Detuned Branes in Randall-Sundrum: Radius Stabilization & SUSY Breaking

Michele Redi

May 2004

In collaboration with: J. Bagger

hep-th/0310086

hep-th/0312220

THE RANDALL-SUNDRUM MODEL

I will consider the geometry of the Randall-Sundrum model,

Five-dimensional AdS space is compactified on an orbifold $S^1/\mathbb{Z}_2$ with two branes located at the fixed points at $\phi = 0$ and $\phi = \pi$.

In the original RS scenario $5D$ metric is $4D$ flat space warped along the fifth dimension,

$$ds^2 = e^{-2\,k\,r_0|\phi|}\eta_{mn}dx^m dx^n + r_0^2 d\phi^2$$

Due to the warping the physical mass scale depends on the position in the extra-dimension,

$$m(\phi) = e^{-kr_0|\phi|}m_0$$

The known hierarchy can be reproduced when,

$$kr_0 \sim 11$$

The crucial point is the stabilization of the distance between the branes. Supersymmetry might still be relevant.

DETUNED RANDALL-SUNDRUM

In the RS model the brane tension T_0 and T_π are tuned with the bulk cosmological constant Tk,

$$T_0 = -T_\pi = T$$

As a consequence r_0 is arbitrary. The ground state is $4D$ flat space.

It is natural to consider the generalization with arbitrary tensions, "The detuned RS-model". The interesting new feature is that r_0 is now fixed,

$$2\,k\,\pi\,r_0 = \log\frac{(T+T_0)(T+T_\pi)}{(T-T_0)(T-T_\pi)}$$

As a bonus supersymmetry can also be broken.

I will first consider the supersymmetric version of the model. The action consists of $5D$ supergravity with cosmological constant plus brane actions. The bosonic part of the action is,

$$S_{bulk} \;=\; -\frac{T}{6k}\int dx\, d\phi\sqrt{-G}\Big(\frac{1}{2}R - 6k^2 + \frac{1}{4}F_{MN}F^{MN} + \ldots\Big)$$

$F = dB$ where B is the graviphoton, a $U(1)$ gauge field.

The brane actions are,

$$S_{branes} = -T_0\int dx\, d\phi\sqrt{-g}\delta(\phi) - T_\pi\int dx\, d\phi\sqrt{-g}\delta(\phi-\pi).$$

For $|T_{0,\pi}| \leq T$, the fermionic brane actions can be chosen such that the full theory is supersymmetric. (Bagger & Belyaev)

For $|T_{0,\pi}| \leq T$ the $4D$ ground state is AdS$_4$.

The $5D$ metric in the vacuum is,

$$\begin{aligned} ds^2 &= F(\phi)^2 \hat{g}_{mn} dx^m dx^n + r_0^2 d\phi^2 \\ B_5 &= b \qquad\qquad B_\mu = 0 \end{aligned}$$

with the warp factor

$$F(\phi) = e^{-kr_0|\phi|} + \frac{1}{4k^2L^2} e^{kr_0|\phi|}$$

The $4D$ metric $\hat{g}_{mn}$ is AdS$_4$ with radius L,

$$\frac{1}{4\,k^2\,L^2} = \frac{T - T_0}{T + T_0}$$

r_0 is fixed by the tensions while B_5 remains arbitrary.

EFFECTIVE THEORY

The $5D$ theory is $N=2$ supersymmetric in the bulk restricted to $N=1$ on the branes. The low energy effective action is $N=1$ supersymmetric.

The bosonic effective theory contains the fluctuations of the $4D$ metric g_{mn} and two scalars associated g_{55} and B_5. B_m and g_{m5} do not produce zero modes because they are odd under $\mathbb{Z}_2$ projection.

The two scalars can be identified with the zero mode of B_5 and the proper distance r between the branes. In the supersymmetric theory they join with Ψ_5 to form a chiral supermultiplet coupled to supergravity.

Supersymmetry in AdS$_4$ requires different masses for the two scalars. In a chiral representation of SUSY in AdS$_4$ we have

$$D(E,0) \oplus D(E+\frac{1}{2},\frac{1}{2}) \oplus D(E+1,0)$$
$$m_0^2 = \frac{E(E-3)}{L^2}$$

Since the VEV b of B_5 is a modulus it must be massless. This leaves only two possibilities,

$$m_r^2 = -\frac{2}{L^2} \qquad\qquad m_r^2 = \frac{4}{L^2}$$

From the linearized analysis one can show that $m_r^2 = 4/L^2$ as required by supersymmetry.

It must be possible to cast the effective action in the standard supersymmetric form,

$$
\begin{aligned}
S_4 &= -\int \sqrt{g}\Big(\frac{1}{2}R + K_{\tau\overline{\tau}}\,\partial^m\tau\partial_m\overline{\tau} + V\Big) \\
V &= e^K(K^{\tau\overline{\tau}}D_\tau P\overline{D_{\overline{\tau}}}\overline{P} - 3P\overline{P})
\end{aligned}
$$

τ is the complex radion field

$$
\tau = r + ib
$$

K (Kähler potential) and P (superpotential) are respectively real and holomorphic functions of τ and

$$
D_\tau P = P_\tau + K_\tau P
$$

Crucial observation: The five-dimensional bosonic theory is invariant under the shift $B_5 \longrightarrow B_5 + c$.

It follows that K (up to Kähler transformation) is function of r only

$$K(\tau, \overline{\tau}) = K(\tau + \overline{\tau})$$

For the same reason,

$$V(\tau, \overline{\tau}) = V(\tau + \overline{\tau})$$

Since P is holomorphic in τ it depends on $Im(\tau) = b$. This implies an infinite number of constraints on K and P.

The most general K and P that satisfy this condition are given by,

$$\begin{aligned} K(\tau,\overline{\tau}) &= -3\log\left[1-c^2\,e^{-a(\tau+\overline{\tau})}\right] \\ P(\tau) &= p_1+p_2\,e^{-3a\tau} \end{aligned}$$

With the change of variables $z = c\,\exp(-a\tau)$ these become,

$$\begin{aligned} K(z,\overline{z}) &= -3\log\left[1-z\,\overline{z}\right] \\ P(z) &= p_1'+p_2' z^3 \end{aligned}$$

This is a generalization to AdS_4 of the no-scale supergravity. The scalars are coordinates of the manifold $SU(1,1)/U(1)$. The $U(1)$ symmetry is broken by the superpotential but it is preserved by the bosonic action.

BOSONIC REDUCTION

To fix the constants in the supersymmetric $\sigma-$model we derive the reduction of the bosonic degrees of freedom.

To compute the effective action we take the ansatz,

$$\begin{aligned} ds^2 &= \Big(F^2(\phi) + A(x)\Big) g_{mn}(x) dx^m dx^n + r_0^2 \Big(\frac{F(\phi)^2}{F(\phi)^2 + A(x)}\Big)^2 d\phi^2 \\ B_5 &= b(x) \end{aligned}$$

The variable A is related to the radius r of the extra-dimension by,

$$r(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi \sqrt{g_{55}} = \frac{r_0}{2\pi}\int_{-\pi}^{\pi} d\phi \frac{F(\phi)^2}{F(\phi)^2 + A(x)}$$

The $4D$ effective action is obtained plugging the ansatz in the $5D$ action and performing the ϕ integral. For the two-derivatives effective action one finds,

$$
\begin{aligned}
S_{\rm eff} \;&=\; -\int d^4x \Big[\frac{M_{Pl}^2}{2} R + 3\pi^2 k^2 M_{Pl}^2 \, \frac{e^{-\pi k(\tau+\overline{\tau})}}{(1-e^{-\pi k(\tau+\overline{\tau})})^2} g^{mn}\partial_m\tau\partial_n\overline{\tau} \\
&\qquad + V(\tau+\overline{\tau})\Big]
\end{aligned}
$$

with the potential given by,

$$
V(\tau+\overline{\tau}) = -\frac{3M_{Pl}^2\,(1-e^{-2\pi k r_0})}{L^2}\left[\frac{1-e^{-2\pi k(\tau+\overline{\tau}-r_0)}}{(1-e^{-\pi k(\tau+\overline{\tau})})^2}\right]
$$

The ground state is AdS$_4$ with radius L and the radion is stabilized by the potential at its VEV r_0.

One finds that the mass of the radion is,

$$m_r^2 = \frac{4}{L^2}$$

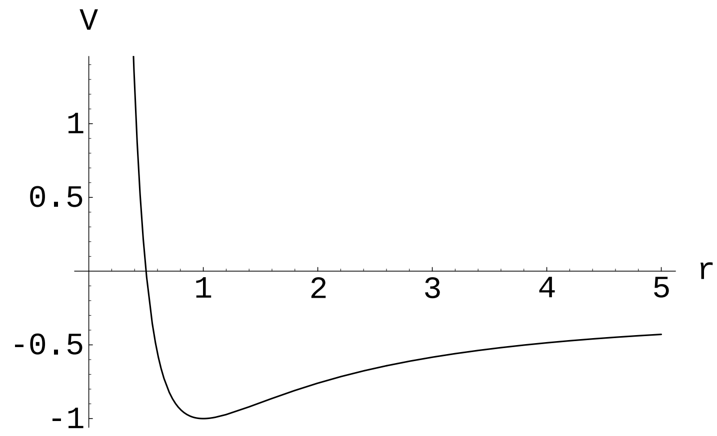
By matching with the supersymmetric effective action we obtain,

$$\begin{aligned} K(\tau,\overline{\tau}) &= -3\log[1 - e^{-\pi k(\tau+\overline{\tau})}] \\ P(\tau) &= \frac{k}{L}\sqrt{\frac{6}{T}}\left(1 - e^{\pi k r_0} e^{-3\pi k \tau}\right) \end{aligned}$$

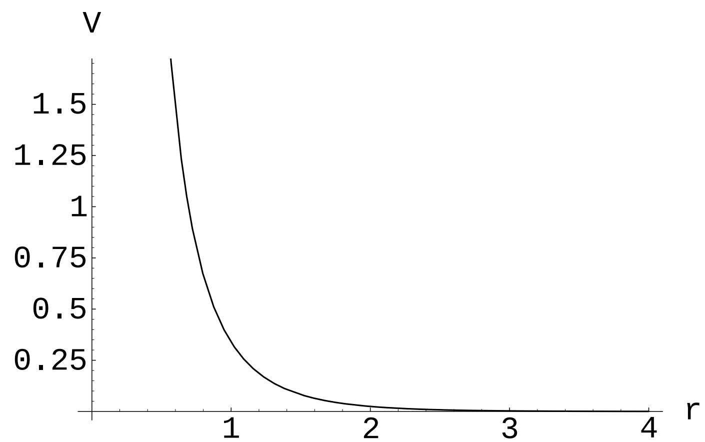
These results are easily extended to the non-supersymmetric case.

The bosonic effective action is the same with $b = 0$. For the case $|T_{0,\pi}| > T$ the effective action is still determined by the supersymmetric action with $L \to iL$.

For generic values of the tensions with $|T_{0,\pi}| < T$ the radius is stabilized.



When $T_0 = T$ and $|T_\pi| < T$ the radion rolls to infinity.



SUPERSYMMETRY BREAKING

The detuned RS scenario allows to break supersymmetry spontaneously.

In the effective theory supersymmetry breaking is signalled by a non-zero VEV of $D_\tau P$. Here

$$D_\tau P|_{r=r_0} \sim \frac{1-e^{i3k\pi b}}{L}.$$

Supersymmetry is broken by a VEV of b, unless $b = 2n/(3k)$.

Supersymmetry cannot be broken in this way in the tuned RS model because $L = \infty$.

This effect is the analog of what happens in no-scale supergravity. There supersymmetry is broken by a constant superpotential and the ground state is flat space.

One can take the limit $k \to 0$, zero bulk cosmological constant. By scaling the appropriately the brane tensions the effective action reduces to no-scale supergravity

$$\begin{aligned} K(\tau,\overline{\tau}) &= -3\log\left[\tau+\overline{\tau}\right] \\ P(\tau) &\sim \sqrt{\frac{T-T_0}{T+T_0}}-\sqrt{\frac{T+T_\pi}{T-T_\pi}} \end{aligned}$$

In this case supersymmetry is broken by constant superpotentials localized on the branes (Bagger, Feruglio and Zwirner).

FIVE DIMENSIONAL THEORY

To see how this happens let's consider the full $5D$ theory,

$$\begin{aligned} S_{\rm bulk} &= \frac{T}{6\,k}\int d^5x\, e_5\Big[-\frac{1}{2}R+6k^2-\frac{1}{4}F^{MN}F_{MN} \\ &+\frac{i}{2}\bar{\Psi}_{Mi}\Gamma^{MNK}D_N\Psi_{Ki}-\frac{3}{2}k\,\bar{\Psi}_{Mi}\Sigma^{MN}\vec{q}\cdot\vec{\sigma}_{ij}\Psi_{Nj} \\ &+\frac{\sqrt{6}}{4}k\,B_N\bar{\Psi}_{Mi}\Gamma^{MNK}\vec{q}\cdot\vec{\sigma}_{ij}\Psi_{Kj}+\ldots\Big] \end{aligned}$$

In AdS supergravity the graviphoton gauges a $U(1)$ subgroup of the $SU(2)_R-$symmetry. The gravitino has charge proportional to the cosmological constant of the bulk.

The shift symmetry of B_5 is violated by the fermionic action. Only in the tuned scenario each value of B_5 is equivalent.

A VEV of B_5 shifts the masses of the $4D$ gravitinos. This breaks SUSY by the Hosotani mechanism. Using non-periodic gauge transformations it can be shown that this is equivalent to Scherk-Schwarz boundary conditions.

When vector- and hyper-multiplets are coupled supersymmetry is also communicated to this sector. The vector multiplet contains

$$V_M, \qquad\qquad \lambda_i, \qquad\qquad \phi$$

The spinors λ_i are charged under the graviphoton so a VEV of B_5 shifts the $4D$ masses. In general this happens for all the states which have non-zero $R-$charge.

QUANTUM CORRECTIONS

At the classical level we have found that the radion is stabilized while the VEV of B_5 is free. Since the shift of B_5 is not a symmetry of the full action we expect,

$$V_{\mathrm{quantum}} = V(r, b)$$

The supersymmetric configuration however is protected,

$$\partial_b V_{\mathrm{quantum}}\Big|_{b=0} = 0$$

It is conceivable (with appropriate matter content) that the true minimum of the potential corresponds to $b \neq 0$. In that case we would have a theory that breaks supersymmetry and stabilizes the radion.

CONCLUSIONS

- I have described the detuned RS-model. In this scenario the distance between the branes is automatically stabilized and the ground state is either AdS$_4$ or dS$_4$.

- The low energy effective action is governed by the "AdS no-scale supergravity".

- The effective action contains a potential for the radion. The mass of the radion is of the order of the curvature of the space. At the classical level, the zero mode of B_5 is a flat direction of the potential but it will be lifted quantum mechanically.

- Supersymmetry can be broken by a VEV for B_5.