Mass Effects For PDF Evolution

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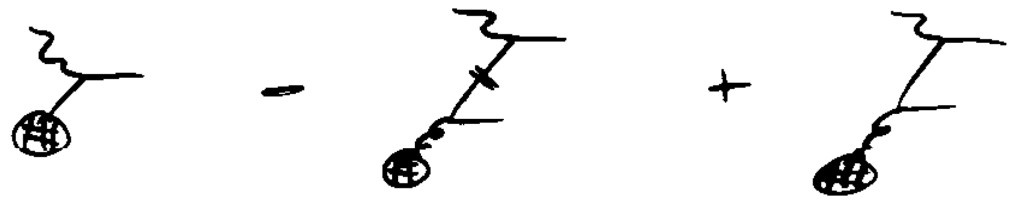
Working
Group I

Heavy Quark Production in DIS

ACOT VFS:

HE: Heavy Excitation — SUB Subtraction — HC Heavy Creation



$$f_H \otimes {}^{0}\sigma \quad - \quad f_g \otimes \tilde{F}_{g\to H} \otimes {}^{0}\sigma \quad + \quad f_g \otimes {}^{1}\sigma$$

- ⇒ SUB removes collinear singularity with $m_H \neq 0$
- ⇒ Retains Full m_H dependence in ${}^{0}\sigma$, ${}^{1}\sigma$
- ⇒ Scheme Formally defined $\begin{cases} \overline{MS} & \mu > m_H \\ BPHZ & \mu < m_H \end{cases}$
  - $f_H = 0$ @ $\mu = m_H$
  - P_{ij} ⇒ Massless $\overline{MS}$ Kernel

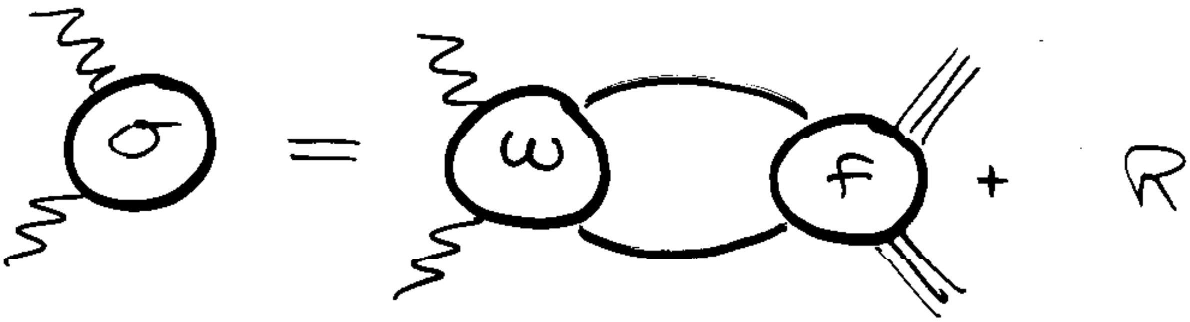
Phenomenology:

- ⇒ Improved μ-scale dep. for $\begin{cases} \text{lepto production} \\ \text{hadro production} \end{cases}$
- ⇒ Improved F_2^c @ HERA (H. Lai + W. Tung

Factorization

$$\sigma = \omega \otimes f + R$$

σ	ω	f	R
Physical X-Section	"Hard" Scattering X-Section	PDF	Remainder

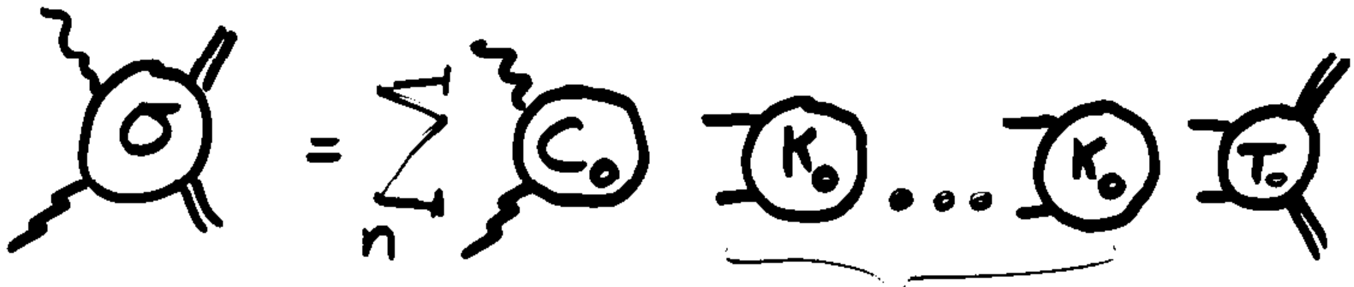


$$R = \left(\frac{\text{Highest Virtuality in } f}{\text{Lowest Virtuality in } \omega} \right)^2$$

For
$$\begin{cases} m_H \lesssim \Lambda & R = \dfrac{\Lambda^2}{Q^2} \\ m_H \gg \Lambda & R = ??? \end{cases}$$

Ingredients of Proof:

J.C. Collins, CERN PP. In prep



n Rungs

$$\sigma = \sum_{n=0}^{\infty} C_0 \,(K_0)^n\, T_0$$

$$\cong \sum_{n=0}^{\infty} C_0 \left[1-(1-Z)K_0\right]^{-1} Z \left[1-K_0\right]^{-1} T_0 \Big\} \text{ Parton Model}$$

$$+ \sum_{n=0}^{\infty} C_0 \,(1-Z)\left[K_0\,(1-Z)\right]^{n} T_0 \Big\{ \text{Remaind.}$$

Where Z is a projection operator: Collinear part

$$Z(K,\ell) = \frac{1}{4}\,\frac{\not{K}+m}{K^+}\,\gamma^+_{\beta\beta}\,(2\pi)^4\,\delta(K^+-\ell^+)\,\delta\!\left(K^- - \frac{m^2}{2K^+}\right)\delta^{(2)}(\vec{K}_T$$

Mass generalized: $Z^2 = Z$; $Z(1-Z) = 0$

<u>Goal:</u>

Show $(1-Z) \Rightarrow$ Suppression $\left(\frac{\Lambda^2}{Q^2}\right)$
$$\left\{\begin{array}{l} Q \gg m_H \\ Q \gtrsim m_H \\ Q < m_H \end{array}\right.$$

Result of Proof:

$$\sigma = \omega \otimes f + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

- Yields all orders definitions of ω and f
  - f: Renormalized Matrix Element of LC Operato
  - ω: IR safe "hard" scattering X-section
- In $m_H \to 0$ limit; $\omega \to$ massless $\overline{MS}$ def.
  - No finite renormalizations

$\boxed{\text{Evolution:}}$

➡ P_{ij} = Massless $\overline{MS}$ Kernels

➡ Evolution $\Longleftrightarrow$ RGE for renorm. PDF

➡ Renormalization Counter terms are mass independent to all orders

$$\boxed{\begin{array}{c}\text{If we want to stick to } \overline{MS} \\ \text{we must use massless Kernels}\end{array}}$$

What about other Schemes ?

- IF $\exists$ factorization proof:

$$\sigma = \omega \otimes f + O\left(\frac{\Lambda^2}{Q^2}\right)$$

- IF $\nexists$, corrections will be larger.

All orders proof is key for consistent definition of ω and f at higher orders

What about a Massive Evolution Scheme

- IF P_{ij} depends on mass $\not\Rightarrow \overline{MS}$

  Must convert Wilson coefficients

- $P_{ij}(x, \mu, M_H)$ requires careful definition

  Ensure Gauge dependence cancels

  Kinematic Ambiguity — MRRS, EHLQ, GR, KO

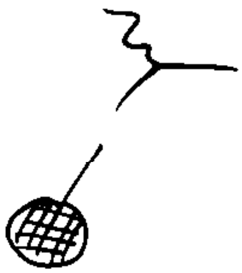
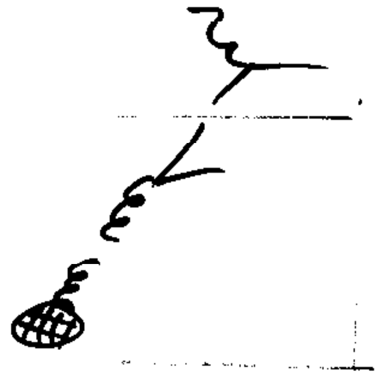
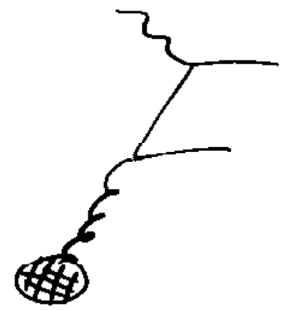
- In a consistent Massive Evolution Scheme

$$\Rightarrow \sigma = \omega \otimes f + O\left(\frac{\Lambda^2}{Q^2}\right)$$

$$\Delta\sigma_{scheme} = 0 + O(\alpha_s^{n+1}) + \underset{\sim 0}{O\left(\frac{\Lambda^2}{Q^2}\right)}$$

How Do $P(m=0)$ and $P(m \neq 0)$ Compare?

Heavy Excitation		Subtraction		Heavy Creation
HE	**−**	**SUB**	**+**	**HC**

$$\sigma_{HE} = f_H \otimes \mathring{\sigma}$$

$$\cong \frac{\alpha_s}{2\pi} f_g \otimes P_{g\to H} \ln\left(\frac{\mu^2}{M_H^2}\right) \otimes \mathring{\sigma} + \alpha_s^2 \left(P \otimes P + {}^{(2)}P \ldots \right)$$

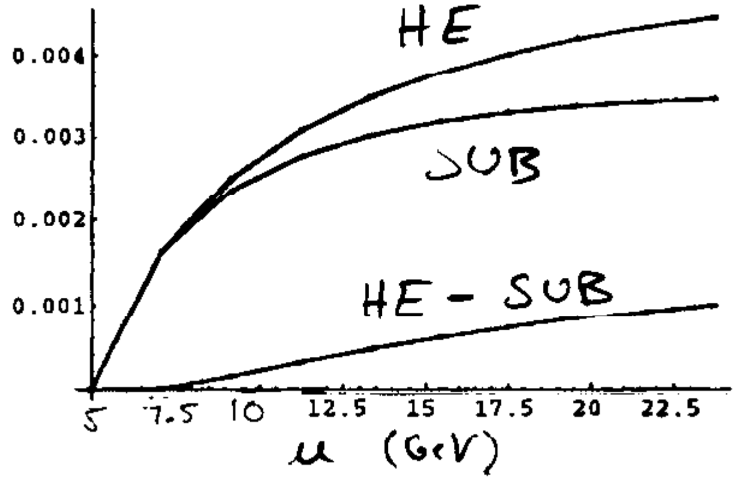
Must Match

$$\sigma_{SUB} \cong \frac{\alpha_s}{2\pi} f_g \otimes P_{g\to H} \ln\left(\frac{\mu^2}{M_H^2}\right) \otimes \mathring{\sigma} + \underbrace{\;0\;}_{\text{Difference is Purely Higher Order}}$$

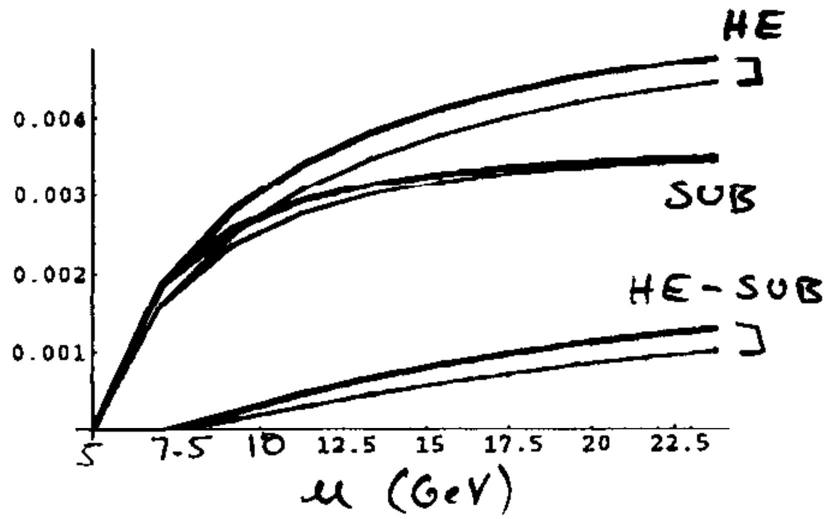
Moral:

> You can use any $P_{g\to H}$
>
> So long as it matches!!!

Compare Massless vs. Massive Evolution



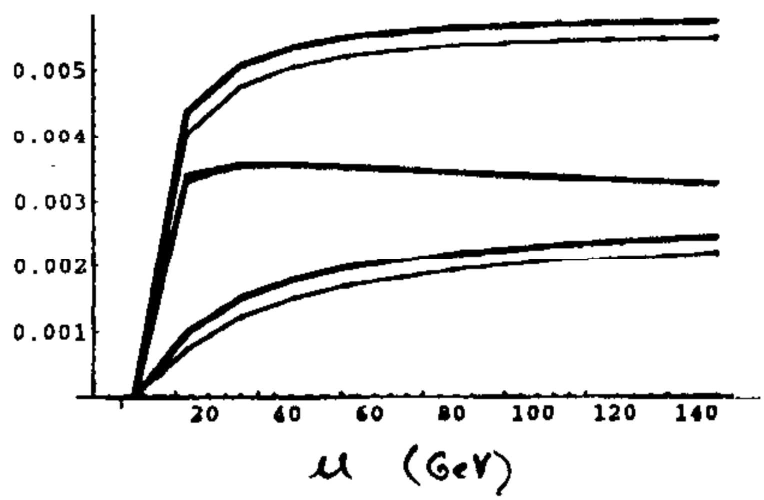
Matching at threshold key to smooth transition



Matching compensat for different Evl.

For $\mu \sim 4 m_H$

$P(m=0) \simeq P(m \neq 0$



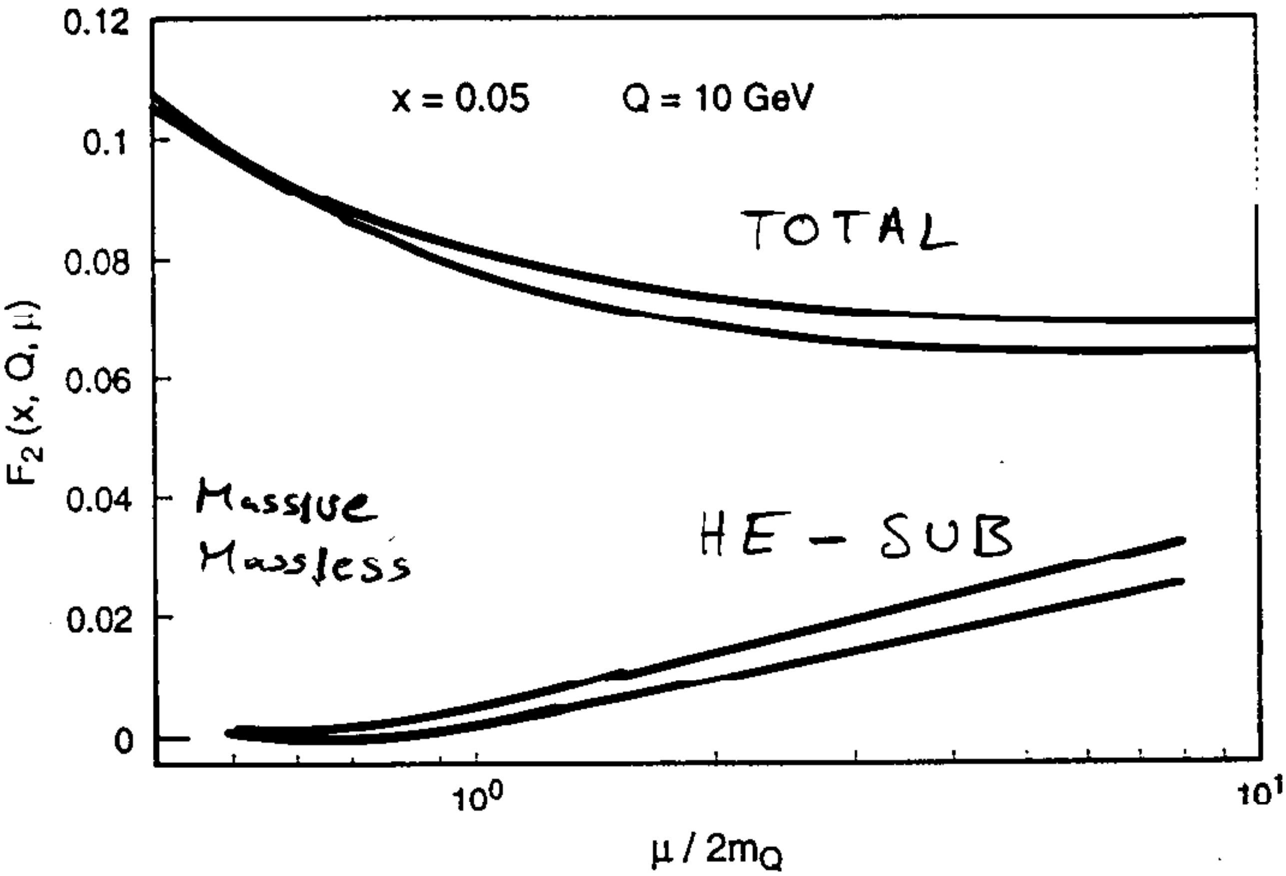
$\Delta \sim 30\%$ for HE - SUB

What about physical X-Section

$$P(x, \mu, m) = \frac{1}{2}\left(x^2 + (1-x)^2 + \frac{2 m_H^2}{\mu^2}\, x(1-x) \right)$$

How does this affect Physical X-Sectio.

B Production:



Variation in TOTAL comparable
to μ-scale variation

Recap:

- ACOT VFS:
  - Factorization valid to $\mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$
  - F, ω, and Evolution P_{ij} are $\overline{MS}$

- Massive Evolution:
  - Requires Careful definition of $P_{ij}(m \neq 0)$
  - Require NLO P_{ij} in Massive Scheme
  - Require ω converted to Massive Scheme

- Renormalization Scheme Dependence

$$\left[\sigma_{\text{Scheme A}} - \sigma_{\text{Scheme B}}\right] \cong \mathcal{O}(\alpha_s^{n+1}) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

  - "Toy" Calculation consistent w/ this result