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A SURVEY OF EIGHT-MAGNET SPIN PRECESSION SNAKES *

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A computer search was done to find new configurations for spin precession snakes. The transformation of proton polarization from the S direction to either the N or L direction within some constraints on magnet and beam properties was used as the basis of the search. The many new solutions found should be useful for external polarized beams and possibly in accelerators.

1. Introduction

There were two motivations for the search for new spin precession snakes:

(1) existing superconducting magnets which might be available for the Fermilab polarized beam do not have an adequate field integral sufficient to precess the proton spin by 90° as required in previous snake designs.

(2) A major problem with using snakes in accelerators is the increase in path length of the beam around the accelerator. This increase in length is momentum dependent in a snake and would require changing the parameters of the acceleration process. Snakes using smaller field integrals and closer spacing would reduce the severity of the problem.

Snakes of eight magnets within the constraints imposed here are capable of reversing the polarization direction or of changing the polarization direction by an arbitrary amount. Snakes utilizing only rotations of 90° in orthogonal planes had been considered previously [1,2]. The object of the search was to find new solutions using smaller rotations (smaller field integrals).

For accelerators, the replacement of an eight-magnet snake by two eight-magnet snakes of smaller magnets leads to a reduction in path length. Steffan [3] † has found a solution of this type utilizing a snake with ten magnets which is equivalent to a sixteen magnet snake with 45° spin rotations.

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† The energy dependence of this configuration is discussed by Turrin [3].

2. Discusssion

The present survey concentrated on transformation of the S type beam (polarization direction in a horizontal plane perpendicular to the beam) to N type or longitudinally polarized beams. Transformation of S to $\overline{S}$ (reversal) was also considered because reversing polarization with a snake reverses the correlation between the polarization and phase space. This correlation can be large for a proton beam from lambda decays. Transforming S to either N or $\overline{N}$ and either L or $\overline{L}$ would be useful for the same reason.

The methods could easily be applied to other cases, including polarization from lambda decays wherein the nominal S type beam has about a seven percent longitudinal component due to relativistic change of frame of reference effects.

The following assumptions were made in the search:

(1) in the high energy limit ,the field integral required for 90° spin rotation is a fixed value of about 27.4 kG M independent of momentum. The magnitude of spin rotation is proportional to the field integral. In the laboratory frame along a fixed direction:

$$\mathrm{d}\boldsymbol{s} = \boldsymbol{s} \times \left(\frac{G}{2}\boldsymbol{B}_{/\!/} + \left[1 + \gamma\ \frac{G-2}{2}\right]\ \boldsymbol{B}_{\perp}\right) \cdot \left(\frac{0.03}{|p|}\right)\Delta l\ .$$

In the laboratory frame along the beam direction:

$$\mathrm{d}\boldsymbol{s} = \boldsymbol{s} \times \left[\frac{G}{2}\boldsymbol{B}_{/\!/} + \gamma\left(\frac{G-2}{2}\right)\boldsymbol{B}_{\perp}\right] \cdot \left(\frac{0.03}{|p|}\right)\Delta l\ .$$

(In m, GeV c^{-1}, kG).

(2) No cases were considered involving spin rotations of more than 90° in a single magnet, although some configurations may have two adjacent magnets with the same field direction.

(3) No cases were considered which produced a net change in the direction of the beam.

(4) Most of the cases considered (ultimately, all of the interesting cases) had complete momentum recombination and no net displacement of the beam.

The following cases were considered: (cases A through G involve orthogonal rotations).

(A) UP, LEFT, DOWN, RIGHT, DOWN, RIGHT, UP, LEFT;

(B) UP, LEFT, DOWN, RIGHT, RIGHT, DOWN, LEFT, UP;

(C) LEFT, UP, RIGHT, DOWN, RIGHT, DOWN, LEFT, UP;

(D) LEFT, UP, RIGHT, DOWN, DOWN, RIGHT, UP, LEFT.

Cases (E), (F) and (G) do not have complete momentum recombination.

(E) LEFT, UP, RIGHT, UP, RIGHT, DOWN, LEFT, DOWN;

(F) LEFT, UP, UP, RIGHT, DOWN, DOWN, RIGHT, LEFT;

(G) UP, UP, LEFT, DOWN, DOWN, RIGHT, RIGHT, LEFT.

Case (S) is the equivalent of half of Steffan's snake and case (T) is the mirror image of (S) as (C) is the image of (A), etc. The differences between (S) and (T) become non-trivial when non-orthogonal bend planes are considered.

(S) LEFT, SPACE, UP, RIGHT, RIGHT, DOWN, DOWN, LEFT, SPACE, UP;

(T) UP, SPACE, LEFT, DOWN, DOWN, RIGHT, RIGHT, UP, SPACE, LEFT.

The bending planes which are designated as LEFT–RIGHT or UP–DOWN can actually have any orientation with respect to each other and with respect to the initial or final polarization direction. In order to investigate all possible angles we could define cases designated as $-Z5$ which would be parameterized by two angles which could be the angles of each bend plane from some reference or the angle between planes and some overall rotation of the system with respect to incident or final polarization.

Investigation of all possible angles even in three degree steps would require more computer time than is warranted at present. We have instead investigated several cases which appear to have reasonable symmetry properties and a single angle parameterization.

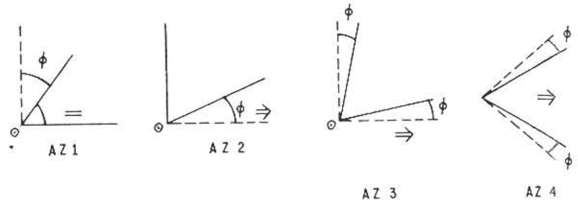

Fig. 1. Definition of angles in configurations $-Z1$, $-Z2$, $-Z3$ and $-Z4$.

In case $-Z1$ (fig. 1) the plane established by the first magnet is rotated about the Z axis by some angle ϕ. In case $-Z2$ the plane established by the second magnet is rotated. Case $-Z3$ is defined such that the planes are symmetric about a 45° plane between S and N directions. In case $-Z4$ the snake is rotated so that the two planes are symmetric about the incident beam polarization (X axis) and the angle between the two planes is adjusted.

In the following example the bend planes were not orthogonal and quotation marks indicate which nominal bend plane was rotated.

(AZ2) UP, "LEFT", DOWN, "RIGHT", ETC.

(BZ2) UP, "LEFT", DOWN, "RIGHT", ETC.

(CZ2) LEFT, "UP", RIGHT, "DOWN", ETC.

(DZ2) LEFT, "UP", RIGHT, "DOWN", ETC.

(SZ2) LEFT, SPANCE, "UP", RIGHT, RIGHT, "DOWN, DOWN", LEFT, SPACE, "UP"

(TZ2) UP, SPACE, "LEFT", DOWN, DOWN, "RIGHT, RIGHT", UP, SPACE, "LEFT"

3. Solutions found

Solutions which are continuous in precession angle are shown in figs. 3–8. Discrete solutions are tabulated.

For 90° rotations in orthogonal directions the following solutions were known previously and found by the program.

Fig. 2. Top and side view of snake C.

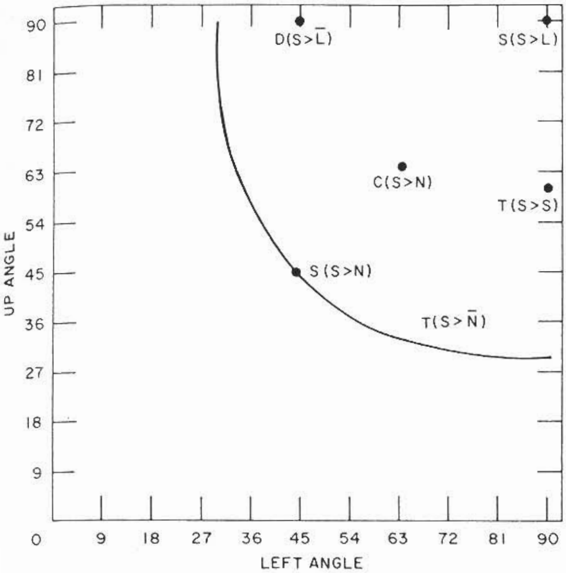

Fig. 3. Solutions for simple S and T snakes and for C and D snakes for precession far from 90°.

Case A transforms S to L;
Case B leaves the direction unchanged;
Case C transforms S to N;
Case D reverses S polarization;
Case E transforms S to N;

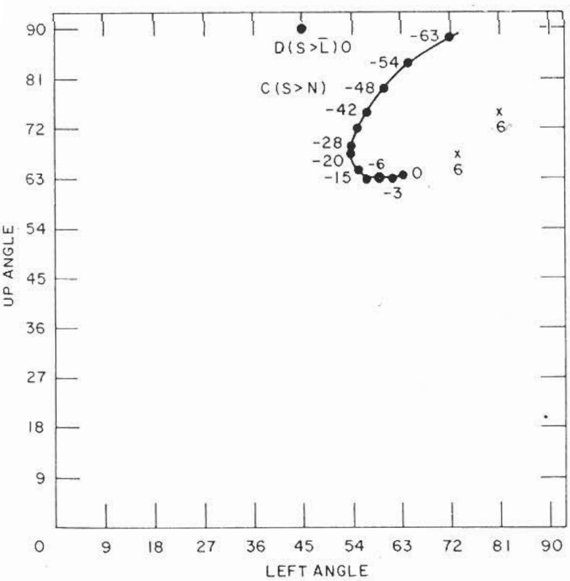

Fig. 4. Solutions for CZ1 and DZ1 snakes. The rotation about the Z axis is noted for several points of the continuum solution. Crosses indicate an area where a solution almost exists.

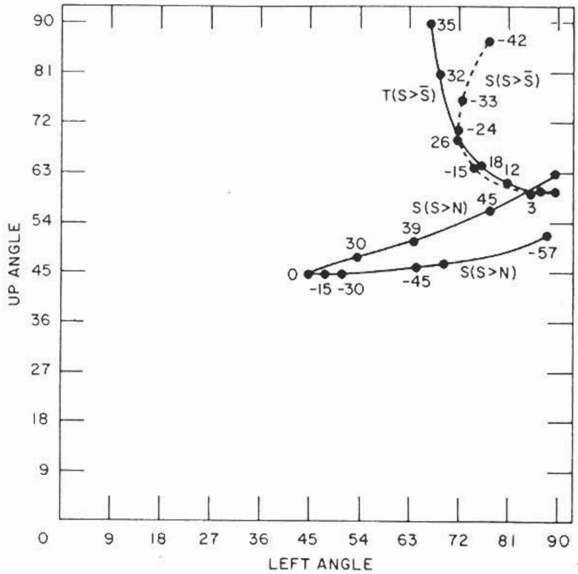

Fig. 5. Solutions for SZ1 and TZ1 snakes. The rotation about the Z axis is noted periodically.

CaseF transforms S to −L;
Case G transforms S to −N;
Case S transforms S to −L;
Case T transforms S to N.
For 45° rotations in orthogonal directions the fol-

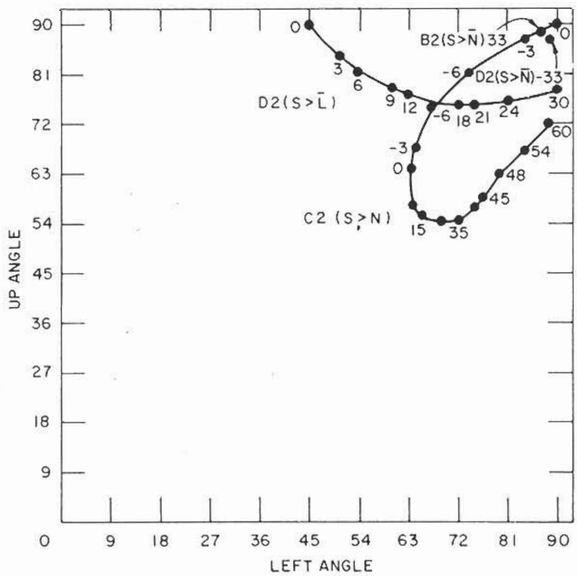

Fig. 6. Solutions for BZ2, CZ2, DZ2 snakes for the S polarization input.

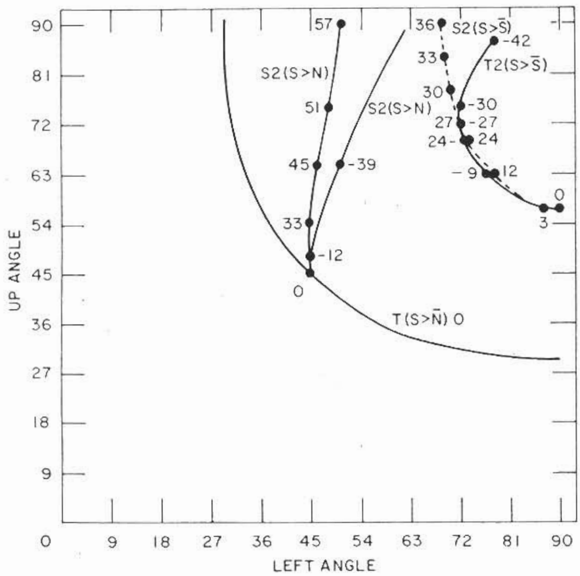

Fig. 7. Solutions for cases SZ2 and TZ2.

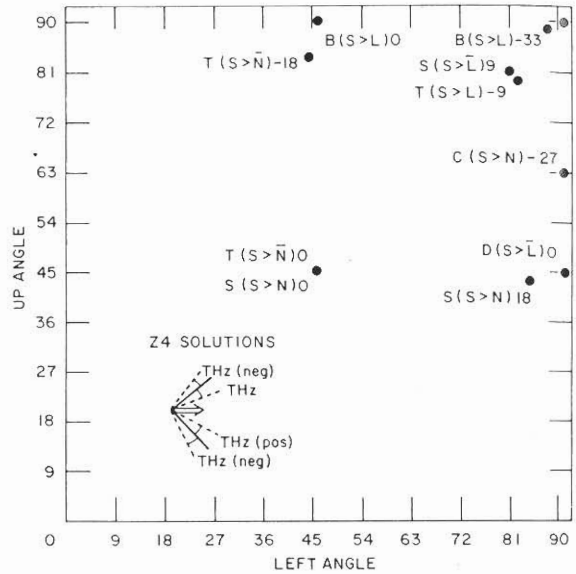

Fig. 8. The $Z4$ solutions with the Z rotation ϕ noted.

lowing solutions were known and found by the program:

Case S transforms S to N;

Case T transforms S to –N.

For orthogonal rotations the following new solutions were found:

Case C (63.6° horz., 63.6° vert.) transforms S to N;

Case D * (45° horz., 90° vert.) transforms S to –L;

Case T (90° horz., 60° vert.) transforms S to –N;

Case S (90° horz., 60° vert.) transforms S to –S;

Case T (90° horz., 60° vert.) transforms S to –S;

Case T (30° horz., 90° vert.) transforms S to –N.

For non-orthogonal bending planes many new solutions were found and are shown in figs. 2 to 8. Some examples are noted here:

(Bend planes rotated clockwise with respect to nominal planes, looking upstream into the beam.)

Case CZ1. Has a continuum of solutions transforming S to N for plane rotations from –63° to 0°. The polarization rotation angles required range from 90° and 78° for the –63° plane rotation from orthogonal to 63° and 63° for the orthogonal Case C.

Case DZ1. Has a solution for a plane rotation of around 20° ± 3° which transforms S to –L. The spin rotations are near 87° and 45°.

Case CZ2. Has a continuum of solutions for plane rotations from 0° to +63° which transforms S to N. The polarization rotations are near 54° and 66° for a bend plane rotation of 20°.

Case DZ2. Has a continuum of solutions which transform S to –L. For a plane rotation near 20° the polarization rotations are near 75° and 72°.

For an external proton beam the only solution which transforms S to L with significantly less than 27 kG M for all magnets in DZ2 which uses about 22.8 kG M maximum. A set of superconducting magnets with half the magnets rotated 72° from the vertical might be difficult to reconfigure for individual experiments but it appears that both N and L polarization directions can be obtained from S polarization with such a system using CZ2 and DZ2 solutions.

Appendix A – path length

Some simplified calculations of path length for a few snake configurations are given.

Path length can be calculated in a simplified manner by assuming that all bends take place at the centers of the magnets and by finding the length of the straight line segments between bend points. Real-world considerations such as the spacing of the magnets necessary to accommodate coils, cryostats and vacuum pipes will be more significant than errors due to the type of calculation.

We consider 82.2 GeV c^{-1} for which the bend necessary for 90° spin precession is 10 mrad.

8 magnet C snake, 90° Rot., lattice space = l_0,
path length increase = $4 \times 10^{-4}\ l_0$.

5 magnet snake plus 2 verniers equivalent lattice space = l_0, momentum not recombined),
path length increase = $3 \times 10^{-4}\ l_0$.

Two 8 magnet C snakes, 63° Rot., lattice space = l_0,
path length increase = $4 \times 10^{-4}\ l_0$.

Two 8 magnet C snakes, 63° Rot., lattice space = $0.707\ l_0$,
path lengths increase = $2.95 \times 10^{-4}\ l_0$.

Two 8 magnet D snakes, 90° and 45° rotations, lattice spaces l_0, $\frac{1}{2}\ l_0$,
path length increase = $3.75 \times 10^{-4}\ l_0$.

Two 8 magnet S snakes, lattice space = $0.5\ l_0$,
path length increase = $1.5 \times 10^{-4}\ l_0$.

A 10 magnet Steffan snake, similar to the above path length increase = $1.69 \times 10^{-4}\ l_0$.

For an accelerator, snake C with 63° rotations plus an identical snake rotated 90° would reverse S type polarization with somewhat less increase in path length than in a single eight magnet snake or five magnet snake and with better properties in terms of momentum recombination and aperture.

For reversing both polarization components transverse to the beam while leaving the L component unchanged, Steffan's snake configuration appears to be the optimum. For reversing the L component and one transverse component while leaving the other transverse component unchanged, a snake made up of two $DZ2$ configurations back to back similar to a solution by A. Turrin [3] would be much better than three 180° rotations at angles 120° around the beam.

Appendix B – arrangement of magnets for a beam line

The minimal useful snake configuration would provide an N or L type beam starting with the S polarization direction as in the beamline utilizing lambda decays. The incident S beam could have the polarization reversed periodically but it would still be useful to be able to provide both N and $\bar{N}$ and L and $\bar{L}$ from S with a snake. Such a use of a snake would reverse the correlation between polarization and phase space in order to control systematic errors. The correlation could be large in a proton beam from lambda decays.

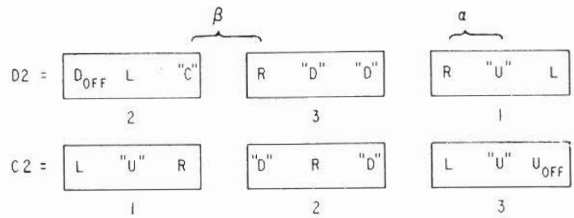

Fig. 9. Arrangement of cryostats for C2 and D2. Note that spacing α must be equal to spacing β for the beam position to remain constant and for momentum recombination.

It appears that no single arrangement of magnets can provide even the minimal function of transforming S to N and L if the magnets are limited to less than 90° spin precession. With an arrangement of 9 magnets in 3 cryostats we can obtain L and N using solutions $DZ2$ and $CZ2$ as shown in fig. 9. Presumably 10 magnets in 4 cryostats would allow $CZ2$, $DZ2$ and $SZ2$ configurations for N, L and $\bar{S}$ beams. We are presently trying to find an arrangement to provide all beam polarizations.

References

[1] D. Underwood et al., A Polarized Beam for the M3 Line, ANL-HEP-PR-78-05 (1978).

[2] Ya.S. Derbenev and A.M. Kondratenko, High Energy Physics With Polarized Beams and Polarized Targets (Argonne, 1978) ed. G.H. Thomas (AIP Conf. Proc. No. 51, 1979) p. 292.

[3] K. Steffen, DESY PET-78/11 (1978); A. Turrin, Polarization Eigenvector in High-Energy Accelerators Equipped With Siberian Snakes LNF/79/44(P), (1979): A. Turrin, Preliminary Analysis of Steffen's Magnet Arrangement for a Siberian Snake with small Orbit Displacement, LNF-78/54(R), (1978).