# A unified BFKL + GLAP description of DIS

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BFKL (leading $\log\frac{1}{x}$) $\rightarrow$ $F_2 \sim x^{-\lambda}$

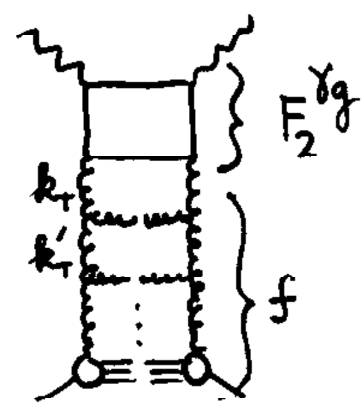
too steep? Ball, Forte
Bojak, Ernst

Thorne: Scheme indep (collinear) analysis of observable
$\rightarrow F_2$ data favour $\log\frac{1}{x}$ summ$^n$ in $\gamma$, coefft. fns.

<u>Natural framework for observables at small $x$</u>

- unintegrated gluon $f(x, k_T^2)$
- $k_T$ factoriz$^n$ theorem

e.g. $F_2 = F_2^{\gamma g} \otimes f$




$$F_2(x, Q^2) = \int_x^1 \frac{dz}{z} \int \frac{dk_T^2}{k_T^2} \, F_2^{\gamma g}(z, k_T^2, Q^2) \, f\left(\frac{x}{z}, k_T^2\right)$$

Starting from BFKL:

solve only in pert. domain, $k_T^2 > k_0^2$
include kinematic constraint
include GLAP

## BFKL eq. for unintegrated gluon:

ONLY SOLVE FOR $k_T^2 > k_0^2$

$$f(x,k_T^2) = f^{(0)} +$$

$$+\bar{\alpha}_s k_T^2 \underbrace{\int_x^1 \frac{dz}{z}\int \frac{dk_T'^2}{k_T'^2}}\left[\frac{f\left(\frac{x}{z},k_T'^2\right) - f\left(\frac{x}{z},k_T^2\right)}{|k_T'^2 - k_T^2|} + \frac{f\left(\frac{x}{z},k_T^2\right)}{\left[4k_T'^4 + k_T^4\right]^{1/2}}\right]$$

$\bar{\alpha}_s \equiv \frac{3}{\pi}$

non-pert. input $\simeq \int_0^{k_0^2} \frac{dk_T'^2}{k_T'^2} f\left(\frac{x}{z},k_T'^2\right) = \frac{x}{z}\, g\left(\frac{x}{z},k_0^2\right)$

kinematic constraint $\Big\}$ $|k'|^2 \simeq k_T'^2 \implies \boxed{k_T'^2 < \frac{k_T^2}{z}}$

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Andersson, Gustafson, Samuelsson

Recipe

(i) Add DGLAP evoln. of gluon

$$\bar{\alpha}_s(k^2)\int_x^1 \frac{dz}{z}\left(\tfrac{1}{6}\, z P_{gg}(z) - 1\right)\underbrace{\frac{x}{z}\, g\left(\frac{x}{z},k^2\right)}_{\frac{x}{z} g\left(\frac{x}{z},k_0^2\right) + \int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2} f\left(\frac{x}{z},k'^2\right)}$$

(ii) Simplify above I.R. $\int dk'^2$

$$k^2\int_0^{k_0^2}\frac{dk'^2}{k'^2|k'^2-k^2|} f\left(\frac{x}{z},k'^2\right) \simeq \int_0^{k_0^2}\frac{dk'^2}{k'^2} f\left(\frac{x}{z},k'^2\right) \equiv \frac{x}{z}\, g\left(\frac{x}{z},k_0^2\right)$$

(iii) Add quark contribution to gluon evolution

$$\frac{\alpha_s(k^2)}{2\pi}\int_x^1 dz\, P_{gq}(z) \underset{\uparrow\ \text{flavor}}{\sum} \left(\frac{x}{z},k^2\right)$$

## Modified eq. for gluon $f(x,k_T)$

input

$$f(x,k_T^2) = \frac{\alpha_S}{2\pi}\int_x^1 dz\, P_{gg}(z)\, \frac{x}{z}\, g\left(\frac{x}{z},k_0^2\right) +$$

$$+\bar{\alpha}_S k_T^2 \int_x^1 \frac{dz}{z}\int_{k_0^2} \frac{dk_T'^2}{k_T'^2}\left[\frac{f\left(\frac{x}{z},k_T'^2\right)\theta\left(\frac{k_T^2}{z}-k_T'^2\right) - f\left(\frac{x}{z},k_T^2\right)}{|k_T'^2-k_T^2|} + \frac{f\left(\frac{x}{z},k_T^2\right)}{[4k_T'^4+k_T^4]^{1/2}}\right]$$

$g \leftarrow g$ (BFKL + k.c)

$$+\bar{\alpha}_S\int_x^1\frac{dz}{z}\left(\frac{z}{6}P_{gg}-1\right)\int_{k_0^2}^{k_T^2}\frac{dk_T'^2}{k_T'^2} f\left(\frac{x}{z},k_T'^2\right) + \frac{\alpha_S}{2\pi}\int_x^1 dz\, P_{gq}(z)\,\Sigma\left(\frac{x}{z},k_T^2\right)$$

$g \leftarrow g$ (GLAP−1) $\qquad\qquad$ $g \leftarrow q$ singlet

1st term

$$\tilde{f}^{(0)}(x,k^2) = \underbrace{f^{(0)}(x,k^2)}_{\text{neglect}} + \frac{\alpha_s}{2\pi}\int_x^1 dz\, P_{gg}(z)\,\frac{x}{z}\, g\left(\frac{x}{z},k_0^2\right)$$

Note (i) Above eqn. involves $f(x,k^2)$ <u>only in the pert. region</u> $k^2 > k_0^2$

Input requires only integrated gluon at $k_0^2$

(ii) Above eqn. reduces to conventional DGLAP evoln. in leading $\log k^2$ approx.

→ e.g. not necessary to parametrise $f(x,k^2)$ in the non-pert. region

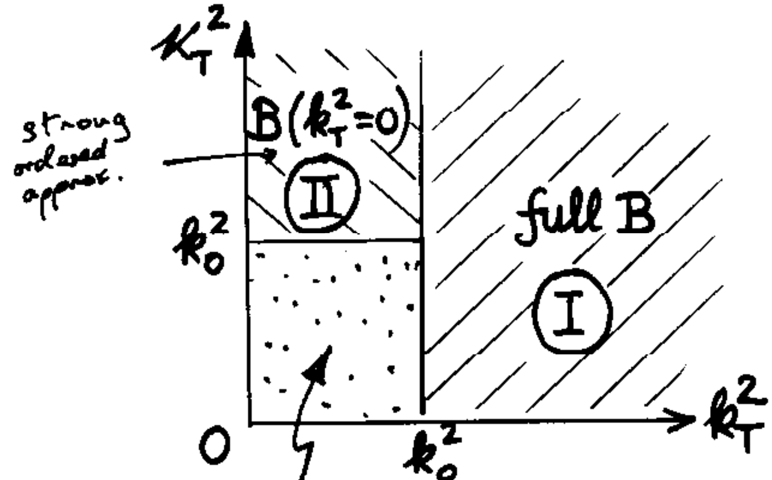
Contrast with previous (AKMS)

$$f(x, k_T^2 < k_0^2) = \frac{k_T^2}{k_T^2+k_a^2} \cdot \frac{k_0^2+k_a^2}{k_0^2}\, f(x,k_0^2)$$

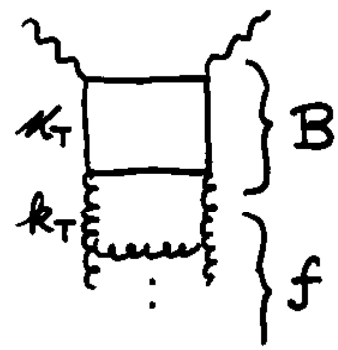
$k^2$ adjustable parameter

## Sea driven by gluon $\to q\bar{q}$

$$S_q(x,Q^2) = \int_x^1 \frac{dz}{z} \underbrace{\int \frac{dk_T^2}{k_T^2}\, B(z,k_T^2,Q^2)}_{k_T^2,\ \kappa_T^2 \text{ integ}^{\underline{n}}} f\left(\frac{x}{z},k_T^2\right)$$




$$S^{(0)}(x) = C_{\mathbb{P}}\, x^{-0.08}(1-x)^8$$



INPUT

$xg(x,k_0^2) = N(1-x)$

## Eq. for quark singlet $\Sigma(x,Q^2)$

$$\Sigma(x,Q^2) = S^{(0)}(x) + \sum_q \int_x^1 \frac{dz}{z} B(z, k_T^2=0, Q^2; m_q^2)\ \frac{x}{z} g\left(\frac{x}{z}, k_0^2\right)$$

sea ← gluon II

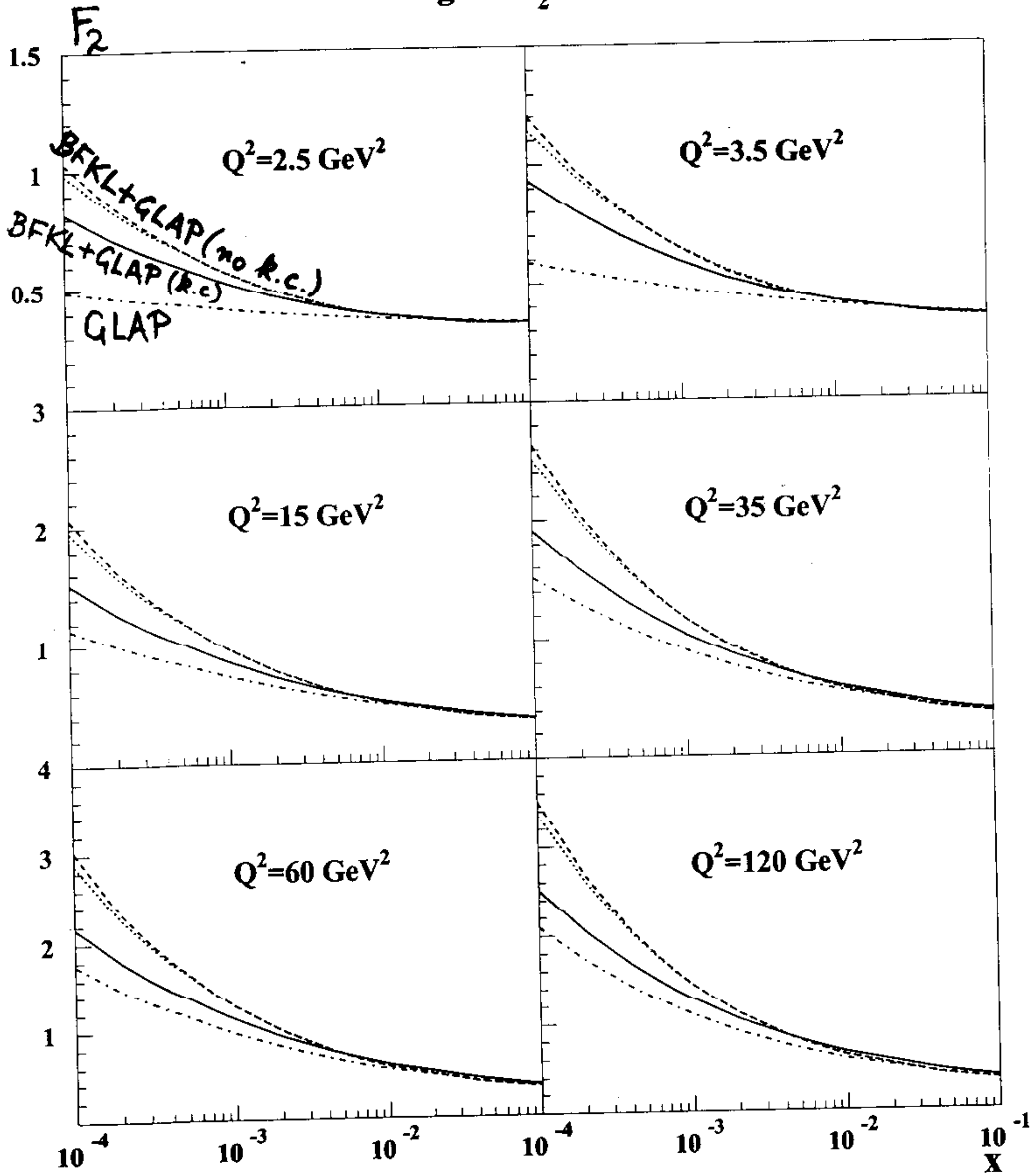
$$+ \sum_q \int_{k_0^2}^{\infty} \frac{dk_T^2}{k_T^2} \int_x^1 \frac{dz}{z} B(z, k_T^2, Q^2; m_q^2)\, f\left(\frac{x}{z}, k_T^2\right)$$

sea ← gluon I

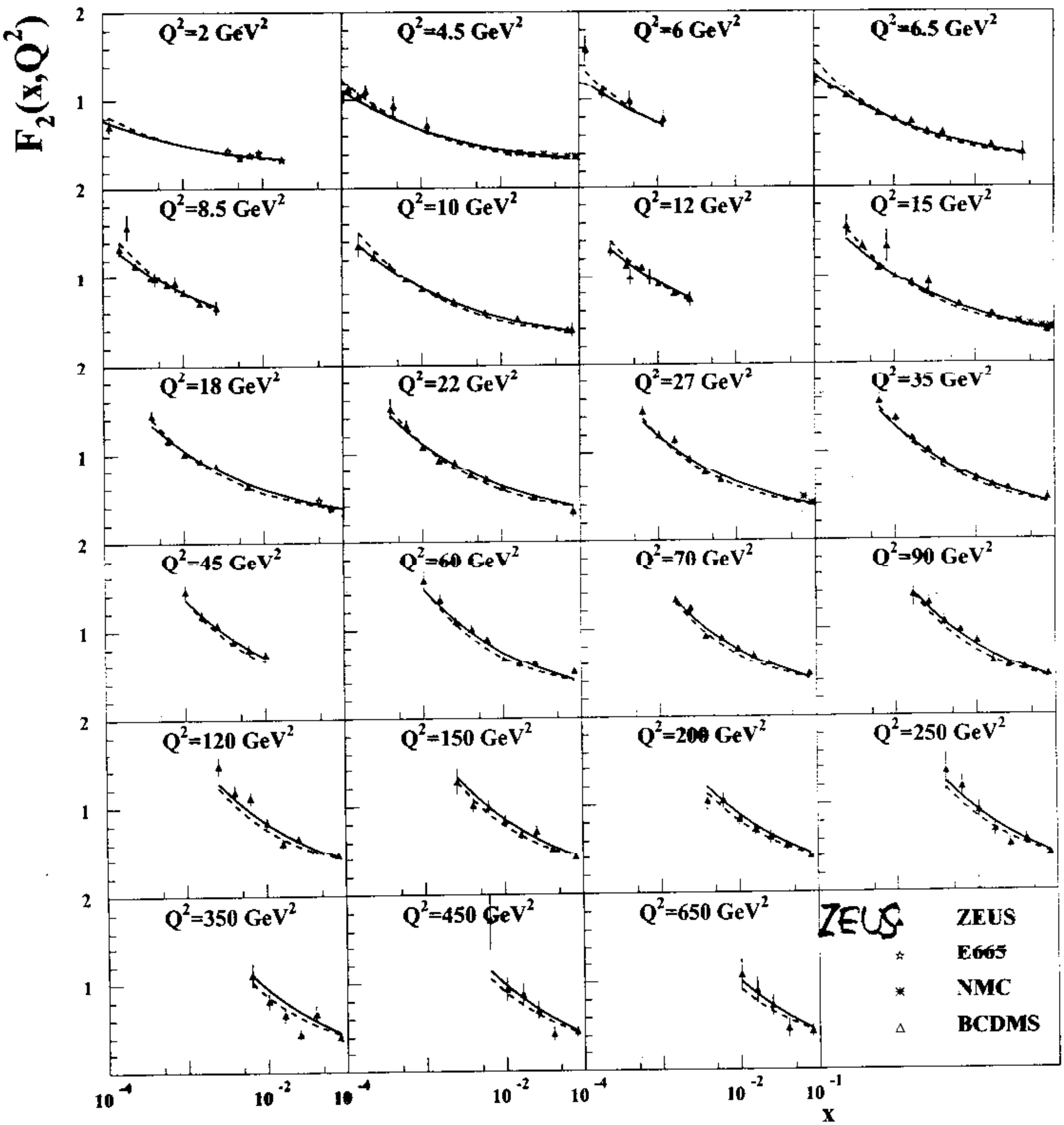
$$+ \sum_q \int_{k_0^2}^{Q^2} \frac{dk_T^2}{k_T^2}\ \frac{\alpha_s}{2\pi} \int_x^1 dz\, P_{qq}(z)\, S_q\left(\frac{x}{z}, k_T^2\right) \quad + \quad V(x,Q^2)$$

sea ← sea $\qquad\qquad$ valence

**Fig.5.: $F_2^{p(l)}$ vs x**




- - - - - BFKL+GLAP (without kin. constraint)
· · · · · · GLAP(g) + full $k_T$-fac (q)
——— BFKL+GLAP (with kin. constraint)
-·-·-·- GLAP



$F_2(x,Q^2)$
$Q^2$=2 GeV$^2$
$Q^2$=4.5 GeV$^2$
$Q^2$=6 GeV$^2$
$Q^2$=6.5 GeV$^2$
$Q^2$=8.5 GeV$^2$
$Q^2$=10 GeV$^2$
$Q^2$=12 GeV$^2$
$Q^2$=15 GeV$^2$
$Q^2$=18 GeV$^2$
$Q^2$=22 GeV$^2$
$Q^2$=27 GeV$^2$
$Q^2$=35 GeV$^2$
$Q^2$=45 GeV$^2$
$Q^2$=60 GeV$^2$
$Q^2$=70 GeV$^2$
$Q^2$=90 GeV$^2$
$Q^2$=120 GeV$^2$
$Q^2$=150 GeV$^2$
$Q^2$=200 GeV$^2$
$Q^2$=250 GeV$^2$
$Q^2$=350 GeV$^2$
$Q^2$=450 GeV$^2$
$Q^2$=650 GeV$^2$
ZEUS
ZEUS
E665
NMC
BCDMS
x

$F_L$

From $k_T$ - factorisation

$$F_L(x,Q^2) = \frac{\alpha_s(Q^2)}{\pi}\left[\frac{4}{3}\int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2 F_2(y,Q^2) + \sum_q e_q^2 \int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2\left(1-\frac{x}{y}\right) y g(y\right.$$

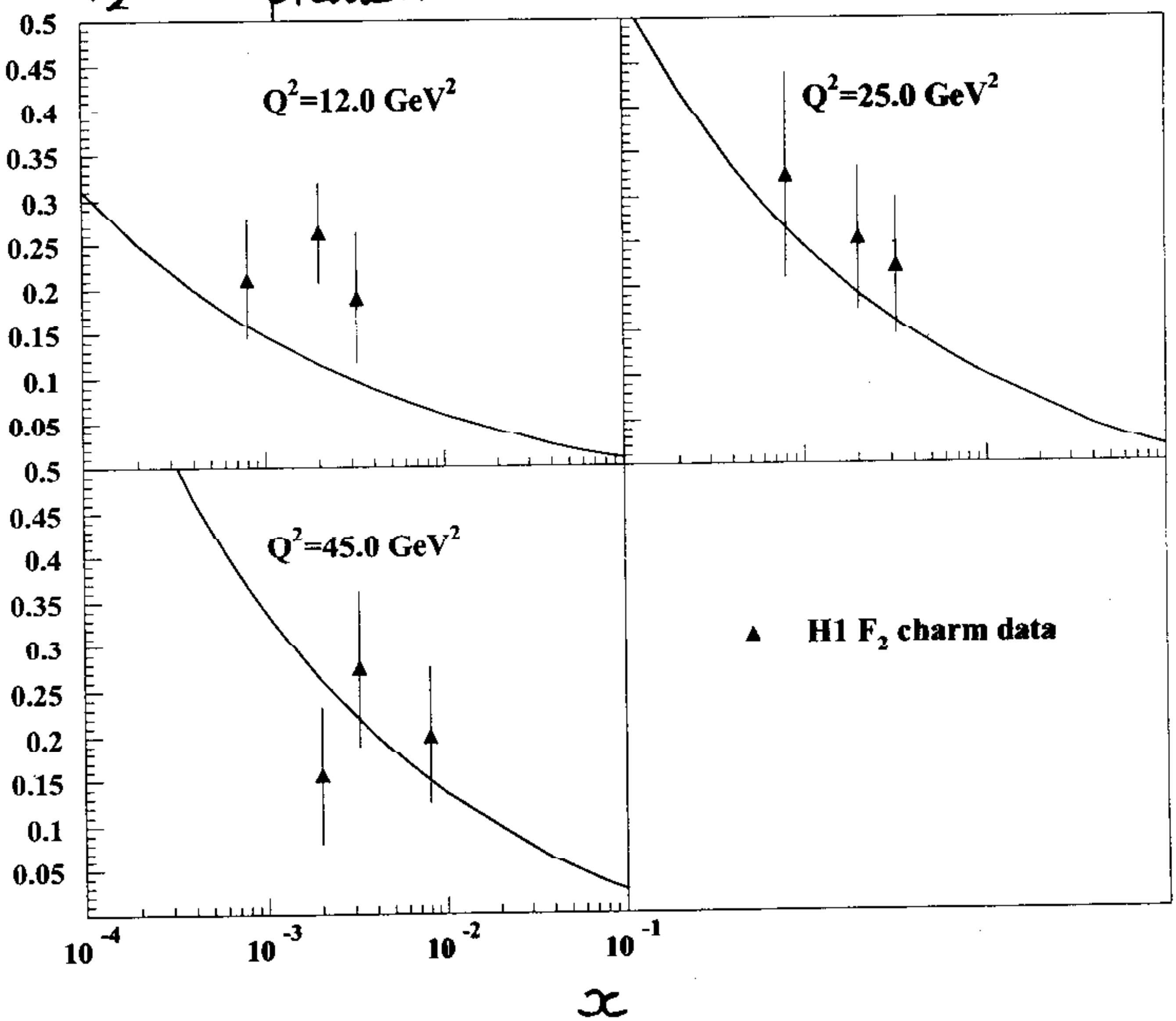
$$+ \sum_q e_q^2 \frac{Q^4}{\pi}\int_{k_0^2} \frac{dk^2}{k^4}\int_0^1 d\beta\, \beta^2(1-\beta)^2 \int d^2\kappa'\, \alpha_s \left[\frac{1}{D_1} - \frac{1}{D_2}\right]^2 f\left(\frac{x}{z}, k^2\right)$$

$D_i$, $\kappa'$ variables depending on $k, \beta, Q^2, x$

Thus predictions for $F_L$

Likewise predictions for $F_2^{charm}$

$F_2^{charm}$ predict$^n$. **Fig.6.: $F_2^c$ vs x**



# Unified BFKL + GLAP descriptions

$k_T$ factorization versus collinear factoriz$^n$

BFKL $f$ + GLAP $\xrightarrow[\text{reduce}]{}$ GLAP with

$\gamma$, coeff. fn. $\to \sum_n c_n \left(\frac{\alpha_s}{\omega}\right)^n$

$\omega = N-1$

- $\boxed{\text{Input}}$

pert. (BFKL) occurs at all $Q^2$

$F_2(x, k_0^2) =$ pert. + non pert

$F_2(x, k_0^2) =$ only non pert.

- $\boxed{\alpha_s}$

fixed $\alpha_s$ in BFKL $\longrightarrow$ $g(\omega) \sim \left(\frac{Q^2}{k_0^2}\right)^{\gamma\left(\frac{\alpha_s}{\omega}\right)}$ OK

theoretical support $\downarrow$ $\qquad$ RG $\downarrow$

(local) running $\alpha_s(k_T^2)$ in BFKL $\xrightarrow[\text{tech.}]{\text{saddle pt}}$ $\exp \int_{k_0^2}^{Q^2} \frac{dq^2}{q^2} \gamma\left(\frac{\alpha_s(q^2)}{\omega}\right)$

?? but not valid at arbitrarily small $x$.

- $\boxed{\text{kinematic constraint}}$

$|k'|^2 \simeq k_T'^2$

V. easy to implement

possible in principle

( [illegible] )

| $k_T$ fac | collinear fac |
| --- | --- |
| • All twists | only leading twists conventionally retained |
| • Simplicity/universality of $f$ | Observable-by-observable |
| $\text{Kernel}^{BFKL}$, $F_2^{\gamma g} \sim O(\alpha_s)$ | $\sum c_n \left(\alpha_s \log \frac{1}{x}\right)^n$ |
| $\int_{\text{phase space}}^{\text{entire } k_T^2} \to \sum \left(\alpha_s \ln \frac{1}{x}\right)^n$ | |

## CONCLUSIONS

- Natural framework for small $x$ observables
  → unintegrated gluon $f(x, k_T^2)$ + $k_T$ factorization

- Solved an equation for $f(x, k_T^2)$ which incorporates
  - (i) BFKL (+ kinematic constraint)
  - (ii) GLAP
  - (iii) 'flat' input

- Excellent 2-parameter description of $F_2$

  Encouraging in having consistent formalism which, with minimum non-pert. (flat) input gives perturbative description of the rise of $F_2$ at small $x$.

  BFKL/GLAP compts. decided by dynamics.

BFKL + GLAP fits to $F_2$

2 parameters

— with kin. constraint, $\chi^2$ comparable to MRS } $xg(x, Q^2=1) = 1.57(1-x)^{2.5}$

--- without kin. c., $\chi^2$ significantly worse } $xg(x, Q^2=1) = 0.85(1-x)^{0.5}$