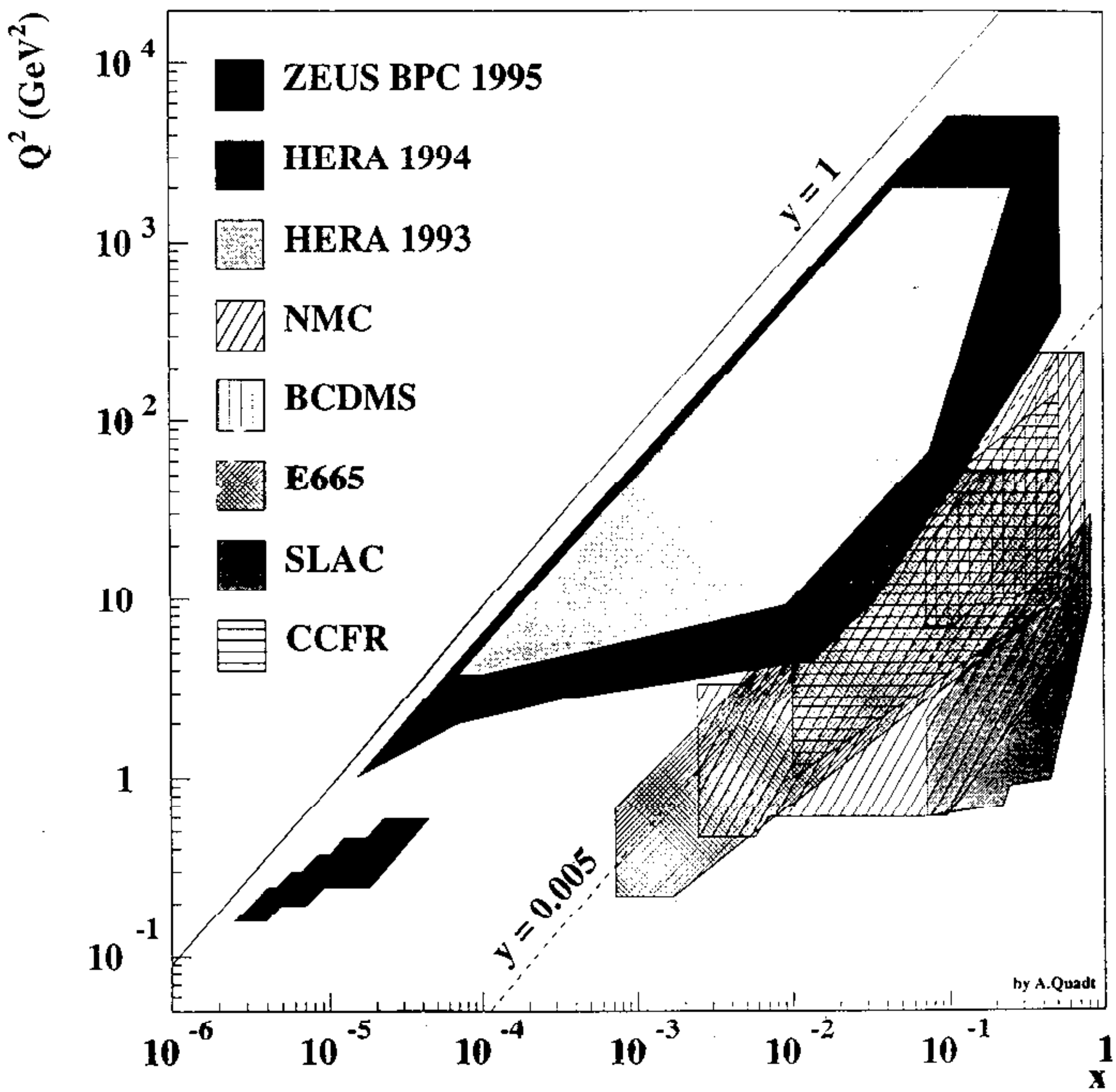
Interpretation of the low Q^2 results

Barbara BADELEK

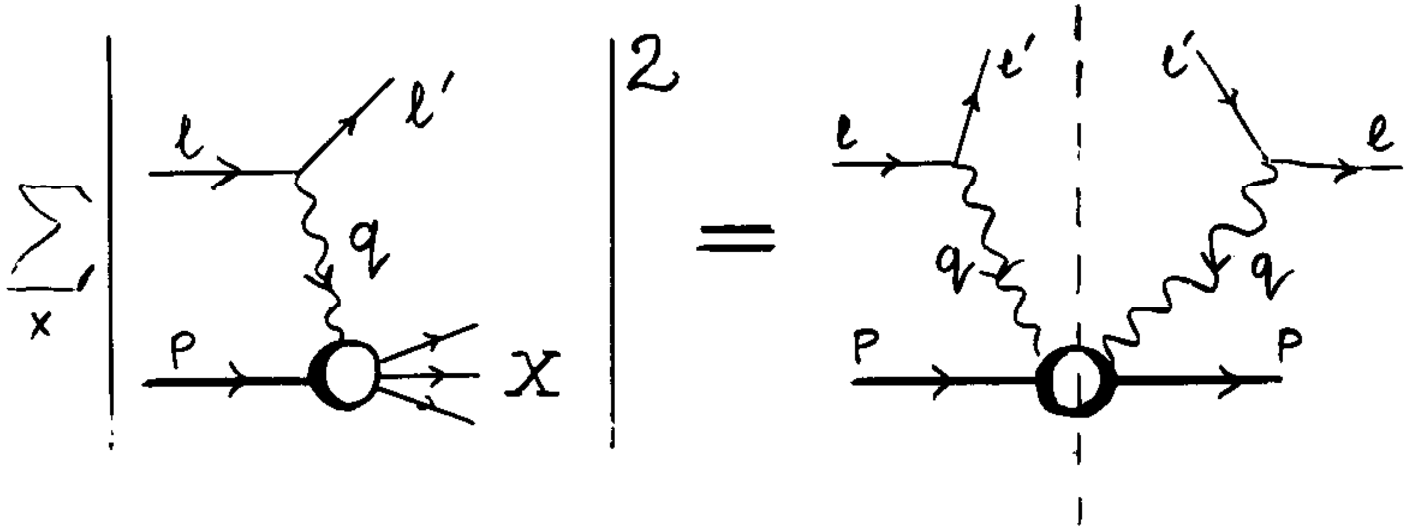
Warsaw / Uppsala

1. Low Q^2 constraints on F_2, R
2. "Higher twists" (GVMD)
3. $F_2^{BK}(x, Q^2)$ vs low Q^2 data
4. Shadowing in ^2D and the new $F_2^d/$ dat
(5. $R^{BKS}(x, Q^2)$)

Kinematic acceptance of F_2^N measurements



Definitions & constraints



$$-q^2 = Q^2, \qquad x = \frac{Q^2}{2pq} = \frac{Q^2}{2M\nu}$$

$$d\sigma \sim L_{\mu\nu} W^{\mu\nu}$$

$$W^{\mu\nu}(p,q) = \frac{F_1(x,Q^2)}{M}\left(-g_{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) +$$

$$\frac{F_2(x,Q^2)}{M\,(p\cdot q)}\left(p^\mu - \frac{pq}{q^2}\,q^\mu\right)\left(p^\nu - \frac{pq}{q^2}\,q^\nu\right)$$

or displaying $W^{\mu\nu}$ singularities at $Q^2=0$

$$W^{\mu\nu}(p,q) = -\frac{F_1}{M}\,g^{\mu\nu} + \frac{F_2}{M(p\cdot q)}\,p^\mu p^\nu +$$

$$\left(\frac{F_1}{M} + \frac{F_2}{M}\,\frac{pq}{q^2}\right)\frac{q^\mu q^\nu}{q^2} - \frac{F_2}{M}\,\frac{p^\mu q^\nu + p^\nu q^\mu}{q^2}$$

Definitions & constraints cont'd

To eliminate singularities we impose:

$$F_2 = \mathcal{O}(q^2)$$

$$\frac{F_1}{M} + \frac{F_2}{M}\,\frac{p\cdot q}{q^2} = \mathcal{O}(q^2)$$

for any ν

$$R = \frac{\sigma_L}{\sigma_T} = \mathcal{O}(q^2)$$

$$F_L = \left(1 + \frac{4M^2x^2}{Q^2}\right)F$$
$$-2xF_1 = \mathcal{O}($$

Surely scaling is not a valid concept at low Q

Theoretical concepts at low Q^2

- In the LLQ^2: $(*)\ F_2(x,Q^2) = x\sum_i e_i^2\left[q_i(x,Q^2)+\bar{q}_i(x,Q^2)\right]$
  + Bjorken scaling ⇒ leading twist

  OPE gives: $(\circledast)\ F_2(x,Q^2) = \sum_{n=0}^{\infty} \frac{C_n(x,Q^2)}{(Q^2)^n}$ · log

  $n=0$ leading "twist" (or twist-2)

  $n\geq 1$ higher "twists" (or twist-$(2n+2)$)

  Higher twists:

  i) are corrections to leading twist $(*)$ at HIGH Q^2

  ii) they DO NOT describe the low Q^2 region $\left(\lim_{Q^2\to 0} \frac{C_n}{(Q^2)^n} \to \infty\right)$

  Individual terms violate: $\lim_{Q^2\to 0} F_2 = O(Q^2)$

  Sum up $(\circledast)$ beforehand and extrapolate to $Q^2=0$.

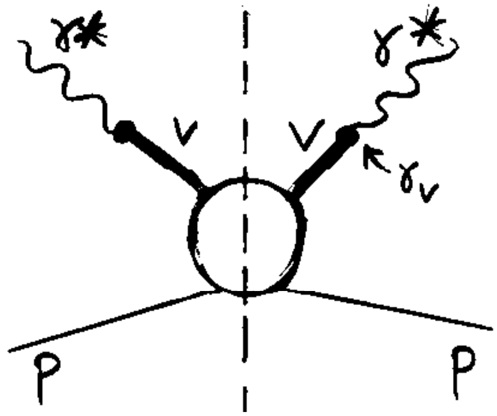
  Done automatically in VMD at hi

- Data analysis: $F_2^{HT} = F_2^{LT}\left(1+\frac{H(x)}{Q^2}\right)$

  ▼ UNJUSTIFIED

VMD cont'd ...

- $$F_2\left(x=\frac{Q^2}{s+Q^2-M^2},Q^2\right)=\frac{Q^2}{4\pi}\sum_V\frac{M_V^4\,\sigma_V(s)}{\gamma_V^2\,(Q^2+M_V^2)^2}$$



$\}\ s \equiv M_X^2 \equiv W$

* if finite number of V $\Rightarrow$ $F_2 \rightarrow$
(ie NO leading twist)

* if infinite number of V $\Rightarrow$ scali

→GVMD←

* $$\frac{Q^2}{(Q^2+M_V^2)^2}=\dots=\frac{1}{M_V^2}\sum_n n(-1)^{n+1}\frac{(M_V^2)^n}{(Q^2)^n}$$

$\uparrow$

$$\frac{1}{(1+x)^2}=\sum n(-1)^{n+1}x^{n-1},\quad |x|<1\quad\left(\frac{M_V^2}{Q^2}\ll 1\right.$$

ALL powers present at $Q^2 \gg M_V^2$

Theoretical concepts... cont'd

- **Phenomenological parametrisations of structure functions**

  i) incorporate $Q^2 \to 0$ constraints

  ii) usually not related to (VMD, QCD) dynamics

- **Dynamical models of F_2**

  i) Glück, Reya & Vogt

  * QCD extension down to $Q^2 \sim 0.3$ with parton distr$^{\underline{s}}$ given at 10 GeV and evolved backwards
    NLO F_2 stable (**LT** F_2 **!**)

  * NMC data at low x, $Q^2 \geq 1$ GeV & HERA at low x, high Q^2 well reproduced

  ii) Kwieciński & B.B.

Theoretical concepts... cont'd

$ii)$ Kwiecinski & BB — Z. Phys $\underline{C43}$ ('89) 25[…]; Phys Lett $\underline{B295}$ ('92) 26[…]

$$(*) \quad F_2(x,Q^2) = \underbrace{\frac{Q^2}{4\pi}\sum_{V}^{\rho,\omega,\phi} \frac{\sigma_{Vp}}{(Q^2+m_V^2)^2}\,\frac{M_V^4}{\gamma_V^2}}_{\text{VMD }(\dagger)} + \underbrace{Q^2\int_{Q_0^2}^{\infty} dQ'^2\,\frac{\Phi(Q'^2,s)}{(Q'^2+Q^2)}}_{\substack{\text{partons}\\(\Delta\Delta)}}$$

$$\Phi(Q'^2,s) = -\frac{\mathrm{Im}}{\pi}\int^{-Q'^2}\frac{dQ''^2}{Q''^2}\,F_2^{AS}(x',Q''^2)$$

* by construction: $F_2(x,Q^2) \xrightarrow[Q^2\to\infty]{} F_2^{AS}(x,Q^2)$
* valid at ARBITRARY Q^2 (incl. $Q^2 = 0$)
* no fit; Q_0^2 well constrained ($Q_0^2 \sim 1.2$ GeV^2)
* photoproduction:

$$\sigma_{\gamma p}(s) = \lim_{Q^2\to 0} 4\pi^2\alpha\,\frac{F_2}{Q^2}$$

– correct shape

$x \lesssim 0.1$

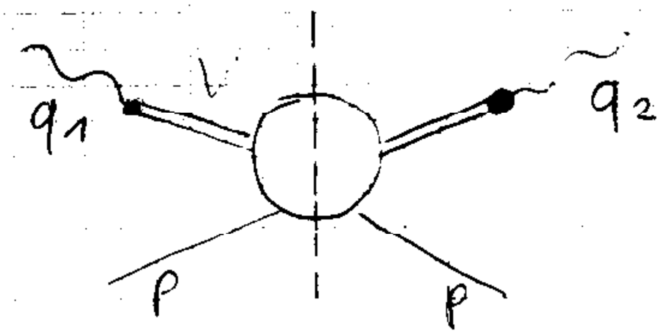
$\nu \gtrsim$ [illegible]

[illegible]

BK model ... cont'd

Representation (*) corresponds to double dispersion relations:

$$F_2 = -q_1 q_2 \int_{4m_\pi^2}^{\infty} dq_1'^2 \int_{4m_\pi^2}^{\infty} dq_2'^2 \; \frac{\rho(q_1'^2, q_2'^2; s)}{(q_1'^2 - q_1^2)(q_2'^2 - q_2^2)}$$



and to $q_1^2 = q_2^2 = -Q^2$. Thus

$$\Phi(s, Q^2) = \int_0^1 d\lambda \int_{4m_\pi^2}^{\infty} dq_1^2 \int_{4m_\pi^2}^{\infty} dq_2^2 \, \delta[\lambda q_1^2 + (1-\lambda) q_2^2 - Q^2] \cdot \rho(q_1^2, q_2^2; s)$$

All (diagonal, non-diagonal) transitions info is hidden in ρ

$\sim \delta(q_1^2 - q_2^2)$

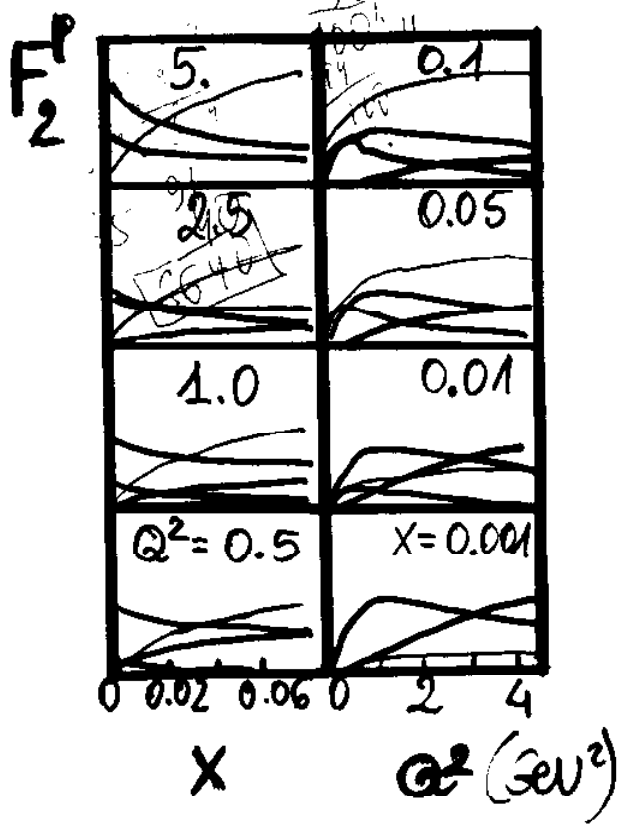
Detailed knowledge of ρ UNNECESSARY!

Sufficient if:

1) leading part of F_2 at $x \to 0$, $Q^2 \to \infty$ give

2) this function is analytic f. of Q^2 (fixed

Contributions to $F_2^p(x,Q^2)$



- sea q's, valence q's } of Δ
- $\mathbb{P}$ part, non-$\mathbb{P}$ part } of Δ

- Onset of partonic (pQCD) mechanism at $Q^2 =$???
- Smooth interplay of perturbative/nonperturbative effects.

CONSTRUCTING THE F_2 ... cont'd

BB & J. Kwieciński (Phys. Lett. B235 ('92)26

- representation (*) revisited & simplified

$$F_2^{(p)}(x,Q^2) = \frac{Q^2}{Q^2+Q_0^2} F_2^{AS}(\bar{x}, Q^2+Q_0^2)$$

$$\bar{x} = \frac{Q^2+Q_0^2}{s+Q^2-M^2+Q_0^2} \equiv \frac{Q^2+Q_0^2}{2M\nu+Q_0^2} \qquad (**)$$

- posesses all main properties of $F_2^{(p)}$ in (*):
  - $F_2^{(p)}(x,Q^2) \rightarrow F_2^{AS}(x,Q^2)$ for large Q
  - preserves analytic properties of $F_2^{(p)}$ in (*) (singularities for $Q^2 < -Q_0^2$)

- photoproduction: $\sigma_{\gamma p}(s) = \lim_{Q^2\to 0} 4\pi^2\alpha \frac{F_2}{Q^2}$
  - correct shape
  - magnitude overestimated by 10-15

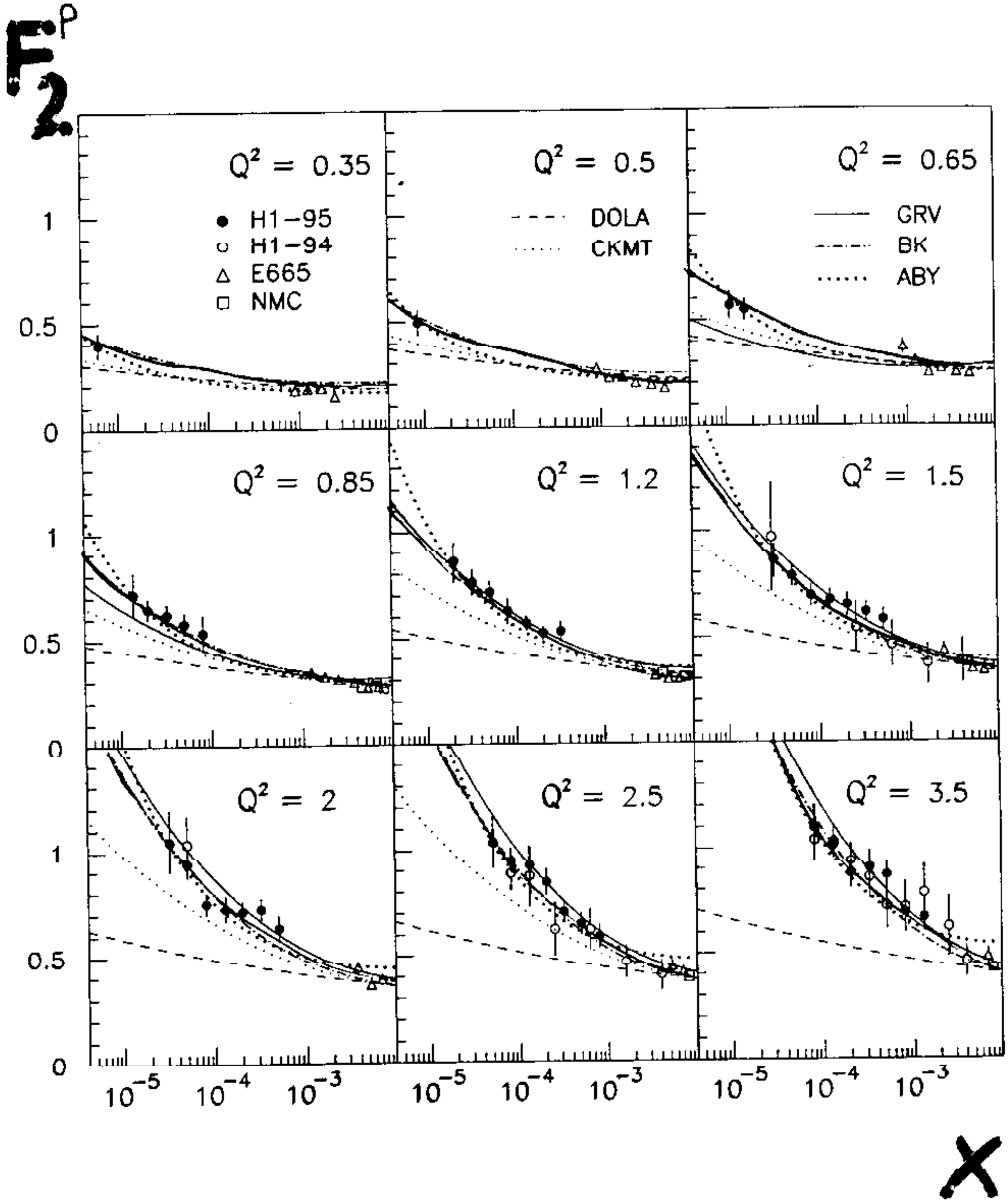
- code for computing $F_2(x,Q^2)$ from (*),(
  available on request from

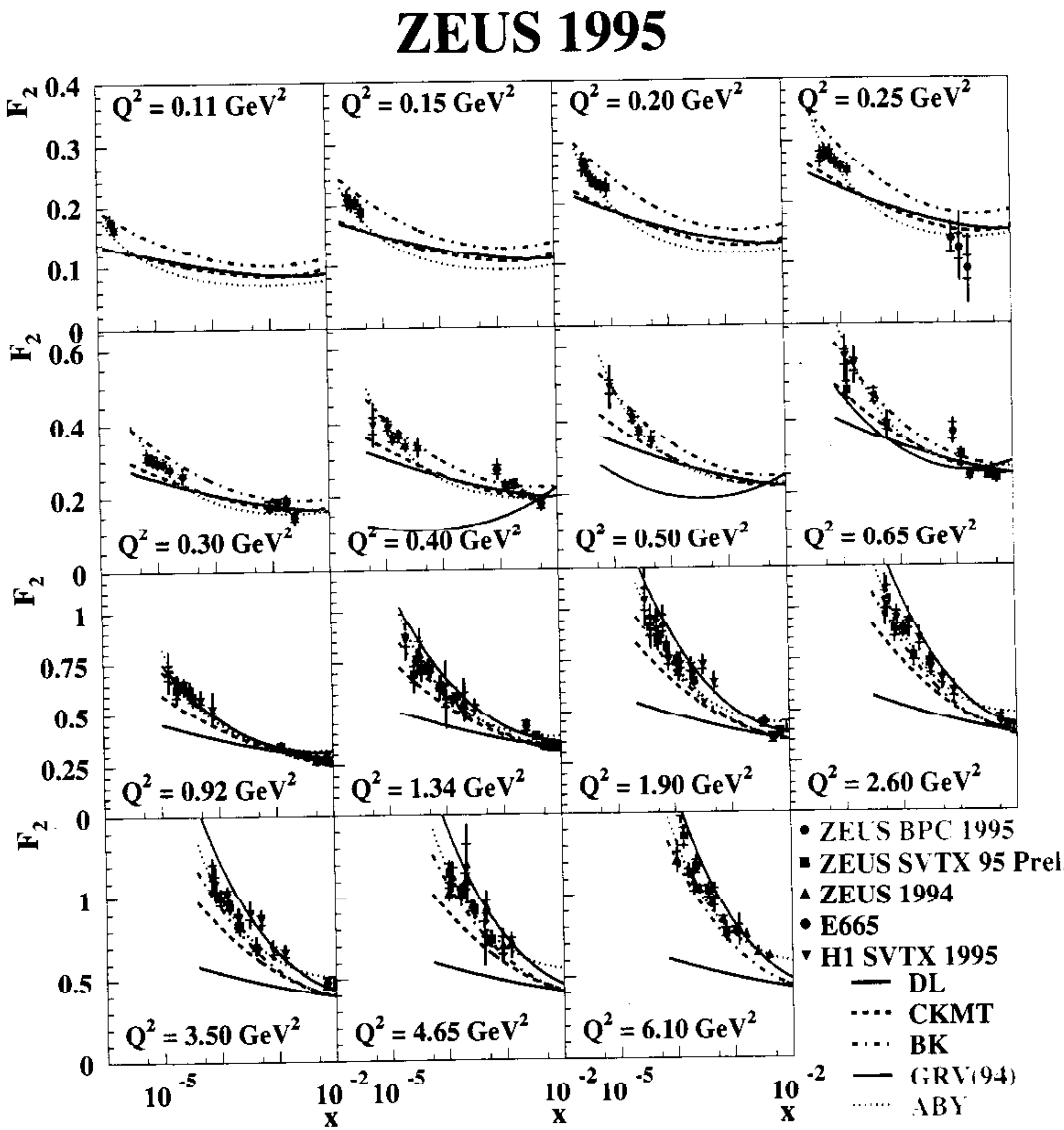
  KWIECINSKIJ @ VSB01.IFJ.EDU.

  or BADELEK @ FUW.EDU.PL

DESY 97-042
hep-ex/970301

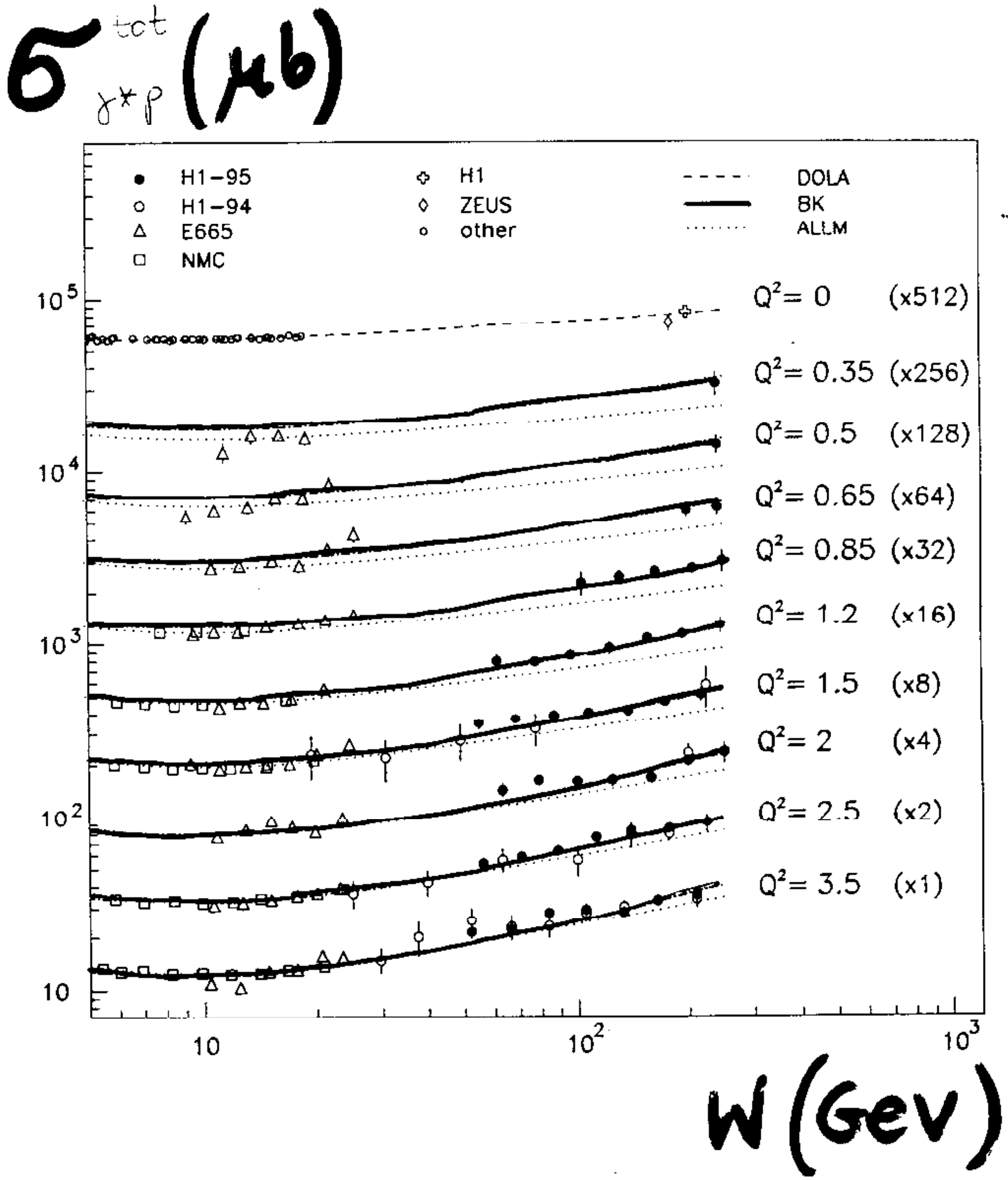
H1




ZEUS 1995
F2
Q² = 0.11 GeV²
Q² = 0.15 GeV²
Q² = 0.20 GeV²
Q² = 0.25 GeV²
Q² = 0.30 GeV²
Q² = 0.40 GeV²
Q² = 0.50 GeV²
Q² = 0.65 GeV²
Q² = 0.92 GeV²
Q² = 1.34 GeV²
Q² = 1.90 GeV²
Q² = 2.60 GeV²
Q² = 3.50 GeV²
Q² = 4.65 GeV²
Q² = 6.10 GeV²
ZEUS BPC 1995
ZEUS SVTX 95 Prel.
ZEUS 1994
E665
H1 SVTX 1995
DL
CKMT
BK
GRV(94)
ABY
x

DESY 97-042
hep-ex/9703013

H1

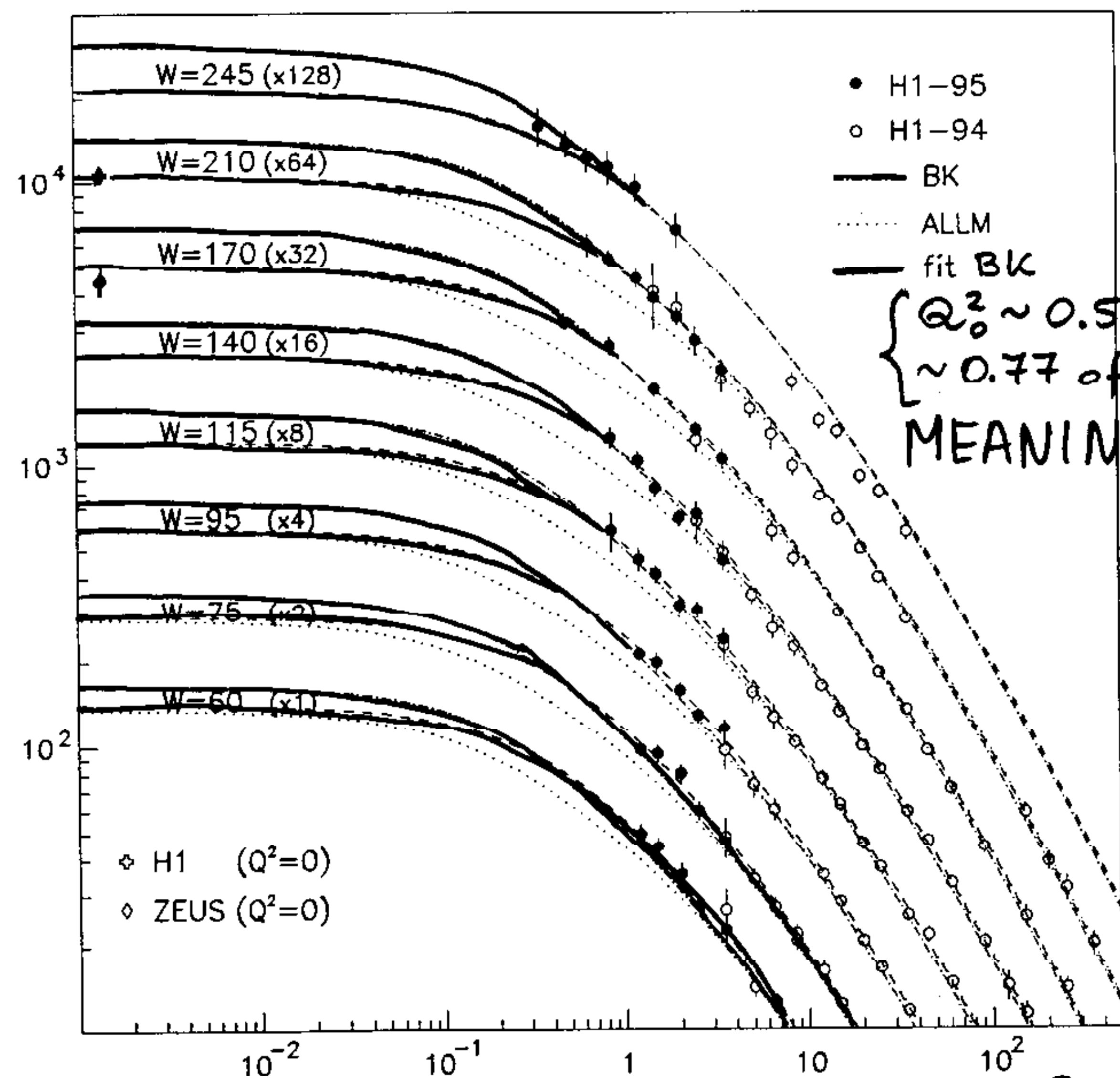


$$\sigma^{tot}_{\gamma^* p} = \sigma_T(x,Q^2) + \sigma_L(x,Q^2) \approx \frac{4\pi^2\alpha}{Q^2} F_2(x,Q^2)$$

DESY-97-04
hep-ex/97030

$\boxed{\text{H1}}$

$\sigma^{eff}_{\gamma^* p}$ (μb)



$\left\{ \begin{array}{l} Q_0^2 \sim 0.5 \text{ GeV} \\ \sim 0.77 \text{ of VMI} \end{array} \right.$

MEANING ??

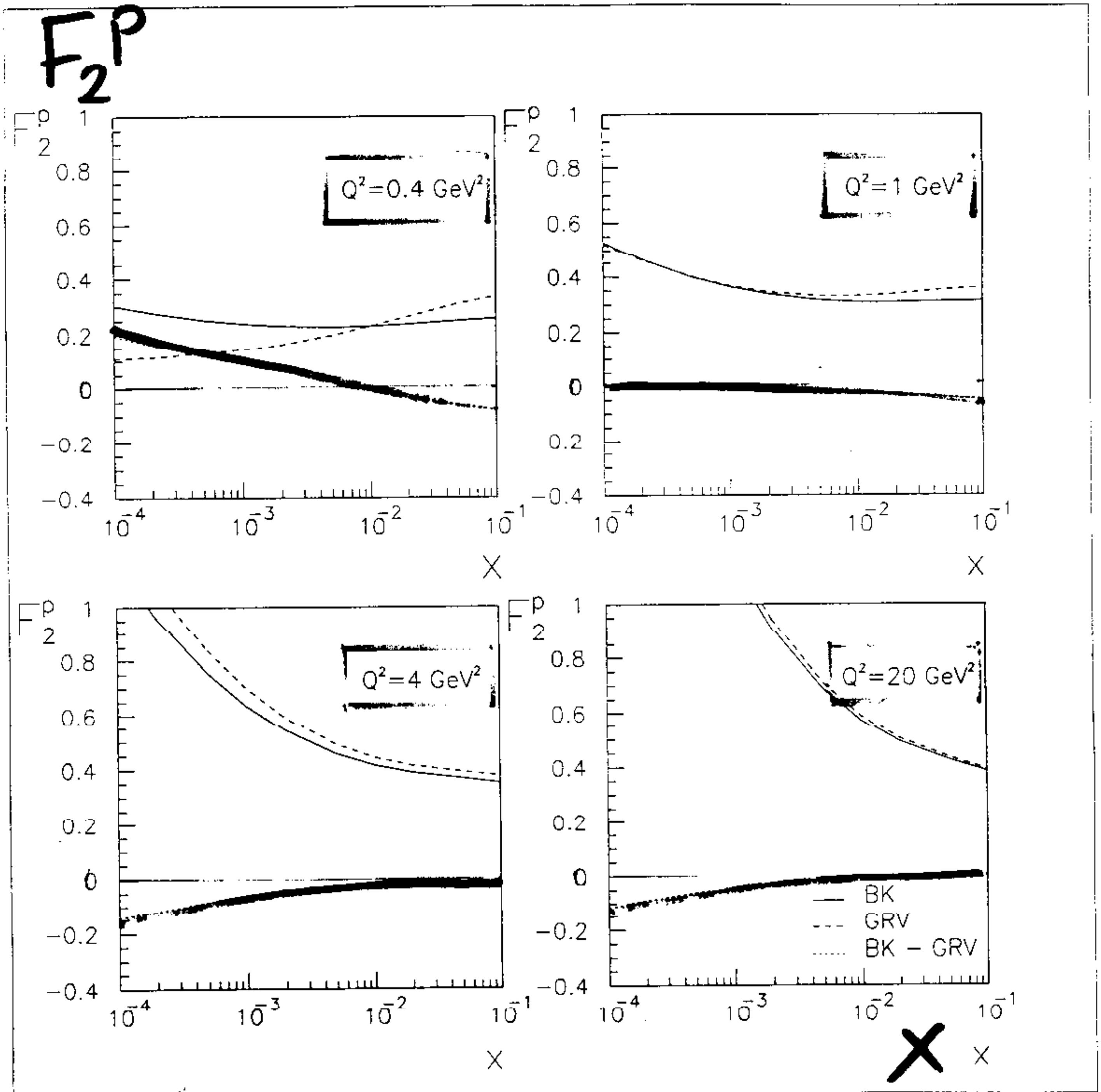
Q^2 (GeV^2)

$$\frac{d^2\sigma}{dx\,dQ^2} \equiv \Gamma\, \sigma^{eff}_{\gamma^* p}(x, y, Q^2)$$

$$\Gamma = \frac{\alpha(2-2y+y^2)}{2\pi Q^2}$$

"Higher twist" (x, Q^2)

$F_2^{HT} = F_2^{BK} - F_2^{GRV}$ (e.g. for a prot



GRV: M. Glück, E. Reya & A. Vogt, Z. Phys. C67 ('95) 4
"GRV94"; NLO, $\overline{MS}$, with charm (Bethe-Heit

BK: B. Badełek & J. Kwieciński, Z. Phys. C43 ('89) 25
Phys. Lett. B2

Shadowing in the deuteron and the F_2^n/F_2^p data

- Data: $2F_2^d \stackrel{\text{df}}{=} F_2^p + F_2^n$

$$\Downarrow$$

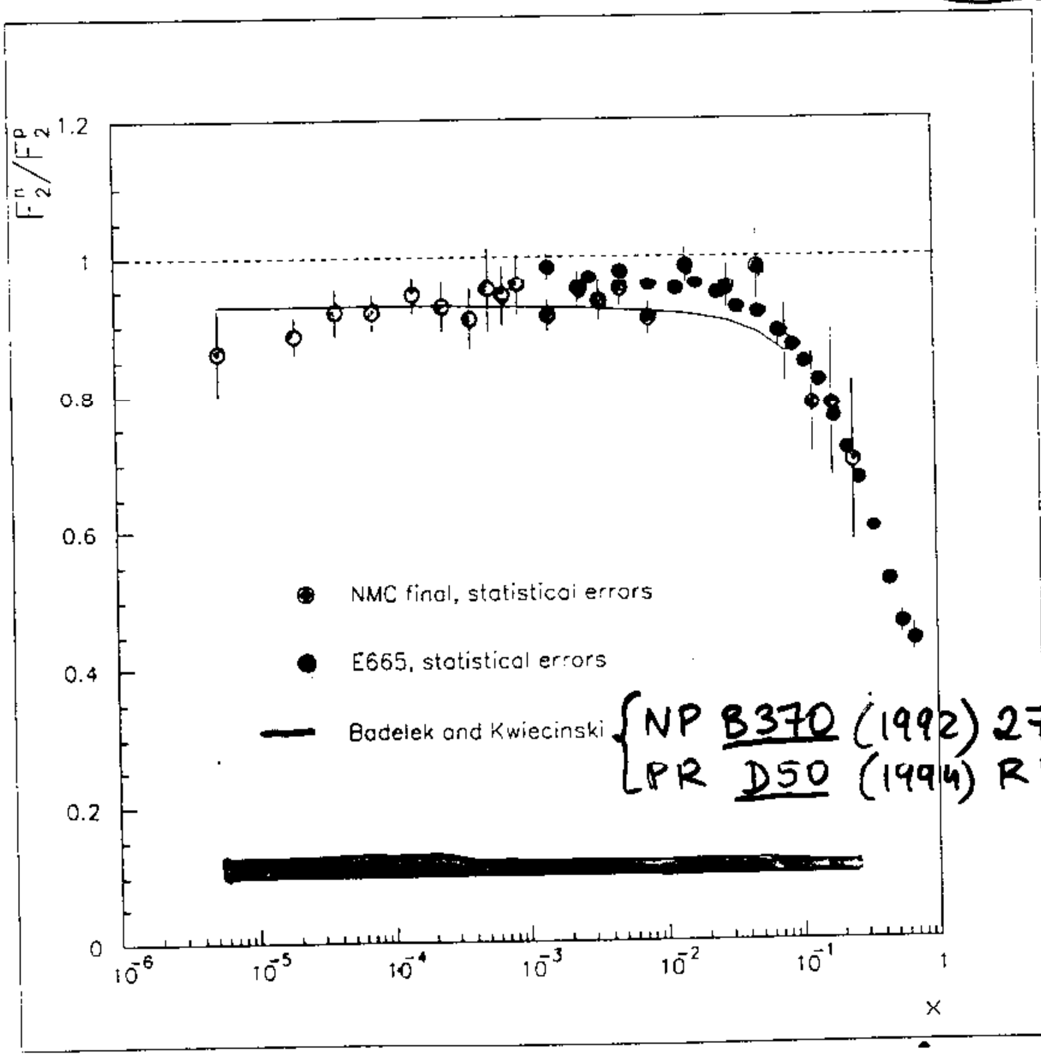
$$\left(\frac{F_2^n}{F_2^p}\right)_{exp} \stackrel{\text{df}}{=} 2\,\frac{F_2^d}{F_2^p} - 1$$

- In reality: $2F_2^d = F_2^p + F_2^n - 2\underbrace{\delta F_2^d}_{>0}$

$$\Downarrow$$

$$\left(\frac{F_2^n}{F_2^p}\right) = \left(\frac{F_2^n}{F_2^p}\right)_{exp} + \frac{2\delta F_2^d}{F_2^p}$$

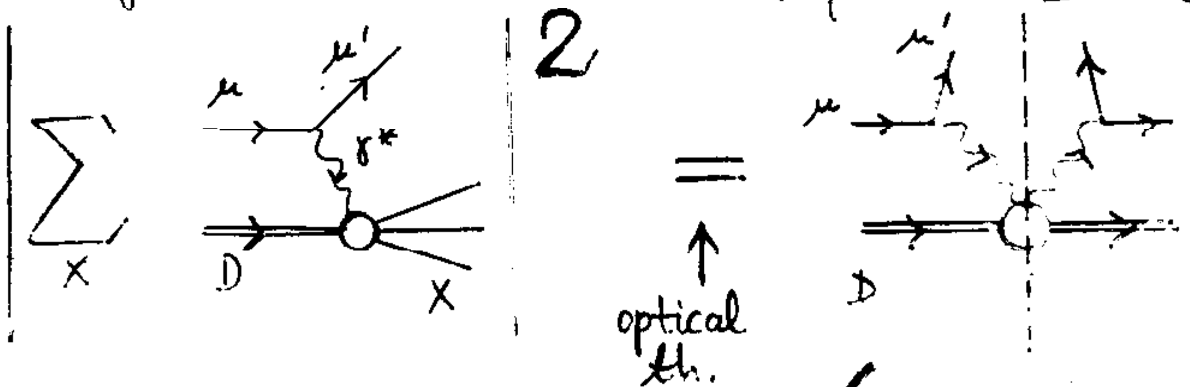
$\dfrac{F_2^n}{F_2^p}$

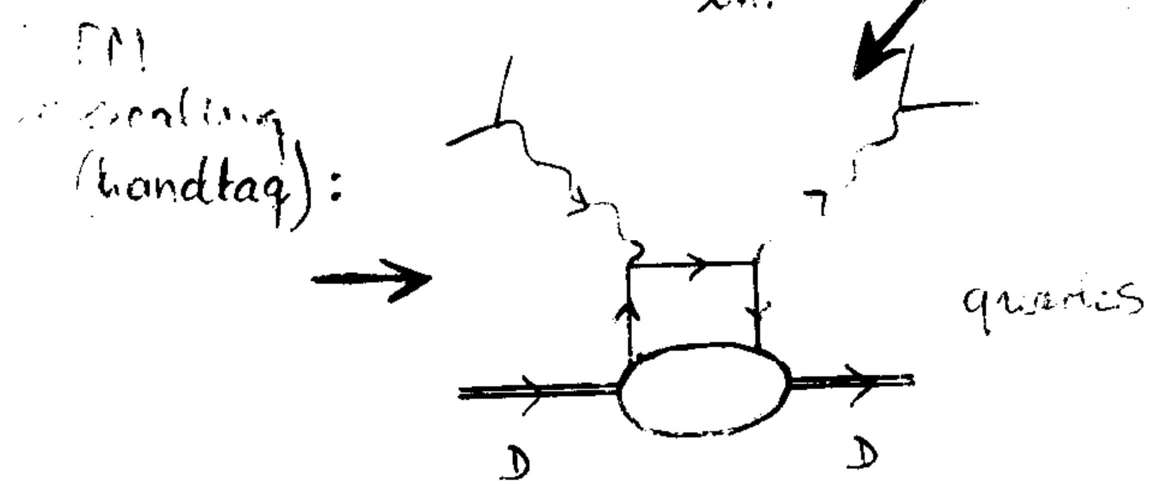


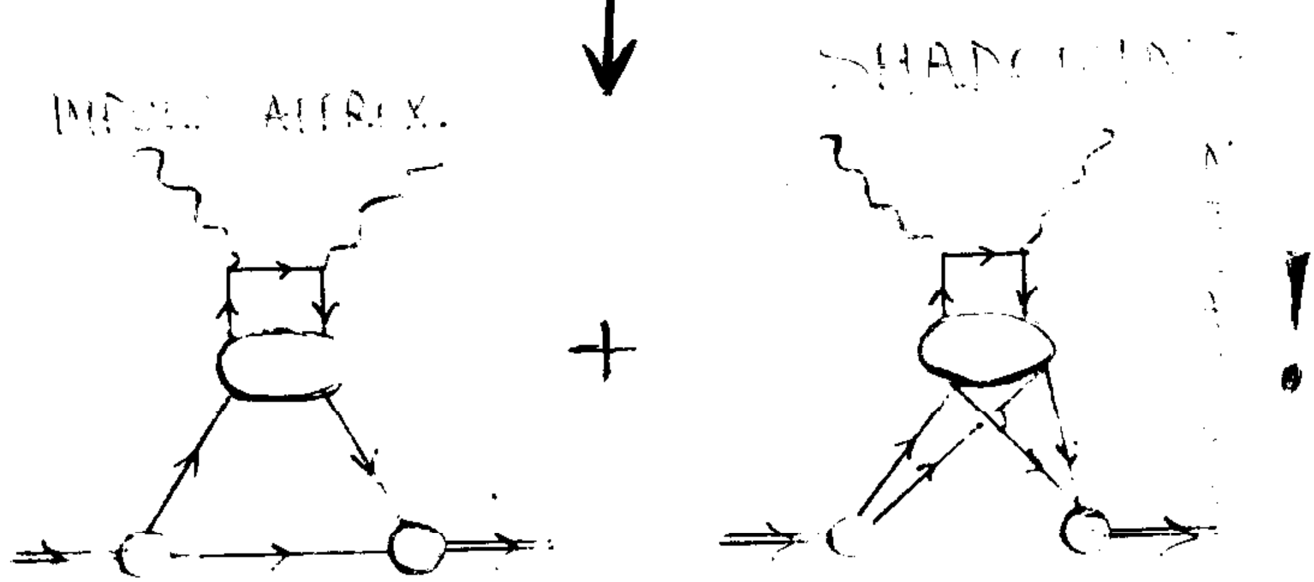
Badelek and Kwiecinski: NP B370 (1992) 278; PR D50 (1994) R4

QUANTITATIVE ESTIMATE OF SHADOWING IN DEUTERON

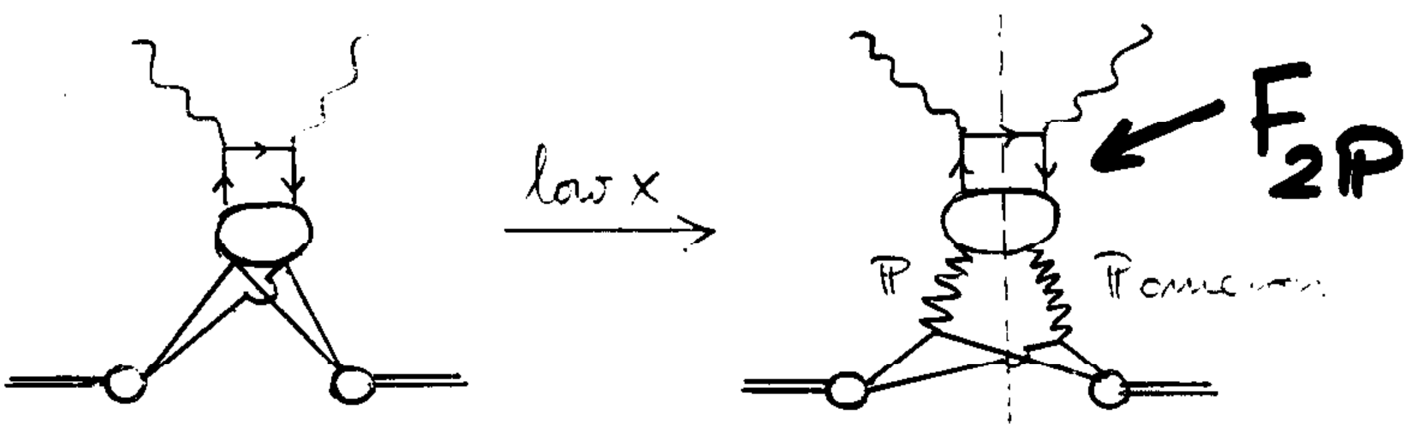
J. Kwieciński & BB, Nucl. Phys. B370 (1992) 275
Phys. Rev. D50 (1994) R4



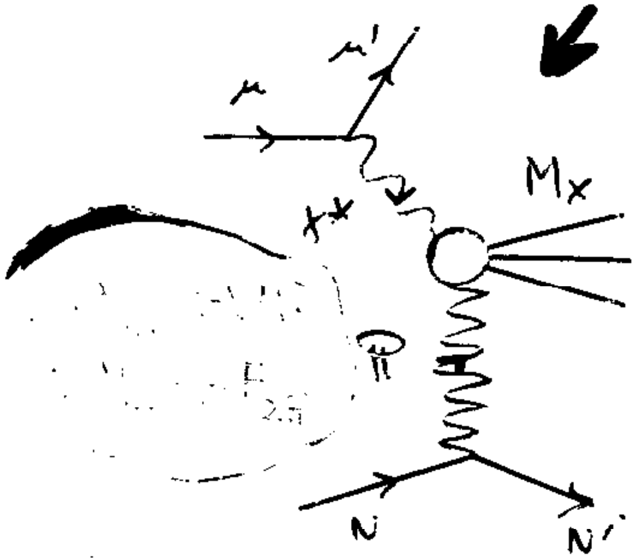




QUANTITATIVE ESTIMATE cont'd

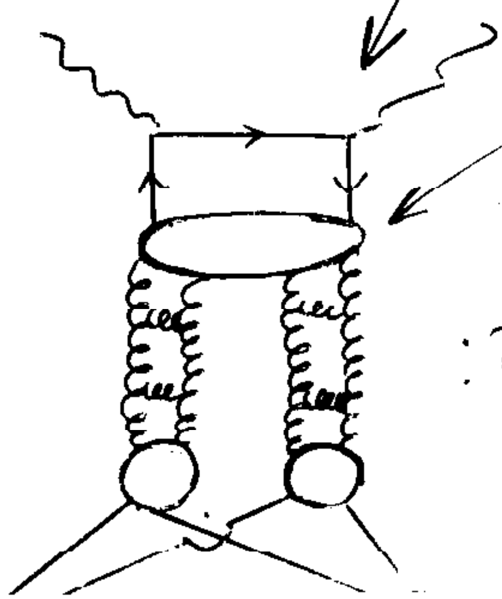


KNOWN! diffractive production dominates



(different cut)

in QCD



different QCD coupling of (gggg) to ($q\bar{q}$)

: $\mathbb{P}$ ≡ two gluon ladder

$$\boxed{\delta F_2^d = \delta F_{2V}^d + \delta F_{2P}^d}$$

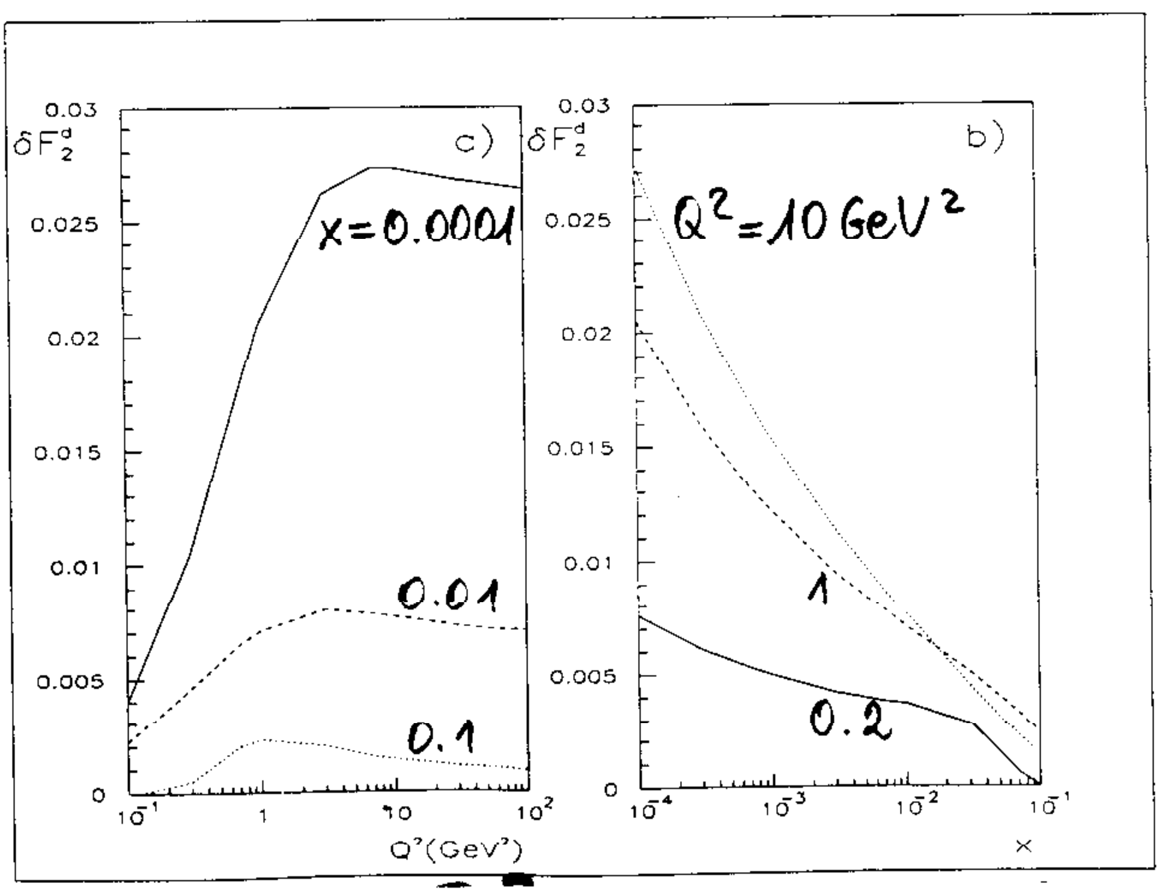
$\delta F_{2P}^d(x) \Leftarrow$ spectrum of diffractively produced masses (small x) and d form factor (hi x)

K. Golec-Biernat + J. Kwieciński
Phys. Lett. B353 (1995) 329

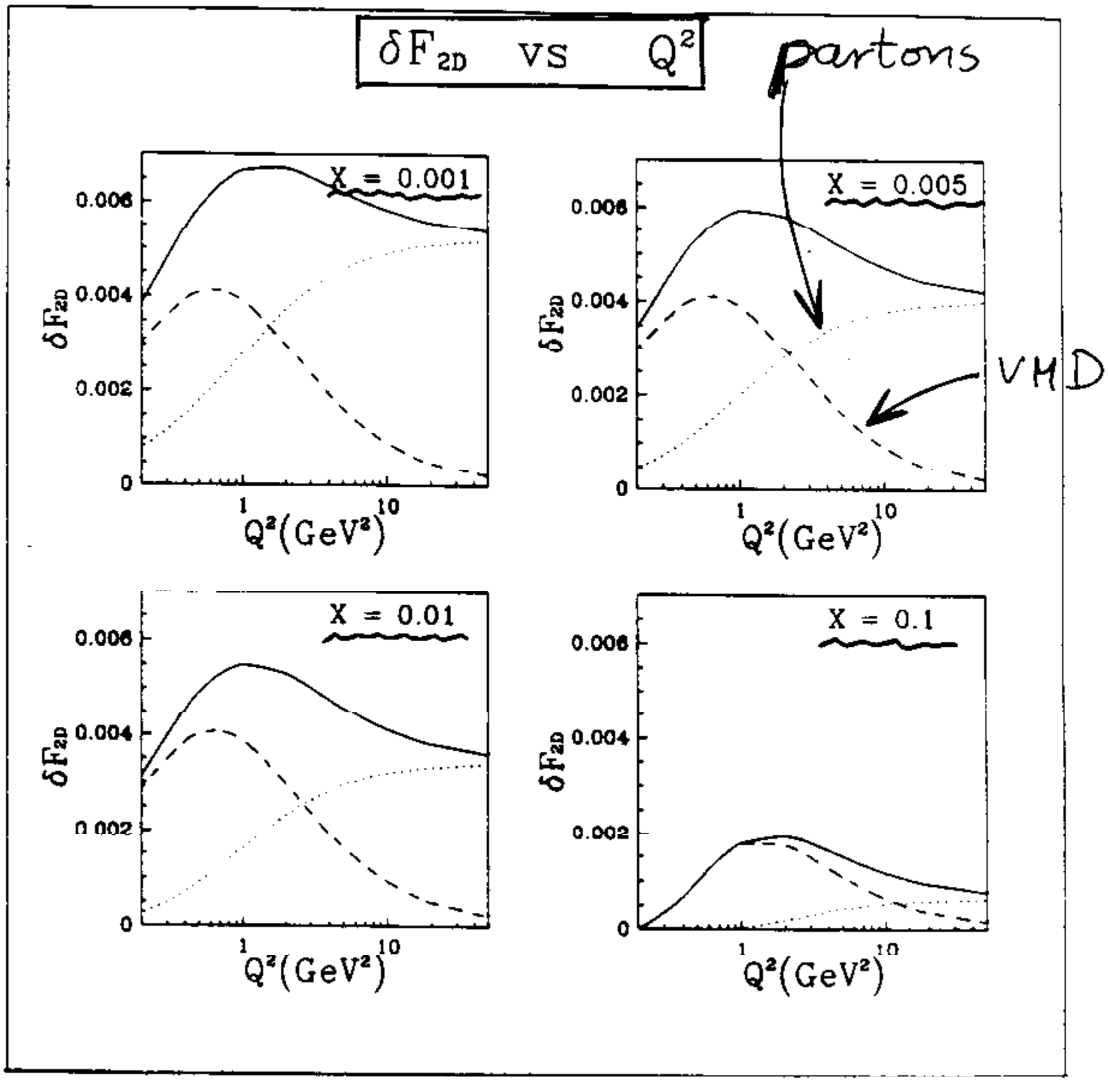
$\delta F_{2V}^d(x) \Leftarrow d$ form factor

$$\delta F_{2V}^d(Q^2) \sim \frac{Q^2}{(Q^2+M_V^2)^2}$$

δF_2^d



$$\boxed{\delta F_2^d = \delta F_{2V}^d + \delta F_{2P}^d}$$

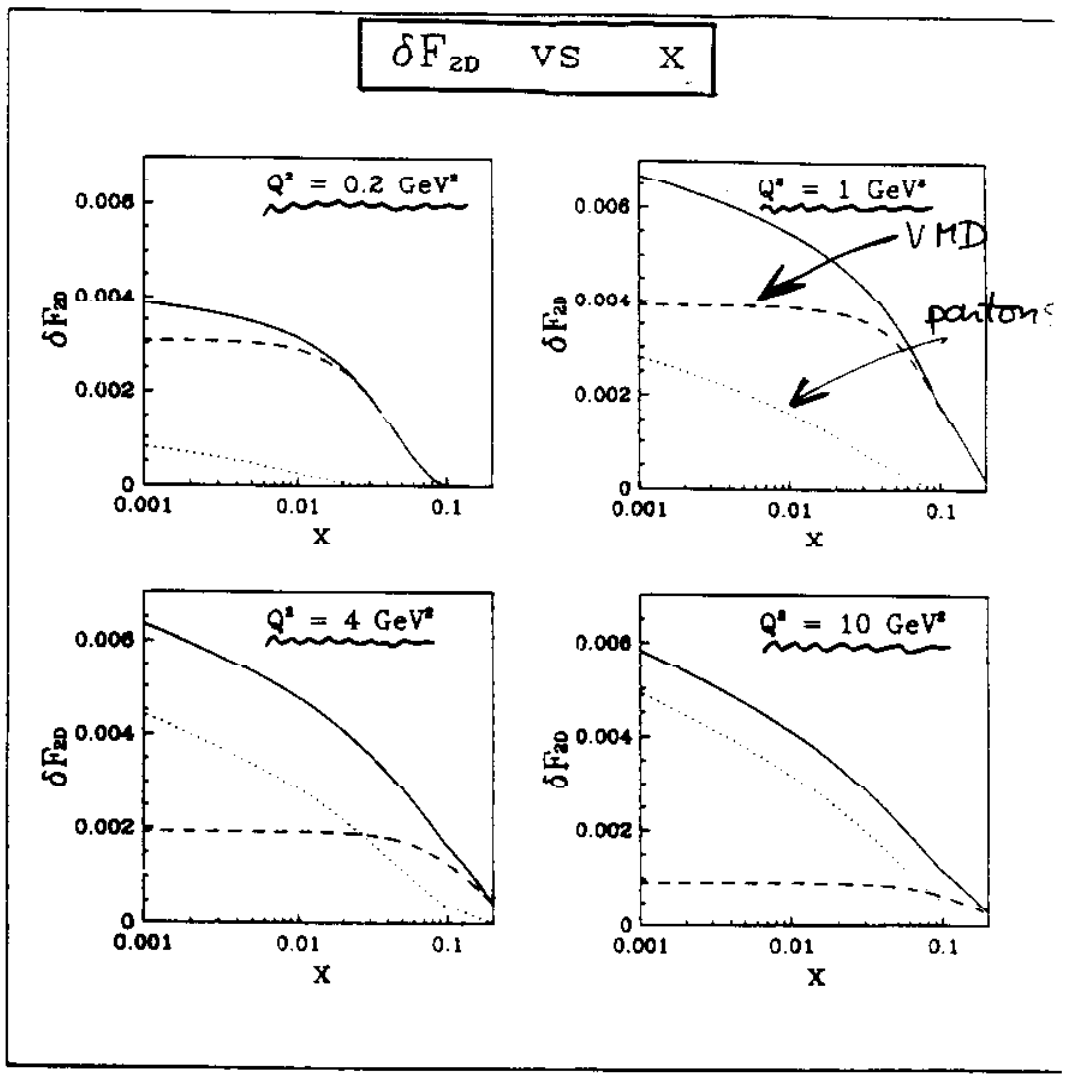


$$\delta F_{2V}^d(Q^2) \sim \frac{Q^2}{(Q^2 + M_V^2)^2}$$

$$\boxed{\delta F_2^d = \delta F_{2V}^d + \delta F_{2P}^d}$$

$\delta F_{2P}^d(x) \Leftarrow$ spectrum of diffractively produced masses (small x) and d form factor (hi x)

$\delta F_{2V}^d(x) \Leftarrow$ d form factor



NMC final

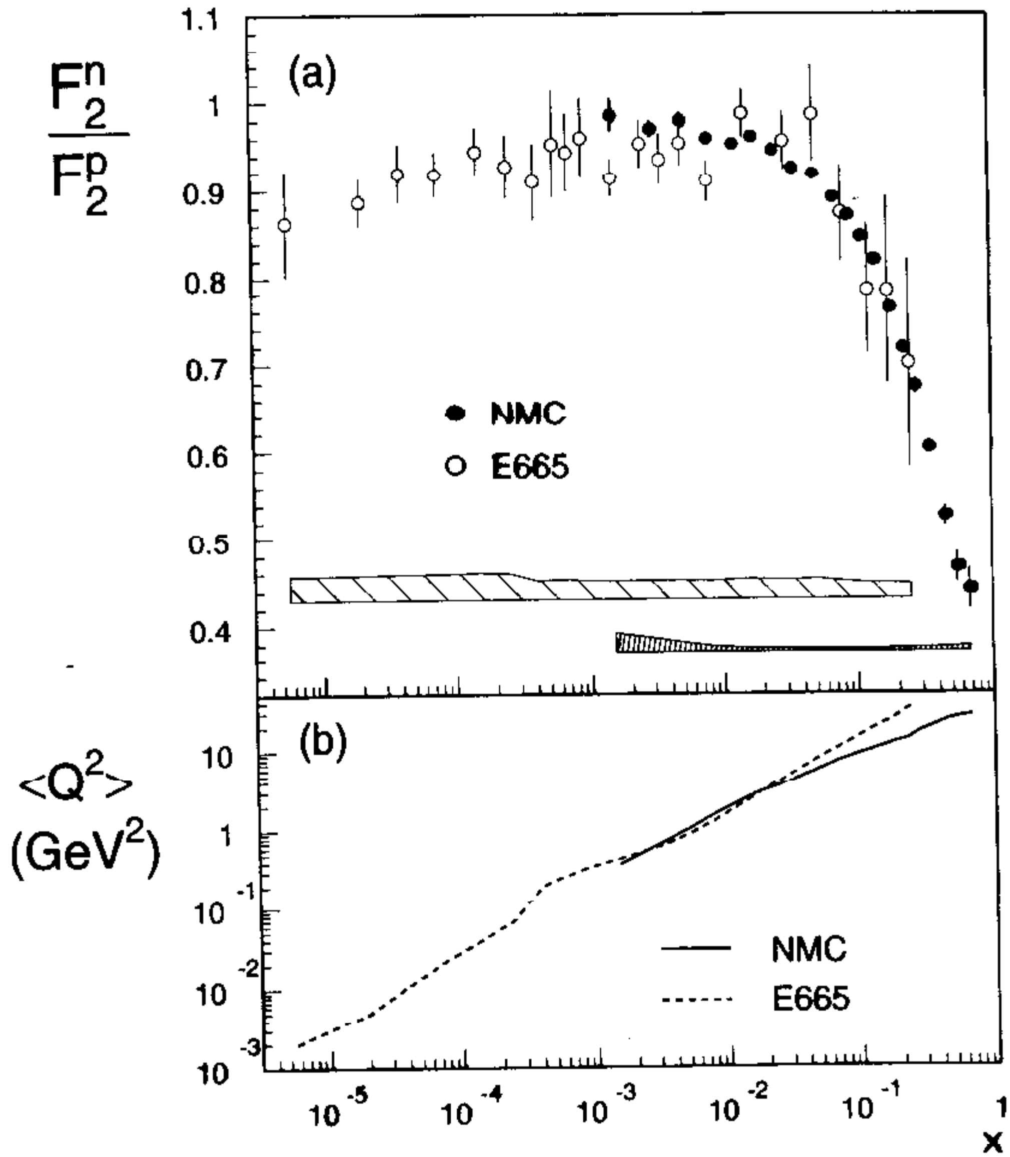


Figure 9: (a) Comparison of the x dependence of the present data for F_2^n/F_2^p with the results from the E665 collaboration [39]. The error bars represent the statistical errors and the bands at the bottom indicate the systematic uncertainties. (b) The average Q^2 of the E665 and the present data versus x.

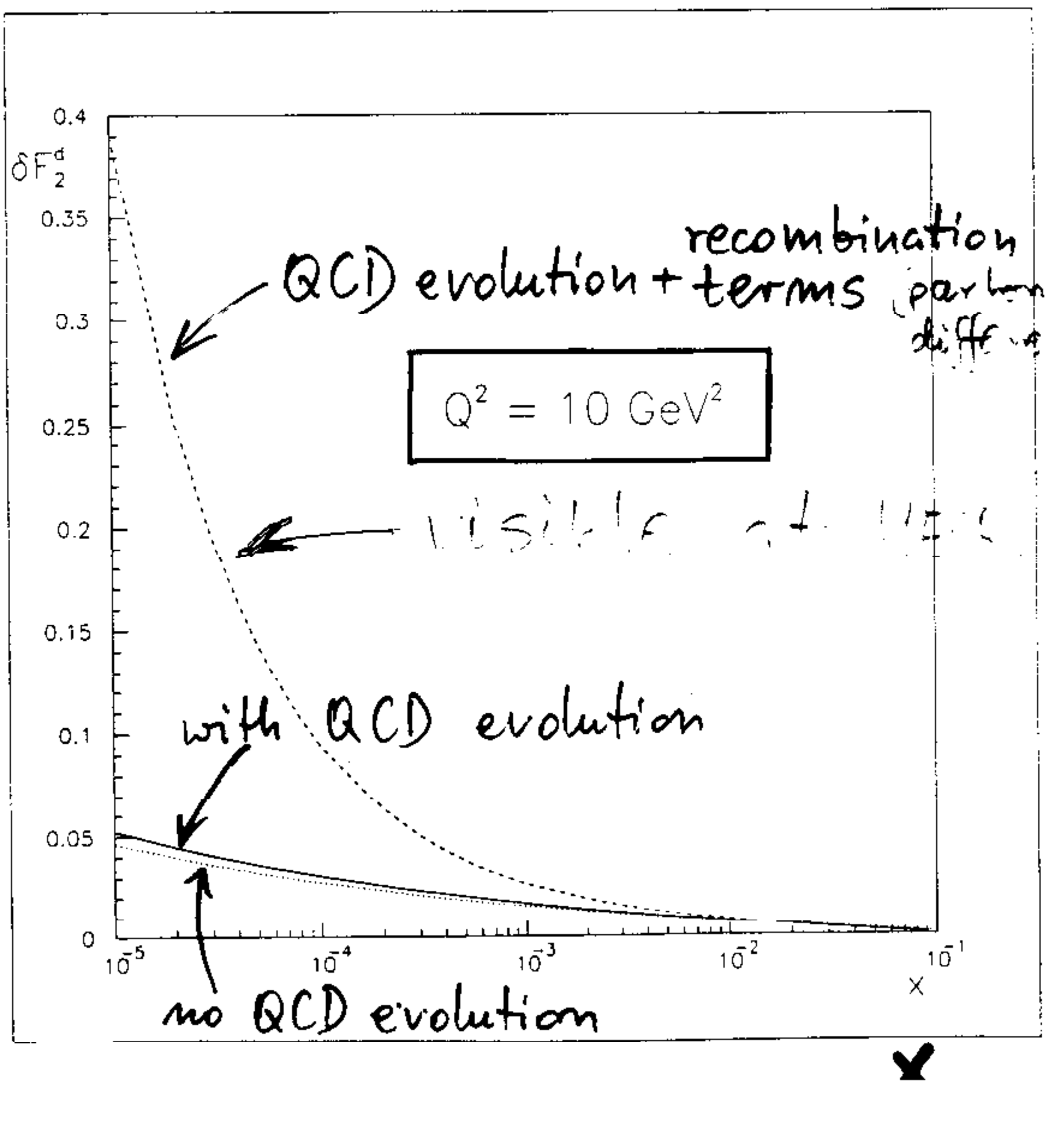
Plans for HERA: e-d scatt.

Important since

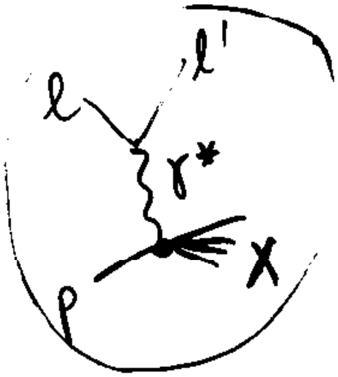
- F_2^n measurement possible
- $F_2^p - F_2^n$ — " — (flavour n-
- Improves QCD analysis of F_2^N dat
  (F_2^d almost entirely flavour singlet)

BUT to this aim: quantification of γ^* shadowing in d needed!

δF_2^d



$$\boxed{R(x,Q^2)} = \frac{\sigma_L}{\sigma_T}$$



$$\frac{d^2\sigma}{dx\,dQ^2} = \Gamma_{\gamma^*}\left\{\sigma_T(x,Q^2) + \varepsilon\,\sigma_L(x,Q^2)\right\}$$

$$\varepsilon = \frac{1 - y - \frac{Q^2}{4E^2}}{1 - y + \frac{y^2}{2} + \frac{Q^2}{4E^2}}$$

- Measure σ for fixed (x,Q^2) and different y, i.e. different ε (or comb $\nu, \bar{\nu}$

- $R = \frac{\sigma_L}{\sigma_T} = \frac{F_2 - 2xF_1}{2xF_1} = \frac{F_L}{2xF_1} = \frac{F_L}{F_T}$

QPM: $F_L = 0 \Rightarrow R = 0$ (ev. effects due to m_q, (k_T

QCD LLA: $R = 0$

- $$R^{QCD}_{NLL} = \frac{1}{2xF_1}\frac{\alpha}{2\pi}x^2\int_0^1 \frac{dy}{y}\left[\frac{8}{3}F_2(y) + 4\sum_i (g_{iV}^2 + g_{iA}^2)\,y\,G\left(1 - \frac{x}{y}\right)\right.$$

glu…

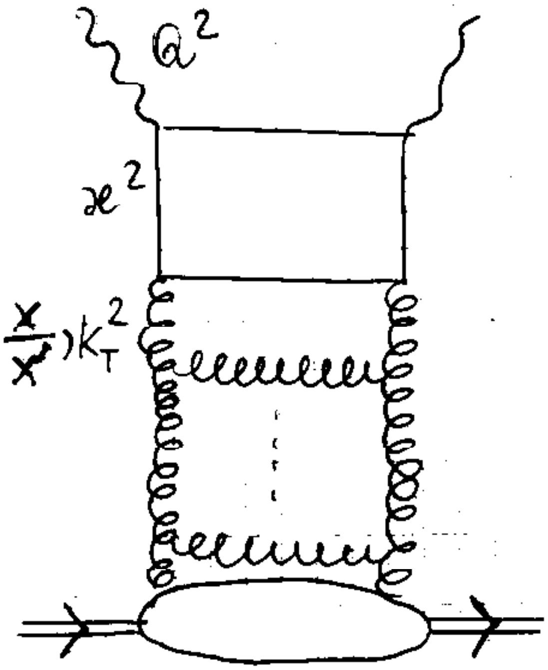
- SLAC fit to all data

- Rad … R(x,Q²) …

A model for F_L & $R = F_L/F_T$ at low x and low Q^2

Kwieciński, Staśto & BB
DTP/96/16; → Z. Phys. C

- "Off-shell" model originated from the high energy factorization formula.



F_L^0 – comes from $\gamma^* g$ fusion calculated perturbatively

f – unintegrated gluon distribution, at small x controlled by BFKL

- $$F_{L,T}(x, Q^2) = \int_x^1 \frac{dx'}{x'} \int \frac{dk_T^2}{k_T^2} \hat{F}^0_{L,T}(x', k_T^2, Q^2)\, f\left(\frac{x}{x'}, k_T^2\right)$$

leading twist: [illegible]

$$F_L(x, Q^2) = \int_x^1 \frac{dx'}{x'}\, C_L(x', Q^2, \alpha_s(Q^2))\, \frac{x'}{x}\, g\left(\frac{x'}{x}, Q^2\right)$$

↑ resummed leading powers $\alpha_s \ln$

- approx

$$f(x, k_T^2) = \frac{\partial x g^{AP}(x, Q^2)}{} \Big|$$ ok for

A model for F_L & R ... cont'd

- Extrapolation to $Q^2=0$
  - have to freeze evolution of $g(x, Q^2)$ and freeze argument of $\alpha_s(Q^2)$

  $\downarrow$ $Q^2 \to Q^2 + 4m_q^2$

  - F_L can be continued to $Q^2=0$ respecting $F_L \sim Q^4$ for $Q^2 \to 0$

- HT in F_L
  - terms vanishing as $\frac{1}{Q^2}$ for $Q^2 \to \infty$
  - integration over $\varkappa_T$ divided into

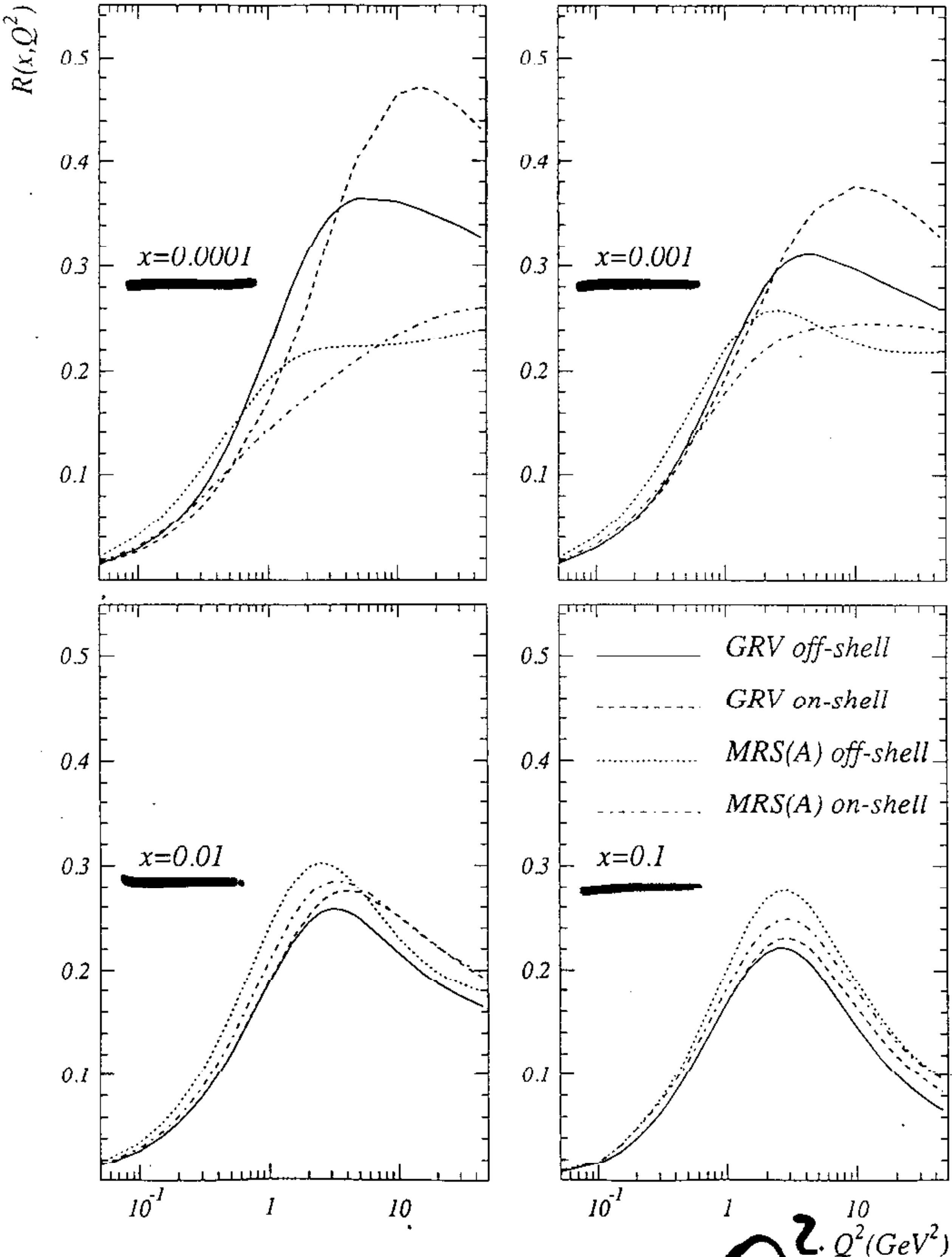
    $0 < \varkappa_T^2 < \varkappa_{0T}^2 \leftarrow$ arbitrary, ~1 Ge

    $\varkappa_T^2 > \varkappa_{0T}^2$
  - at low $\varkappa_T^2$ ("soft" physics):

    $\alpha_s \, z \, g(z, Q^2) \to A$ ($\approx$ const)
  - get A from F_2: nonpert. part at $Q^2 \to \infty$ originates from $\varkappa_T^2$ lo (soft $\mathbb{P}$ contrib; intercept = 1)
  - $F_L^{HT} \to \frac{1}{Q^2}$ at $Q^2 \to \infty$ OK.

Kwiecinski, Stasto, BB (DTP/96/16 and → Z. Phys. C)

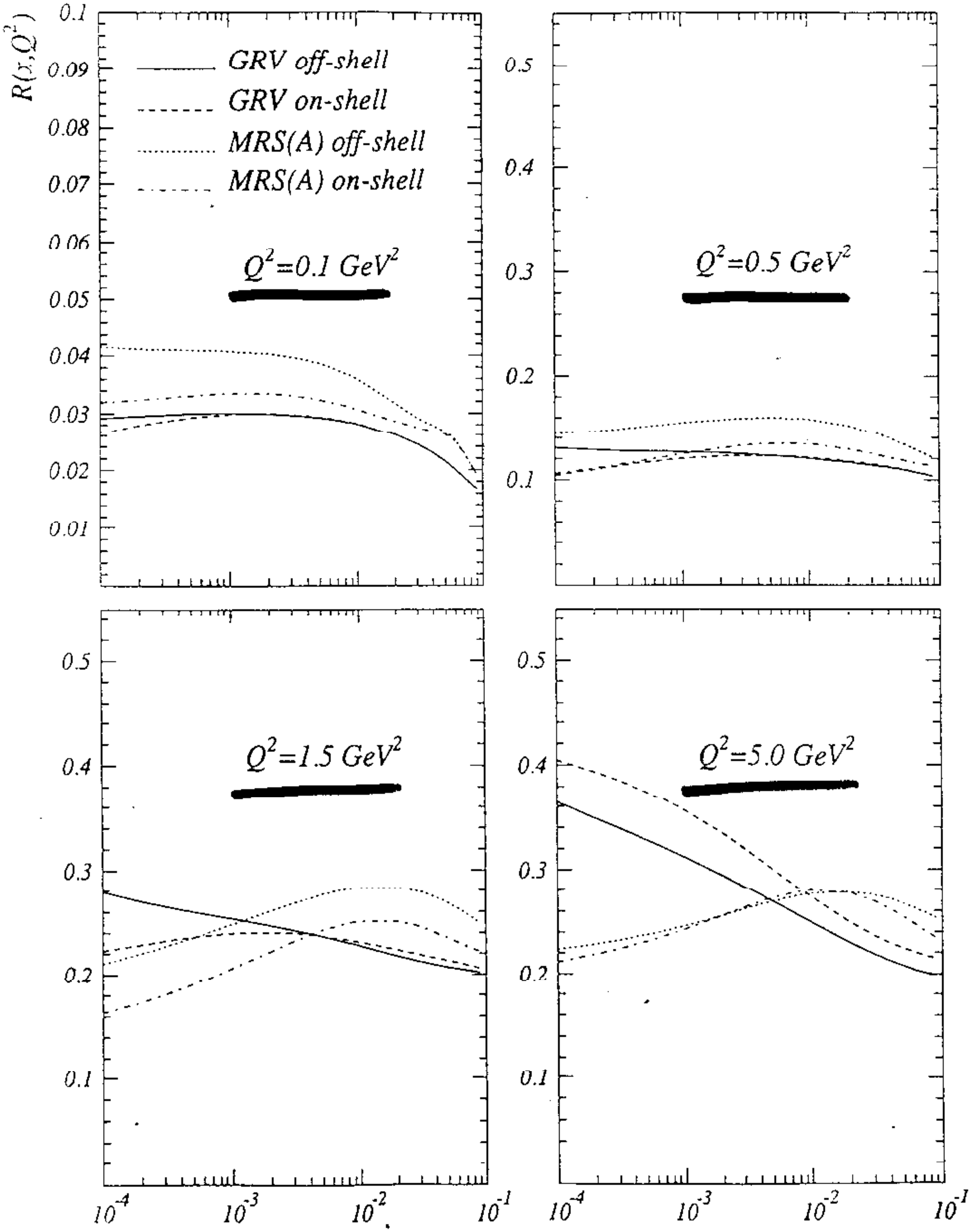
R.



Q^2.

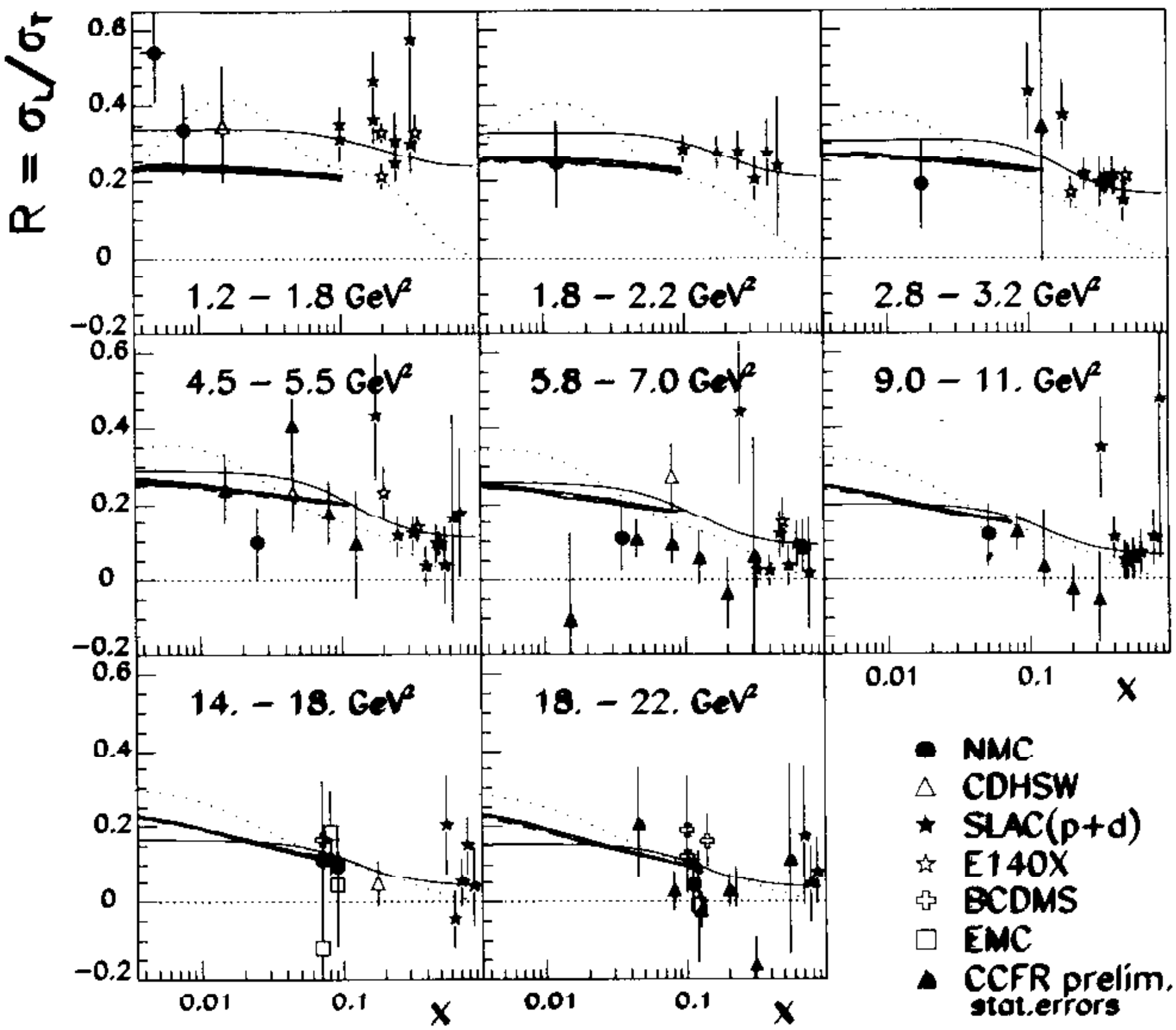
Kwieciński, Staśto, BB (DTP/96/16 and → Z. Phys. C)

R.



X

R - data & parametrisations



lines: solid – R(1990) – SLAC param.
dotted – QCD predictions with MRS param.
dashed – model Badelek, Kwiecinski, Stasto
(Durham preprint DTP/96/16 and to appear in Z. Phys. C)

Σmary

- Importance of non-perturbative effects for understanding physics + analysis
- Limiting conditions for s.f. formulat
- Parametrisations (models) collected
- Nonperturbative manifestations –
  - F_2^p at $Q^2 \lesssim 1\,GeV^2$
  - R –‖–

-quantified (preliminarily)

- γ^* shadowing in the deuteron – quantified; possible test of a ne mechanism at HERA

- First attempts to understand non-perturbative physics – DONE!