THEORY II:

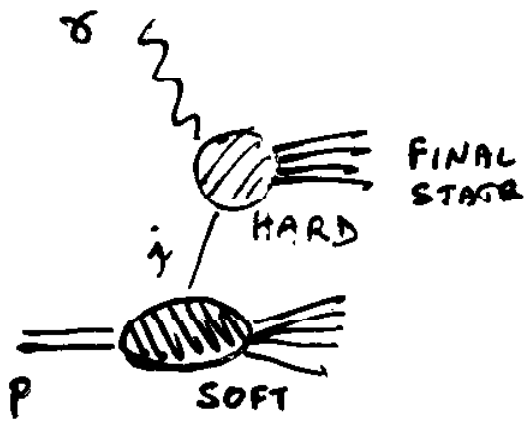
THE FRONTIERES OF PERTURBATIVE QCD

STEFANO FORTE

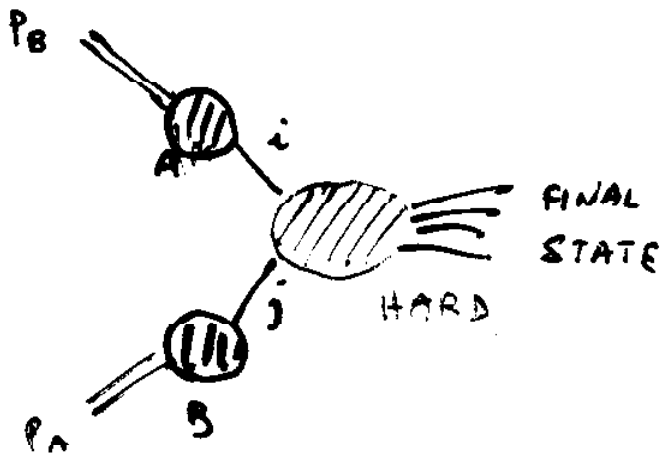
INFN TORINO
&
BARCELONA U.

ANATOMY OF A CALCULATION

DIS



$p\bar{p}$



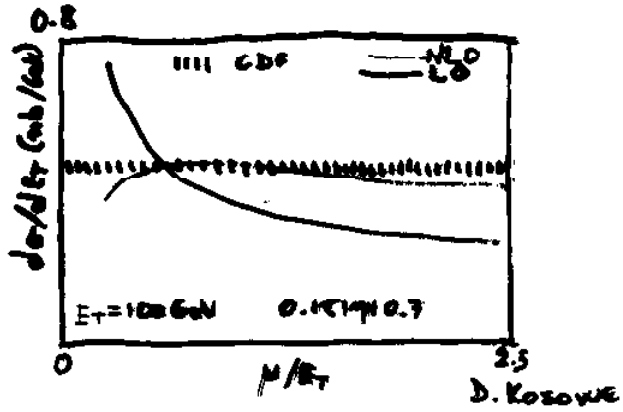
PROGRESS

- HARD AMPLITUDES WITH MANY PARTONS — Kosower, Bern, Magnea, del Duca
  - ↳ "STRING INSPIRED" METHODS
- HARD AMPLITUDES AT HIGH ORDERS — Beneke, Akhoury, Marchesini, Sterman
  - ↳ BEYOND LOG ACCURACY (RENORMALONS)
- OBSERVABLES & FINAL STATES — Tung, Rijken, Frixione
  - ↳ JETS, HEAVY QUARKS
- HARD & SOFT RESUMMATION — Berger, Sterman (Kidonakis), del Duca, Bell
  - ↳ LOGS $(1-x)$, LOGS $\frac{1}{x}$

MATRIX ELEMENT CALCULATIONS

- NEED NLO (AT LEAST!) TO REDUCE SCALE UNCERTAINTY

NL MATRIX ELEMENT COMPENSATES μ-DEP. DUE TO $\alpha(\mu)$



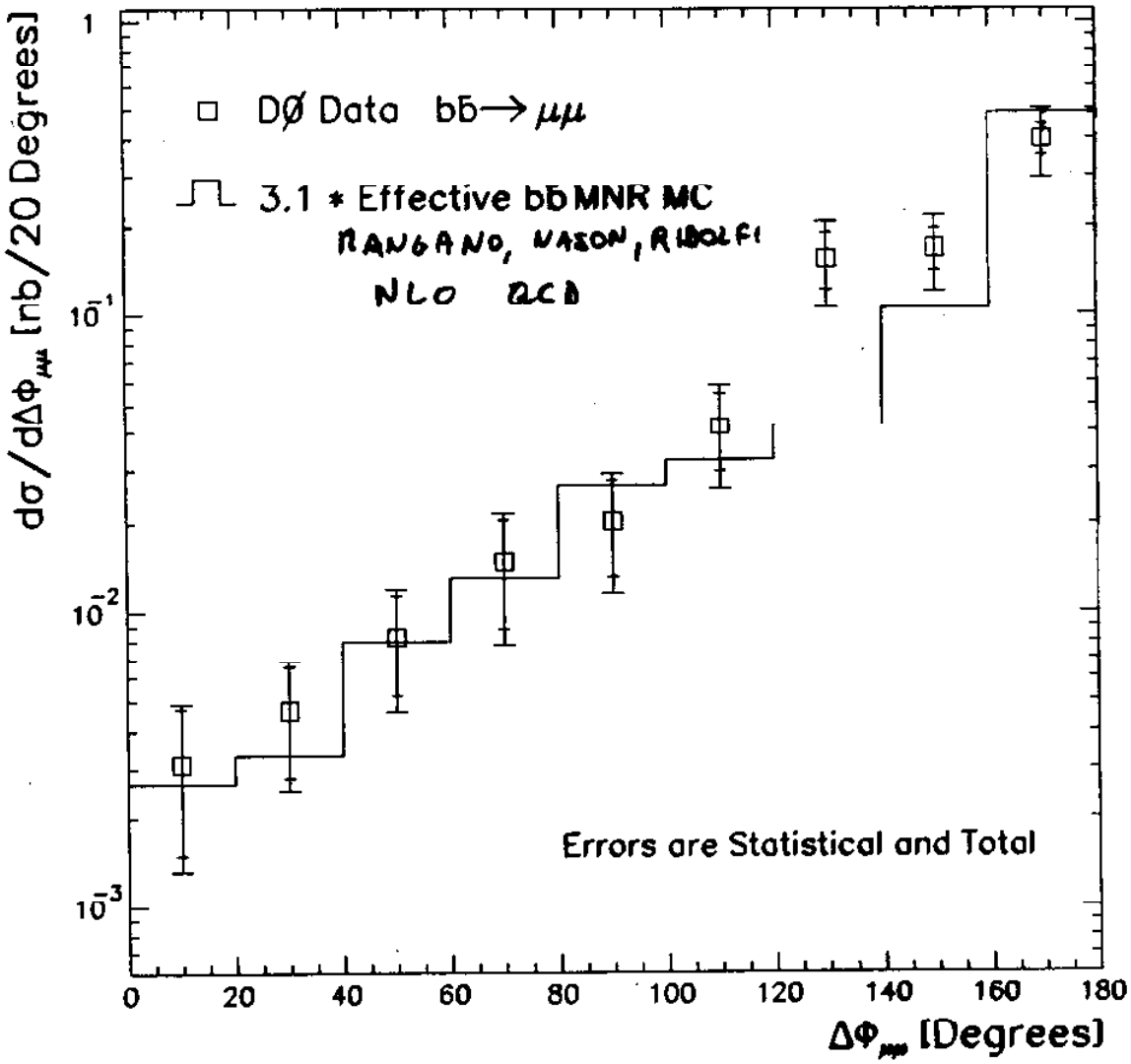
- AND TO DEFINE NONTRIVIALLY SOME OBSERVABLES (e.g. ANGULAR CORRELATION IN HQ PRODUCTION)

- NEED MANY PARTONS FOR APPLICATIONS (e.g. W, Z + jet production ⇝ background to top in $t\bar{t} \to 6$ jets)

→ MANY TERMS EVEN AT TREE LEVEL

→ MANY DIAGRAMS INCREASE RAPIDLY W/ PERTURBATIVE ORDER

DØ: $b\bar{b}$ AZIMUTHAL CORRELATION



$$
\begin{aligned}
&+ 4i\sigma_1^2 \cdot e_3^2 \cdot e_4 \cdot e_5^2 + 4i\sigma_1^2 \cdot e_3^2 \cdot e_4^2 \cdot e_5^2 - 4i\sigma_1^2 \cdot e_3^2 \cdot e_5^4 + 4i\sigma_1^2 \cdot e_3 \cdot e_4^3 \cdot e_5^2 - 4i\sigma_1^2 \cdot e_3 \cdot e_4^2 \cdot e_5^3 \\
&+ 4i\sigma_1^2 \cdot e_4^3 \cdot e_5^2 - 4i\sigma_1^2 \cdot e_4^2 \cdot e_5^3 \\
&+ 4i\sigma_1^2 \cdot e_3^2 \cdot e_4^2 \cdot e_5^2 + 4i\sigma_1^2 \cdot e_3^2 \cdot e_4 \cdot e_5^3 - 4i\sigma_1^2 \cdot e_3^2 \cdot e_5^4 + 4i\sigma_1^2 \cdot e_3 \cdot e_4^3 \cdot e_5^2 - 4i\sigma_1^2 \cdot e_3 \cdot e_4^2 \cdot e_5^3 \\
&+ 4i\sigma_1^2 \cdot e_4^3 \cdot e_5^2 - 4i\sigma_1^2 \cdot e_4^2 \cdot e_5^3 \\
&+ 4i\sigma_1^2 \cdot e_3^2 \cdot e_4^2 \cdot e_5^2 + 4i\sigma_1^2 \cdot e_3^2 \cdot e_4 \cdot e_5^3 - 4i\sigma_1^2 \cdot e_3^2 \cdot e_5^4 + 4i\sigma_1^2 \cdot e_3 \cdot e_4^3 \cdot e_5^2 - 4i\sigma_1^2 \cdot e_3 \cdot e_4^2 \cdot e_5^3 \\
&+ 4i\sigma_1^2 \cdot e_4^3 \cdot e_5^2 - 4i\sigma_1^2 \cdot e_4^2 \cdot e_5^3 \\
&- 4i\sigma_1^2 \cdot e_3^2 \cdot e_4^2 \cdot e_5^2 + 4i\sigma_1^2 \cdot e_3^2 \cdot e_4 \cdot e_5^3 - 4i\sigma_1^2 \cdot e_3^2 \cdot e_5^4 + 4i\sigma_1^2 \cdot e_3 \cdot e_4^3 \cdot e_5^2 - 4i\sigma_1^2 \cdot e_3 \cdot e_4^2 \cdot e_5^3 \\
&- 4i\sigma_1^2 \cdot e_4^3 \cdot e_5^2 - 4i\sigma_1^2 \cdot e_4^2 \cdot e_5^3 \\
&+ 4i\sigma_1^2 \cdot e_3^2 \cdot e_4^2 \cdot e_5^2 + 4i\sigma_1^2 \cdot e_3^2 \cdot e_4 \cdot e_5^3 - 4i\sigma_1^2 \cdot e_3^2 \cdot e_5^4 + 4i\sigma_1^2 \cdot e_3 \cdot e_4^3 \cdot e_5^2 - 4i\sigma_1^2 \cdot e_3 \cdot e_4^2 \cdot e_5^3 \\
&+

WER

$$A_5(+\,+\;+\;+\,+) = 0$$

$$A_5(-\,+\;+\;+\,+) = 0$$

$$A_5(-\,-\;+\;+\,+) = \frac{i\,\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}$$

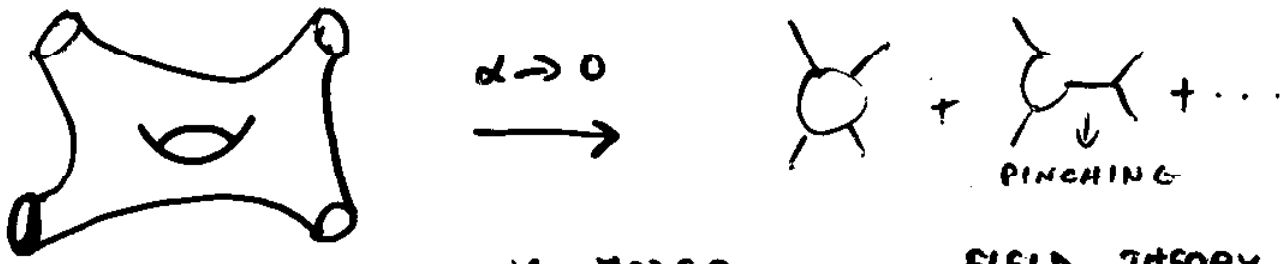
$$A_5(-\,+\;-\;+\,+) - \frac{i\,\langle 1\,3\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}$$

STRING-BASED METHODS

- FEYNMAN DIAGRAMS ARE NOT EFFECTIVE IN ORGANIZING A PERT. COMPUTATION

Bern, Kosower

STRING IDEA



MASSIVE STRING MODES DECOUPLE AS $M_{Pl} \to \infty$ → FIELD THEORY RECOVERED IN THE LIMIT

⇝ FT RULES FOR HELICITY AMPLITUDES

- NUMBER OF TERMS MINIMIZED
- CANCELLATIONS PERFORMED @ EARLY STAGE
- SYSTEMATIC STRUCTURE

→ UNITARITY BASED RULES AT ONE LOOP

Bern
Kosower
Dixon
Dunbar

- RECONSTRUCT AMPLITUDES FROM CUTS
- SOW TREE AMPLITUDES (NOT TREE DIAGRAMS

STRING USEFUL AS GUIDING ORGANIZATIONAL PRINCIPLE

RESULTS ...

TREE LEVEL IS A CLOSED CHAPTER:

- ANALYTIC RESULTS AVAILABLE TO ALL N (FOR CERTAIN HELICITIES) — Kosower, Parke-Taylor-Mangano, Berends-Giele
- NUMERIC RECURRENCE RELNS. TO ALL N

ONE LOOP:
- THE ONCE HEROIC 4-PARTON AMPLITUDE (Ellis + Sexton) IS NOW AVAILABLE IN POLARIZED CASE (Bern + Kosower; Kunszt, Signer, Trócsa)
- 5 PARTON AMPLITUDES DONE WITH "STRING METHODS
- NL LOG 1/x N-GLUON PRODUCTION (REAL LOOPS TO LIPATOV VERTEX) — del Duca

AND PROGRESS

- V + 4 PARTONS IN PROGRESS — Bern, Dixon, Dunbar, Weinzierl
  ($V + q\bar{q}q'\bar{q}'$ AVAILABLE)

TWO LOOPS
- TOY MODEL: N=4 SUSY QCD: — Bern
  EXPLICIT CONSTRUCTION OF AMPLITUDE IN TERMS OF SCALAR INTEGRALS (FROM CUTTING METHODS)
- CONJECTURE TO ALL LOOPS

FROM STRING
- TWO-POINT AMPLITUDE ($\to$ β-FUNCTION) ALMOST AVAILABLE — Magnea (+ di Vecchia, Lerda, Russo)

$\rightsquigarrow$ TWO LOOP RULES

STRING THEORY $\to$ STRUCTURE OF FIELD THEORY

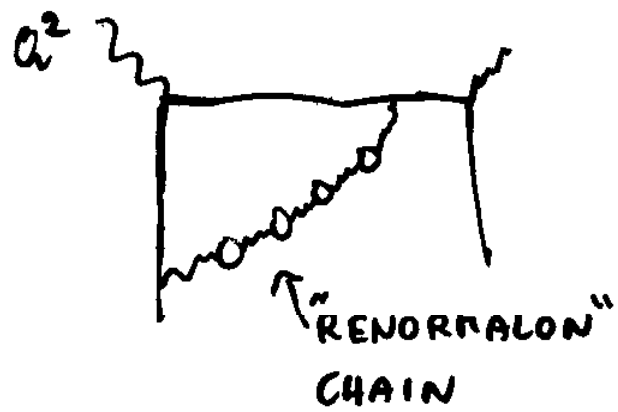
HIGH ORDERS →

- ONLY NLO AVAILABLE FOR QUANTITIES WITH MORE THAN ONE KIN. VARIABLE (SUCH AS STRUCTURE FUNCTIONS)

- BUT HIGHER ORDERS AVAILABLE FOR SINGLE OBSERVABLES (e.g. BJORKEN SUM RULE KNOWN TO NNNLO)

⇝ BUT PERTURBATIVE EXPANSION DIVERGES!

→ POWER CORRECTION.



$$C(\alpha) \sim \sum_n \alpha_s \int d^4k \left(\alpha_s(Q^2)\beta_0 \ln\frac{k}{a}\right) \times F(k, a^2)$$

$$\sim \sum c_n \alpha^{n+1}(a^2)$$

c_n GROW FACTORIALLY

⇝ $B(u) \equiv \sum \frac{c_n}{n!}\left(\frac{u}{-\beta_0}\right)^n$ BOREL RESUMMATION

$$C(\alpha) = \left(-\frac{1}{\beta_0}\right)\int_0^\infty dt\, e^{-t/\beta_0 \alpha_s}\, B(u) \quad ; t = \ln a^2/\Lambda^2$$

$B(u) \sim$ GEOMETRIC SERIES ⇝ $\frac{1}{1-u}$

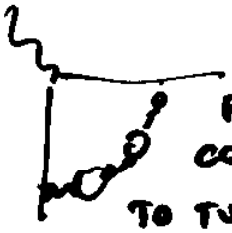
Λ^2 AMBIGUITY OF PERTURBATION THEORY

PHYSICS OF RENORMALONS

- AMBIGUITY MUST CANCEL IN PHYSICAL OBSERVABLES $\rightsquigarrow$ HANDLE ON HIGHER TWISTS

$$F_2(x,Q^2) = \int_x^1 \frac{dy}{y}\, C\left(\frac{x}{y}, Q^2\right) f(y,Q^2)$$

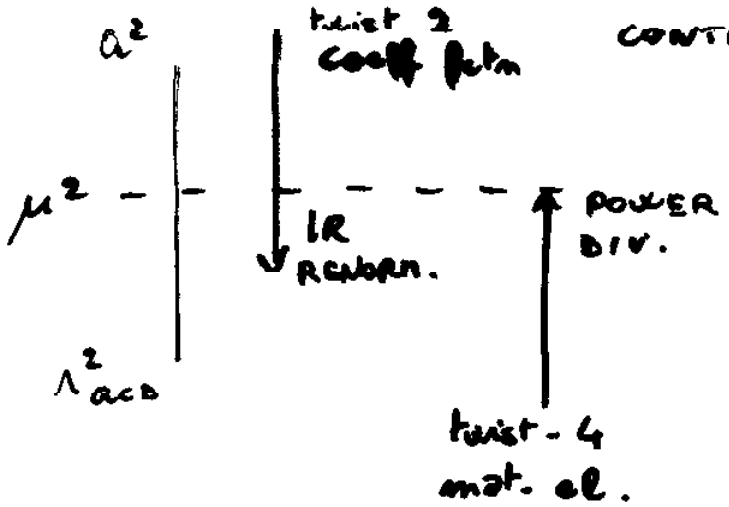
$$+ \frac{1}{Q^2} \int d\xi_1 d\xi_2\, C\,(x,\xi_1,\xi_2,Q^2)\, T(x,\xi_1,\xi_2,$$



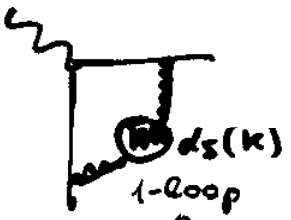
RENORM. CONTRIBUTION (IR) TO TWIST 2 $\longleftrightarrow$

POWER (UV) DIVERGENT CONTRIB. TO TWIST 4

Beneke



- RELATION TO IR REGION $\rightsquigarrow$ DISPERSIVE APPROACH
Marchesini
(+Dokshitzer, Webb



DISP. RELATION FOR COUPLING

$$\frac{\alpha(k^2)}{k^2} = -\int_0^\infty d\lambda^2 \frac{\alpha_{eff}(\lambda^2)}{(\lambda^2+k^2)^2} \rightsquigarrow \text{IR FINITE}$$

$$\hookrightarrow C(\alpha) = \int_0^\infty d\lambda^2\, \mathcal{F}(\lambda^2)\, \alpha_{eff}(\lambda^2)$$

$\hookrightarrow$ 1-loop diagram with g. mass

NON-ANALYTIC TERMS IN $\mathcal{F}$ $\leftrightarrow$ POLES IN $\mathcal{M}$

PHYSICS FROM RENORMALON.

- ASSUME X-DEP OF RENORMALON MATCHES THAT OF H.T. (BUT LOG. DEPENDENCE IS DIFFERENT: CAN ONLY BE TRUE AT 1 SCALE)

→ CAN PREDICT TWIST-4 SHAPE

- ASSUMPTION MEANS HT IS DOMINATED BY ITS UV CUTOFF PIECE

  ⇝ NO INSIGHT ON HADRON STRUCTURE

OR LEADING X-DEP. OF F_L AT LARGE-X Akhoury FROM THAT OF F_2 BUT NEGLECTED TERMS AS IMPORTANT !!

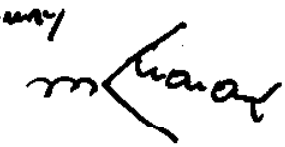
→ CAN CALCULATE POWER CORRNS. TO PROCESSES WITHOUT OPE (DRELL-YAN, EVENT-SHAPES)

DIS, DRELL-YAN : $1/Q^2$ (OR LESS)

EVENT SHAPES SET OBSERVABLES : $1/Q$

→ UNIVERSALITY? CORRNS. TO EVENT SHAPES
→ MULTIPLE PARTON EMISSION
→ UNIVERSAL (COUNTEREXAMPLES...)

Akhoury



$\to k_s \frac{\Lambda}{Q}$

k_{s_1}/k_{s_2} = calculable

APPROXIMATELY JUSTIFIABLE IN DISPERSIVE APPROACH Marchesi

Ex: DIFFERENTIAL THRUST DISTRIBUTION

Dokshitzer Webber

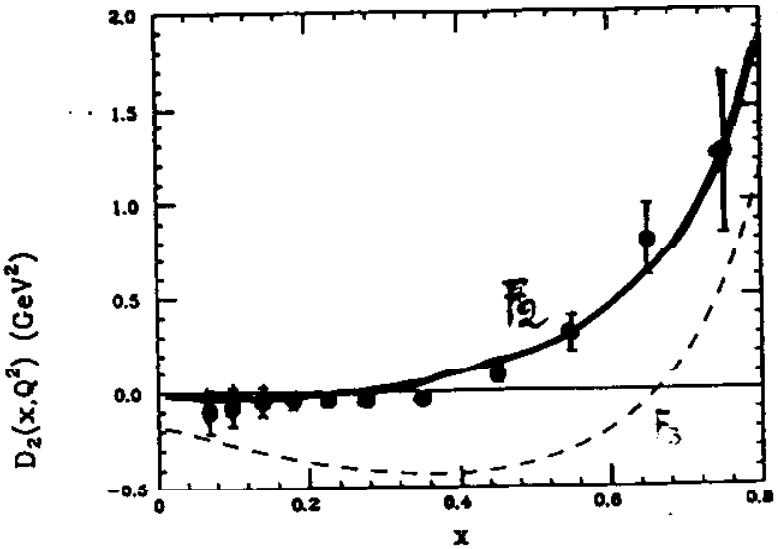
$$\frac{1}{\sigma}\frac{d\sigma}{dT}(T) = \frac{1}{\sigma}\frac{d\sigma}{dT}(T-\delta T) \; ; \quad \delta T = \frac{A}{\chi Q}$$

A ← KNOWN

χ ← CALCULABLE IN DISP. APPROA

MIRACLES...

Dokshitzer, Marchesini, Webber;
Dasgupta, Webber



renormalon calculation
↓ fit to data

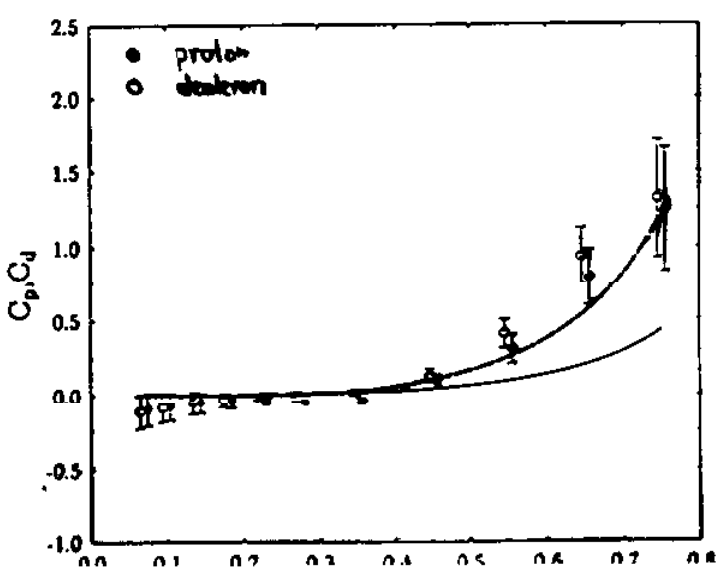
Figure 1: Coefficients of $1/Q^2$ contributions to F_2 (solid), and to F_1 and F_3 (dashed).

x - dependence of twist-4 corrections normalized to twist-2

$$F_i(x,Q^2) = F_i^{\text{twist-2}}(x,Q^2)\left[1+\frac{C_i(x)}{Q^2}\right]$$

DIS

Maul et al.



PROGRESS ...

- SUCCESFUL PHENOMENOLOGY DESPITE INSECURE FOUNDATION — Beneke, Akhoury, Marchesini

- FRAGMENTATION PROCESSES — Beneke (+ Braun, Magnea)
  - $\frac{\Lambda}{Q}$ CORRNS. (FROM RESUMMATION OF FORMAL $\left(\frac{\Lambda}{Qx}\right)^2$ EXPANSION)
  - DISPERSIVE & HO METHODS DISAGREE (BUT DIFFERENCE SMALL IN PRACTICE)
  - QUARK/GLUON FRAGMENTATION DISAGREE IN NORM. : BAD FOR UNIVERSALITY

- RENORMALONS FROM THE LATTICE — Marchesini (+ di Renzo, Onofri)

$$\langle \alpha_s F^2 \rangle_{LGT} = \langle \alpha_s F^2 \rangle^{PT}_{LGT} + \Lambda^4/Q^4$$

↑ from MC simulation ↑ from computed PT to $O(\alpha_s^8)$

(But also evidence of $\frac{\Lambda^2}{Q^2}$: not in OPE : from $\alpha_s(Q)$?

- NONLOCAL OPERATORS — Sterman (+ Korchemsky, Oderd.)

  "WEB" STRUCTURES

  → SYSTEMATIC IDENTIFICATION OF REGIONS ASSOCIATED TO STRONG COUPLING → POWER CORRNS.

HEAVY QUARKS IN THE FINAL STATE

THE QUARK MASS AS A LARGE SCALE

- ALLOWS PERTURBATIVE DETERMINATION OF VARIOUS OBSERVABLES — Ridolfi
- REQUIRES UNDERSTANDING OF TWO-SCALE PROCESSES — Tung

HQ PRODUCTION

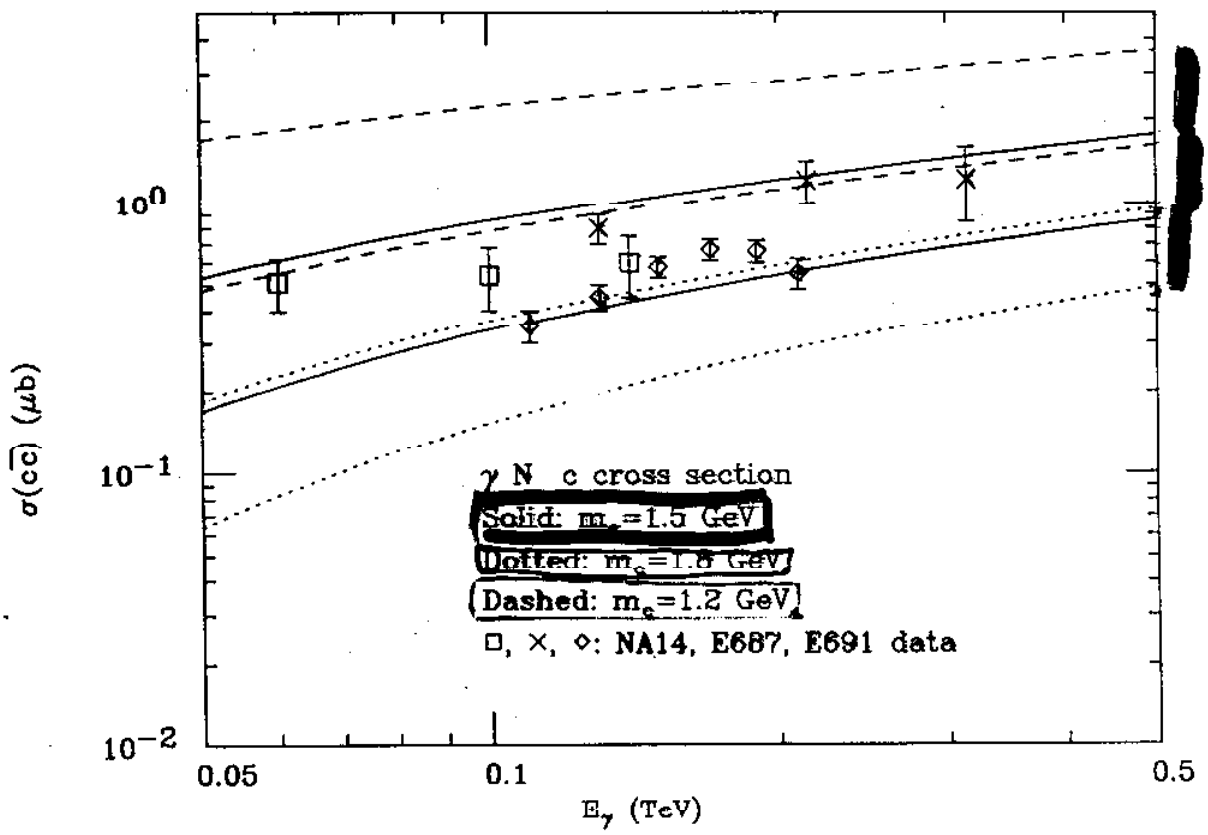
Ridolfi (+ Mangano, Frixione, Nason)

DETAILED PHENOMENOLOGY AVAILABLE AT NLO FOR TOTAL X-SECT, SINGLE-INCLUSIVE p_T, AZIMUTH CORR., ...

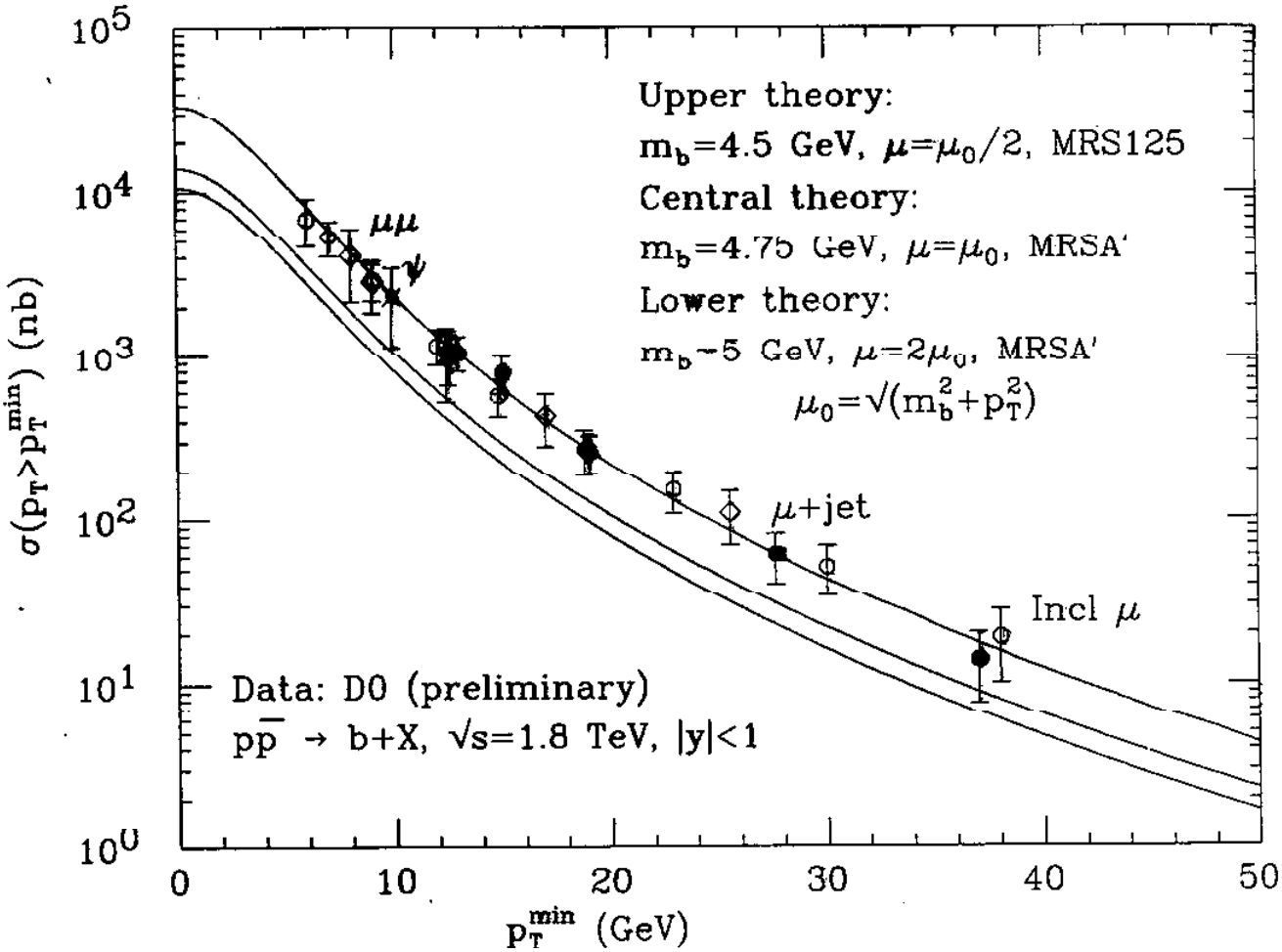
CHARM: GOOD AGREEMENT BUT LARGE UNCERTAINTIES (STILL ...) FOR TOTAL X-SECT

BOTTOM: GOOD AGREEMENT WITH p_T SHAPE BUT LARGE NORMALIZATION UNCERTAINTY

TOTAL $c\bar{c}$ CROSS-SECTION in γN COLLISIONS



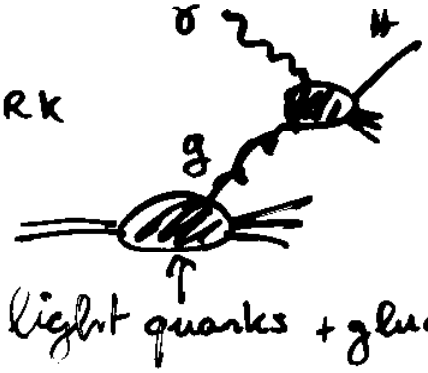
DØ DATA vs. THEORY



THE HQ PARTON OXYMORON

Tung
Roberts

HEAVY QUARK



BAD IF $m_H \gg Q$
logs of $\frac{Q}{m_H}$ are present

PARTON

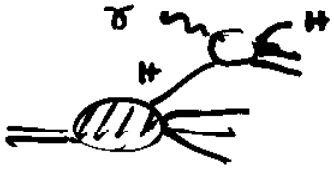
$m_H = 0$
$\overline{MS}$ subtr.

BAD IF $m_H \sim Q$
threshold poorly described

all quarks with $m_H < Q$ + gluons

$m \neq 0$ FACTORIZATION

Witten
Collins, Wilczek, Zee
Aivazis, Collins,
Olness, Tung



$m_H \neq 0$
mass subtr.

- PDFS EVOLVE ACCORDING TO USUAL ($\overline{MS}$) EV. EQ.
- NO $\ln \frac{Q}{m_H}$ AFTER MASS SUBTR.
- REDUCES TO HQ PARTON WHEN $m_H \sim Q$, $m_H \gg Q$

MOST EFFICIENT SCHEME DEPENDS ON PROCESS, OBSERVABL

DEFINING QCD OBSERVABLES

Frixione

- CROSS SECTION IS PLAGUED BY IR DIVERGENCES DUE TO MASSLESS PARTICLES WHICH CANCEL, BUT PATTERN COMPLICATED

  ⇝ DIFFICULT TO IMPLEMENT:
  - CALCULATE DIVERGENCE ANALYTICALLY
  - CANCEL IT & INTEGRATE NUMERICALLY FINITE PART

SLICING : ISOLATE SINGULAR REGION IN PHASE-SPACE
INTEGRATION ⇝ FINITE PART: LARGE CANCELLATIONS

SUBTRACTION: DECOMPOSE INTEGRALS INTO SINGLE-SINGULAR PARTS & SUBTRACT ⇝ CUMBERSOME WITH MANY PARTON.

IMPROVED SUBTRACTION X-SECT. IS EXPRESSED IN TERMS OF S-FUNCTIONS, CAN BE SUBTRACT WITHOUT FURTHER DECOMPOSITION
(Frixione, Kunszt, Signer; Catani, Seymour)

also Klasen, Kramer
Mirkes, Zeppenfeld ⇝ CALCN. OF $e^+e^- \to 4$ jets (Signer, Dixon)

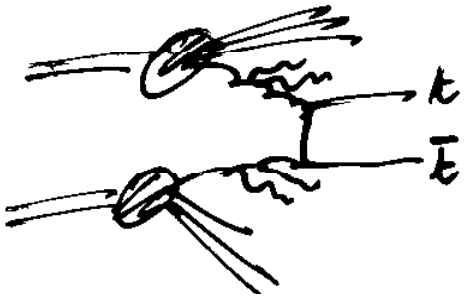
$pp \to 3$ parton IR safe
$ep, \gamma p \to$ " " " " Frixione, Ridolfi, Nason
$\bar{p}p \to$ jets w. HQ

RESUMMATION

Berger
Sterman
(Kidonakis)

SOFT GLUONS

- EMISSION OF SOFT PARTICLES CAN HAVE LARGE EFFECTS CLOSE TO THRESHOLD for top prod.



$s \to 4m_t^2$

BECAUSE EVEN SMALL ENERGY SPENT CAN SUPPRESS THE X-SECT SIGNIFICANTLY

- SUPPRESSION CAN BE NEGATIVE (AND IT IS IN $\overline{MS}$ & DIS SCHEME) BECAUSE PART OF THE RADIATION IS ALREDY INCLUDED IN STR. FCTN.

- LOGS OF $1 - \frac{4m^2}{s}$ ARE PRODUCED & MUST BE RESUMMED

- RESUMMATION IS PERFORMED BY EXPONENTIATION IN MOMENT SPACE (Berger, Contopanagos; Catani, Mangano, Nason, Laenen, Smith, van Neerven Trentadue)

TREATMENT OF SUBLEADING CORRNS. & DEFINITION OF THE PERT. REGION DIFFER BETWEEN AVAILABLE COMPUTATIO

EFFECT IS SMALL ($\gtrsim 10\%$) AT TEVATRON

BUT GROWS RAPIDLY WITH ENERGY

$\sigma^{t\bar{t}}$ @

1.8 TeV	$5.52^{+0.03}_{-0.42}$ pb	~10% enh.
2.0	7.56	~40% enh
4.0	46.0	

- EFFECT IS SMALL AT TEVATRON, BUT GROWS RAPIDLY WITH ENERGY

$\sqrt{s}$ (TeV)	σ (pb)	enh
1.8	5.52	~10%
2.0	7.56	~40%
3.0	22.40	

- NLO CORRECTIONS COMPUTED IN MOMENT SPACE FOR $qq \to Q\bar{Q}$ Kidonakis, Sterman

MOMENT INVERSION & COMPUTATION OF $gg \to Q\bar{Q}$ IN PROGRESS

RESUMMATION

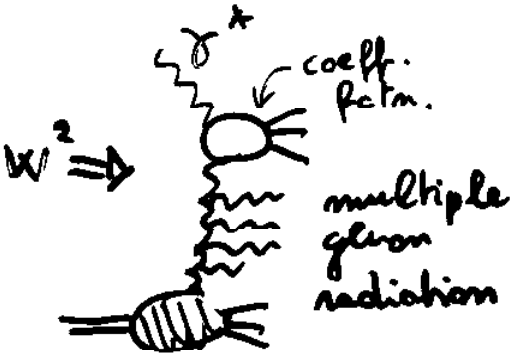
HIGH ENERGY

del Duca
Ball

- HIGH C.M. ENERGY $\rightsquigarrow$ NEW LARGE SCALE

  $\rightsquigarrow$ NEW LOGS

IN D.I.S. $W^2 = Q^2 \frac{(1-x)}{x} \rightsquigarrow \ln \frac{1}{x}$



SPLITTING FUNCTIONS ACQUIRE EXTRA $\ln \frac{1}{x}$ AT EACH ORDER

$$P_{gg} \sim P_{gq} \sim \frac{1}{x} \ln \frac{1}{x}^{m} \frac{1}{m!} \alpha^{m}$$

$$P_{qg} \sim P_{qq} \sim \frac{1}{x} \ln \frac{1}{x}^{m} \frac{1}{m!} \alpha^{m+1} \text{ subleading}$$

- ALL LEADING NONTRIVIAL ORDER KNOWN

  FOR SPL. FCTN. + COEFF. FCTN. THANKS TO $k_\perp$ FACTORIZATION (Ciafaloni, Catani, Hautmann)

$$F_2(x, Q^2) = \int \frac{dy}{y} \int \frac{dk_\perp^2}{k_\perp^2} C\left(\frac{x}{y}, \frac{Q^2}{k_\perp^2}; \alpha(\mu^2)\right) f(y, k_\perp$$

  + BFKL EQN

  (SUMS leading $\ln \frac{1}{x}$) $\rightsquigarrow$ GENERALIZ. TO QUARK (Catani + Hautmann

- NL ANOMALOUS DIMENSION IN GLUON SECTOR

  (ALMOST) AVAILABLE (Fadin et al., Lipatov + Fadin, del Duca, Camici + Ciafaloni)

  BUT COEFF. FUNCTION MISSING!

STRETCHING THE BOUNDARIES

Ball + S.F.

- LEADING $\ln\frac{1}{x}$ SUMMED "BY HAND" IF JUST ADDED TO AP SPLITTING FUNCTIONS
- RESULT PHENOMENOLOGICALLY USELESS! (AT BEST)

⇝ RG IMPROVEMENT OF DOUBLE FACTORIZATION

$$s^2 \simeq \frac{\Lambda^2}{x} \qquad F(Q^2, s^2) = \int\frac{dl^2}{l^2}\int\frac{dk^2}{k^2}\, c\left(\frac{s^2}{l^2}, \frac{Q^2}{k^2}; \alpha(\mu^2)\right) f\left(\frac{l^2}{\mu^2}, \frac{k^2}{\mu^2}; \mu^2\right)$$

MASS FACT. (LO) ENERGY FACT.

$$\sigma_N\left(\frac{Q^2}{\mu^2}; \mu^2\right) = C_N\left(\frac{Q^2}{\mu^2}; \alpha(\mu^2)\right) F_N(\mu^2); \quad \sigma_n\left(\frac{s^2}{\mu^2}; \mu^2\right) = C_n\left(\frac{Q^2}{\mu^2}; \alpha(\mu^2)\right) F_n($$

$$+ O(1/Q^2) \qquad + \text{const} + O(1/s^2)$$

$$\sigma_N = \int\frac{dx}{x}\, x^N \sigma(x) \qquad \sigma_n = \int\frac{dk^2}{k^2}\left(\frac{\Lambda^2}{Q^2}\right)^n \sigma(Q^2)$$

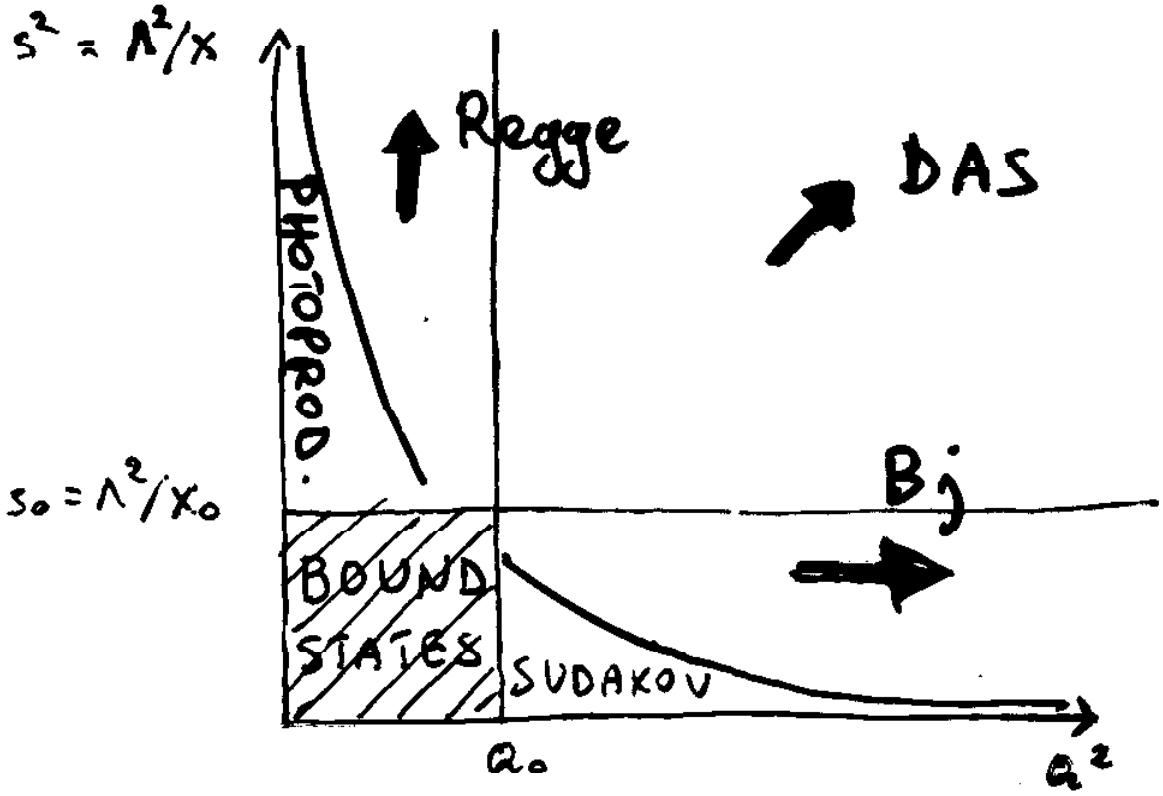
$$F_N = \int\frac{dk_\perp^2}{k_\perp^2} f_N \qquad F_n = \int\frac{dl^2}{l^2} f_n$$

⇝ LONGITUDINAL EVOLUTION EQUATION

$$\frac{d}{d\ln s^2/\Lambda^2} F_n(s^2) = \gamma_n(\alpha(s^2))\, F_n(s^2)$$

$$\gamma_n = \frac{C_A\, \alpha_s}{\pi}\, \chi(n) \rightsquigarrow P(x) = \frac{1}{|1-x|_+}$$

- SPLITTING FCTN. COINCIDES WITH LIPATOV KERNEL



- HIGH ENERGY ASYMPTOTICS UNIVERSAL (ONLY ONE PARTON)

- REGGE LIMIT → LOG BEHAVIOR $\left(\ln 1/x\right)^{4\alpha^2 \ln 2}$
- PHOTOPROD. LIMIT → SOFT (UNITARY) SOFTER THAN ANY POWER OF $\ln \frac{1}{x}$
- DAS LIMIT → SMOOTH MATCHING TO DAS

EVOLUTION REMAINS PERTURBATIVE AT SMALL Q^2 IF S IS LARGE (?!!)

"IMAGINE A

WORLD

WHERE THERE IS

NO POMERON" R. Ball

ONLY PQCD