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The Evolution of the Singlet Structure Functions
of the Proton and the Photon at Small x

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1. Basic Formalism
2. Non-Singlet Combinations
3. $F_2^p(x, Q^2)$ and $F_L^p(x, Q^2)$
4. $F_2^\gamma(x, Q^2)$
5. Conclusions

1. Basic Formalism

$$
\begin{aligned}
q_{\mathrm{NS},i}^{\pm} &= q_i \pm \bar{q}_i - \frac{1}{N_f}\sum_{r=1}^{N_f}(q_r \pm \bar{q}_r)\,, \\
q_{\mathrm{NS}}^{\mathrm{val}} &= \sum_{r=1}^{N_f}(q_r - \bar{q}_r)\,,
\end{aligned}
$$

$$
\boldsymbol{q}_S = \begin{pmatrix} \Sigma \\ g \end{pmatrix} \;,\; \Sigma \equiv \sum_{r=1}^{N_f}(q_r + \bar{q}_r)\,.
$$

$$
F_i(x,Q^2) = \sum_{r=1}^{2N_f} a_{ir}\, c_{i,r}(x,Q^2) \otimes q_r(x,Q^2) + a_{ig}\, c_{i,g}(x,Q^2) \otimes g(x,Q^2)\,,
$$

$$
\begin{aligned}
\frac{\partial\, q_{\mathrm{NS}}^{\pm}(x,Q^2)}{\partial \ln Q^2} &= P_{\mathrm{NS}}^{\pm}(x,\alpha_s) \otimes q_{\mathrm{NS}}^{\pm}(x,Q^2)\,, \\
\frac{\partial\, \boldsymbol{q}_{\mathrm{S}}(x,Q^2)}{\partial \ln Q^2} &= \boldsymbol{P}_{\mathrm{S}}(x,\alpha_s) \otimes \boldsymbol{q}_{\mathrm{S}}(x,Q^2)\,.
\end{aligned}
$$

$$
\frac{da_s}{d\ln Q^2} = -\sum_{k=0}^{\infty} a_s^{k+2}\beta_k\,.
$$

$$
\begin{aligned}
P^{\pm}(x,a_s) &= \sum_{l=0}^{\infty} a_s^{l+1} P_l^{\pm}(x)\,, \\
\boldsymbol{P}(x,a_s) &\equiv \begin{pmatrix} P_{qq}(x,a_s) & P_{qg}(x,a_s) \\ P_{gq}(x,a_s) & P_{gg}(x,a_s) \end{pmatrix} = \sum_{l=0}^{\infty} a_s^{l+1} \boldsymbol{P}_l(x)\,, \\
c_{i,j}(x,Q^2) &= \delta(1-x)\delta_{jq} + \sum_{l=1}^{\infty} a_s^l c_{ij,l}(x)\,.
\end{aligned}
$$

$$
\begin{aligned}
&\int_0^1 dx\, P_l^{-}(x) = 0\,, \\
&\int_0^1 dx\, x \sum_i P_{ij,l}^{\mathrm{unpol.}}(x) = 0\,,
\end{aligned}
$$

$$\gamma_L(N, a_S) = -2 \begin{pmatrix} 0 & 0 \\ C_F/C_A & 1 \end{pmatrix} \gamma_L(N, \alpha_s) \,.$$

$$\gamma(N, a_s) = -2 \int_0^1 dx \, x^{N-1} \boldsymbol{P}(x, a_s) \,.$$

$$\rho \equiv \frac{N}{\overline{\alpha}_s} = 2\psi(1) - \psi(\gamma_L) - \psi(1 - \gamma_L) \equiv \chi[\gamma_L],$$ LIPATOV et al. eq.

$$\gamma_L(N, a_s) = \sum_{k=1}^{\infty} g_k^{(0)} \left(\frac{\overline{\alpha}_s}{N} \right)^k \,.$$

$$\gamma_{\rm NL}(N, \alpha_s) = -2 \begin{pmatrix} \frac{C_F}{C_A}[\gamma_{\rm NL} - \frac{8}{3} a_s T_F] & \gamma_{\rm NL} \\ \gamma_{gq,\rm NL} & \gamma_{gg,\rm NL} \end{pmatrix},$$

$$\begin{aligned} \gamma_{\rm NL}^{\rm DIS}(N, a_s) &= \gamma_{\rm NL}^{Q_0}(N, a_s) R(\gamma_L) = T_F \frac{\alpha_s}{3\pi} \frac{2 + 3\gamma_L - 3\gamma_L^2}{3 - 2\gamma_L} \frac{[B(1 - \gamma_L, 1 + \gamma_L)]^3}{B(2 + 2\gamma_L, 2 - 2\gamma_L)} R(\gamma_L) \\ &= 2 \frac{\alpha_s}{3\pi} T_F \sum_{k=1}^{\infty} g_k^{qg,(1)} \left(\frac{\overline{\alpha}_s}{N} \right)^k, \end{aligned}$$ CATANI HAUTMANN 94

with $B(x, y)$ denoting the Beta function and

$$R(\gamma) = \left[\frac{\Gamma(1 - \gamma)\chi(\gamma)}{\Gamma(1 + \gamma)\{-\gamma\chi'(\gamma)\}} \right]^{1/2} \exp \left[\gamma\psi(1) + \int_0^\gamma d\zeta \, \frac{\psi'(1) - \psi'(1 - \zeta)}{\chi(\zeta)} \right] \,.$$

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$$\gamma_{gg,\mathrm{NL}}^{q\bar{q},Q_0} = -\alpha_s \frac{\chi_1^{(\mathrm{s})}(\gamma_L) + \chi_1^{(\mathrm{ns})}(\gamma_L)}{\chi'(\gamma_L)} - \frac{C_F}{C_A}\gamma_{qg,\mathrm{NL}}^{Q_0} .$$

$$\begin{aligned}
\alpha_s\chi_1^{(\mathrm{s})}(\gamma_L) &= \frac{2}{\gamma_L^2}\frac{C_F}{C_A}\gamma_{qg,\mathrm{NL}}^{Q_0}(\gamma_L) \\
\alpha_s\chi_1^{(\mathrm{ns})}(\gamma_L) &= \frac{\alpha_s N_f}{6\pi}\left\{\frac{1}{2}\left[\chi'(\gamma_L) + \chi^2(\gamma_L)\right] - \frac{5}{3}\chi(\gamma_L)\right\} - \frac{1}{\gamma_L^2}\frac{C_F}{C_A}\gamma_{qg,\mathrm{NL}}^{Q_0}(\gamma_L) .
\end{aligned}$$

$$\begin{aligned}
\gamma_{gg,\mathrm{NL}}^{q\bar{q},Q_0} = & -\frac{N_f\alpha_s}{6\pi}\Bigg\{1 + \frac{23}{6}\frac{\bar{\alpha}_s}{N} + \left[\frac{71}{18} - \frac{\pi^2}{6}\right]\left(\frac{\bar{\alpha}_s}{N}\right)^2 + \left[\frac{233}{27} - \frac{13}{36}\pi^2 - 8\zeta(3)\underline{\frac{C_F}{C_A}}\right]\left(\frac{\bar{\alpha}_s}{N}\right)^3 \\
& + \left[\frac{1276}{81} - \frac{71}{108}\pi^2 + \frac{79}{3}\zeta(3) - \frac{7}{120}\pi^4 - \frac{52}{3}\zeta(3)\frac{C_F}{C_A}\right]\left(\frac{\bar{\alpha}_s}{N}\right)^4 \quad \uparrow ! \\
& + \Bigg[\left(\frac{8384}{243} - \frac{233}{162}\pi^2 + \frac{284}{9}\zeta(3) - \frac{91}{720}\pi^4 + 2\zeta(5) - \frac{4}{3}\zeta(3)\pi^2\right) \\
& + \left(\frac{4}{3}\zeta(3)\pi^2 - \frac{284}{9}\zeta(3) - 16\zeta(5)\right)\frac{C_F}{C_A}\Bigg]\left(\frac{\bar{\alpha}_s}{N}\right)^5 \\
& + \Bigg[\Bigg(\frac{45928}{729} - \frac{638}{243}\pi^2 - \frac{65}{18}\zeta(3)\pi^2 - 2\zeta(3)^2 - \frac{497}{2160}\pi^4 + \frac{125}{3}\zeta(5) + \frac{2330}{27}\zeta(3) \\
& - \frac{31}{3024}\pi^6\Bigg) + \left(\frac{26}{9}\pi^2\zeta(3) - \frac{104}{3}\zeta(5) - \frac{1864}{27}\zeta(3) - 80\zeta(3)^2\right)\frac{C_F}{C_A}\Bigg]\left(\frac{\bar{\alpha}_s}{N}\right)^6 \\
& + O\left(\frac{\bar{\alpha}_s}{N}\right)^7\Bigg\} .
\end{aligned}$$

In the DIS–$\overline{\mathrm{MS}}$ scheme $\gamma_{gg,\mathrm{NL}}^{q\bar{q}}$ is given by, cf. [15],

$$\begin{aligned}
\gamma_{gg,\mathrm{NL}}^{q\bar{q},\mathrm{DIS}} &= \gamma_{gg,\mathrm{NL}}^{q\bar{q},Q_0} + \beta_0\alpha_s^2\frac{d\ln R(\alpha_s)}{d\alpha_s} + \frac{C_F}{C_A}\left[1 - R(\alpha_s)\right]\gamma_{qg,\mathrm{NL}}^{Q_0} \\
&\equiv \alpha_s\sum_{k=1}^{\infty}\left[\frac{N_f}{6\pi}\left(\underline{d_{gg,k}^{q\bar{q},(1)}} + \frac{C_F}{C_A}\underline{d_{gg,k}^{q\bar{q},(2)}}\right) + \frac{\beta_0}{4\pi}\underline{\hat{r}_k}\right]\left(\frac{\bar{\alpha}_s}{N}\right)^{k-1} .
\end{aligned}$$

$\uparrow$ DIS.

F_L :

$$\begin{aligned}
C_L^g &= \frac{\alpha_s}{3\pi}T_F h_L(\gamma_L)R(\gamma_L) = \frac{2\alpha_s}{3\pi}T_F\sum_{k=1}^{\infty}c_k^L\left(\frac{\bar{\alpha}_s}{N}\right)^k , \\
C_L^q &= \frac{C_F}{C_A}\left[C_L^g - \frac{2\alpha_s}{3\pi}T_F\right] ,
\end{aligned}$$

$$h_L(\gamma) = \left(\frac{1-\gamma}{3-2\gamma}\right)\frac{\left[B(1-\gamma,1+\gamma)\right]^3}{B(2-2\gamma,2+2\gamma)} .$$

k	$g_k^{(0)}$	$g_k^{qg,1}$	c_k^L
1	1.000000000E+0	1.000000000E+0	1.000000000E+0
2	0.000000000E+0	2.166666666E+0	−3.333333333E−1
3	0.000000000E+0	2.299510377E+0	2.132843711E+0
4	2.404113806E+1	8.271090891E+0	2.272314949E+0
5	0.000000000E+0	1.492492689E+1	0.434344619E+0
6	2.073855510E+1	2.922678142E+1	2.026429511E+1
7	1.733928958E+1	1.028116363E+2	2.303149346E+1
8	2.016698555E+0	1.948865992E+2	3.464487971E+1
9	3.988627732E+1	4.851003221E+2	2.650042403E+2
10	1.687465187E+2	1.524438954E+3	3.300383179E+2
11	6.998811177E+1	3.114512279E+3	8.503712548E+2
12	6.612526445E+2	8.583754389E+3	3.908491758E+3
13	1.945314152E+3	2.475708601E+4	5.674329386E+3
14	1.717675538E+3	5.474354877E+4	1.776800580E+4
15	1.064326485E+4	1.561954918E+5	6.219822925E+4
16	2.556678709E+4	4.269798325E+5	1.070279253E+5
17	3.678133240E+4	1.011107922E+6	3.514745865E+5
18	1.716848319E+5	2.893976929E+6	1.050584301E+6
19	3.753792748E+5	7.690417370E+6	2.103406452E+6
20	7.360245729E+5	1.919194565E+7	6.807468863E+6
k	$d_{gg,k}^{q\bar{q},(1)}$	$d_{gg,k}^{q\bar{q},(2)}$	$\dot{r}_k$
1	−1.000000000E+0	0.000000000E+0	0.000000000E+0
2	−3.833333333E+0	0.000000000E+0	0.000000000E+0
3	−2.299510376E+0	0.000000000E+0	0.000000000E+0
4	−5.065605818E+0	3.205485075E+0	9.616455224E+0
5	−3.523670351E+1	8.568702514E+0	−3.246969702E+0
6	−3.218245315E+1	1.835447655E+1	2.281241061E+1
7	−1.060268680E+2	8.632838009E+1	1.654162989E+2
8	−4.853159484E+2	1.924088636E+2	−2.469139930E+0
9	−5.806186371E+2	4.962344972E+2	7.458249428E+2
10	−2.176371931E+3	1.794742819E+3	2.784859262E+3
11	−7.553679737E+3	4.023320193E+3	1.505001272E+3
12	−1.158215080E+4	1.136559381E+4	1.818320928E+4
13	−4.328579102E+4	3.589638820E+4	4.899274185E+5
14	−1.269309428E+5	8.412529889E+4	6.109247725E+5
15	−2.392549581E+5	2.456097133E+5	3.984470167E+5
16	−8.469557573E+5	7.168572021E+6	9.205515787E+5
17	−2.262541206E+6	1.764587230E+6	1.783326920E+6
18	−4.974873276E+6	5.167844173E+6	8.347774614E+6
19	−1.648990863E+7	1.443009883E+7	1.842662795E+7
20	4.222994214E+7	3.702246358E+7	4.535538189E+7

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Table 1: The expansion coefficients in eqs. (29,34,38) and (40).

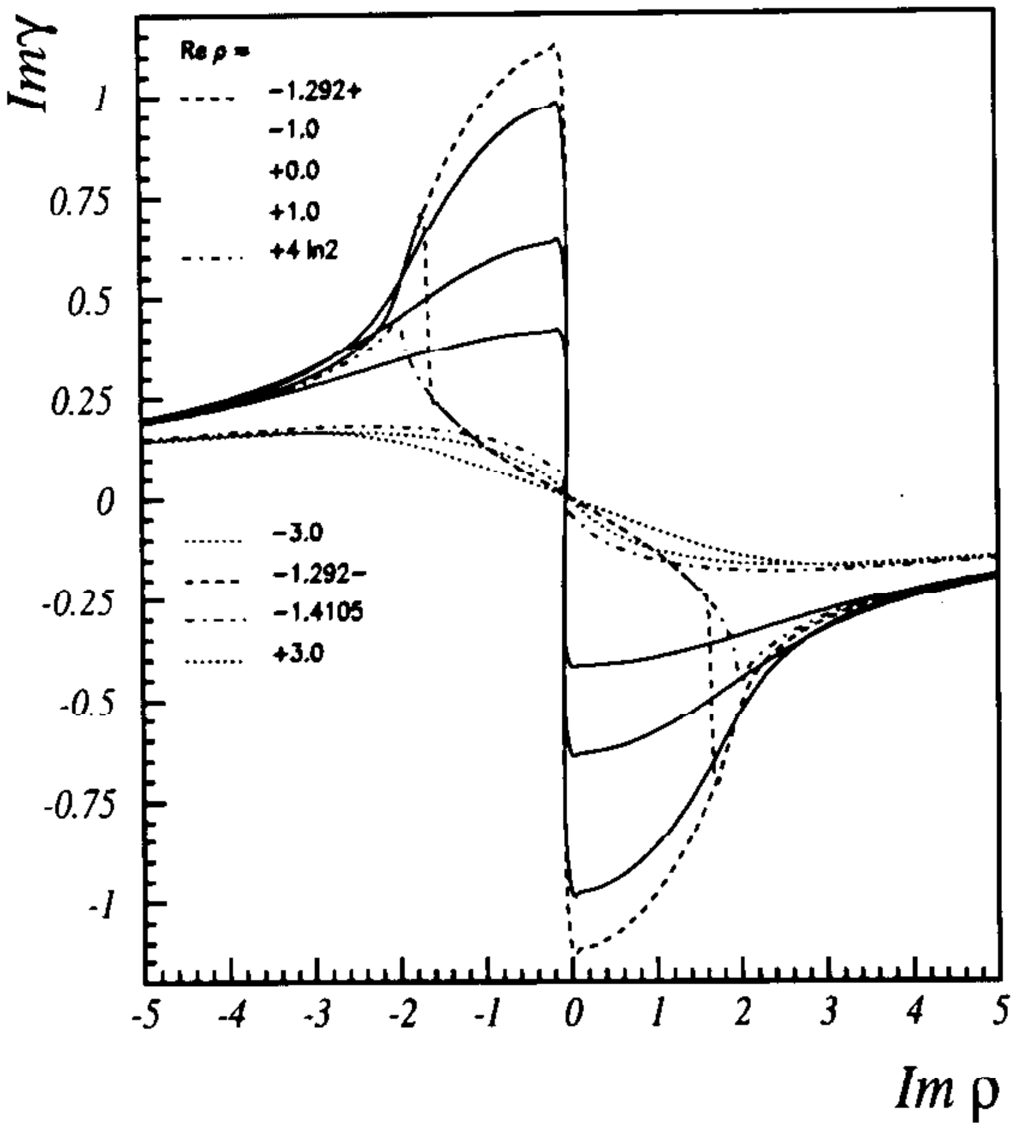

Figure 1b: Imaginary part of $\gamma_L(\rho)$ for $-3 \leq \mathrm{Re}\rho \leq 3$ and $-5 \leq \mathrm{Im}\rho \leq 5$. The notations are the same as in figure 1a.

ALL THE ABOVE QUANTITIES ARE FUNCTIONS OF $\gamma_L(N)$.

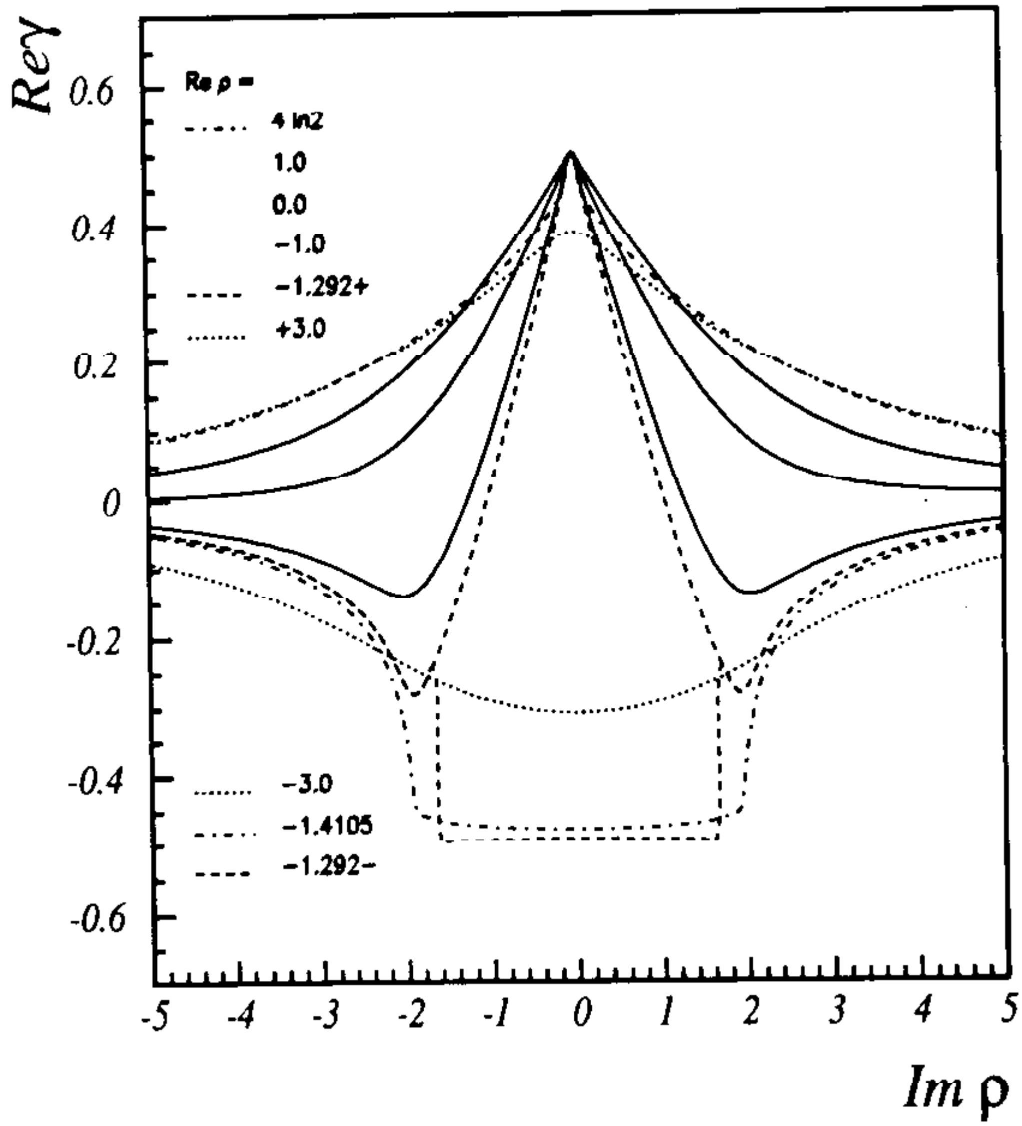

Figure 1a: Real part of $\gamma_L(\rho)$ for $-3 \leq \mathrm{Re}\rho \leq 3$ and $-5 \leq \mathrm{Im}\rho \leq 5$. The labels of the curves $\gamma_L|_{\mathrm{Re}\rho}$ refer to the order of their values for $|\mathrm{Im}\rho| \sim 0$. The dash-dotted lines show the behavior of $\mathrm{Re}\gamma_L$ at the singularities of the perturbative branch, $\mathrm{Re}\rho = 4\ln 2$ and -1.4105. The branch points mainfest by the 'roof' at $\gamma_L = 0.5$ and the corner points of $\gamma_L(\mathrm{Re}\rho = -1.4105)$. The dashed lines mark a transition in γ_L for smaller values of $\mathrm{Im}\rho$ at $\mathrm{Re}\rho = 1.292$. The dotted lines illustrate the behavior of $\mathrm{Re}\gamma_L$ outside the range of the singularities.

SOLUTION OF THE EVOLUTION eqs.

$$\begin{aligned}\frac{\partial \boldsymbol{q}(a_s,N)}{\partial a} &= -\frac{1}{\beta_0 a_s}\Big[\boldsymbol{P}_0(N) + a_s\Big(\boldsymbol{P}_1(N) - \frac{\beta_1}{\beta_0}\boldsymbol{P}_0(N)\Big) \\ &\qquad + a_s^2\Big(\boldsymbol{P}_2(N) - \frac{\beta_1}{\beta_0}\boldsymbol{P}_1(N) + \Big\{\Big(\frac{\beta_1}{\beta_0}\Big)^2 - \frac{\beta_2}{\beta_0}\Big\}\boldsymbol{P}_0(N)\Big) + \ldots\Big]\,\boldsymbol{q}(a_s,N) \\ &\equiv -\frac{1}{a_s}\Big[\boldsymbol{R}_0(N) + \sum_{k=1}^{\infty} a_s^k \boldsymbol{R}_k(N)\Big]\,\boldsymbol{q}(a_s,N)\,.\end{aligned}$$

$$\begin{aligned}\boldsymbol{R}_0 &= \frac{1}{\beta_0}\boldsymbol{P}_0\,, \\ \boldsymbol{R}_k &= \frac{1}{\beta_0}\boldsymbol{P}_k - \sum_{i=1}^{k}\frac{\beta_i}{\beta_0}\boldsymbol{R}_{k-i}\end{aligned}$$

$$[\boldsymbol{R}_{k\geq 1}, \boldsymbol{R}_0] \neq 0$$

$$\boldsymbol{q}^{\mathrm{LO}}(a_s,N) = \Big(\frac{a_s}{a_0}\Big)^{-\boldsymbol{R}_0(N)}\boldsymbol{q}(a_0,N) \equiv \boldsymbol{L}(a_s,a_0,N)\,\boldsymbol{q}(a_0,N)\,,$$

$$\begin{aligned}\boldsymbol{q}(a_s,N) &= \boldsymbol{U}(a_s,N)\,\boldsymbol{L}(a_s,a_0,N)\,\boldsymbol{U}^{-1}(a_0,N)\,\boldsymbol{q}(a_0,N) \\ &= \Big[1 + \sum_{k=1}^{\infty} a_s^k \boldsymbol{U}_k(N)\Big]\,\boldsymbol{L}(a_s,a_0,N)\Big[1 + \sum_{k=1}^{\infty} a_0^k \boldsymbol{U}_k(N)\Big]^{-1}\boldsymbol{q}(a_0,N)\,.\end{aligned}$$

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$$
\begin{aligned}
[\boldsymbol{U}_1, \boldsymbol{R}_0] &= \boldsymbol{R}_1 + \boldsymbol{U}_1 , \\
[\boldsymbol{U}_2, \boldsymbol{R}_0] &= \boldsymbol{R}_2 + \boldsymbol{R}_1 \boldsymbol{U}_1 + 2\boldsymbol{U}_2 , \\
&\vdots \\
[\boldsymbol{U}_k, \boldsymbol{R}_0] &= \boldsymbol{R}_k + \sum_{i=1}^{k-1} \boldsymbol{R}_{k-i} \boldsymbol{U}_i + k\,\boldsymbol{U}_k \equiv \widetilde{\boldsymbol{R}}_k + k\,\boldsymbol{U}_k .
\end{aligned}
\tag{52}
$$

These equations can be solved recursively by applying the eigenvalue decomposition of the LO splitting function matrix, completely analogous to the NLO case of only $\boldsymbol{U}_1$ in ref. [4]. One writes

$$
\boldsymbol{R}_0 = r_- \boldsymbol{e}_- + r_+ \boldsymbol{e}_+ , \tag{53}
$$

where r_- (r_+) stands for the smaller (larger) eigenvalue of $\boldsymbol{R}_0$,

$$
r_\pm = \frac{1}{2\beta_0}\left[P_{qq}^{(0)} + P_{gg}^{(0)} \pm \sqrt{\left(P_{qq}^{(0)} - P_{gg}^{(0)}\right)^2 + 4 P_{qg}^{(0)} P_{gq}^{(0)}}\,\right] . \tag{54}
$$

The matrices $\boldsymbol{e}_\pm$ denote the corresponding projectors,

$$
\begin{aligned}
\boldsymbol{e}_- &= \frac{1}{r_- - r_+}\left[\boldsymbol{R}_0 - r_+ \boldsymbol{I}\right] , \\
\boldsymbol{e}_+ &= \frac{1}{r_+ - r_-}\left[\boldsymbol{R}_0 - r_- \boldsymbol{I}\right] ,
\end{aligned}
\tag{55}
$$

with $\boldsymbol{I}$ the 2×2 unit matrix. Hence the LO evolution operator in eq. (50) can be written as

$$
\boldsymbol{L}(a, a_0, N) = \boldsymbol{e}_-(N)\left(\frac{a_s}{a_0}\right)^{-r_-(N)} + \boldsymbol{e}_+(N)\left(\frac{a_s}{a_0}\right)^{-r_+(N)} . \tag{56}
$$

Inserting the obvious identity

$$
\boldsymbol{U}_k = \boldsymbol{e}_- \boldsymbol{U}_k \boldsymbol{e}_- + \boldsymbol{e}_- \boldsymbol{U}_k \boldsymbol{e}_+ + \boldsymbol{e}_+ \boldsymbol{U}_k \boldsymbol{e}_- + \boldsymbol{e}_+ \boldsymbol{U}_k \boldsymbol{e}_+ \tag{57}
$$

into eq. (52), one finally obtains the expansion coefficients in eq. (51) as

$$
\boldsymbol{U}_k = -\frac{1}{k}\left[\boldsymbol{e}_- \widetilde{\boldsymbol{R}}_k \boldsymbol{e}_- + \boldsymbol{e}_+ \widetilde{\boldsymbol{R}}_k \boldsymbol{e}_+\right] + \frac{\boldsymbol{e}_+ \widetilde{\boldsymbol{R}}_k \boldsymbol{e}_-}{r_- - r_+ - k} + \frac{\boldsymbol{e}_- \widetilde{\boldsymbol{R}}_k \boldsymbol{e}_+}{r_+ - r_- - k} . \tag{58}
$$

SUBLEADING TERMS:

$$\gamma_{qq,1}^{DIS} \underset{N_f=4}{=} -\frac{123.3}{N} + 405.9$$

$$\gamma_{qg,1}^{DIS} = -\frac{277.3}{N} + 846.2$$

$$\gamma_{gq,1}^{DIS} = \frac{91.26}{N} - 453.5$$

$$\gamma_{gg,1}^{DIS} = \frac{245.3}{N} - 988.3$$

MODEL:

A: $\gamma_{ij} \to \gamma_{ij} - \delta_{ij}\ \gamma_{ij}(1, a_s)$

B: $\gamma_{ij} \to \gamma_{ij} \cdot (1-N)$

C: $\gamma_{ij} \to \gamma_{ij}\ (1-N)^2$

D: $\gamma_{ij} \to \gamma_{ij}\ (1-2N+N^3)$.

2. Non-Singlet Combinations

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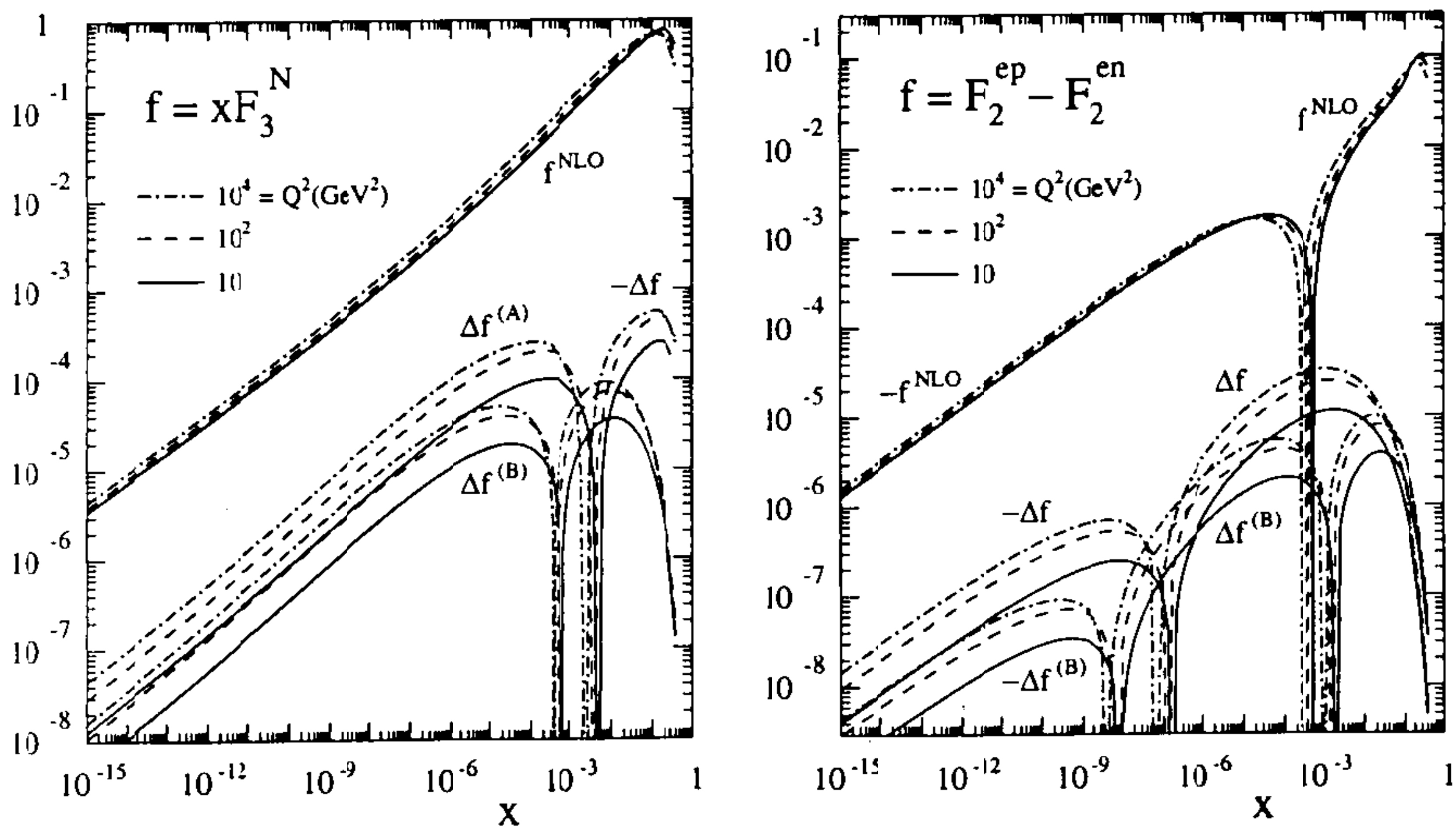
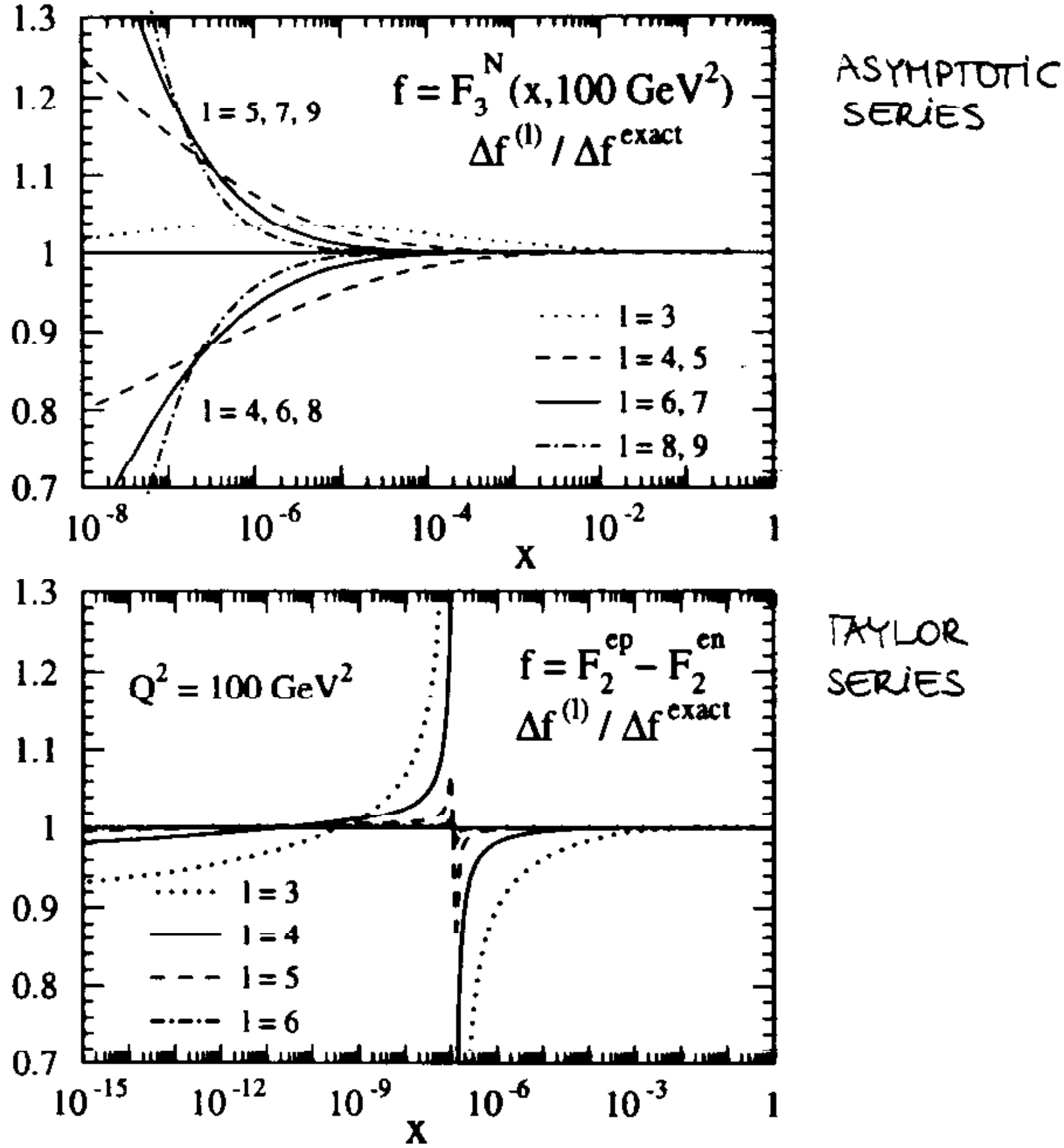


CONVERGENCE OF THE SERIES
IN α_s:



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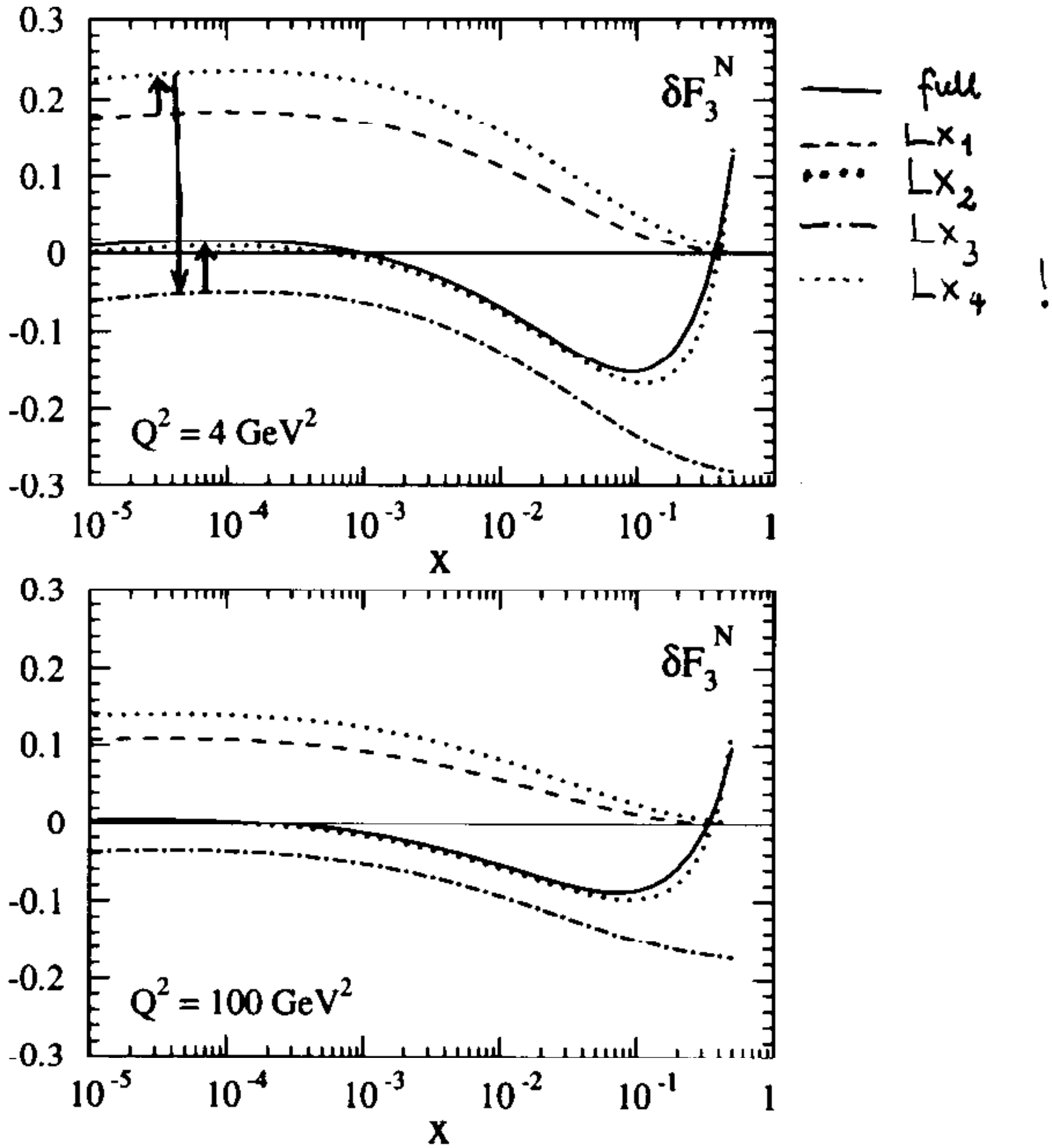
AN EXAMPLE IN NLO:

$$\delta F_3^{NLO} = \frac{a_s \cdot C_3 \otimes x(u_v + d_v)}{x(u_v + d_v)}$$



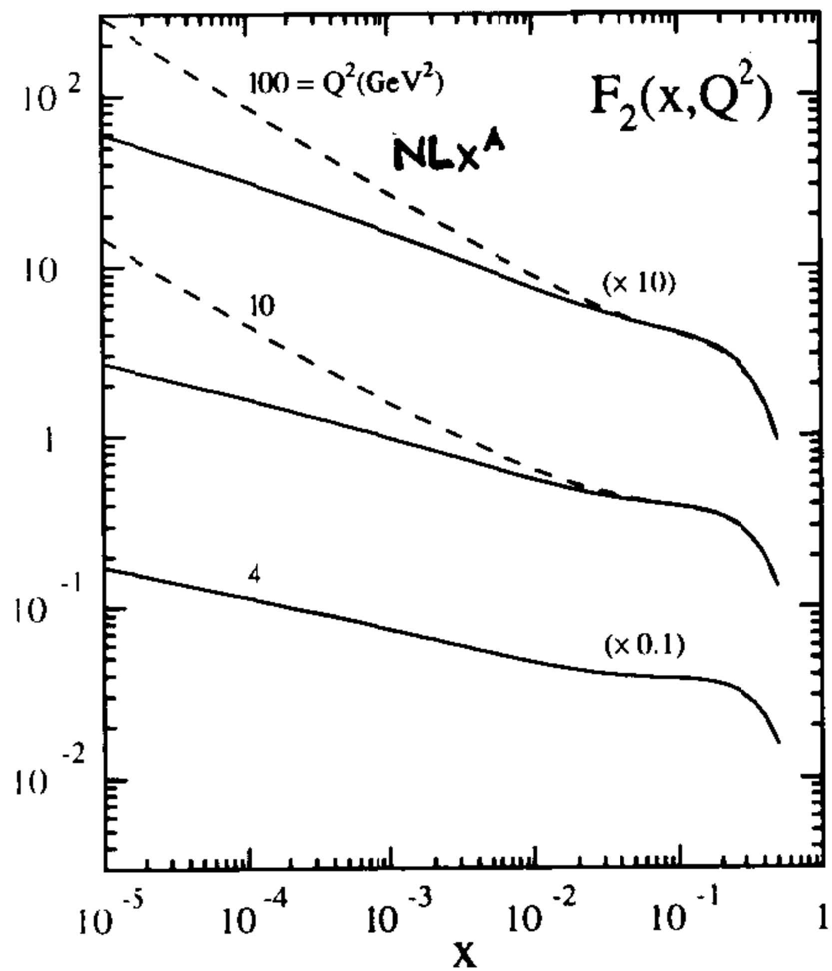
$Lx_1 = Lx$

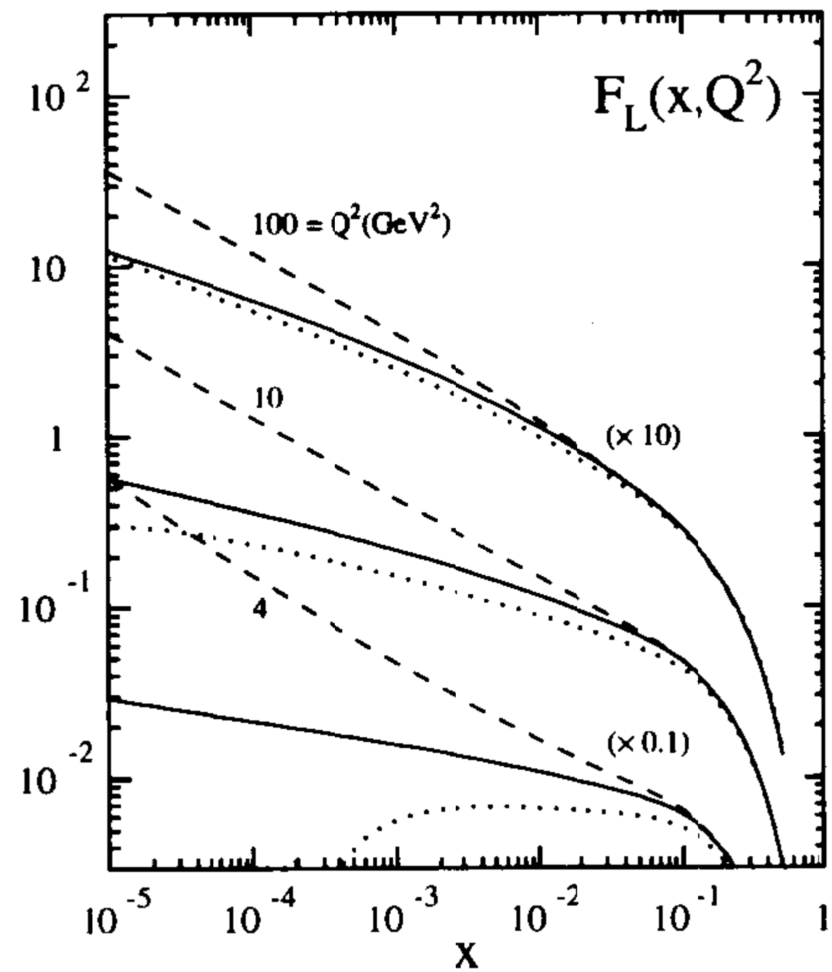
$Lx_2 = Lx + NLx$

$Lx_3 = Lx + NLx + NNLx$

⋮

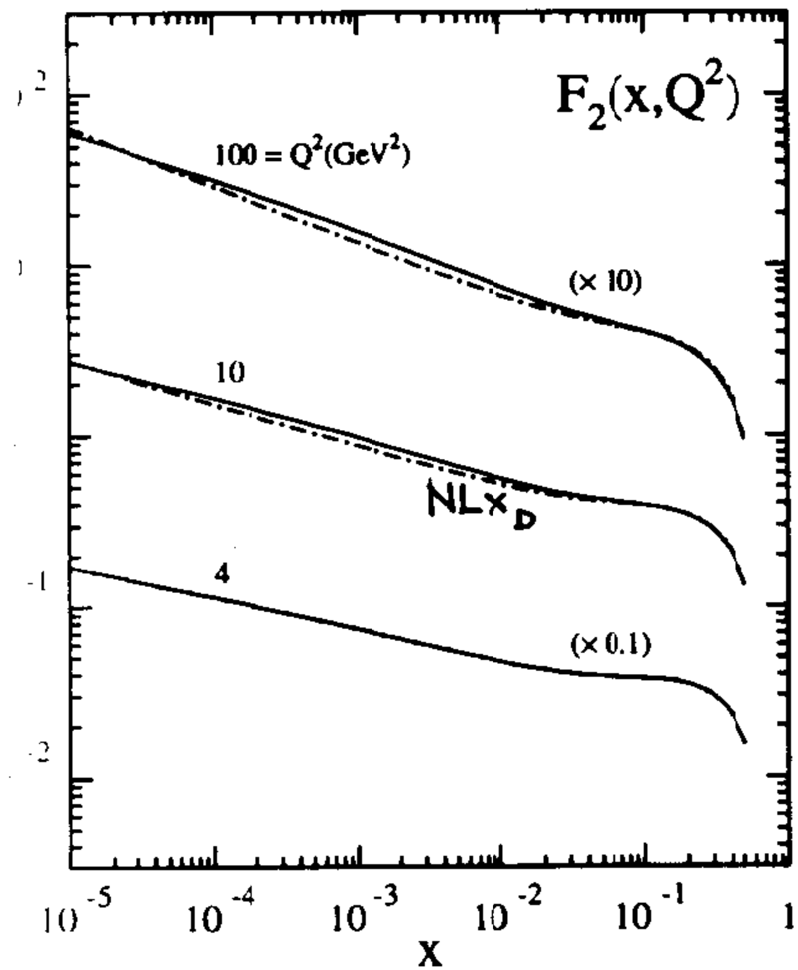
3. $F_2^p(x,Q^2)$, $F_L^p(x,Q^2)$

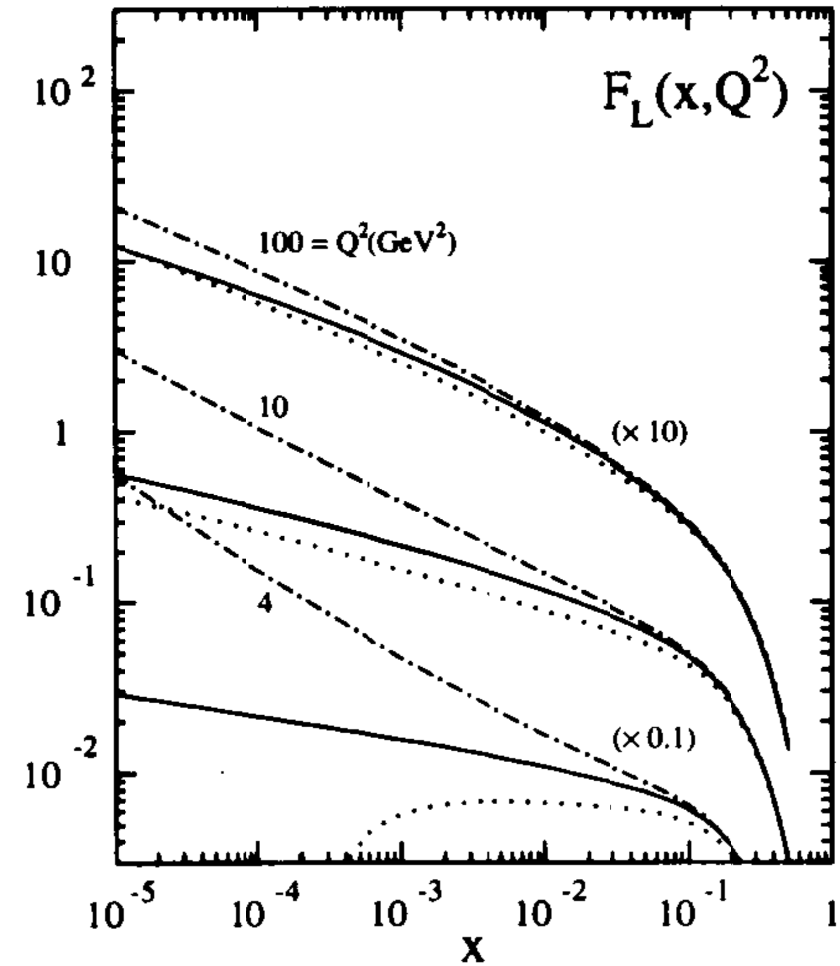




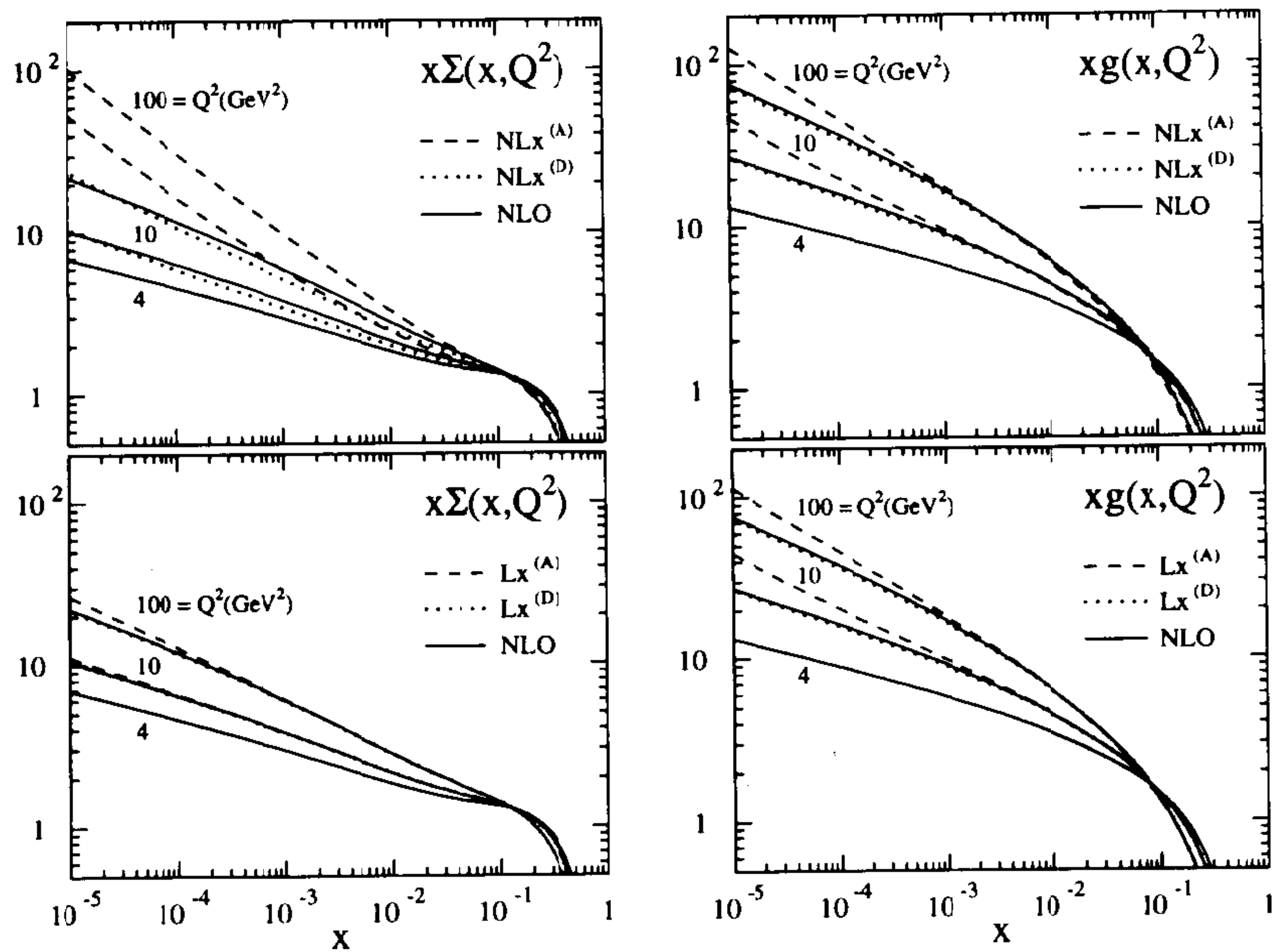
.... $C_L^{NLO} + C_L^{>\alpha^2} \cdot (1-2N)$

F_L IS DRIVEN BY THE COEFFICIENT FUNCTION (NOT BY THE PDF'S).

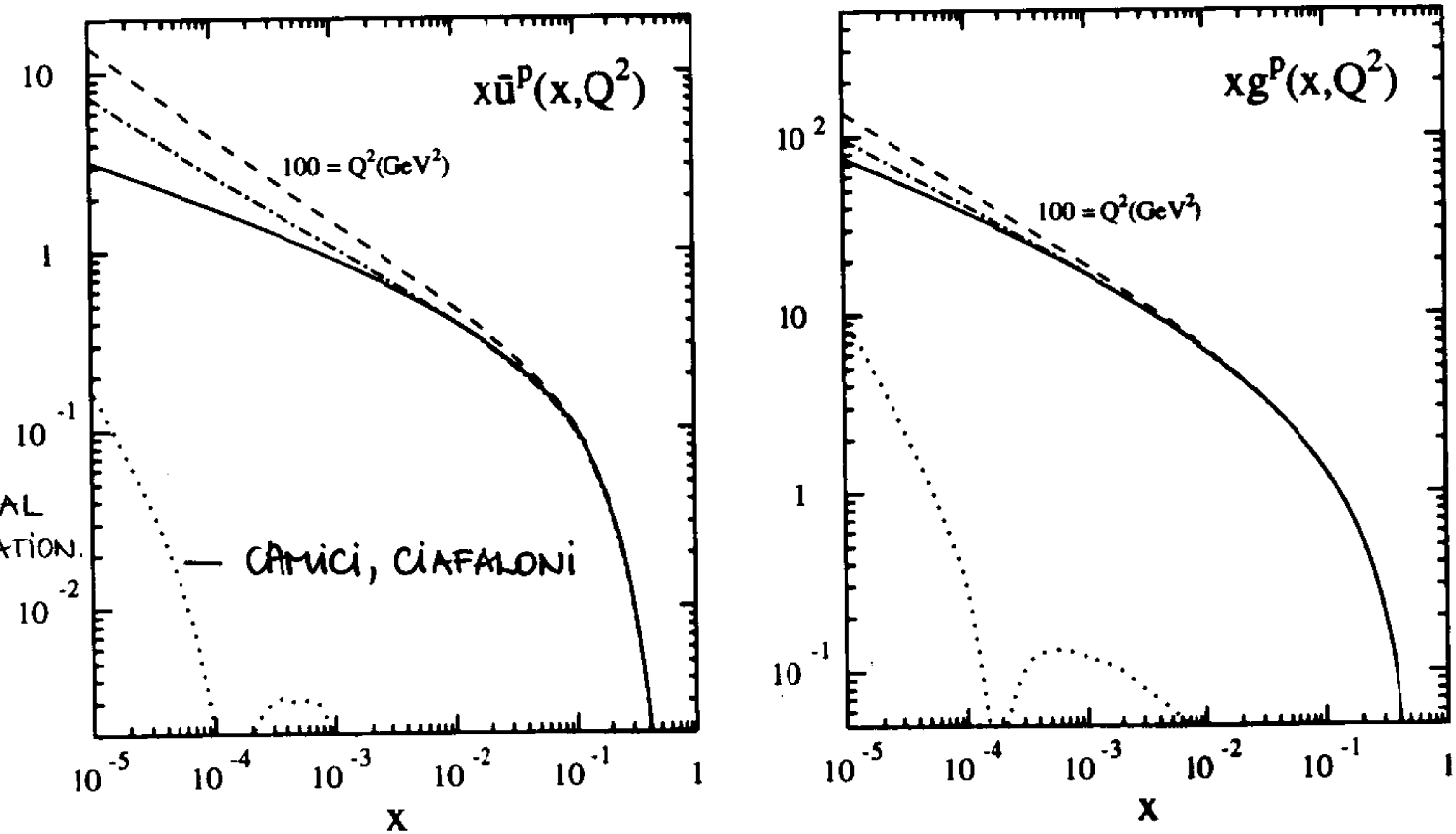




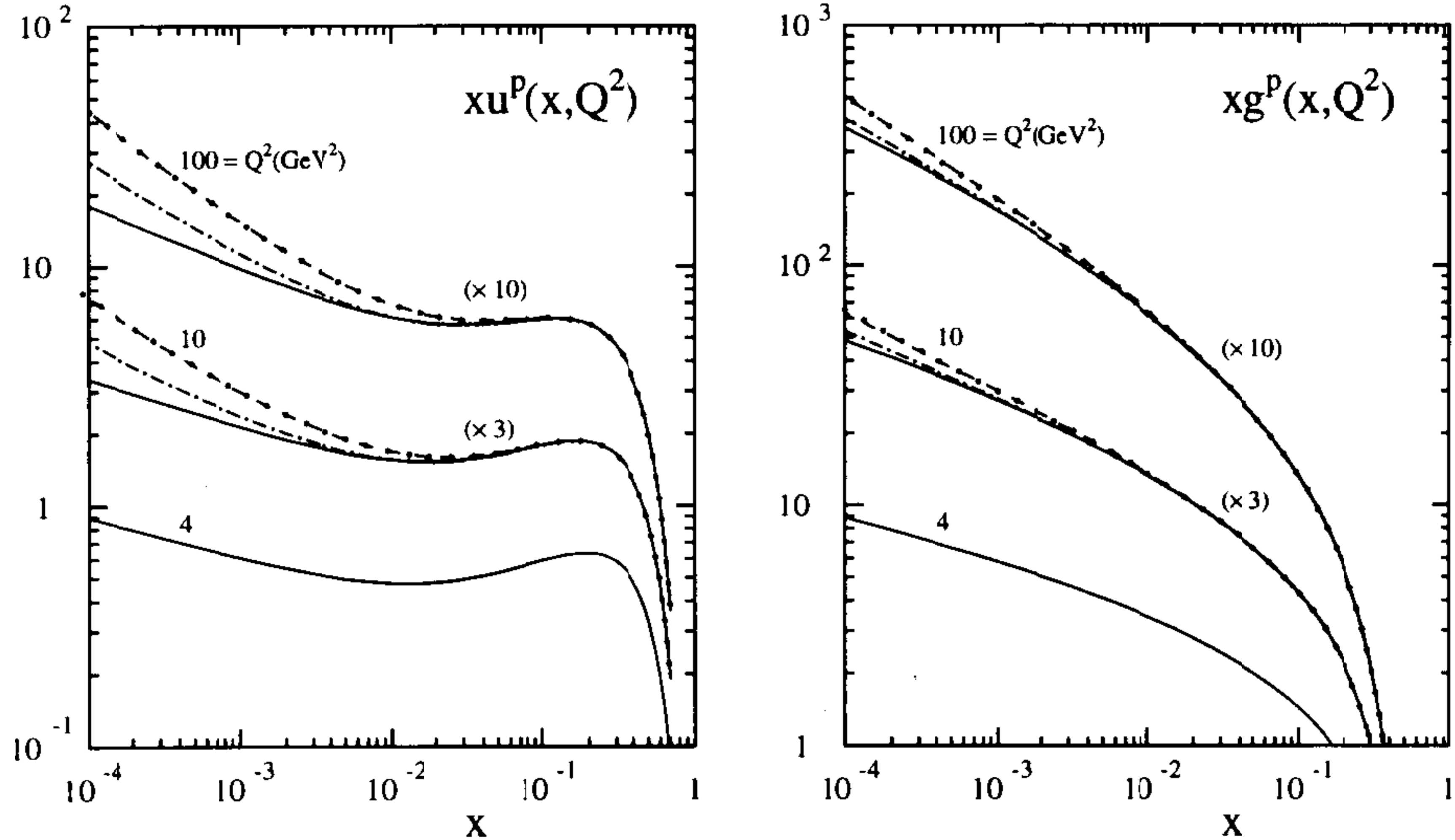
$\gamma \cdot (1 - 2N + N^3)$


xΣ(x,Q²)
100 = Q²(GeV²)
10
4
NLx (A)
NLx (D)
NLO
xg(x,Q²)
Lx (A)
Lx (D)
x

NO
ESSENTIAL
MODIFICATION.



.... + Ciafaloni/Camici



4. $F_2^\gamma(x, Q^2)$

$$
\begin{pmatrix} \dot{q}^\gamma \\ \dot{\bar{q}}^\gamma \\ \dot{G}^\gamma \\ \dot{\Gamma}^\gamma \end{pmatrix} = \frac{\alpha}{2\pi} \begin{pmatrix} P_{q\gamma} \\ P_{\bar{q}\gamma} \\ P_{G\gamma} \\ P_{\gamma\gamma} \end{pmatrix} \otimes \Gamma^\gamma + \begin{cases} \frac{\alpha_s}{2\pi} \mathbb{P}_{ij} \otimes P_j \\ \\ \frac{\alpha}{2\pi} P_{\gamma j} \otimes P_j \end{cases} \qquad i,j = 1,3.
$$

$$\Gamma^{\gamma,0} = \delta(1-x) + O(\alpha). \qquad q^\gamma = \bar{q}^\gamma$$

<u>SOLVE</u>:

$$\dot{q}^\gamma = \frac{\alpha}{2\pi} k_q + \frac{\alpha_s}{2\pi} \left[\sum_{k=1}^{N_f} (P_{qq} + P_{q\bar{q}}) \otimes q^\gamma + P_{qg} \otimes G^\gamma \right]$$

$$\dot{G}^\gamma = \frac{\alpha}{2\pi} k_g + \frac{\alpha_s}{2\pi} \left[\sum_{k=1}^{N_f} (P_{gq} + P_{g\bar{q}}) \otimes q^\gamma + P_{gg} \otimes G^\gamma \right.$$

INHOM. EVOLUTION EQUS.

$$k_i = \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^k k_{i,k}$$

$$\mathbb{P}_{ij} = \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^k P_{ij}^{(k)}$$

$\longrightarrow$ EXPRESS F_2^γ & EXPAND.

- SCHEME INDEP. REQUESTS:

$$k_0 \qquad = \text{inv.}$$

$$k_0 C_1 + k_1 - \delta \cdot P_0 C_{\gamma 1} \qquad = \text{inv.}$$

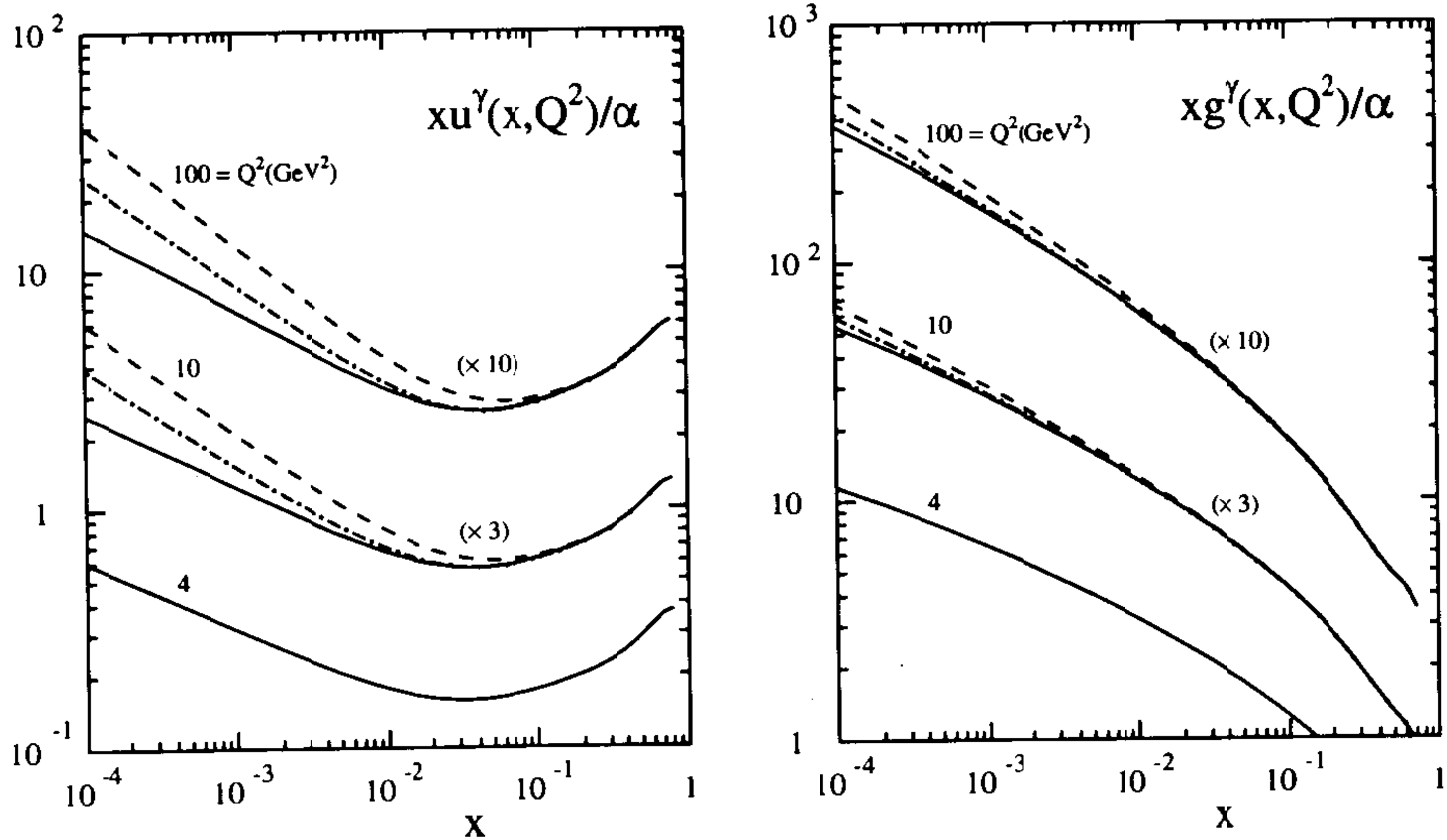
$$k_2 + C_2 k_0 + C_1 k_1 - \delta \cdot (P_0 C_{\gamma 2} + P_1 C_{\gamma 1} + [C_1, P_0] C_{\gamma 1})$$

$$+ \delta \beta_0 C_1 C_{\gamma 1} - \delta \beta_0 C_{\gamma 2} \qquad = \text{inv}$$

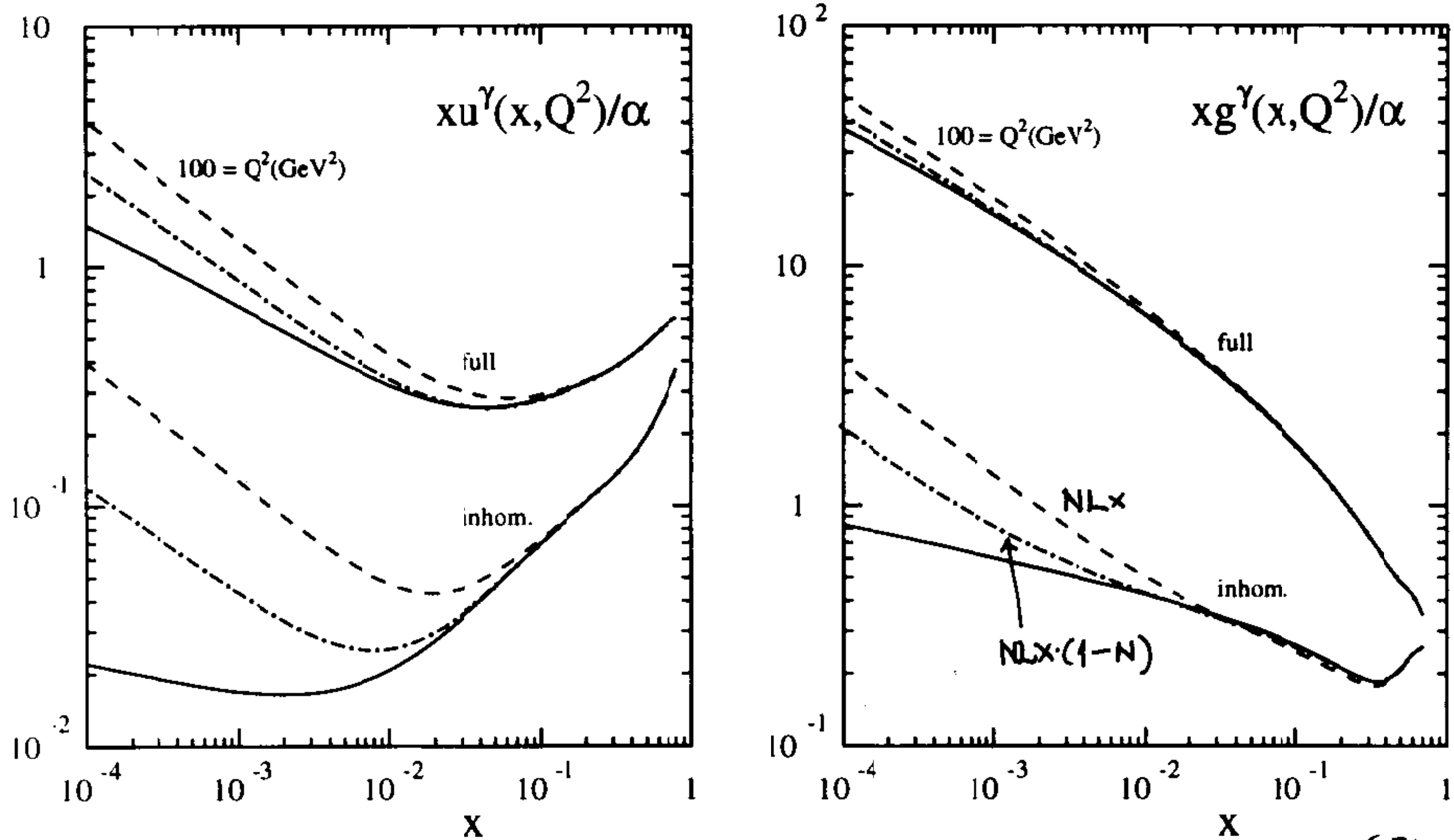
etc.

DIS γ: <u>NO NLx</u> terms in k_n $\left(C_i \sim \frac{1}{(N-1)^{i-1}} \right)$

4. $F_2^{\gamma}(x,Q^2)$



--- NLX
—·—· NLX_B [· (1-



$Q_0^2 = 4\,\mathrm{Ge}$

GRV$_\gamma$; DIS

5. Conclusions

- AN UPDATE OF THE SMALL x RESUMMATION FOR F_2 and F_L WAS PERFORMED.

- SUBLEADING TERMS IN $\left(\frac{\alpha_s}{N}\right)^k (\alpha_s N)^a$ ARE AS IMPORTANT. $\longrightarrow$ NEED TO BE CALCULATED.
  $\longrightarrow$ ANOMALOUS DIMENSIONS & COEFFICIENT FCTS.
- DIS SCHEME: F_L DEPENDS ON C_L^g <u>MOSTLY</u>)
  g - 'RATHER' STABLE!

  $\delta C_L^g \leftrightarrow \delta F_L$!
- THE TERMS $\propto N_f$ IN γ_{gg} WERE INCLUDED. THEIR NUMERICAL EFFECT IS SMALL ($< 1\%$). STILL TO WAIT UNLESS THE COMPLETE NLO RESUMMED γ_{gq} & γ_{gg} BECOME AVAILABLE.

- THE SMALL x CONTRIBUTIONS TO F_2^γ WERE CALCULATED.

  THE POINTLIKE TERM DOES NOT RECEIVE RESUMMED CORRECTIONS IN NLx IN THE DIS SCHEME.
  THE CORRECTIONS ARE DUE TO THE HADRONIC PARTS.

$$C_{\gamma i} \sim \frac{1}{(N-1)^{i-1}} \quad \rightarrow$$ PRODUCTS $k_i C_j$, $C_i C_j$: NO CONTRIB. IN NLx.

NLx:

$$\underset{x \to 0}{inv} = k^n - \delta \, (\underbrace{P_0 \, C_{\gamma,n} + \ldots}_{\text{not NLX}} \; P_{n-1} \, C_{\gamma,1}) - \delta \, \beta_0 \, \theta \, (N-1) \, C_{n,\gamma}$$

$C_{\gamma,n}$ ONLY FOR $N=1$, NLx

$$inv \underset{x \to 0}{=} k_n - \delta \cdot P^{q\gamma}_{n-1} \, C_{\gamma,1}$$

$\uparrow$ NO HADRONIC IMPACT , IF $\overline{MS} \rightarrow$ DIS

$$k_{\overline{MS},n}\big|_{x \to 0} = \frac{1}{2} P_{q\gamma,0}(N=1) \cdot P_{gq,n-1}\big|_{x \to 0}$$

constant !

$$k_{DIS_\gamma,n}\big|_{x \to 0} = 0.$$

I.E. ONLY THE HADRONIC CONTR. ARE LEFT IN THIS WIDELY USED SCHEME.