Gauge Coupling Unification in Minimal SUSY SU(5) and Its Prediction

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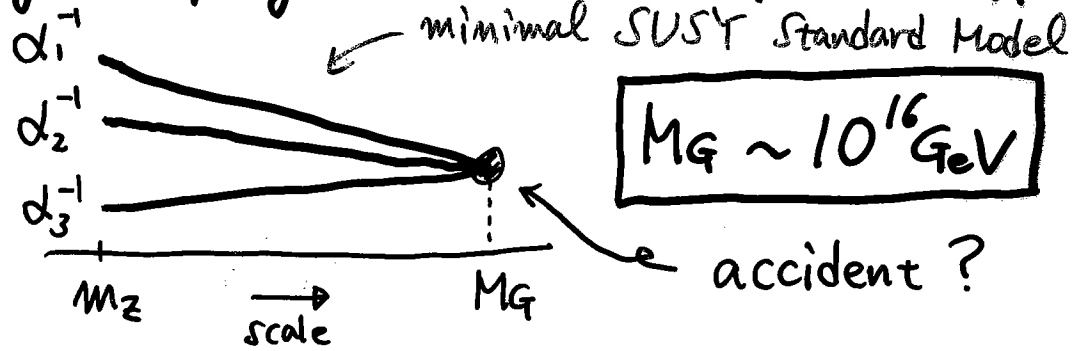
Reference

hep-ph/0312159
Physics Letter B.

K. Tobe and James D. Wells

Introduction Motivation of SUSY GUT

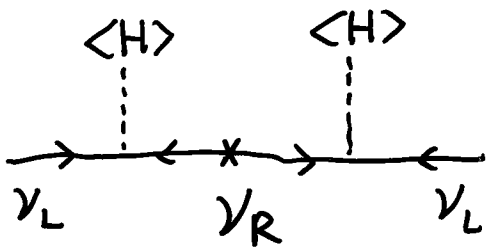
* gauge coupling unification ~ why $g_3 > g_2 > g_1$ at M_Z ?



$$\boxed{M_G \sim 10^{16}\,\mathrm{GeV}}$$

* non zero ν masses (atmospheric ν, solar ν ...)

seesaw mechanism (heavy ν_R)



$$m_\nu = \frac{y_\nu^2 \langle H \rangle^2}{M_R}$$

$$\delta m^2_{atm} = 2\times 10^{-3}\,\mathrm{eV}^2 \sim m^2_{\nu_3} \Rightarrow \boxed{M_R \sim 10^{15}\,\mathrm{GeV}}$$

$$\underline{\underline{M_R \sim M_G\ !}} \leftarrow \text{accident ??}$$

* unified picture of quarks and leptons

SM	$Q,\ u_R,\ e_R$	$d_R,\ L$	ν_R
SU(5)	10	$\bar{5}$	1
SO(10)	16		

$$\boxed{\text{theoretical studies (model building, analysis ...) are very important !!}}$$

Problem of SUSY GUT.

See also Goto and Nihei PRD 59, 115009 (1999)

PHYSICAL REVIEW D, VOLUME 65, 055009

Not even decoupling can save the minimal supersymmetric SU(5) model

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(Received 19 October 2001; published 14 February 2002)

We make explicit the statement that the minimal supersymmetric (SUSY) SU(5) model has been excluded by the SuperKamiokande search for the process $p \to K^+ \bar{\nu}$. This exclusion is made by first placing limits on the colored Higgs triplet mass, by forcing the gauge couplings to unify. We also show that taking the superpartners of the first two generations to be very heavy in order to avoid flavor changing neutral currents, the so-called "decoupling" idea, is insufficient to resurrect the minimal SUSY SU(5). We comment on various mechanisms to further suppress proton decay in SUSY SU(5). Finally, we address the contributions to proton decay from gauge boson exchange in the minimal SUSY SU(5) and flipped SU(5) models.

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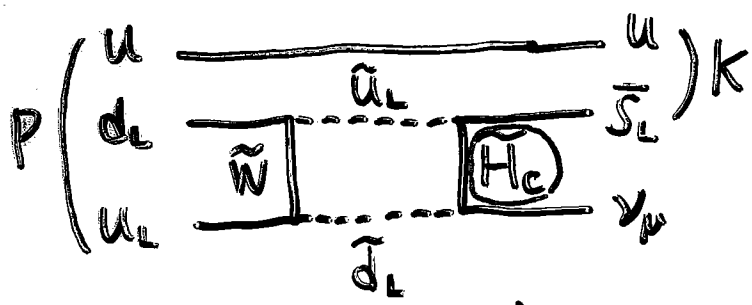
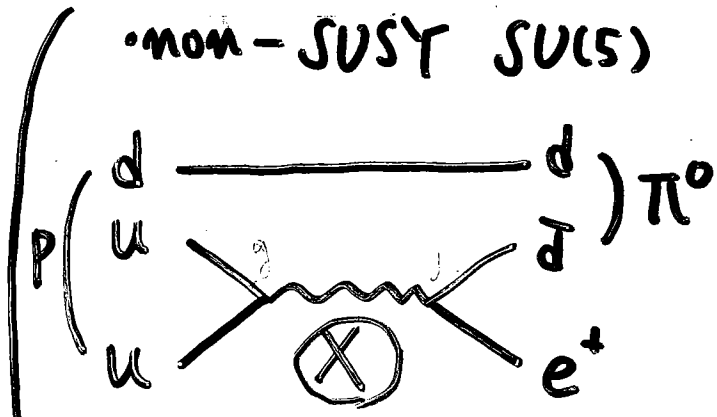
• minimal SUSY SU(5)



$$H_5 = (H, H_c)$$

$$\sim \frac{1}{M_{H_c}} \quad \frac{1}{m_{SUSY}}$$

• non-SUSY SU(5)



$$\sim \frac{1}{M_X^2}$$

Proton decay limit constrains colored Higgs mass M_{H_c}

$$\boxed{M_{H_c} > 2 \times 10^{17}\,\mathrm{GeV}}$$

On the other hand,

gauge coupling unification also constrains M_{H_c}:

$$\boxed{M_{H_c} < 10^{16}\,\mathrm{GeV}}$$ ← I will discuss this later.

Contents

1. Introduction

2. Constraint on M_{H_C}
from gauge coupling unification

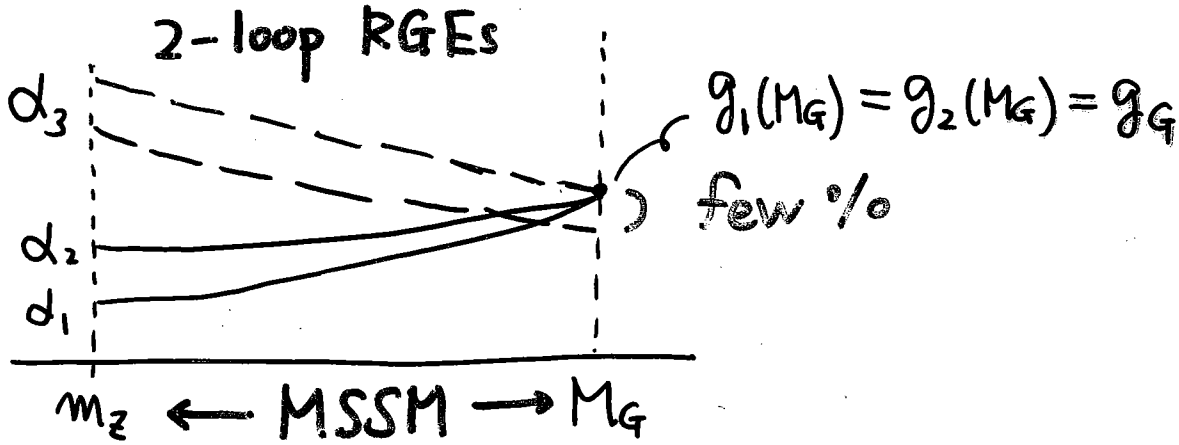
3. Solution to the proton decay problem
in minimal SUSY SU(5)

~ effect of non-renormalizable operator :
$$\frac{\Sigma}{M_{Pl}} F_{\mu\nu} F^{\mu\nu}$$

4. Prediction in gaugino mass spectrum.
$$\sim \frac{F_\Sigma}{M_{Pl}} \tilde{G}\tilde{G}$$

5. Summary

2. Constraint on M_{H_C} from gauge coupling unification



Without GUT scale threshold corrections, gauge coupling unification predicts

$$\alpha_3(m_Z) > \underline{0.126} \qquad (m_{SUSY} < 1\,\text{TeV})$$

Note : $\alpha_3(m_Z)_{exp} = \underline{0.117} \pm 0.002$

$$\boxed{\text{GUT scale threshold corrections are very important to realize the gauge coupling unification}}$$

☆ minimal SUSY SU(5) (M_{H_C}, M_Σ, M_V)

- Quarks and Leptons

$$10 = (Q, u_R, e_R)\ ,\ \bar{5} = (\bar{d}_R, L)$$

- Higgs $\quad 5_H = (H_2, \underline{H_C})\ ,\ \bar{5}_{\bar{H}} = (\bar{H}_2, \underline{\bar{H}_C})$

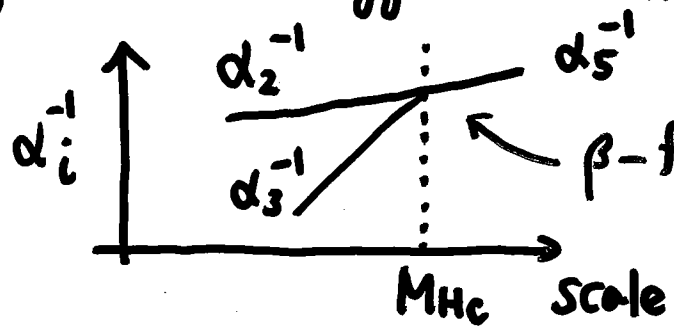
$$\Sigma(24) = \{\underline{(8,1) + (1,3) + (1,1) + (3,2) + (\bar{3},2)}\}$$

- Gauge

$$V(24) = \{G(8,1) + W(1,3) + B(1,1) + \underline{X(3,2) + (\bar{3},2)}\}$$

* GUT scale threshold corrections

e.g. colored Higgs mass M_{H_c} threshold



$H = (H_2, \underline{\underline{H_c}})$

β-function of α_3 changes at M_{H_c}

At $Q \sim M_G$ ← calculated from $g_i(m_Z)$

$$\frac{1}{g_G^2(Q)} = \frac{1}{g_i^2(Q)} - \frac{1}{8\pi^2} \sum_{a=H_c,V,\Sigma} \underline{b_{ai}} \ln \frac{Q}{M_a}$$

↑ unified coupling

↑ 1-loop GUT threshold

b_a : β-function of GUT particles

$$\begin{cases} -\dfrac{1}{g_1^2(Q)} + \dfrac{3}{g_2^2(Q)} - \dfrac{2}{g_3^2(Q)} = \dfrac{3}{10\pi^2} \ln \dfrac{M_{H_c}}{Q} \\[2ex] \dfrac{5}{g_1^2(Q)} - \dfrac{3}{g_2^2(Q)} - \dfrac{2}{g_3^2(Q)} = \dfrac{9}{2\pi^2} \ln \dfrac{M_U}{Q} \end{cases}$$

$$M_U^3 \equiv M_V^2 M_\Sigma$$

$g_i^2(Q)$ can be calculated from $g_i^2(m_Z)$ via RGEs

$$10^{14}\,\text{GeV} \le M_{H_c} \le \underline{\underline{10^{16}\,\text{GeV}}}$$

$$9 \times 10^{15}\,\text{GeV} \le M_U \le 2 \times 10^{16}\,\text{GeV}$$

$$\left(0.115 \le \alpha_3(m_Z) \le 0.119, \quad 200\,\text{GeV} \le m_{SUSY} \le 3\,\text{TeV}\right)$$

Note : Proton decay constraint

$$\underline{\underline{M_{H_c} \ge 2 \times 10^{17}\,\text{GeV}}}$$ Problem !

3. Solution to the proton decay problem in minimal SUSY SU(5)

we need $M_{H_C} > 2\times 10^{17}$ GeV

★ different type of GUT scale correction.

→ non-renormalizable operator (NRO)

$$\mathcal{L} = -\frac{1}{2}\frac{1}{g_G^2}\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) + \frac{\lambda\,\mathrm{Tr}(\Sigma\, G_{\mu\nu}G^{\mu\nu})}{M_{Pl}}$$

- Hill
- Shafi and Wetterich
- ⋮ (1984)

SU(5) breaking vev

$$\langle \Sigma(24)\rangle = V_G \begin{pmatrix} 2/3 & & & & 0 \\ & 2/3 & & & \\ & & 2/3 & & \\ & & & -1 & \\ 0 & & & & -1 \end{pmatrix}$$

$$V_G \sim 10^{16}\ \mathrm{GeV}$$

$$\longrightarrow \quad \mathcal{L} = -\frac{1}{4}\left(\frac{1}{g_G^2} + \underline{\underline{C_i\varepsilon}}\right) G^{Ai}_{\mu\nu}\, G^{Ai\,\mu\nu}$$

$$C_i = \left\{-\frac{1}{3}, -1, \frac{2}{3}\right\} \text{ for } \{U(1)_Y, SU(2)_L, SU(3)\}$$

$$\varepsilon = 8\lambda\frac{V_G}{M_{Pl}} \sim (1-10)\%, \quad M_{Pl} = 2\times 10^{18}\ \mathrm{GeV}$$

<u>this NRO correction saves</u>

<u>the minimal SUSY SU(5) model !!</u>

$$\frac{1}{g_G^2(\Theta)} + \boxed{C_i \varepsilon} = \frac{1}{g_i^2(\Theta)} - \underbrace{\frac{1}{8\pi^2}\sum_a b_{ai} \ln\frac{Q}{M_a}}_{\text{1-loop GUT correction}}$$

NRO correction

Interesting relation

$$\boxed{\begin{aligned} C_i &= -\{1,1,1\} + \frac{5}{3} b_{Hci} \\ b_{Hci} &= \left\{\frac{2}{5}, 0, 1\right\} : \beta\text{-function of } H_c \end{aligned}}$$

$$\left(\frac{1}{g_G^2(\Theta)} - \underline{\underline{\varepsilon}}\right) = \frac{1}{g_i^2(\Theta)} - \frac{1}{8\pi^2} b_{Hci}\left(\ln\frac{Q}{M_{H_c}} + \underline{\underline{\frac{40\pi^2}{3}\varepsilon}}\right) - \frac{1}{8\pi^2}\sum_{a\neq H_c} b_{ai} \ln\frac{Q}{M_a}$$

$$1/\bar{g}_G^2(\Theta) \equiv 1/g_G^2(\Theta) - \varepsilon \quad , \quad \underline{M_{H_c}^{eff} = M_{H_c} e^{-\frac{40\pi^2}{3}\varepsilon}}$$

$$\underline{\frac{1}{\bar{g}_G^2(\Theta)}} = \frac{1}{g_i^2(\Theta)} - \frac{1}{8\pi^2}\sum_{a=\underline{M_{H_c}^{eff}}, M_V, M_\Sigma} b_{ai} \ln\frac{Q}{M_a}$$

gauge coupling unification

$$\longrightarrow 10^{14}\,\text{GeV} \leq \underline{M_{H_c}^{eff}} \leq 10^{16}\,\text{GeV}$$

$$\underline{M_{H_c}^{eff}} = M_{H_c} e^{-\frac{40\pi^2}{3}\varepsilon}$$

If $\varepsilon > 0$ and $\varepsilon \sim O(\text{few }\%)$

M_{H_c} is large enough to avoid the proton decay problem !!

M_{H_c} constraints

from gauge coupling unification

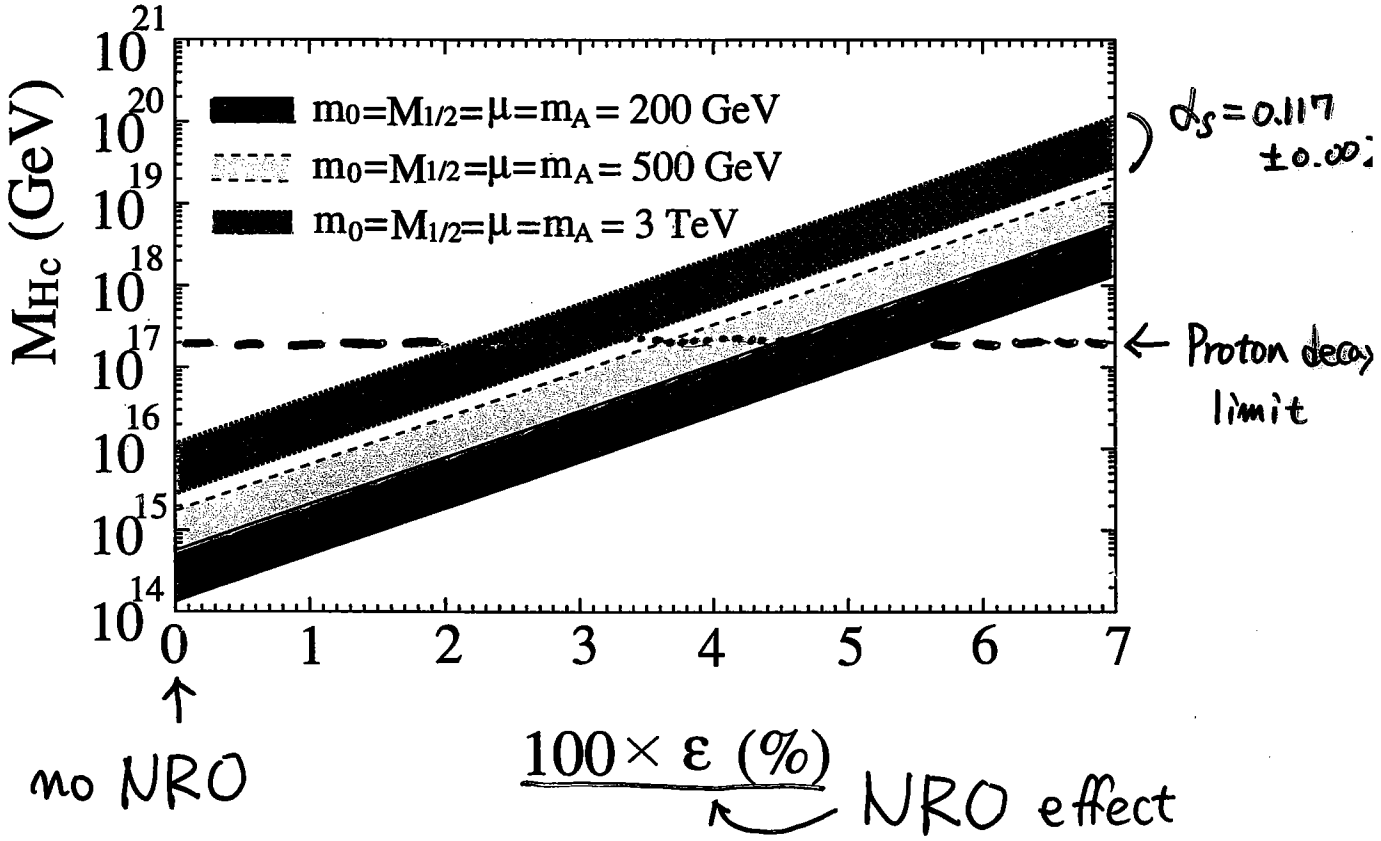


Figure 1: Fit for the heavy triplet Higgsino mass as a function of adjoint-Higgs corrections (ϵ) in order to accomplish gauge coupling unification. Here we define m_0 (universal scalar mass) and $M_{1/2}$ (universal gaugino mass) at the GUT scale, and μ and m_A at the weak scale without imposing a radiative electro-weak symmetry breaking condition. One expects $\epsilon \sim$ few%, and thus $M_{H_c} > 10^{17}$ GeV can be naturally achieved as is required by proton decay constraints. The width of each band is primarily due to the current uncertainty in $\alpha_s(m_Z)$.

Next question :

Can we test this solution ?

4. Prediction in gaugino mass spectrum.

* NRO correction to gaugino masses

$$\frac{\Sigma G_{\mu\nu} G^{\mu\nu}}{M_{Pl}} \overset{SUSY}{\Longleftrightarrow} \frac{F_\Sigma \hat{\tilde{G}} \tilde{G}}{M_{Pl}}$$

$\hat{\tilde{G}}$: SU(5) gaugino

F_Σ : F-term of $\Sigma(24)$

$$\langle F_\Sigma \rangle \simeq \langle \Sigma \rangle (A_\Sigma - B_\Sigma)$$

$$A_\Sigma \sim B_\Sigma \sim m_{SUSY}$$

gaugino masses

$$M_i = \underbrace{M_{1/2}}_{\text{universal mass}} - \underbrace{\frac{1}{2} c_i \varepsilon g_G^2 (A_\Sigma - B_\Sigma)}_{\text{NRO correction}}$$

$$c_i = \{-\tfrac{1}{3}, -1, \tfrac{2}{3}\}, \quad \varepsilon \sim \frac{M_G}{M_{Pl}}$$

U(1), SU(2), SU(3)

Including 1-loop GUT threshold corrections ....

at $Q = \Lambda_U$ where $g_1(\Lambda_U) = g_2(\Lambda_U) = g_U$

$$M_1(\Lambda_U) = g_U^2 \bar{M} + g_U^2 \left[\frac{\varepsilon}{6}(A_\Sigma - B_\Sigma) - \frac{1}{16\pi^2}\left(10 g_U^2 \bar{M} + 10(A_\Sigma - B_\Sigma) + \frac{2}{5} B_5\right) \right]$$

$$M_2(\Lambda_U) = g_U^2 \bar{M} + g_U^2 \left[\frac{\varepsilon}{2}(A_\Sigma - B_\Sigma) - \frac{1}{16\pi^2}\left(6 g_U^2 \bar{M} + 6 A_\Sigma - 4 B_\Sigma\right) \right]$$

$$M_3(\Lambda_U) = g_3^2 \bar{M} + g_U^2 \left[\underbrace{-\frac{\varepsilon}{3}(A_\Sigma - B_\Sigma)}_{\text{NRO}} - \underbrace{\frac{1}{16\pi^2}\left(4 g_U^2 \bar{M} + 4 A_\Sigma - B_\Sigma + B_5\right)}_{\text{1-loop}} \right]$$

$$\delta_{1-2} \equiv \frac{M_1(\Lambda_U) - M_2(\Lambda_U)}{M_2(\Lambda_U)}$$

$$\delta_{3-2} \equiv \frac{M_3(\Lambda_U) - M_2(\Lambda_U)}{M_2(\Lambda_U)}$$

$\Lambda_U \simeq$ GUT scale

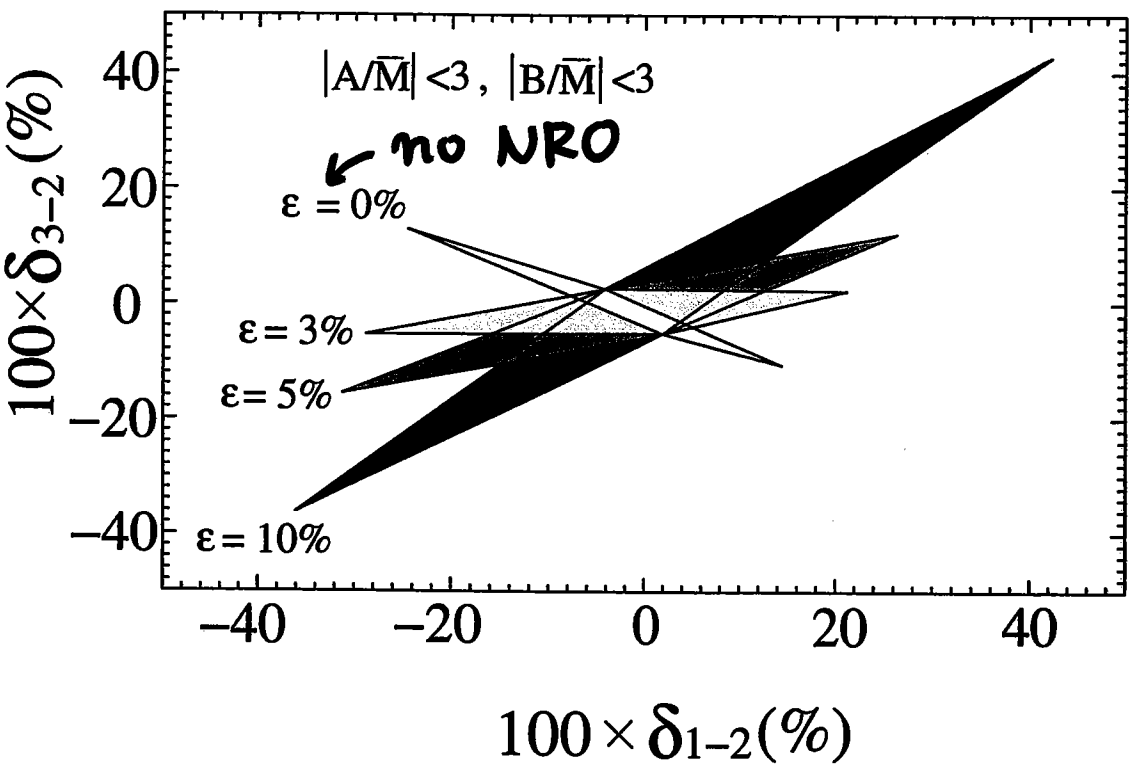


Figure 1: δ corrections to the gaugino masses at the scale Λ_U where $g_1(\Lambda_U) = g_2(\Lambda_U)$. We have defined $\delta_{1-2} = (M_1(\Lambda_U) - M_2(\Lambda_U))/M_2(\Lambda_U)$ and $\delta_{3-2} = (M_3(\Lambda_U) - M_2(\Lambda_U))/M_2(\Lambda_U)$. Here we have assumed universal A-terms ($A_\Sigma = A_\lambda \equiv A$) and B-terms ($B_\Sigma = B_5 \equiv B$) and varied them over the ranges $|A/\overline{M}| < 3$ and $|B/\overline{M}| < 3$.

$\varepsilon = 0$ $\quad \delta_{1-2} \times \delta_{3-2} < 0$

$\varepsilon \neq 0$ $\quad \delta_{1-2} \times \delta_{3-2} > 0$

precision measurement of SUSY parameters are very important to test this solution.

In order to determine GUT-scale gaugino masses,

* Experiments $\boxed{\text{superpartner masses and couplings}}$

$\Downarrow$

* Reconstruction of SUSY parameters at weak scale

$$M_1, M_2, M_3, \mu, m_{\tilde{q}}^2, m_{\tilde{\ell}}^2, \cdots\cdots$$

$\Downarrow$

* Renormalization group equation analysis

$$\mu \frac{d}{d\mu} M_i = \beta^{(1)}(M_i) + \beta^{(2)}(M_1, M_2, M_3, A_i) + \cdots$$

$\Downarrow$

* Determination of GUT-scale parameters

$$M_1(\Lambda_U), \; M_2(\Lambda_U), \; M_3(\Lambda_U)$$

at Λ_U where $g_1(\Lambda_U) = g_2(\Lambda_U)$

hep-ph/0403133
Allanach, Blair, Kraml, Martyn, Polesello, Porod, Zerwas

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* Measurements

	Mass, ideal	"LHC"	"LC"	"LHC+LC"
$\tilde{\chi}_1^\pm$	179.7		0.55	0.55
$\tilde{\chi}_2^\pm$	382.3	–	3.0	3.0
$\tilde{\chi}_1^0$	97.2	4.8	0.05	0.05
$\tilde{\chi}_2^0$	180.7	4.7	1.2	0.08
$\tilde{\chi}_3^0$	364.7		3-5	3-5
$\tilde{\chi}_4^0$	381.9	5.1	3-5	2.23
$\tilde{e}_R$	143.9	4.8	0.05	0.05
$\tilde{e}_L$	207.1	5.0	0.2	0.2
$\tilde{\nu}_e$	191.3	–	1.2	1.2
$\tilde{\mu}_R$	143.9	4.8	0.2	0.2
$\tilde{\mu}_L$	207.1	5.0	0.5	0.5
$\tilde{\nu}_\mu$	191.3	–		
$\tilde{\tau}_1$	134.8	5-8	0.3	0.3
$\tilde{\tau}_2$	210.7	–	1.1	1.1
$\tilde{\nu}_\tau$	190.4	–	–	–
$\tilde{q}_R$	547.6	7-12	–	5-11
$\tilde{q}_L$	570.6	8.7	–	4.9
$\tilde{t}_1$	399.5		2.0	2.0
$\tilde{t}_2$	586.3		–	
$\tilde{b}_1$	515.1	7.5	–	5.7
$\tilde{b}_2$	547.1	7.9	–	6.2
$\tilde{g}$	604.0	8.0	–	6.5
h^0	110.8	0.25	0.05	0.05
H^0	399.8		1.5	1.5
A^0	399.4		1.5	1.5
$H^\pm$	407.7	–	1.5	1.5

(GeV)

Table 1: *Accuracies for representative mass measurements at "LHC" and "LC", and in coherent "LHC+LC" analyses for the reference point SPS1a [masses in* GeV*]. $\tilde{q}_L$ and $\tilde{q}_R$ represent the flavours $q = u, d, c, s$ which cannot be distinguished at LHC. Positions marked by bars cannot be filled either due to kinematical restrictions or due to small signal rates; blank positions could eventually be filled after significantly more investments in experimental simulation efforts than performed until now. The "LHC" and "LC" errors have been derived in Ref. [9] and Ref. [17], respectively, in this document.*

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* Reconstruction of SUSY parameters at M_Z

	Parameter, ideal	"LHC+LC" errors
M_1	101.66	0.08
M_2	191.76	0.25
M_3	584.9	3.9
μ	357.4	1.3
$M^2_{L_1}$	$3.8191 \cdot 10^4$	82.
$M^2_{E_1}$	$1.8441 \cdot 10^4$	15.
$M^2_{Q_1}$	$29.67 \cdot 10^4$	$0.32 \cdot 10^4$
$M^2_{U_1}$	$27.67 \cdot 10^4$	$0.86 \cdot 10^4$
$M^2_{D_1}$	$27.45 \cdot 10^4$	$0.80 \cdot 10^4$
$M^2_{L_3}$	$3.7870 \cdot 10^4$	360.
$M^2_{E_3}$	$1.7788 \cdot 10^4$	95.
$M^2_{Q_3}$	$24.60 \cdot 10^4$	$0.16 \cdot 10^4$
$M^2_{U_3}$	$17.61 \cdot 10^4$	$0.12 \cdot 10^4$
$M^2_{D_3}$	$27.11 \cdot 10^4$	$0.66 \cdot 10^4$
$M^2_{H_1}$	$3.25 \cdot 10^4$	$0.12 \cdot 10^4$
$M^2_{H_2}$	$-12.78 \cdot 10^4$	$0.11 \cdot 10^4$
A_t	$-497.$	9.
$\tan\beta$	10.0	0.4

→ RGE analysis

Table 1: *The extracted SUSY Lagrange mass and Higgs parameters at the electroweak scale in the reference point SPS1a [mass units in* GeV*].*

* Determination of GUT scale parameters

	Parameter, ideal	"LHC+LC" errors
M_1	250.	0.15
M_2	*ditto*	0.25
M_3		2.3
M_{L_1}	100.	6.
M_{E_1}	*ditto*	12.
M_{Q_1}		23.
M_{U_1}		48.
M_{L_3}		7.
M_{E_3}		14.
M_{Q_3}		37.
M_{U_3}		58.
M_{H_1}	*ditto*	8.
M_{H_2}		41.
A_t	$-100.$	40.

←

Table 2: *Values of the SUSY Lagrange mass parameters after extrapolation to the unification scale where gaugino and scalar mass parameters are universal in mSUGRA [mass units in* GeV*].*

* Renormalization Group Equation Analysis.

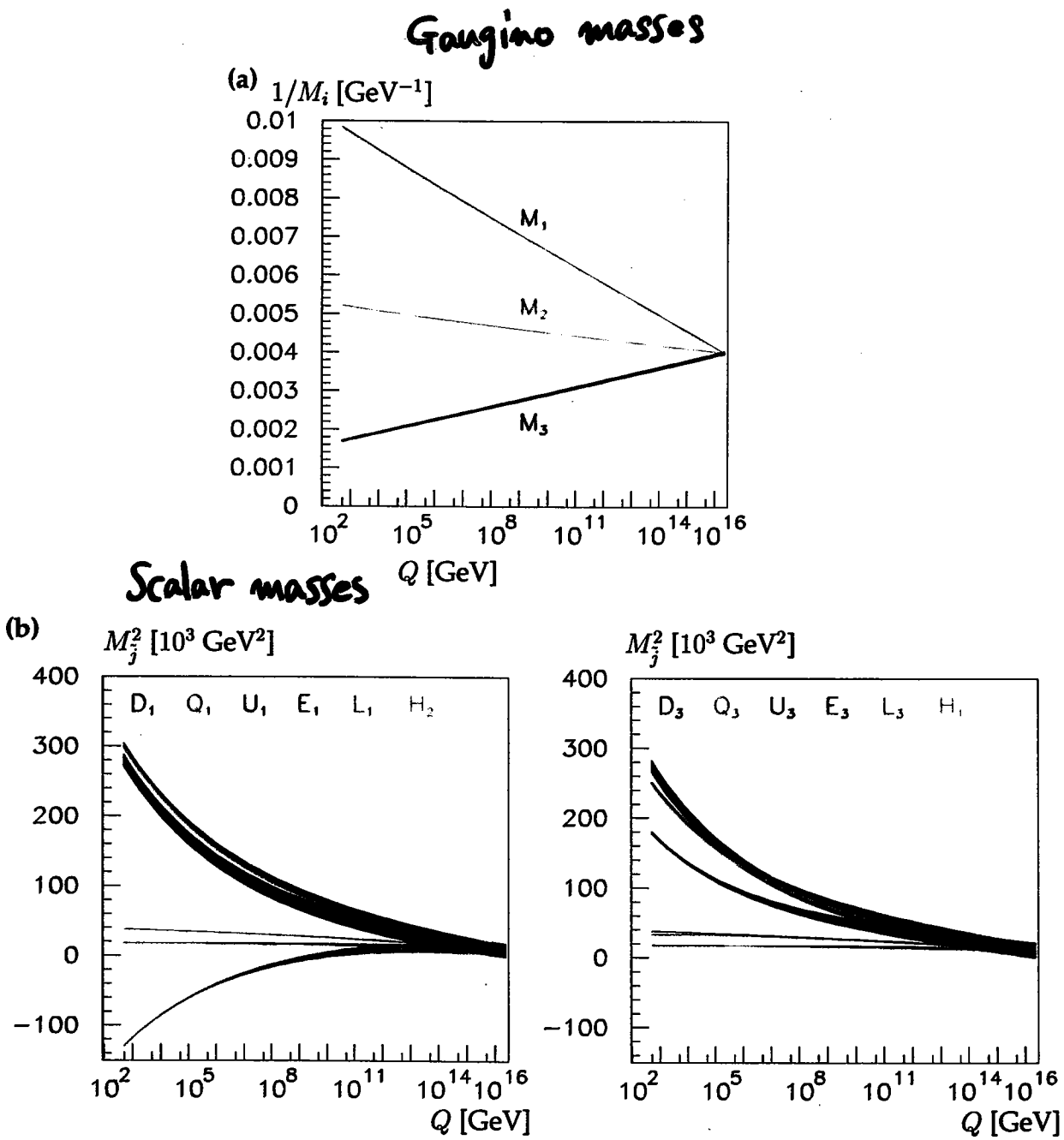


Figure 1: *Evolution, from low to high scales, (a) of the gaugino mass parameters for "LHC+LC" analyses; (b) left: of the first–generation sfermion mass parameters (second generation, dito) and the Higgs mass parameter $M^2_{H_2}$; right: of the third–generation sfermion mass parameters and the Higgs mass parameter $M^1_{H_2}$.*

5. Summary and discussion

In "minimal SUSY SU(5)".

Proton decay	(naive) gauge coupling unificatio
$\rightarrow M_{H_C} > 2\times 10^{17}$ GeV	$\rightarrow M_{H_C} < 10^{16}$ GeV

problem.

✱ Simple solution to this proton decay problem

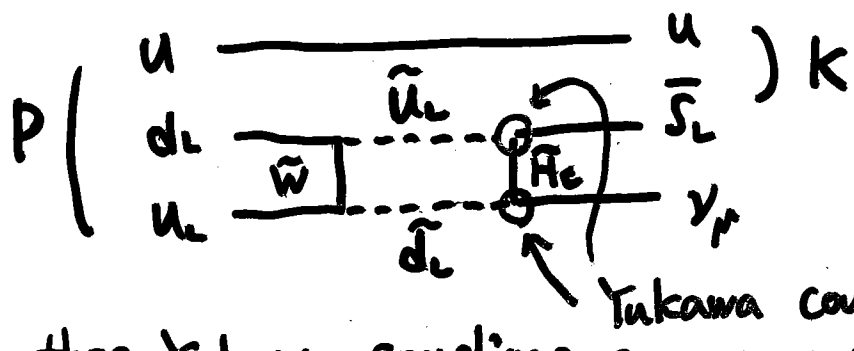
NRO $\quad \mathrm{Tr}(\Sigma(24)\, G_{\mu\nu} G^{\mu\nu})/M_{Pl}$

- the NRO exists naturally in SUGRA ...

  $\varepsilon \sim M_G/M_{Pl} \sim$ few % natural! $\quad M_V, M_Z \uparrow \varepsilon > 0 \quad M_{H_C}$

- this solution works in any SUSY SU(5) where $\Sigma(24)$ breaks SU(5) general!

NOTE : Other solutions



$\left(\begin{array}{c} SU(5) \\ y_e = y_d \end{array}\right)$

these Yukawa couplings can receive the corrections from NROs, so that the proton decay limit on M_{H_C} can be lowered.

see recent works by

Bajc, Perez, Senjanovic hep-ph/0204311
Bajc, Melfo, Senjanovic hep-ph/0304051
Emmanuel-Costa, Wiesenfeldt hep-ph/0302272
Rakshit, Raz, Roy Shadmi hep-ph/0309318
Kakizaki, Yamaguchi hep-ph/0203192 .....

* prediction of our scenario

$$\frac{\Sigma\, G_{\mu\nu}G^{\mu\nu}}{M_{Pl}} \overset{SUSY}{\Longleftrightarrow} \frac{F_\Sigma \tilde{G}\tilde{G}}{M_{Pl}}$$

$$\delta_{1-2} = \frac{M_1(\Lambda_U) - M_2(\Lambda_U)}{M_2(\Lambda_U)}, \quad \delta_{3-2} = \frac{M_3(\Lambda_U) - M_2(\Lambda_U)}{M_2(\Lambda_U)} \quad \text{at } \Lambda_U \sim M_G$$

$\rightarrow$ non-zero ε $\qquad \delta_{1-2} \times \delta_{3-2} > 0$

$(\delta_{1-2} \sim 10\%,\ \delta_{2-3} \sim 10\% \qquad \varepsilon > \text{few}\%)$

precision measurement of SUSY parameters are very important to test this solution !!

We need <u>LHC + LC</u> !!

and <u>understanding of the theoretical uncertainties</u>

hopefully testable!

Hopefully .....

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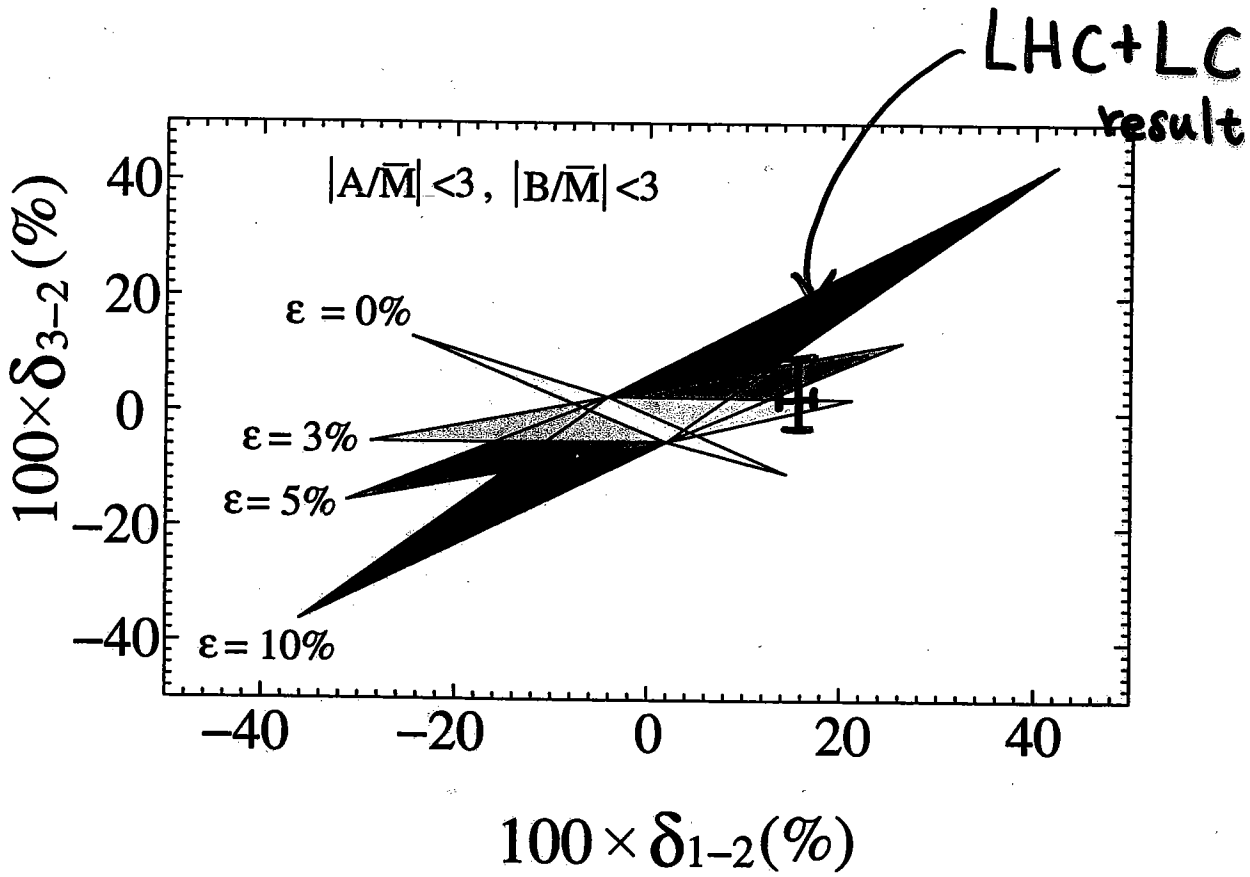

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