^{\prime d_i}(0) = [\lambda, 1, 1]
$$

$$
f_0^{\prime L_i}(0) = [\lambda, 1, 1] \qquad f_0^{\prime e_i}(0) = [\lambda^4, \lambda^2, 1]
$$

Solved to get the quantized 5D masses for the SM fermions

$$
m_{Q_i}^{bm} = [-6.5, -4.5, 0] \qquad m_{u_i}^{bm} = [-10.5, -4.5, 0]
$$

$$
m_{d_i}^{bm} = [-2.5, 0, 0]
$$

$$
m_{L_i}^{bm} = [-2.5, 0, 0] \qquad m_{e_i}^{bm} = [-8.5, -4.5, 0]
$$

Gauge Coupling Unification

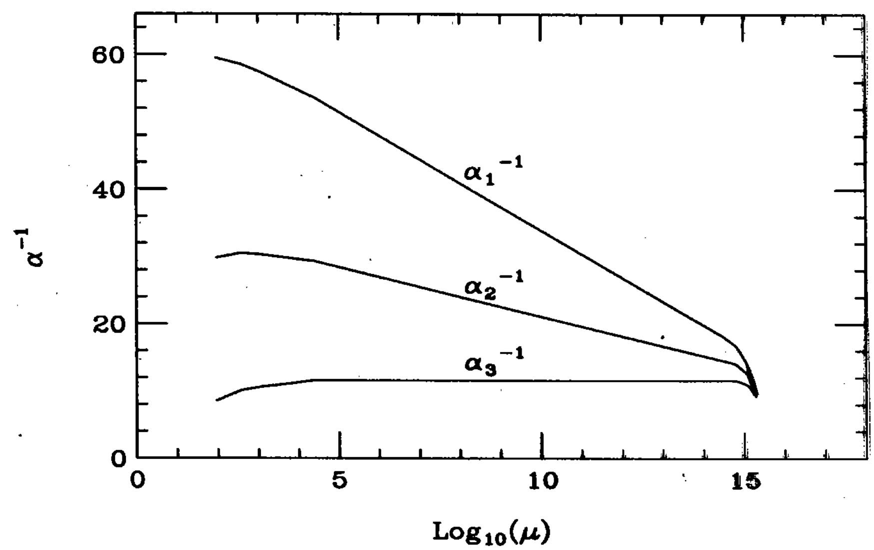
The one-loop renormalization group equations are

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \sum_{\phi} \frac{b_i^{\phi}}{2\pi} \ln\frac{\mu}{m_{\phi}}\, \theta(\mu - m_{\phi})$$

For simplicity, we assume

$$m \simeq 1 \text{ TeV}\ ,\ m' = M_f = m_{D_1} = m_{D_2} \simeq 25 \text{ TeV}$$
$$m_{susy} \simeq 360 \text{ GeV}\ ,\ m_{\rm spec} = m_{D_3} \simeq 1 \text{ TeV}$$

$$M_{\rm GUT} \geq \frac{1}{R_2} \geq \sqrt{M_{\rm GUT}^2 - \frac{1}{r_1^2}}$$



With $1/R_1 = 1.0 \times 10^{14}$ **GeV,**

$$M_{\rm GUT} \simeq 2 \times 10^{15}\ \text{GeV}$$

$$m_{T\varphi}^{bm} \simeq 0.955 \times 10^{15}\ \text{GeV} \qquad m_{PQ}^{bm} \simeq 1.05 \times 10^{15}\ \text{GeV}$$

If $p = q = 1$, $m_{T\varphi}^{bm} \sim m_{PQ}^{bm} \sim 3 \times 10^{15}$ GeV.

Some Comments:

(1) Either $SU(2)_H$ gauge interaction is in the 6D space-time or on the 4-brane at $z = 0$, the RGE running for $SU(2)_H$ is similar to that in Ref.[a].

(2) No proton decay problem in the model because we can define a continuous $U(1)_R$ symmetry.

[a] R. Harnik, G. D. Kribs, D. T. Larson and H. Murayama.

(3) We can generate common 5D mass $m^{bm}_{T\varphi}$ for T^7, $\varphi_{u,d}$, D_1 and D_2; $-m^{bm}_{T\varphi}$ for T^8, $\bar{\varphi}_{u,d}$, $\overline{D}_1$ and $\overline{D}_2$; m^{bm}_{PQ} for T^5, P^2, Q^2 and D_3; and $-m^{bm}_{PQ}$ for T^6, P^1, Q^1 and $\overline{D}_3$ by introducing and breaking of extra $U(1)'$ gauge symmetry.

(4) Anomaly Cancellation via a Chern-Simons term on the 4-brane at $z = \pi R_2/2$ or a 6D topological term.

(5) Charge Quantization. The charge quantization can be obtained due to: gauge invariance of the localized superpotential $\varphi_u D^c \Phi^{\hat{a}}$ on the 3-brane at $(y = \pi R_1/2, z = \pi R_2/2)$ and Yukawa superpotentials, and Anomaly cancellation.

VI. CONCLUSION.

We embed the MSFHM into the 6D $N = 2$ SUSY **orbifold** $SU(5)$ model:

- The correct mass scales for the vector-like particles.
- The SM fermion masses and mixings.
- Charge Quantization.
- No proton decay problem.