M. Botje, UU94 10.[illegible].94

Future measurements of α_s from scaling violations at HERA

– Report on studies done during the Workshop 'Future physics at HERA', 1995-6.

■ 'Tools'

A Detailed Comparison of NLO QCD Evolution Codes

J. Blümlein[a], M. Botje[b], C. Pascaud[c], S. Riemersma[a], W.L. van Neerven[d], A. Vogt[e,f], and F. Zomer[c]

■ 'Experimental errors'

Future Precision Measurements of $F_2(x,Q^2)$, $\alpha_S(Q^2)$ and $xg(x,Q^2)$ at HERA

M. Botje[a], M. Klein[b], C. Pascaud[c]

■ 'Theoretical errors'

Theoretical Uncertainties in the Determination of α_s from F_2^p at HERA

J. Blümlein[a], S. Riemersma[a], W.L. van Neerven[b], and A. Vogt[c,d]

Comparison of NLO evolution codes

- Altarelli Parisi evolution:

$$\frac{\partial}{\partial \ln Q^2} \begin{bmatrix} \Sigma \\ g \end{bmatrix} = \frac{\alpha_s}{4\pi} \begin{bmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{bmatrix} \otimes \begin{bmatrix} \Sigma \\ g \end{bmatrix}$$

QCD splitting functions, known up to NLO

$$\frac{\partial \alpha_s}{\partial \ln Q^2} = -\frac{\beta_0}{4\pi}\,\alpha_s^2 - \frac{\beta_1}{(4\pi)^2}\,\alpha_s^3 + \cdots$$

- Solution of AP: evolve $\Sigma(x, Q_0^2)$, $g(x, Q_0^2)$, α_s given $\alpha_s(\mu_0)$ or Λ.

'x-space' Evolve numerically on a grid in x and Q^2.

- Conceptually simple.
- Worry about numerical accuracy.

'N-space' Take moments: $f^N \equiv \int_0^1 dx\, x^{N-1} f(x)$

⇨ System of ordinary differential equations.

⇨ Analytical solution; transform back to x-space.

- Mathematically more involved.
- Accuracy $\sim 10^{-5}$: no problem.

- <u>Aim</u> for an accuracy $\lesssim 0.1\%$ over a wide kinematic range

  $\sim 10^{-5} < x < 1$

  $\sim 4 < Q^2 < 10^4 \text{ GeV}^2$

- <u>HERA workshop</u>: 4 (+2) NLO codes compared:
  - Simple 'toymodel' parametrisations of pdf's at input scale $Q^2 = 4\,\text{GeV}^2$. $10^{-5} < x < 1$.
  - Evolve α_s with $\Lambda = 250$ MeV and $f = 4$.
  - Evolve pdf's up to $Q^2 = 10^4\,\text{GeV}^2$ and $f = 4$.
  - Compare results.

- <u>Evolution codes</u>:

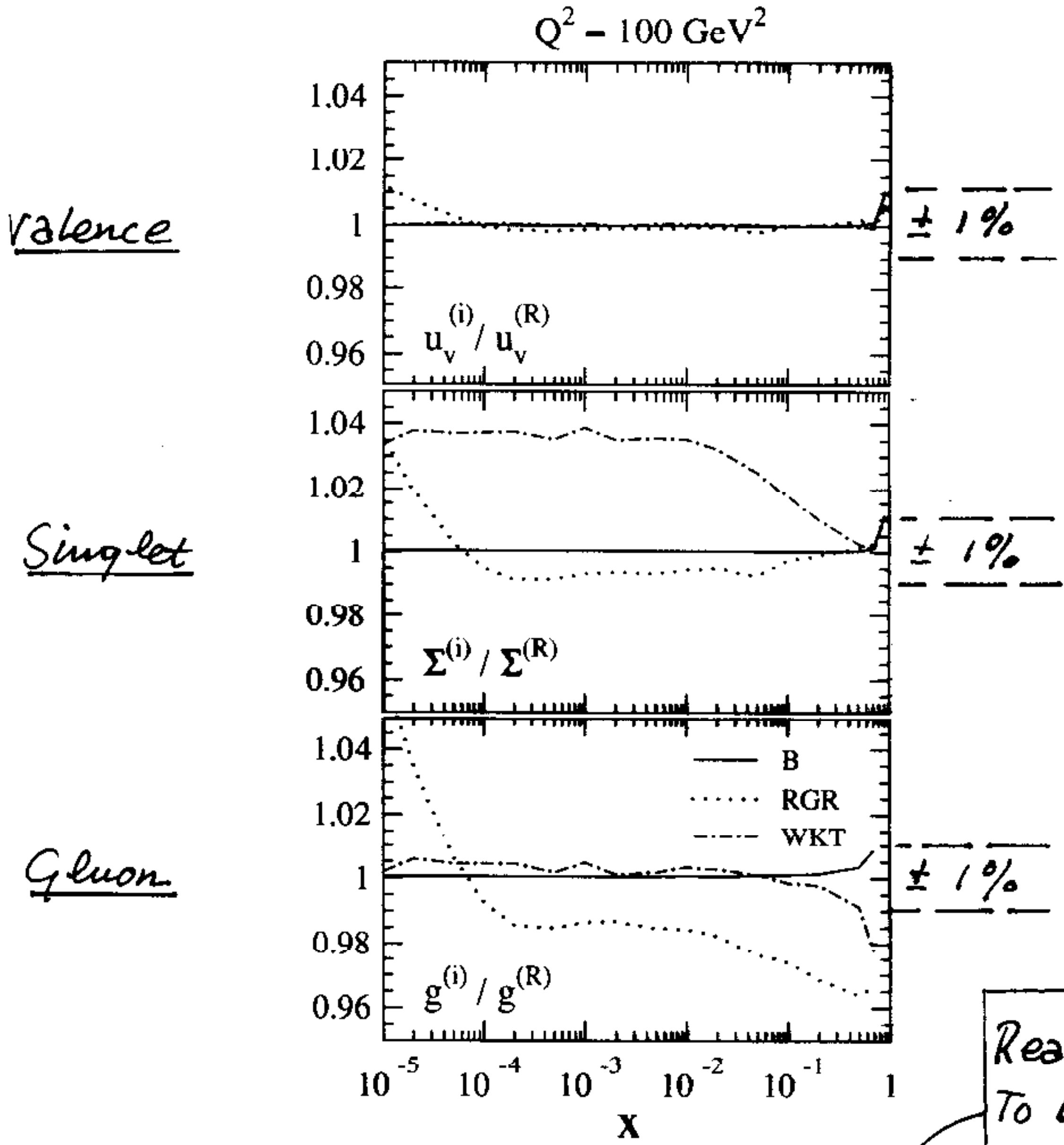
(B)	Botje / ZEUS	x - space
(PZ)	Pascaud Zomer / H1	x - space
(V)	Vogt	N - space
(R)	Riemersma	N - space

+

(WKT)	W.K. Tung	x - space
(RGR)	R.G. Roberts	x - space

■ Evolve to $Q^2 = 100$ GeV

o Compare Botje (B), Tung (WKT), Roberts (RGR) to Riemersma.

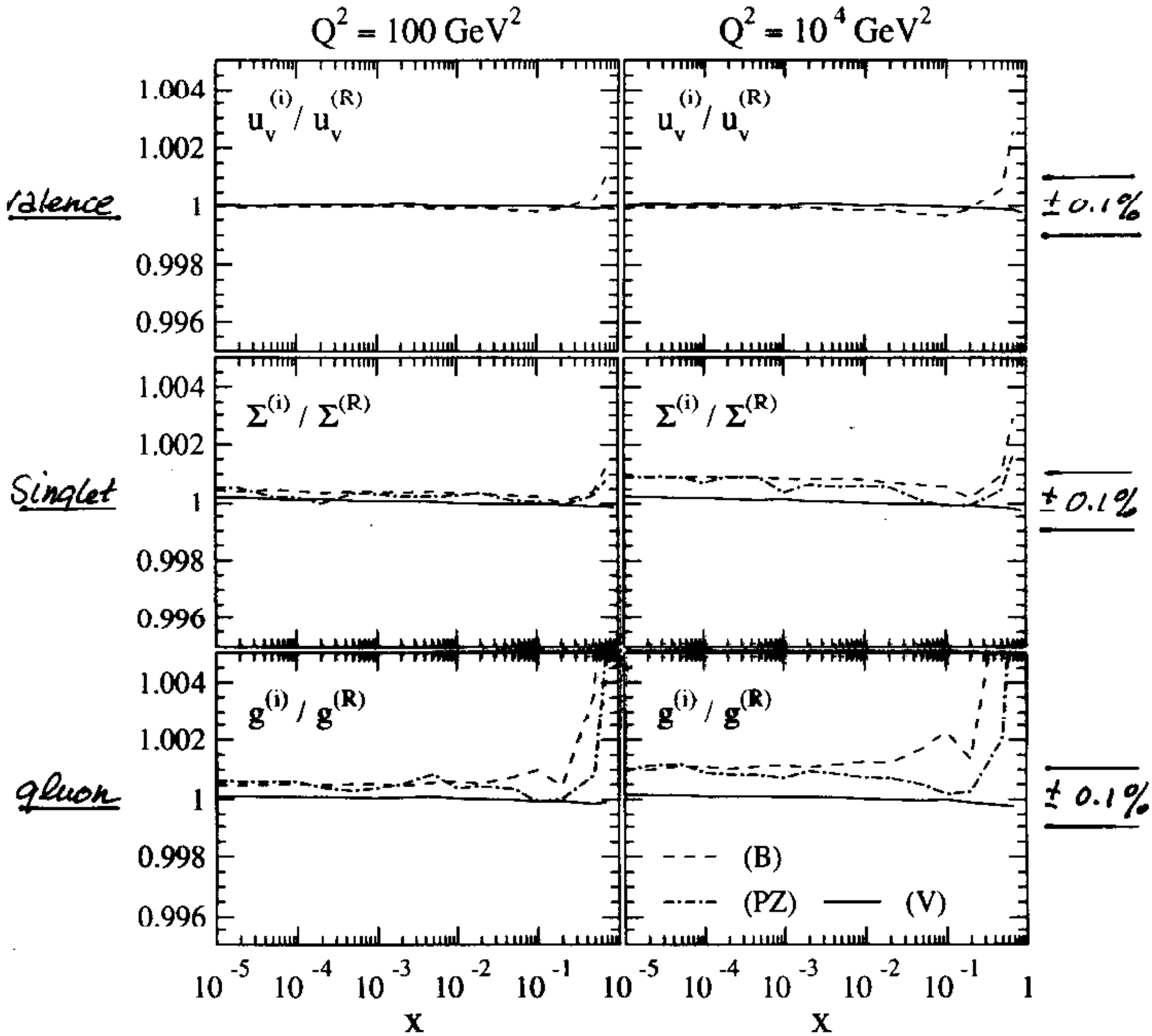


Real ?
To be sorted out...

➡ WKT, RGR : $\frac{\Delta f}{f}$ up to ~ 4%.

B : see next plot ($\frac{\Delta f}{f} < 0.05\%$)

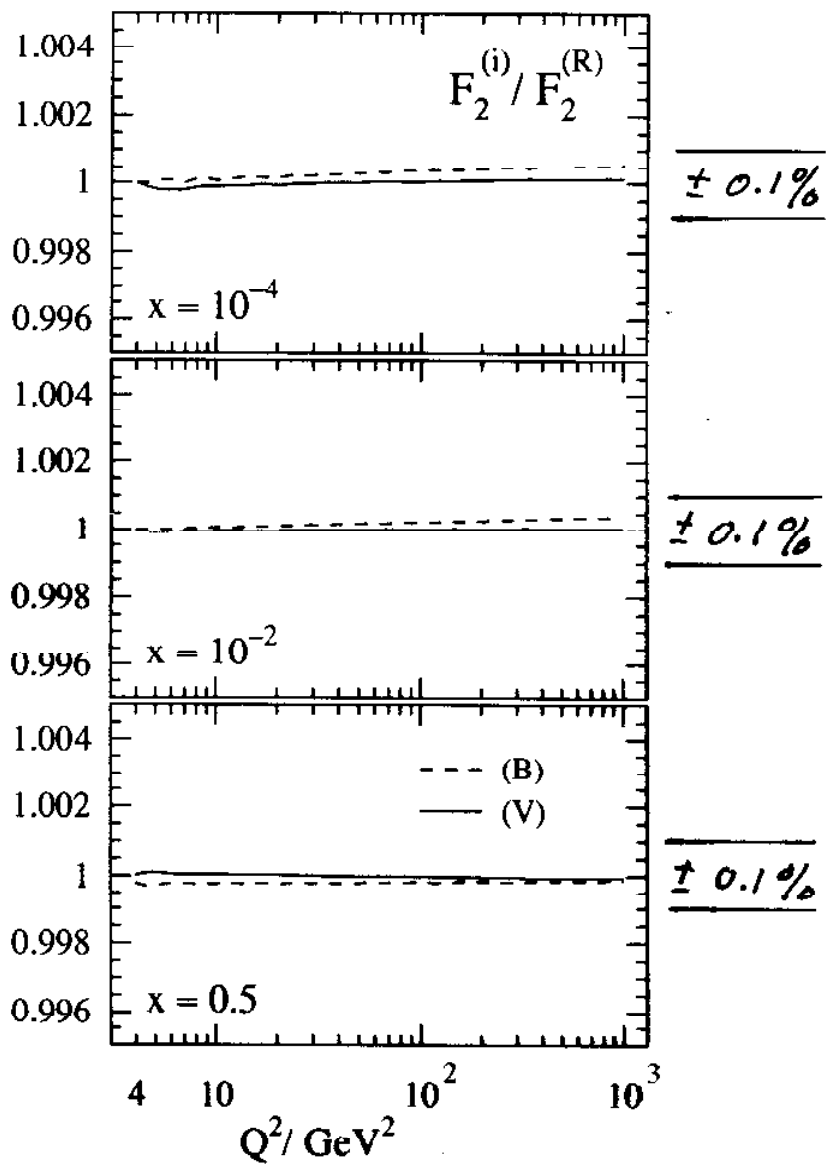
• Compare Botje (B), Pascaud & Zomer (PZ), Vogt (V) to Riemersma.



⇨ $Q^2 = 100\ GeV^2$: $\Delta f / f \lesssim 0.05\%$
$Q^2 = 10^4\ GeV^2$: $\Delta f / f \lesssim 0.1\%$
Except for B, PZ (x-space programs) at very high x (where pdf's vanish).

F_2 vs Q^2

- Compare Botje (B), Vogt (V) to Riemersma.



$\Rightarrow \quad \frac{\Delta F_2}{F_2} < 0.05\%$

■ Experimental error on α_s

- Standard dataset

ep: 27.6×820 GeV $Q^2 < 100$ GeV^2 $\mathcal{L} = 10\,pb^{-1}$
$Q^2 > 100$ GeV^2 $\mathcal{L} = 500\,pb^{-1}$

$Q^2 > 0.5$ GeV^2
$\theta_e < 177°$ $1.5 \cdot 10^{-5} < x < 0.7$
$\theta_h > 8°$ $0.5 < Q^2 < 5 \cdot 10^4$ GeV^2
$y < 0.8$

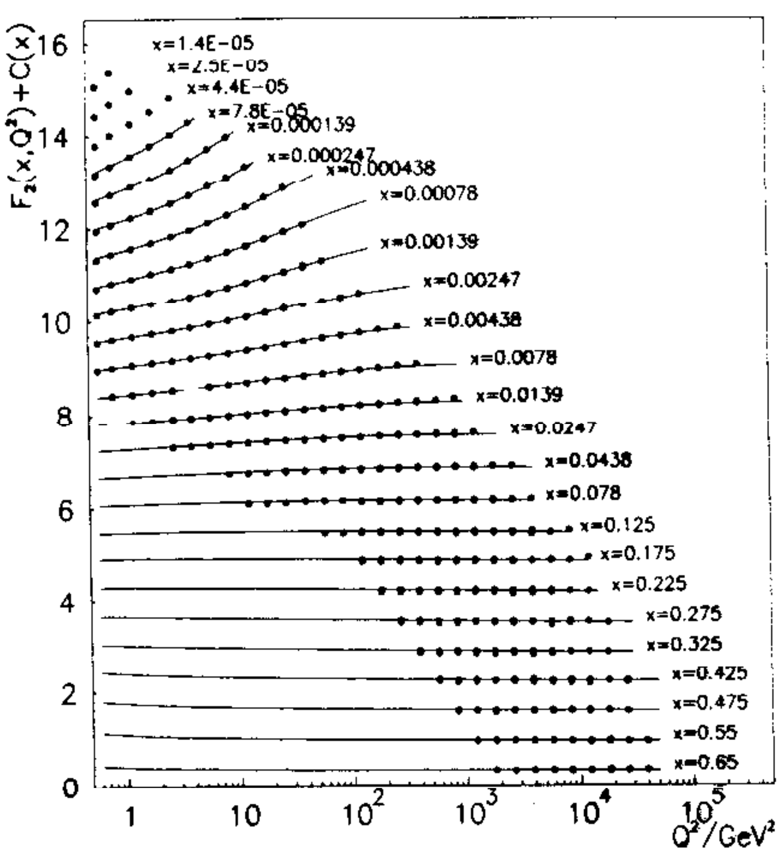
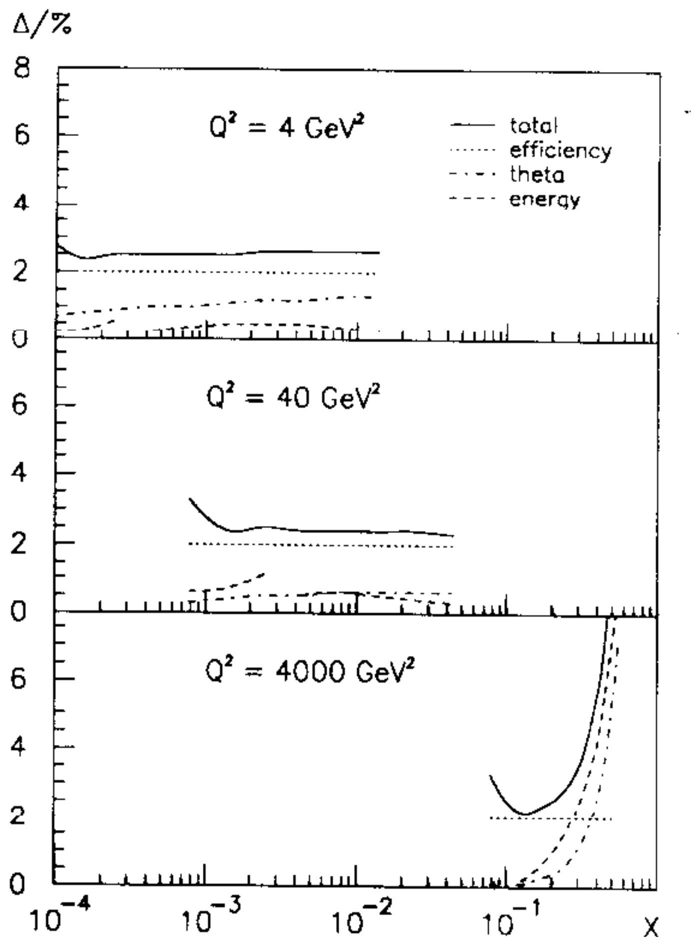


Figure 1: Simulated structure function data sets. The huge luminosity of 1 fb^{-1} will lead to precise data even at very high Q^2. For $Q^2 \geq 10000$ GeV^2 about 2000 events may be available. The largest bins shown are made with 20-50 events. With $\mathcal{L}$ = 10 pb^{-1} for $Q^2 \geq 0.5$ GeV^2 about 10^7 events are occuring which will be prescaled at lowest Q^2. The curve represents a NLO QCD fit. The high x, low Q^2 region can not be accessed with HERA but is almost completely covered by the fixed

• Systematic errors

$E_{e'}$	$\simeq 0.5-1\%$	rad. corrections	$\simeq 1-2\%$
θ_e	$\simeq 1$ mrad	efficiency	$\simeq 2\%$
E_h	$\simeq 2\%$	$\mathcal{L}$	$\simeq 1\%$
γ-backg.	$\simeq 1-2\%$	'random' syst. error	$\simeq 1\%$

$\Delta F_2/F_2$



$\Rightarrow$ Total syst error $(\Delta F_2/F_2)_{syst} \simeq 3\%$ over most of the kinematic range.

factor 2~5 better than presently achieved.

Analysis Method

- Incorporate syst. errors in the model:

$$F_2(p,s) = F_2^{QCD}(p)\left(1 - \sum_\ell s_\ell \Delta_\ell\right)$$

* $F_2^{QCD}(p)$ obeys NLO DGLAP evolution eqn^s

* $\{p\} = \alpha_s$ + parameters of input parton dist^{ns}

$$Q_0^2 = 4\,GeV^2 \quad \begin{cases} xg = A_g\, x^{\delta_g}(1-x)^{\eta_g} \\ x\Sigma = A_s\, x^{\delta_s}(1-x)^{\eta_s}(1+\epsilon_s\sqrt{x}+\gamma_s x) \\ x\Delta_{ud} = A_{ns}\, x^{\delta_{ns}}(1-x)^{\eta_{ns}} \end{cases}$$

* Δ_ℓ = relative systematic error of source ℓ

* s_ℓ = 'syst parameter' = deviation in units of s.d.

- $$\chi^2 = \sum_{\substack{\text{data}\\ \text{points}}} \left(\frac{F_2(p,s) - F_2^{meas}}{\sigma_{stat}^2}\right)^2 - \sum_\ell s_\ell^2$$

⇒ Two types of fit:

1) Fit p and s $\longrightarrow \Delta\alpha_s^{fit}$
   O.k. if kinematic dependences of the Δ_ℓ are well known.
2) Set $s_\ell \equiv 0$ and fit only p : $\Delta\alpha_s^{stat}$
   Propagate the syst. errors : $\Delta\alpha_s^{syst}$
   $\Big\}\ \Delta\alpha_s^{tot}$

III Fits to HERA F_2^p data alone

* ep: $Q^2 < 100\ GeV^2$ $\mathcal{L} = 10\ pb^{-1}$
  $Q^2 > 100\ GeV^2$ $\mathcal{L} = 500\ pb^{-1}$
* Fit α_s, $x\Sigma$, xg ; fix $x\Delta_{ud}$ → $\Delta\alpha_s(M_Z^2) \lesssim 0.004$
* $Q^2_{cut} > 3\ GeV^2$

$\Delta\alpha_s(M_Z^2)$

	syst. fitted	syst. fixed
standard	.006	.012
Q_c^2 $3 \to 1\ GeV^2$	.002	.006
Q_c^2 $3 \to 8\ GeV^2$	.007	.012
L $500 \to 1000\ pb^{-1}$	.006	.012
L $500 \to 10\ pb^{-1}$	.010	.015
* Add deuterium	.004	.009
** Add low energy	.005	?

(*) $10\ pb^{-1}$ ed $Q^2 < 100\ GeV^2$
$50\ pb^{-1}$ ed $Q^2 > 100\ GeV^2$
fit $x\Delta_{ud}$ → $\Delta\alpha_s = 0.004$ is eliminated

(**) $200\ pb^{-1}$ ep $27.6 \times 400\ GeV$
$10\ pb^{-1}$ ep $15.0 \times 820\ GeV$ } important for systematics!

Inclusion of fixed target data

SLAC / BCDMS / NMC $\quad W^2 > 10\ GeV^2$

$Q^2 > 3\ GeV^2$

BCDMS main syst. error left free in the fit.

(1) Check:

Fit to SLAC / BCDMS alone gives

$$\Delta\alpha_s(M_Z^2) = 0.0030 \text{ (as published)}$$

(2) HERA + fixed target:

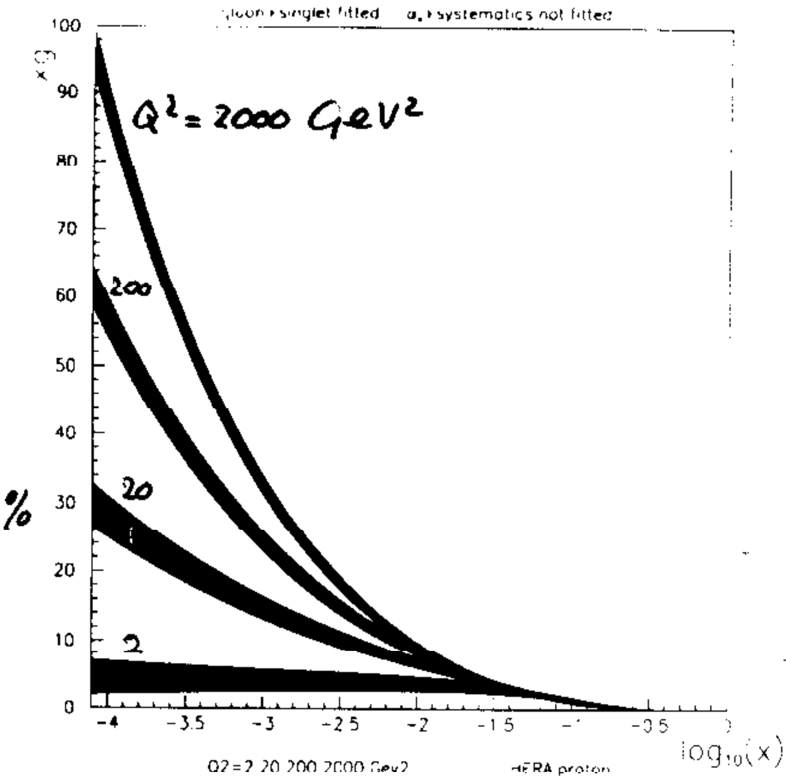
- Much less sensitivity to Q_c^2 and L.
- Systematics fitted:

$$\Delta\alpha_s(M_Z^2) = .0015 - .0020$$

- Systematics fixed:

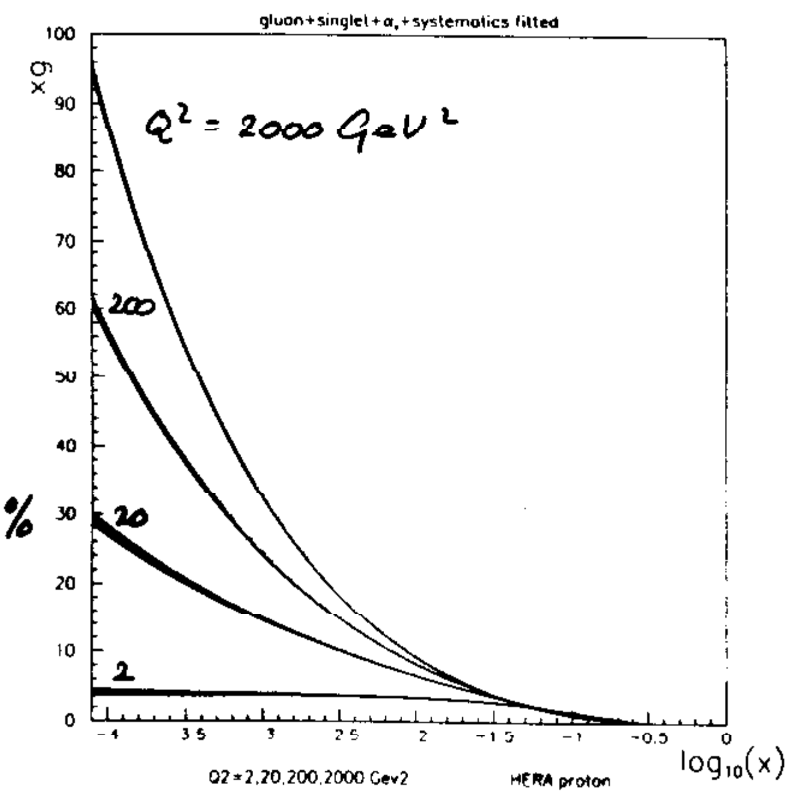
$$\Delta\alpha_s(M_Z^2) = .0025 - .0035$$

Aside: $xg(x)$ from HERA F_2?



Systematics fixed.

$\Delta g/g \simeq 10\%$



Systematics fitted.

$\Delta g/g \simeq 3\%$

Theoretical uncertainties

Three sources investigated:

① Representation of α_s $\quad (a \equiv \alpha_s/4\pi)$

$$* \quad \frac{1}{a_s} = \frac{1}{a_0} + \beta_0 \ln\frac{Q^2}{Q_0^2} - \frac{\beta_1}{\beta_0}\ln\left\{\frac{a_s(\beta_0+\beta_1 a_0)}{a_0(\beta_0+\beta_1 a_s)}\right\}$$

$$* \quad a_s = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)} - \frac{\beta_1}{\beta_0^3}\frac{\ln\ln Q^2/\Lambda^2}{\ln^2 Q^2/\Lambda^2}$$

$$\Longrightarrow \quad \Delta\alpha_s(M_Z^2) \simeq 0.001$$

② Different NLO evolution prescriptions.

$$\frac{\partial f^N(a_s)}{\partial a_s} = -\frac{a_s P_0^N + a_s^2 P_1^N + \cdots}{\beta_0 a_s^2 + \beta_1 a_s^3 + \cdots}\cdot f^N(a_s) \quad (*$$

ⓐ Solve (*) directly (∴ x-space approach)

ⓑ Expand (*) in a_s and truncate:

$$\frac{\partial f^N(a_s)}{\partial a_s} = -\frac{1}{\beta_0 a_s}\left[P_0^N + a_s\left(P_1^N - \frac{\beta_1}{\beta_0}P_0^N\right) + \cdots\right] f^N(a_s)$$

⇒ ⓐ and ⓑ differ in terms NNLO and beyond

⇒ Differences between ⓐ and ⓑ found to be unexpectedly large (→ fig.)

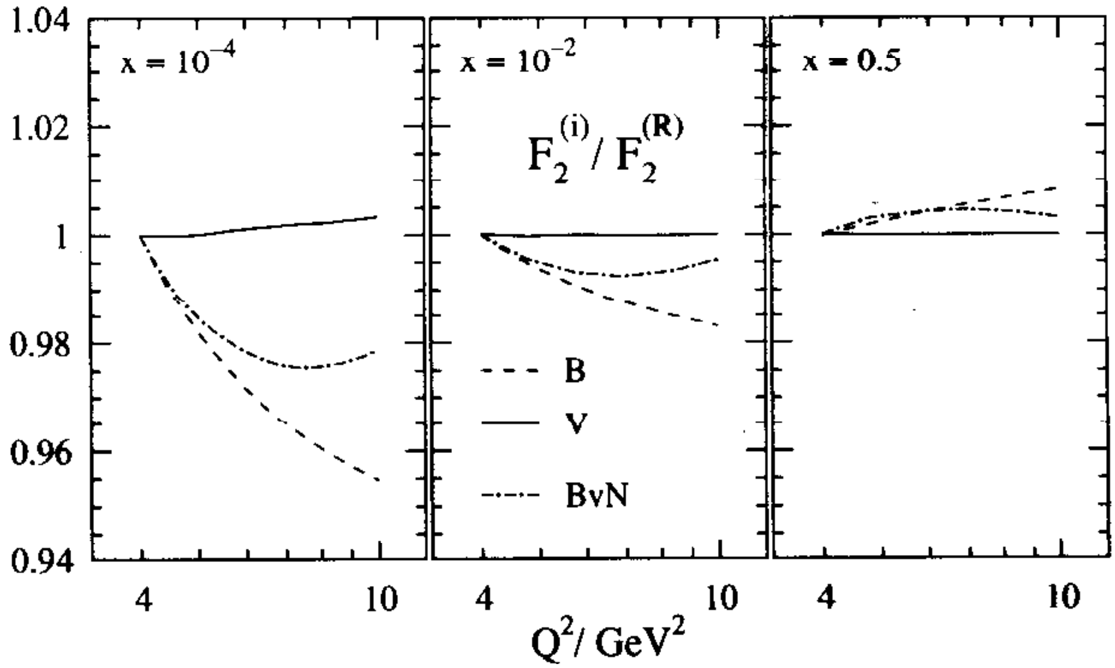
$$\Longrightarrow \quad \Delta\alpha_s(M_Z^2) \simeq 0.003$$

Different NLO evolution prescriptions

R = Riemersma (N-space), truncation # 1.

V = Vogt (N-space), truncation # 2.

B = Botje (x-space), no truncation.



⇨ Differences due to terms NNLO and beyond.

Buildup fast close to the input scale $Q_0^2 = 4\ GeV^2$ (where α_s is large) reaching up to 6% at $Q^2 = 20\ GeV^2$ @ low x.

NB. BvN = Blumlein, van Neerven: taylor expansion only valid close to Q_0^2: used to check 'x-space' approach.

③ Renormalisation / factorisation scale uncertainties

$$\frac{\partial f^N(M^2, R^2)}{\partial \ln M^2} = a_s(R^2)\left[P_0^{\,N} + a_s(R^2)\, P_1^{\,N} + a_s(R^2)\,\beta_0\, P_0^{\,N} \ln \frac{R^2}{M^2}\right] \cdot f^N(M^2, R^2)$$

M^2 = mass factorisation scale.
R^2 = renormalisation scale.

ⓐ Create reference dataset in the HERA kinematic range with $Q^2 = M^2 = R^2$ and $\alpha_s(M_Z^2) = 0.112$.

ⓑ Perform one parameter fits to $\alpha_s(M_Z^2)$

- fix $M^2 = Q^2$, vary $Q^2/4 < R^2 < 4Q^2$
- fix $R^2 = M^2$, vary $Q^2/4 < M^2 < 4Q^2$

⇒ For $Q^2_{cut} > 50\ GeV^2$

$$\Delta\alpha_s(M_Z^2) = \left.\begin{matrix} +0.004 \\ -0.006 \end{matrix}\right|_R \quad \left.\begin{matrix} -0.003 \\ +0.003 \end{matrix}\right|_M$$

⇒ For $Q^2_{cut} > 20\ GeV^2$

$$\Delta\alpha_s(M_Z^2) = \left.\begin{matrix} +0.005 \\ -0.007 \end{matrix}\right|_R \quad \left.\begin{matrix} -0.004 \\ +0.007 \end{matrix}\right|_M$$

➡ $\Delta\alpha_s$ increases with decreasing Q^2_{cut}

Summary

- □ 4 independent NLO evolution codes are in agreement $\sim 0.05-0.1\%$ over a large kinematic range $10^{-5} < x$ $4 < Q^2 < 10^4$ GeV^2.

- ■ Combining Hera data with fixed target data may give:

$$\Delta\alpha_s(M_Z^2)\big|_{exp} \simeq 0.0015$$

  This requires good knowledge of the systematic errors. Extension of the fit to Q^2 as low as possible is desirable.

- ■ Different representations of the NLO evolution yields effects much larger than expected.

  For α_s: $\Delta\alpha_s(M_Z^2)\big|_{evol} \simeq 0.003$

- ● Largest th. error due to scale dependence:

$$\Delta\alpha_s(M_Z^2) \simeq \pm .005\,(R) \pm .003\,(M)$$

  for $Q^2 > 50$ GeV^2.

- □ NNLO needed to reduce the theoretical error and fully exploit the region of low Q^2 (~ 3 GeV^2)