QFT 3: PROBLEM SET 3

(1) Rewrite gauge-fixing term as:

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}\sum_{i=1}^{3}\left(\partial_\mu A^\mu_i - g\xi\, 2\,\mathrm{Im}\left(\phi_0^\dagger T_i \phi_1\right)\right)^2$$
$$-\frac{1}{2\xi}\left(\partial_\mu B^\mu - g'\frac{\xi}{2}\, 2\,\mathrm{Im}\left(\phi_0^\dagger \phi_1\right)\right)$$

where $\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} = \langle\phi\rangle$

$$\phi_1 = \phi - \phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}-i(\varphi^1 - i\varphi^2)\\ h + i\varphi^3\end{pmatrix} = \begin{pmatrix}-i\phi^+\\ h+i\phi^3\end{pmatrix}$$

and generators $T^i = \frac{1}{2}\sigma^i \quad T^0 = \mathbb{1}$

$$\phi_0^\dagger T^1 \phi_1 = -\frac{iv}{4}(\varphi^1 - i\varphi^2)$$
$$\phi_0^\dagger T^2 \phi_1 = \frac{v}{4}(\varphi^1 - i\varphi^2)$$
$$\phi_0^\dagger T^3 \phi_1 = -v/4\,(h + i\varphi^3)$$

Summing up all the above in $\mathcal{L}_{gf}$ mass terms

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}\left(\left(\frac{g\xi v}{2}\right)^2(\phi_1^2 + \phi_2^2) + \frac{(g^2+g'^2)\xi v}{2}\phi_3^2\right)$$

$$m_{\phi_1} = m_{\phi_2} = \frac{gv\sqrt{\xi}}{2} \qquad m_{\phi_3} = \frac{\sqrt{g^2+g'^2}\,v\sqrt{\xi}}{2}$$

$$m_h = 0$$

No mass term from any $V(\phi)$ since must be gauge invariant.

QFT 3: PROBLEM SET 2

Ghost mass: quadratic terms in ghost action:

$$M_{ab}^2 = g_a g_b \xi \, \phi_0^\dagger \{T^a, T^b\} \phi_0$$

$$= \frac{1}{4} g^2 \xi \, \phi_0^\dagger \{\sigma^i, \sigma^j\} \phi_0 + \frac{1}{2} g'^2 \xi \, \phi_0^\dagger \phi_0 \delta_{00}$$

$$+ \frac{1}{2} g g' \xi \left(\phi_0^\dagger \sigma^i \phi_0 \delta_0^j + \phi_0^\dagger \sigma^j \phi_0 \delta_0^i \right)$$

$$= \frac{1}{2} g^2 \xi \quad v^2 \delta^{ij} + \frac{1}{2} g'^2 \xi \, v^2 \delta^{00} - \frac{1}{2} g g' \xi \, v^2 \left(\delta^{03} + \delta^{30}\right)$$

$$\Rightarrow \quad M_{ab}^2 = \begin{pmatrix} g^2 v^2 \xi & & & \\ & g^2 v^2 \xi & & \\ & & g^2 v^2 \xi & -g g' \xi v^2 \\ & & -g g' \xi v^2 & g'^2 v^2 \xi \end{pmatrix}$$

Diagonalise $\Rightarrow$

$$m_{1,2} = g v \sqrt{\xi}$$

$$m_3 = \sqrt{g^2 + g'^2} \, v \sqrt{\xi}$$

$$m_4 = 0.$$

propagators are just:

Goldstones: $\dfrac{i \delta_{ab}}{k^2 - m_{ab}^2}$ $\quad a - - \rightarrow - - - b$

Ghosts : $\dfrac{i \delta_{ab}}{k^2 - M_{ab}^2}$ $\quad a \cdots \rightarrow \cdots b$

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1 ctd

Ghost - gauge vertex:

$$\mathcal{L}_{ghost} = \bar{c}^a \left(-\partial^2 \delta^{ab} - g f^{acb} A^c_\mu \right) c^b$$

$$\longrightarrow \quad -g f^{acb} p^\mu$$

(for $SU(2)\times U(1)$ $f^{acb} \to \varepsilon^{acb}$, $a \in \{1,2,3\}$, 0 otherwise)

Goldstone - gauge - ~~from ϕ kinetic terms~~

Get the same terms as usual from $D_\mu\phi^\dagger D^\mu\phi$ - let's not deal with those.

Interested in terms you get as result of gauge fixing.

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$$-\frac{1}{2\xi}\left\{\left[\partial_\mu A^{\mu a} - 2\,\mathrm{Im}\left(\phi_0^\dagger T^a \phi'\right) g a \xi\right]^2\right\}_i = \mathcal{L}_{gf}$$

$$= -\frac{1}{2\xi}\Big(\partial_\mu A^{\mu i}\partial_\nu A^{\nu i} + \partial_\mu B\,\partial^\mu B$$ ← gauge kinetic

$$- 4g\xi\,\partial_\mu A^{i\mu}\,\mathrm{Im}\left(\phi_0^\dagger T^i \phi'\right)$$
$$- 2g'\xi\,\partial_\mu B^\mu\,\mathrm{Im}\left(\phi_0^\dagger \phi'\right)$$ new gauge-goldstone interactions (*)

$$+ 4g^2\xi^2\left(\mathrm{Im}\left(\phi_0^\dagger T^i \phi'\right)\right)^2$$
$$+ g'^2\xi^2\left(\mathrm{Im}\left(\phi_0^\dagger \phi_i\right)\right)^2$$

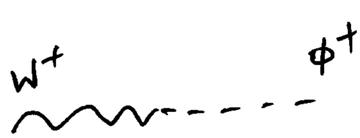
$$\mathrm{Im}\left(\phi_0^\dagger\phi'\right) = \frac{1}{2}v\phi^3$$
$$\mathrm{Im}\left(\phi_0^\dagger T^i\phi'\right) = -\frac{v}{4}\phi^i$$

$$(*) \rightarrow = -\frac{1}{2\xi}\left(-4g\xi\,\partial_\mu A^{i\mu}\left(-\frac{v}{4}\phi^i\right) - g'\xi\,\partial_\mu B^\mu\phi^3\right)$$

$$= \frac{1}{2}gv\,\partial_\mu A^{i\mu}\phi^i + \frac{1}{2}vg'\,\partial_\mu B^\mu\phi^3$$

$$= \frac{1}{2}gv\left[(\partial_\mu W^{+})\phi^- + (\partial_\mu W^-)\phi^+\right] + \frac{1}{2}v\sqrt{g^2+g'^2}\,\partial_\mu Z^\mu\phi^3$$

$\Rightarrow$



$\frac{1}{2}gv\,p_\mu$

$\frac{1}{2}v\sqrt{g^2+g'^2}\,p^\mu$

are our two new interaction vertices

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(2)
$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} - \mu^2 g^2 \mathrm{Tr}\left[A_\mu A^\mu\right]$$

Start with partition fn:

$$Z = \int DA\, e^{iS[A]} \overset{\text{fix gauge}}{=} \int \mathcal{D}\alpha\, \delta\left(G(A^\alpha)\right) \left|\frac{\delta G(A)}{\delta\alpha}\right| \int \mathcal{D}A\, e^{iS[A]}$$

where $A^\alpha = U\left(A + \frac{i}{g}\partial_\mu\right)U^+$

Want to have no U's in $\int DA^\alpha$ - term for decoupling.

- $\mathcal{D}A^\alpha = \mathcal{D}A$ by translation invariance
- $\mathcal{L}_{\text{mass}}[A^\alpha] = -\mu^2 g^2 \mathrm{Tr}\left[A^\alpha_\mu A^{\mu\alpha}\right] = -\mu^2 g^2 \mathrm{Tr}\left[\left(UA_\mu U^+ + \frac{i}{g} U\partial_\mu U^+\right) \times \left(UA^\mu U^+ + \frac{i}{g} U\partial^\mu U^+\right)\right]$

$$= -\mu^2 g^2 \mathrm{Tr}\left(A_\mu A^\mu - \frac{i}{g} UA_\mu \partial^\mu U^+ \mp \frac{i}{g}\partial_\mu U^+ U A^\mu - \frac{1}{g^2} U\partial_\mu U^+ U\partial^\mu U^+\right)$$

$$= -\mu^2 \mathrm{Tr}\left(g^2 A_\mu A^\mu - 2ig A_\mu \partial^\mu U^+ U + \partial_\mu U \partial^\mu U^+\right)$$

by $(UU^+) = \mathbb{1}$
$\Rightarrow \partial_\mu U U^+ + U\partial_\mu U^+ = 0$

$\Rightarrow$ we get the interaction terms and can't separate the ~~ga~~ gauge terms completely

(8)

QFT3: PROBLEM SET 2

2 cHI

SU(2) + Higgs doublet; $\lambda \to \infty$ VEV $\begin{pmatrix} 0 \\ v \end{pmatrix}$, $|v| < \infty$

$\longrightarrow$ gauge fields mass $gv/\sqrt{2}$

$\longrightarrow$ scalars $\phi = U \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$, $U \in SU(2)$, $U = \exp(i \xi^a \sigma^a/2)$

Higgs potential $V(\phi) = \lambda/4 \left(\phi^\dagger \phi - v^2/2 \right)^2$

$= \lambda/4 \left(h^2/2 + 2vh \right)^2$

$\Rightarrow \quad m_h^2 = 2\lambda v \longrightarrow \infty$

$m_{\xi^a}^2 = 0$

$\Rightarrow$ h is non-dynamical $\Rightarrow$ Integrate out

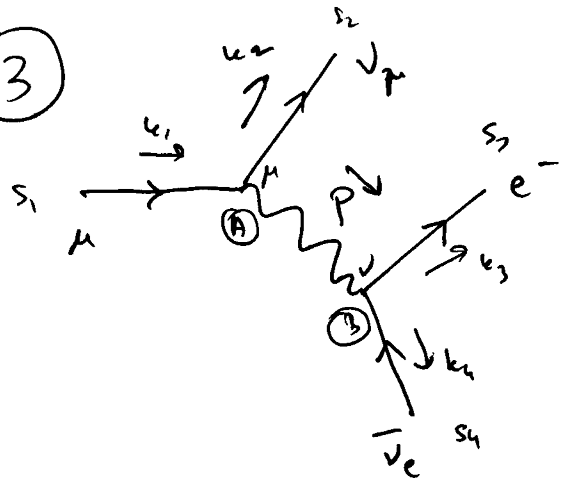
cf: Proca mass Lagrangian

- 3 massive ~~gauge~~ vectors, mass^2 $\mu^2 g^2$
- 3 massless scalars

$\Rightarrow$ same field content and masses if $\mu^2 = v^2/2$

QFT 3: PROBLEM SET 3

(3)



with vertices

(A) $\dfrac{ig}{\sqrt{2}} \gamma^\mu \dfrac{1-\gamma^5}{2}$

(B) $\dfrac{ig}{\sqrt{2}} \gamma^\nu \dfrac{(1-\gamma^5)}{2}$

$\overset{\vec{p}}{\mu \sim\sim \nu} = -\dfrac{i g_{\mu\nu}}{p^2 - m_W^2}$

$$\mathcal{M} = \left(\bar{u}^{s_1}(k_1) \frac{ig}{\sqrt{2}} \gamma^\mu \frac{(1-\gamma^5)}{2} u^{s_2}(k_2)\right) \frac{-i g_{\mu\nu}}{p^2 - m_W^2} \cdot$$
$$\cdot \left(\bar{u}^{s_3}(k_3) \frac{ig}{\sqrt{2}} \gamma^\nu \frac{1-\gamma_5}{2} v^{s_3}(k_4)\right)$$

Sum over spins

$$|\mathcal{M}|^2 = \frac{g^4}{4 \cdot 64} \sum_{s_i} \bar{v}_a^{s_3}(k_4) \left(\gamma^\mu (1-\gamma^5)\right)_{ab} u_b^{s_3}(k_3)\, \bar{u}_d^{s_2}(k_2) \left(\gamma_\mu (1-\gamma^5)\right)_{de} u_e^{s_1}(k_1)$$
$$\bar{u}_g^{s_1}(k_1) \left(\gamma^\nu (1-\gamma^5)\right)_{gh} u_h^{s_2}(k_2)\, \bar{u}_l^{s_3}(k_3) \left(\gamma_\nu (1-\gamma^5)\right)_{lm} v_m^{s_3}(k_4) \times \frac{1}{(p^2 - m_W^2)^2}$$

$$= \frac{g^4 \mathrm{Tr}}{4 \cdot 64}\left[(\not{k}_1 + m_\mu)\gamma^\nu (1-\gamma^5) \not{k}_2\, \gamma_\mu (1-\gamma^5)\right] \times \mathrm{Tr}\left[\not{k}_4 \gamma^\mu (1-\gamma^5)(\not{k}_3 + m_e) \times \gamma_\nu (1-\gamma^5)\right]$$
$$\cdot \frac{1}{(p^2 - m_W^2)^2}$$

(Assuming $m_\nu = 0$)

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- Drop masses
- Take $k_{1\alpha} k_{2\beta} \, \mathrm{Tr}\left[\gamma^\alpha \gamma^\nu (1-\gamma^5) \gamma^\beta \gamma^\mu (1-\gamma^5)\right]$

$$= 8\left[k_1^\mu k_2^\nu - (k_1 \cdot k_2) g^{\mu\nu} + k_1^\nu k_2^\nu - i k_{1\alpha} k_{2\beta} \varepsilon^{\alpha\beta\mu\nu}\right]$$

Taking
- $\varepsilon^{\alpha\beta\mu\nu} \varepsilon_{\alpha\beta\rho\sigma} = -2(\delta^\mu{}_\rho \delta^\nu{}_\sigma - \delta^\mu{}_\sigma \delta^\nu{}_\rho)$
- $\varepsilon^{\alpha\beta\mu\nu} k^1_\mu k^2_\nu k^3_\alpha k^4_\beta = 0$ since not linearly dependent

get $$|\mathcal{M}|^2 = \frac{g^4}{(p^2 - m_W^2)^2} (k_2 \cdot k_4)(k_1 \cdot k_3)$$

$$\approx \frac{g^4}{m_W^4} (k_2 \cdot k_4)(k_1 \cdot k_3)$$

In SM, $m_W = gv/2$

$$\Rightarrow \quad v \approx \frac{1}{2}\left(\frac{|\mathcal{M}|^2}{(k_1 \cdot k_3)(k_2 \cdot k_4)}\right)^{1/4}$$

$\Rightarrow$ measure $|\mathcal{M}|^2$, get v!