Physics 323. Problem set II. Thursday, April 7, 2005

DUE THURSDAY, APRIL 14, 2005

Problem 1. (10 points)

Demonstrate that the metric tensor $g_{\mu\nu}$ is invariant under Lorentz transformations.

Problem 2. (50 points)

Imagine an infinitely long conductor, with uniform positive charge density ρ and a vector current-density $j_x = \rho v$, as measured by an observer in a reference frame K, where the x-axis concides with the center of the conductor. The density ρ may be thought as a succession of positive charges q_n, with $x_{n+1} - x_n = L$, where x_n denotes the location of the n-th charge. If a is the area of the transverse section of the conductor, $\rho \simeq q/(L\ a)$. The current density j_x is a result of the motion of the positive charges, which move with velocity v in the x-direction. The conductor is neutral, since there is also a similar succession of negative charges, with $x_{n+1} - x_n = L$. The negative charges are at rest.

The circuit is closed by a similar infinitely long conductor, parallel to the first one, and separated by a distance D from it. The positive and negative charge-densities are the same as above, but the current goes in the opposite direction: $j_x = -\rho v$.

a) What are the values of the electric and magnetic fields measured by an observer on the plane determined by the conductors ? What is the force acting upon a particle moving with velocity v in the x direction on the same plane ? Does the result depend on where the charge is located ?

Imagine now an observer in a system K', where the positive charges in the first conductor are at rest.

b) Use the Lorentz transformation laws of the distance between the charges and obtain the positive and negative charge-densities in the two conductors as measured by an observer in K' (Hint: use the proper law of addition of velocities).

c) Solve problem b) by simply using the transformation properties of the four-vector J^μ. Ignoring the small sections at infinity connecting the two conductors, show that the above results are consistent with charge conservation.

d) Find the electric and magnetic fields in the system K' for an arbitrary point on the plane determined by the conductors and demonstrate that they are consistent with what is obtained from a Lorentz transformation of the components of the electromagnetic field tensor $F^{\mu\nu}$. What is the force upon the same particle as in problem a), as seen by an observer in the system K' ?

Problem 3. (40 points)

Consider the first of the conductors described in problem 2.

What are the values of the charge-current and density observed by an observer in a system K'', which moves at a constant velocity v' in the y-direction (perpendicular to x) with respect to the reference frame K ? Demonstrate this result by using the Lorentz transformation laws of the four-vector current density J^μ, as well as the transformation laws of the distances between charges (Hint: take into account the variation of the transverse area of the conductors, and the proper velocities' addition law).