# Two-loop self-energies in SUSY

Argonne '04

**Stephen P. Martin**
**Northern Illinois University and Fermilab**

Supersymmetry predicts

- Lots of new particles to discover at LHC*
- A light Higgs†
- An era of perturbative control‡

I will discuss Strategy and Tactics for computing physical masses at two loops and beyond, and their implementation for the Higgs scalars.

*almost certainly
†probably
‡hopefully

Masses are key observables in SUSY. Predictions of specific models allow/require precise calculations.

Sparticle mass corrections are known at one loop order. (Probably not adequate for gluino and squarks, at least.)

Higgs sector: Full 2-loop effective potential + partial 2-loop self-energy. $\sim$ 1 GeV?

At LHC, $\Delta m_{h^0} \sim$ 100-200 MeV.
At LC, $\Delta m_{h^0} \sim$ 50 MeV.

Theory calculations must advance so as not to be an obstacle to understanding.

If all other quantities held constant:

$$\frac{\partial m_{h^0}}{\partial m_t} \approx 0.4 \left(\frac{120 \text{ GeV}}{m_{h^0}}\right) \left(\frac{m_t}{175 \text{ GeV}}\right)^3 \log\left(\frac{m_{\tilde{t}}}{m_t}\right)$$

Need both accurate measurement of top mass, and two-loop threshold corrections to top mass.

To calculate physical masses

Evaluate self-energy = sum of 1-particle irreducible Feynman diagrams:

$$\Pi(s) = \Pi^{(1)}(s) + \Pi^{(2)}(s) + \ldots$$

The complex pole mass

$$s_{\text{pole}} = M^2 - i\Gamma M$$

is the solution of:

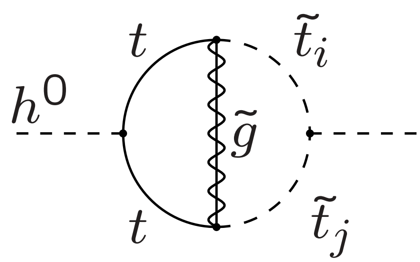
$$\det[m^2_{tree} + \Pi(s) - s] = 0.$$

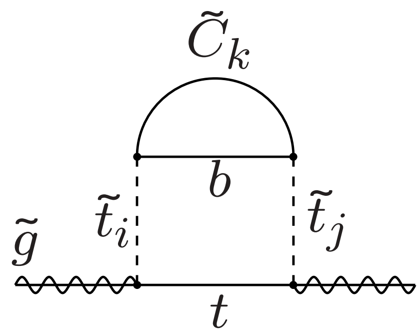
The pole mass is gauge invariant at each order in perturbation theory, can be related to kinematic masses as measured at colliders.

There are a finite number of two-loop, two-point Feynman diagrams. Why not just do them once, store the results, and get it over with?

A key feature of the problem: <u>many</u> distinct particles.

- **2-loop diagrams involve many different mass scales simultaneously.**
  For example:




  etc.
  Large, diverse hierarchies of ratios of squared masses.

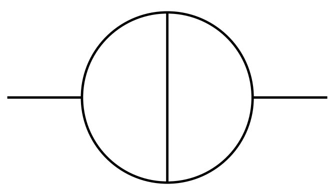
- **Method should be generic, reuseable from start to finish.**
  Do calculations for scalars, fermions, vectors in a <u>general</u> field theory. Then apply to Higgs, squarks, sleptons, and quarks, gluino, charginos, neutralinos, . . .

Strategy:

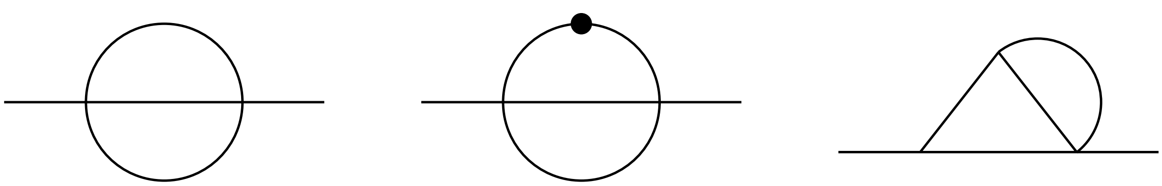
- Reduce all self-energies in general theory to a few basis integrals
- Basis integrals contain $\overline{\text{DR}}'$ (or $\overline{\text{MS}}$) counter-terms, so finite.
- Numerically evaluate basis integrals quickly and reliably for arbitrary values of masses.

Tarasov's basis and recurrence relations:

$M$



"Master integral"

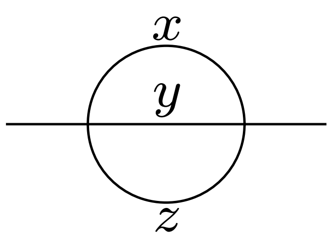


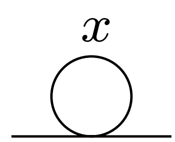
$S$ $T$ $U$

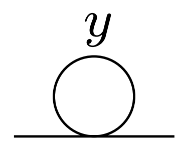
Can always reduce 2-loop self-energies to a linear combination of these, with coefficients rational functions of:

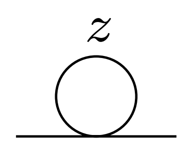
- $s = p^2 =$ external momentum invariant
- $x, y, z, \ldots =$ internal propagator masses

Inclusion of counterterms:

$S(x, y, z) =$ 

$-\frac{1}{\epsilon}\Big[$ 

$+$ 

$+$ 

$\Big]$

$$+\frac{1}{2\epsilon^2}(x+y+z)+\frac{1}{2\epsilon}(s/2-x-y-z)$$

and similar for others.

These are finite as regulator removed ($\epsilon \to 0$).

Tarasov's recurrence relations are complicated, but implemented in `Tarcer` (Mertig+Scharf).

Only need to be evaluated once for each diagram in a general renormalizable field theory.

108 diagrams for a <u>general</u> scalar self-energy.

Half are done: `hep-ph/0312092`.

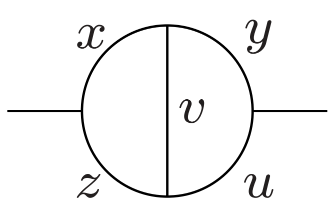
To evaluate basis integrals:

Values at $s = 0$ are known analytically, in terms of logs, dilogs.

$$\frac{\partial}{\partial s}\text{(basis integral)} = \text{(another self-energy integral)}$$
$$= \text{(linear combination of basis integrals)}$$

So, we have a set of coupled, first-order, linear differential equations.

Consider the Master integral $M(x, y, z, u, v)$:




and the basis integrals obtained from it by removing propagator(s):

$$U(x, z, u, v),\ U(y, u, z, v),\ U(z, x, y, v),\ U(u, y, x, v),$$
$$S(x, u, v),\ T(x, u, v),\ T(u, x, v),\ T(v, x, u),$$
$$S(y, z, v),\ T(y, z, v),\ T(z, y, v),\ T(v, y, z)$$

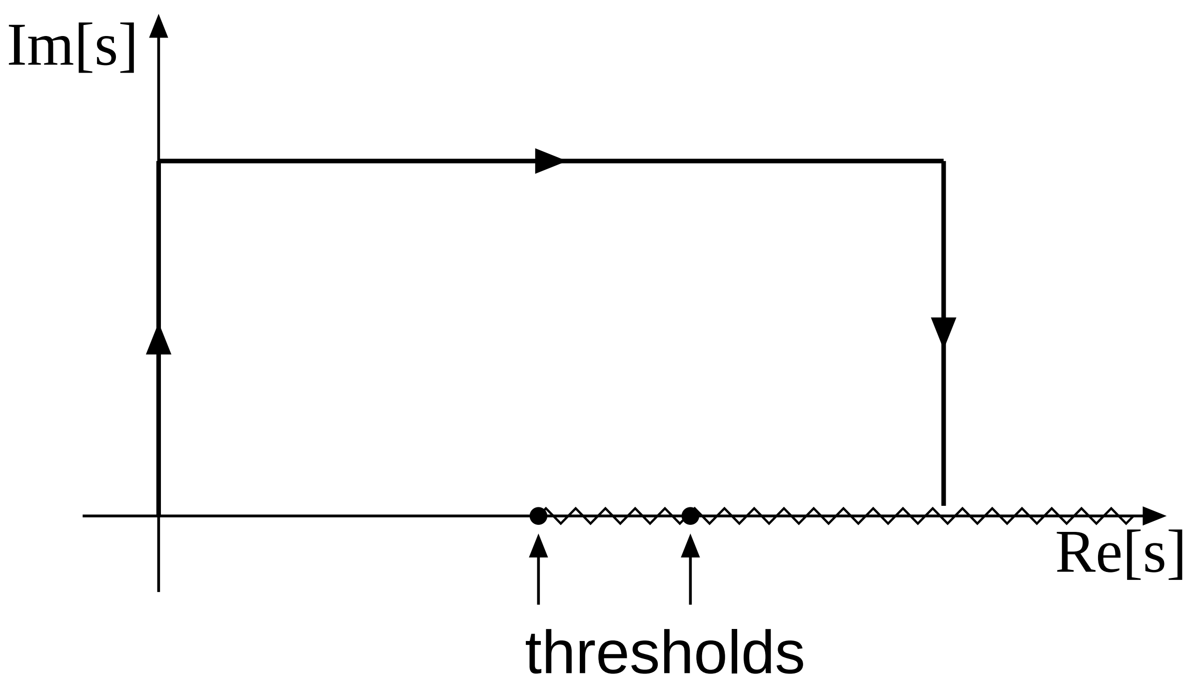
Call these 13 integrals $I_n$, ($n = 1, \ldots, 13$).

## Differential equations method for basis integrals

$$\frac{d}{ds} I_n = \sum_m K_{nm} I_m + C_n$$

Here $K_{nm}$ are rational functions of $s$ and $x, y, z \ldots$, and $C_n$ are one-loop integrals. These are obtained by using Tarasov's recursion relations.

Solve for basis integrals $I_n$ using Runge-Kutta integration in the complex $s$-plane, starting from known values at $s = 0$.

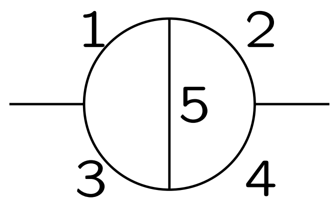


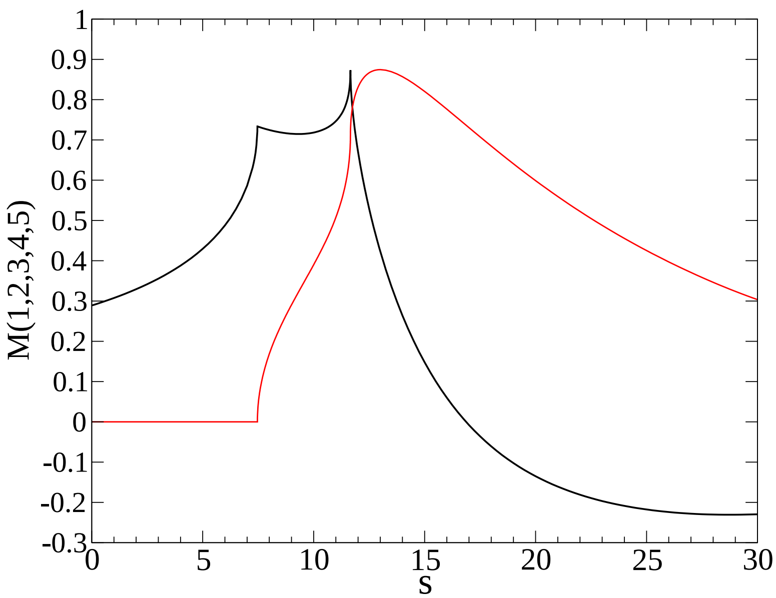

Method implemented for $S, T, U$ type integrals by Caffo, Czyz, Laporta, Remiddi.

I have extended the method to also work for $M$.

## Examples:

$$M(1,2,3,4,5) =$$




Black = real part

Red = imag. part

Note sharp behavior near thresholds!

Takes $\lesssim$ 1 second on a modern workstation to compute $M(x, y, z, u, v)$ and 12 subordinate basis integrals $U$, $S$, $T$ for generic masses.

Takes few seconds for some (pathological) special cases.

Advantages of the method:

- Basis integrals can be computed for any values of all masses and $s$, to arbitrary accuracy.

- All necessary basis integrals are obtained simultaneously in a single numerical computation.

- Branch cuts automatically dealt with correctly by choosing integration contour in upper-half complex $s$ plane.

- Simple checks on the numerical accuracy follow from changing choice of contour.

SPM, `hep-ph/0307101`
SPM and D.G. Robertson, C program, to appear.

Mathematica program available now by request.
Slooooow. NO WARRANTY.

## Applications

2-loop squark pole masses, in progress.
(Almost done.)

2-loop top-quark, gluino pole masses.
(This summer?)

Partial 2-loop self-energies, pole masses for all MSSM Higgs scalars ($h^0, H^0, A^0, H^\pm$).

All contributions to Higgs $\Pi(s)$ of order:

$$\alpha_s y_t^2,\ \alpha_s y_b^2,\ \alpha_s y_t y_b,$$
$$\alpha_s g^2,\ \alpha_s g g',\ \alpha_s g'^2$$
$$y_t^4,\ y_t^3 y_b,\ y_b^2 y_t^2,\ y_b^3 y_t,\ y_b^4,\ y_\tau^4,\ y_b^2 y_\tau^2$$

are now included.

Previous results used the effective potential approximation, in which $\Pi^{(2)}(s)$ is approximated by $\Pi^{(2)}(0)$ when computing the pole mass. (That approximation relies on $m_{h_0}^2 \ll m_t^2, m_{\tilde{t}}^2$.)

Technical notes:

- I use the supersymmetric version of dimensional reduction, $\overline{\text{DR}}'$. (Epsilon-scalar masses are removed by redefinition of scalar masses.)

- I expand around minimum of the two-loop effective potential, not the tree-level potential. (Therefore, tadpole graphs need not be included.)

- I use Landau gauge for electroweak gauge bosons, general gauge for gluons.

To find Higgs scalar pole masses:

$$\det[\mathbf{m}^2_{\mathbf{tree}} + \Pi(s) - s\mathbf{1}] = 0$$

Solutions to this eigenvalue equation are

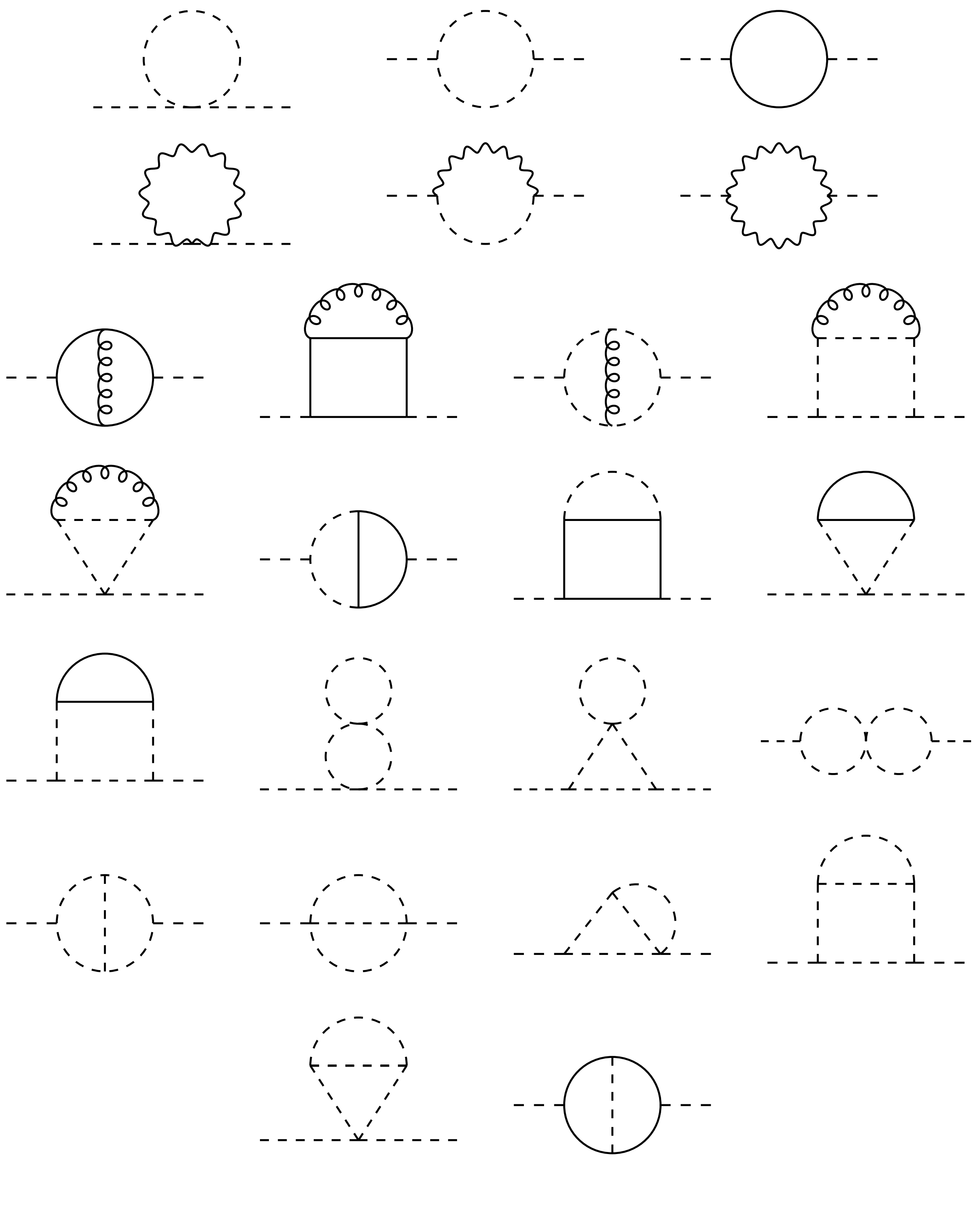
$$s_p = \text{complex pole masses} = M^2 - i\Gamma M.$$

Here $\Pi(s)$ is a:

- $4\times4$ matrix for neutral scalars $h^0, H^0, G^0, A^0$.
- $2\times2$ matrix for charged scalars $G^\pm, H^\pm$.

Note: CP conservation NOT assumed anywhere.

The diagrams:



40 diagram topologies (counting fermion mass insertions)

## A Simple Limit

Let $M_{\mathrm{SUSY}}$ = common squark, gluino mass, and assume

$$M^2_{\mathrm{SUSY}} \gg m_t^2 \gg m^2_{h^0}$$

Keep only leading term in $\alpha_s$.

Result:

$$\begin{aligned} m^2_{h^0,\mathrm{pole}} &= m_Z^2 \cos^2(2\beta) \\ &\quad + \frac{y_t^2}{16\pi^2}[m_t^2 \Delta_1 + m^2_{h^0}\Delta_1' + \ldots] \\ &\quad + \frac{g_3^2 y_t^2}{(16\pi^2)^2}[m_t^2 \Delta_2 + m^2_{h^0}\Delta_2' + \ldots] \end{aligned}$$

where, with $L \equiv \log(M^2_{\mathrm{SUSY}}/m_t^2)$,

$$\begin{aligned} \Delta_1 &= 12L, \\ \Delta_1' &= 2 - 3L, \\ \Delta_2 &= 96L^2 + 32L - 32, \\ \Delta_2' &= -12L^2 + 12L + 44/3. \end{aligned}$$

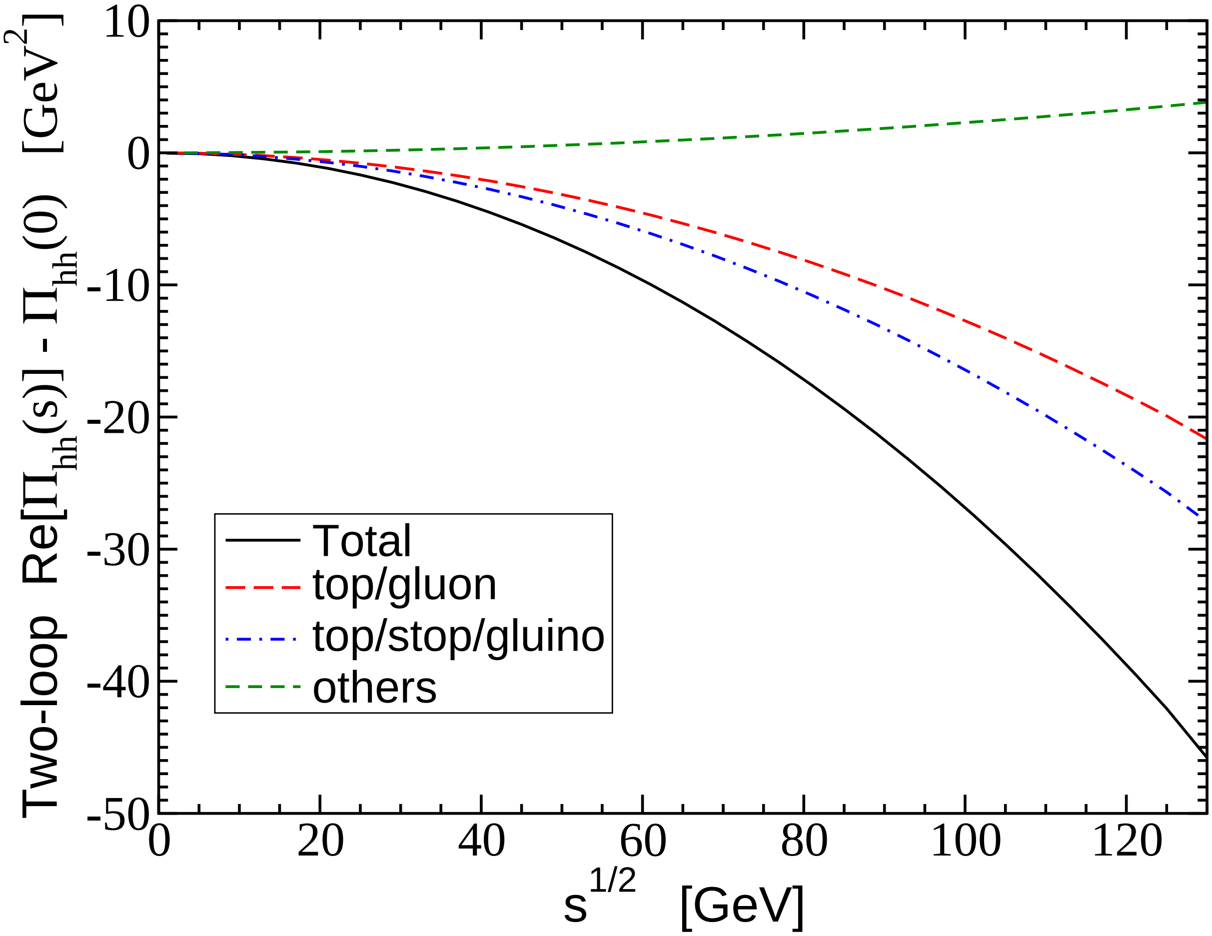
**NEW!**

Note: $\Delta_2'$ is smaller than naive estimate. Leading log approximation is not great.

Two-loop contribution to self-energy:

$$Re[\Pi_{h^0h^0}(s)] - \Pi_{h^0h^0}(0)$$

Turns out to have significant cancellations, so smaller than naive expectation:



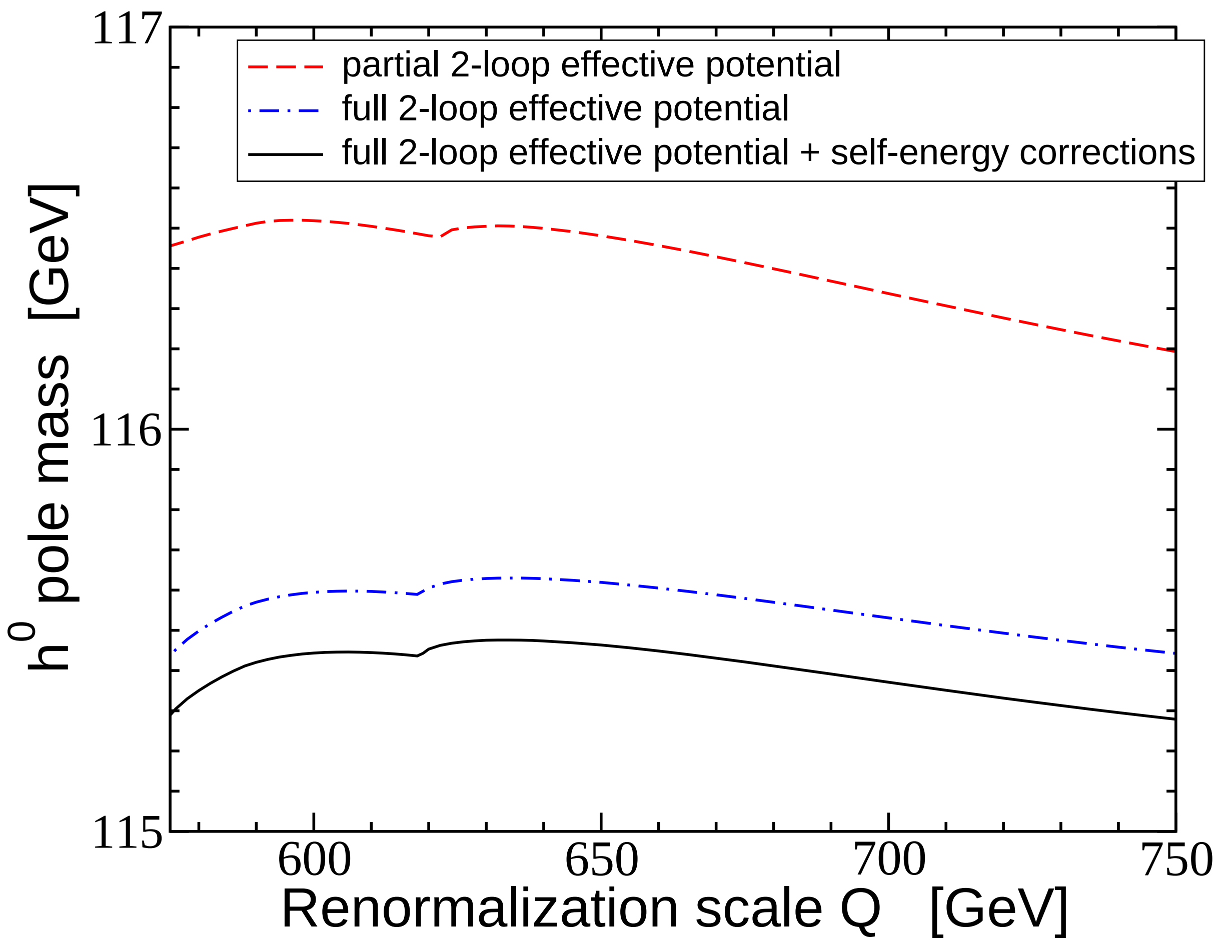

Diagrams with top-quark loop and strong interactions dominates the momentum-dependent contribution to the self-energy.

Use this to obtain the pole mass:

To obtain pole mass, full 2-loop effective potential + partial momentum-dependent contributions to self-energy:

$$\Pi^{(2)}(s) \approx \Pi^{(2)}_{\rm par}(s) - \Pi^{(2)}_{\rm par}(0) + \Pi^{(2)}(0)$$

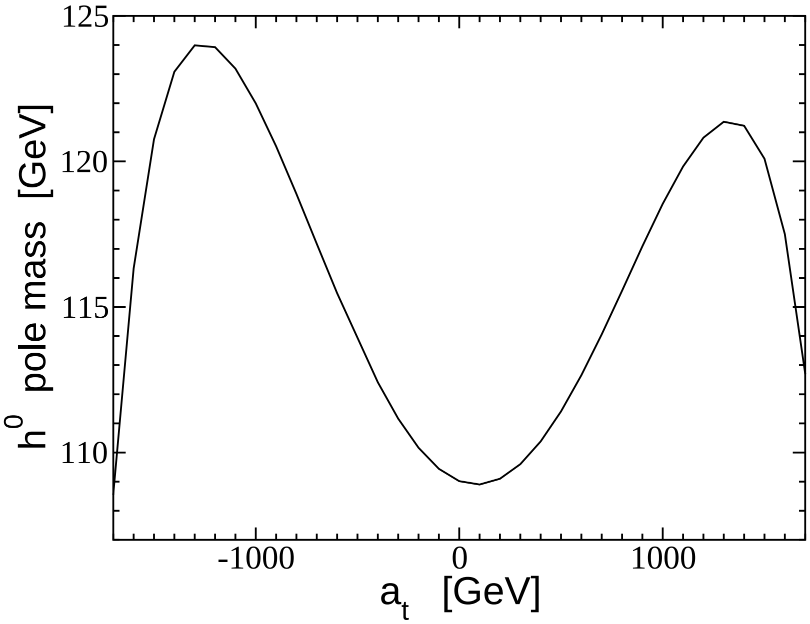
The last term is obtained numerically to arbitrary accuracy from the derivatives of the full 2-loop effective potential (SPM, `hep-ph/0206136`).

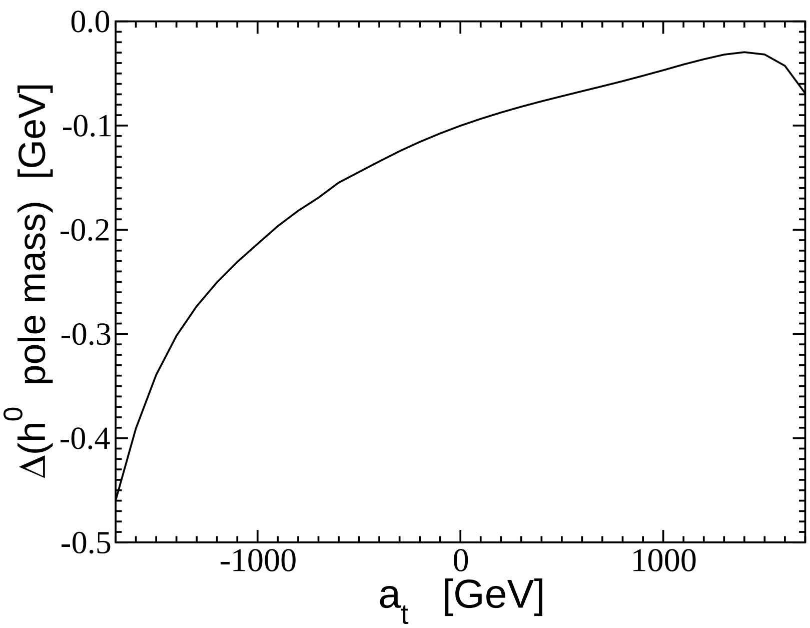



Effect on pole mass is to lower by 160 MeV, compared to full two-loop effective potential approximation.

I have checked that effect is similar for a variety of MSSM models, including large $\tan\beta$.

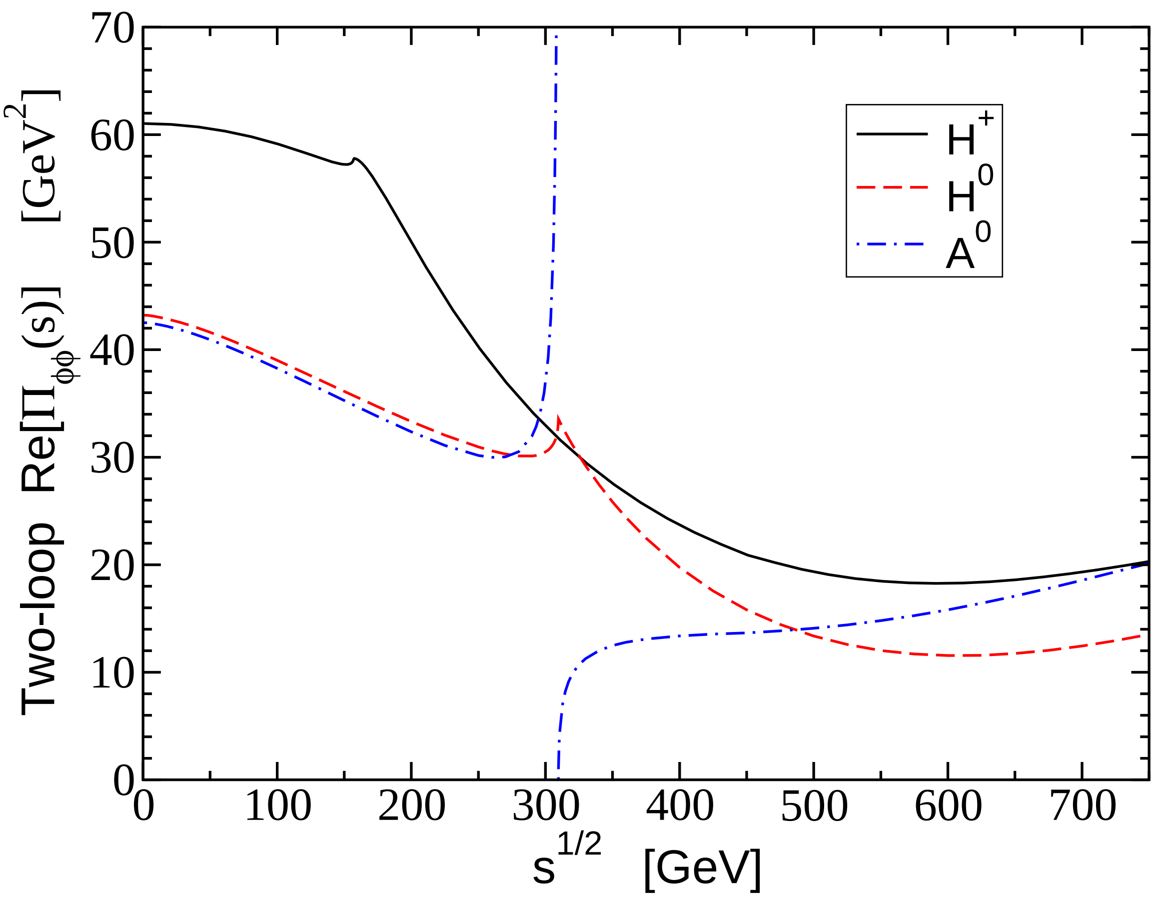
For example, varying the top squark mixing:




This is the difference between the pole mass calculated with (partial) momentum-dependent contributions to the self-energy, and the 2-loop effective potential approximation.

In the same model, 2-loop contributions to the self-energy functions for $H^0$, $A^0$, $H^\pm$:

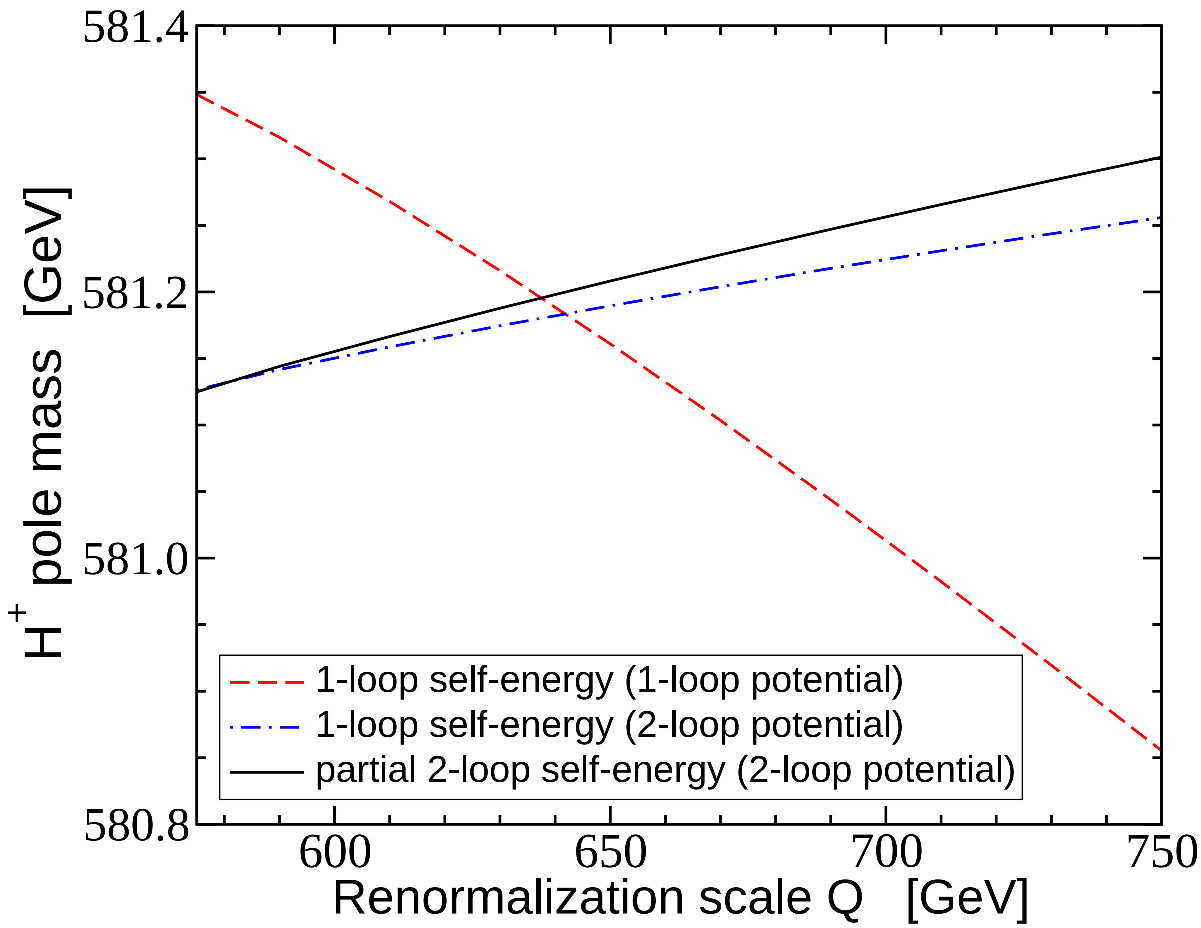



Self-energy for $A^0$ has $\sqrt{4m_t^2 - s}$ and $\ln(4m_t^2 - s)$ singularities from top-quark loops with gluon exchange.

$H^0$ and $H^\pm$ are continuous but non-differentiable at threholds $\sqrt{s} = 2m_t$ and $\sqrt{s} = m_t + m_b$, respectively.

There is significant cancellation between $\alpha_s y_t^2$ and $y_t^4$ contributions.

(Note: effective potential approximation is wrong by $>100$ per cent for heavy Higgs scalars!)




Use of the 2-loop effective potential to relate VEVs to Lagrangian parameters is important in reducing the scale dependence.

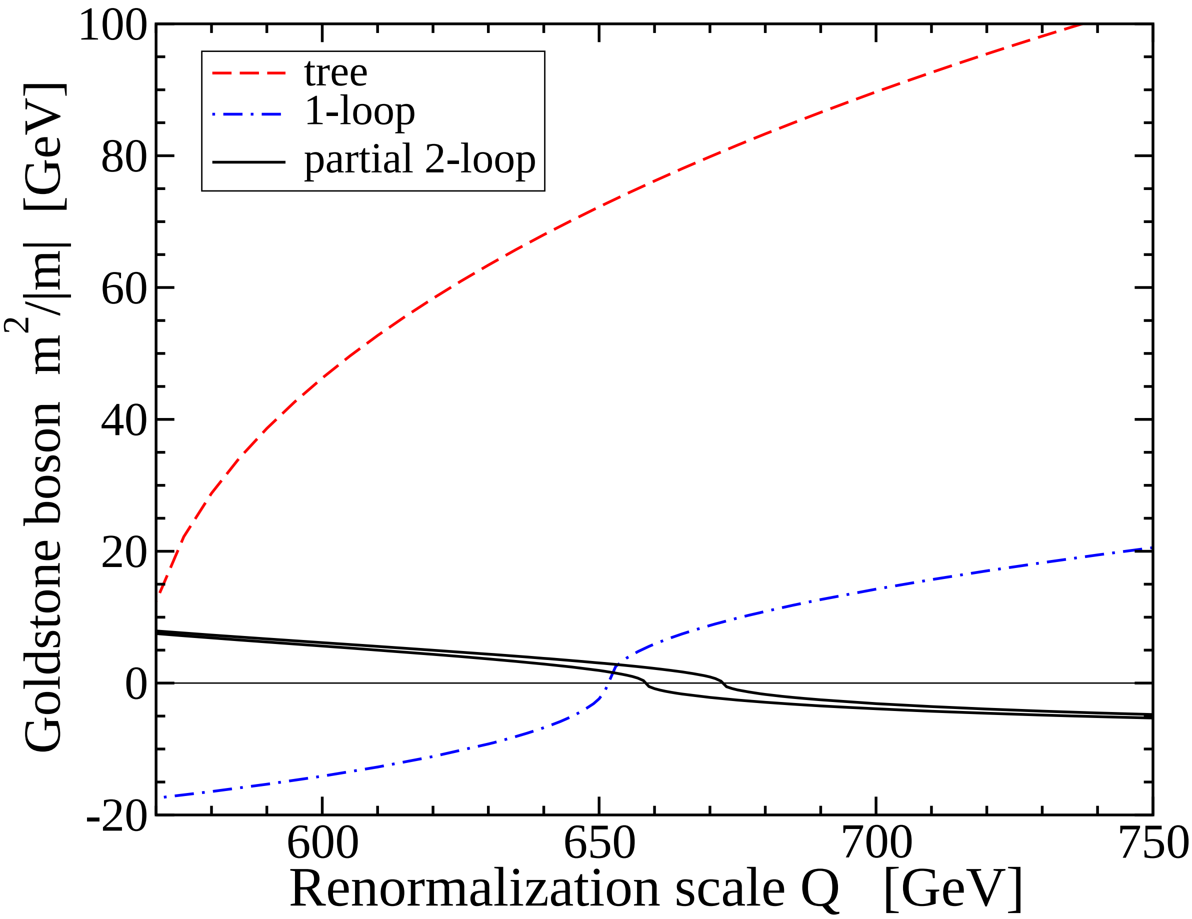
Use of the $\alpha_S y_t^2$ and $y_t^4$ 2-loop self-energy terms makes only a small difference compared to the 1-loop self-energy.

(Difference is smaller than the scale dependence!)

This is because heavier Higgs are mostly $H_d$, don't have large couplings to the top (s)quarks.

Remaining scale dependence will require calculating the rest of the two-loop diagrams, including all electroweak effects.

A check: The Goldstone boson (mass)$^2$ should vanish.




Including all two-loop effects would give $m^2_{G^0} = m^2_{G^\pm} = 0$, exactly.

In any given model, this can be used to estimate rough size of two-loop errors (but not three-loop errors!) and to choose a renormalization scale $Q$.

## Outlook

- Two-loop calculations for self-energies in the MSSM are necessary and possible

- I favor a Strategy based on:

  - $\overline{\text{DR}}'$ scheme (complementary to on-shell)

  - Reusable, generic calculations

  - Fast computations of basis two-loop integrals

- Some 3-loop calculations (e.g. for $h^0$) will eventually be necessary

- Progress continues