Diffractive Factorization and NDPE - dijets

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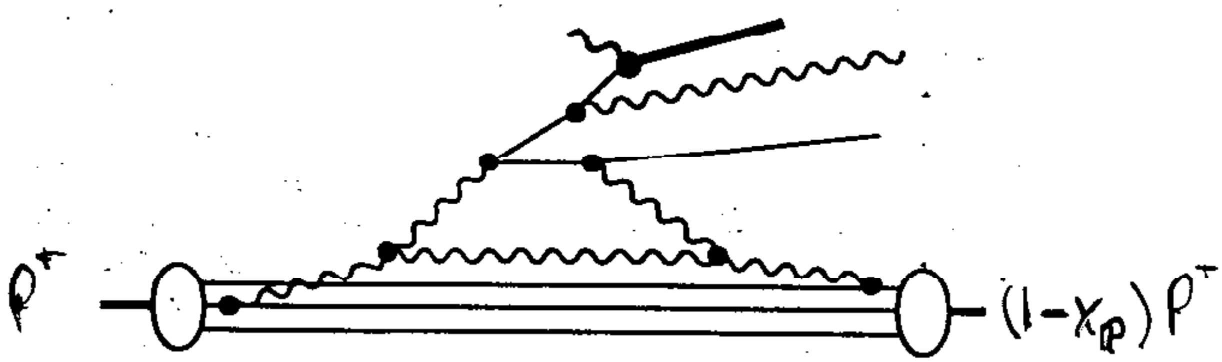


FIG. 1. A graph for $e + p \to p + X$.

Structure Function F_2:

$$F_2(x,Q^2) = \sum_a \int_x^1 d\xi\ f_{a/A}(\xi,\mu)\ \hat{F}_{2,a}(x/\xi,Q^2;\mu). \tag{1}$$

$f_{a/A}(\xi,\mu)$ is the distribution of partons of kind a in hadron A
where

$$\hat{F}_{2,a}(x/\xi,Q^2;\mu) = e_a^2\ \delta(1-x/\xi) + \mathcal{O}(\alpha_s). \tag{2}$$

Diffractive Structure Function:

$$\frac{dF_2^{\rm diff}(x,Q^2;x_{I\!P},t)}{dx_{I\!P}\,dt} = \sum_a \int_x^{x_{I\!P}} d\xi\ \frac{d\,f_{a/A}^{\rm diff}(\xi,\mu;x_{I\!P},t)}{dx_{I\!P}\,dt}\ \hat{F}_{2,a}(x/\xi,Q^2;\mu). \tag{3}$$

Hypothesize that $d\,f_{a/A}^{\rm diff}(x_a,\mu)/dx_{I\!P}\,dt$ has a particular form:

$$\frac{d\,f_{a/A}^{\rm diff}(\xi,\mu;x_{I\!P},t)}{dx_{I\!P}\,dt} = \frac{1}{8\pi^2}\,|\beta_A(t)|^2 x_{I\!P}^{-2\alpha(t)}\ f_{a/I\!P}(\xi/x_{I\!P},t,\mu)\,. \tag{4}$$

$\beta_A(t)$ is the pomeron coupling to hadron A
$\alpha(t)$ is the pomeron trajectory

Model of Ingelman and Schlein, applied to the case of deeply inelastic scattering:

$$\frac{dF_2^{\rm diff}(x,Q^2;x_{I\!P},t)}{dx_{I\!P}\,dt} = \frac{|\beta_A(t)|^2}{8\pi^2}\,x_{I\!P}^{1-2\alpha(t)}\ \sum_a \int_\beta^1 d\tilde{\beta}\,f_{a/I\!P}(\tilde{\beta},t,\mu)\,\hat{F}_{2,a}(\beta/\tilde{\beta},Q^2;\mu), \tag{5}$$

where $\beta = x/x_{I\!P}$.

Inclusive Distribution Functions

(Standard Case)

QUARKS:

$$f_{j/A}(x,\mu) \equiv \frac{1}{4\pi}\frac{1}{2}\sum_s \int dy^- e^{-ixP_A^+ y^-} \langle P_A, s|\bar{\tilde{\psi}}_j(0,y^-,\mathbf{0})\gamma^+\tilde{\psi}_j(0)|P_{A,}$$

type $j \in \{u, \bar{u}, d, \bar{d}, \ldots\}$ in a hadron of type A

GLUONS:

$$\begin{aligned} f_{g/A}(x,\mu) \equiv\ & \frac{1}{2\pi x P_A^+}\frac{1}{2}\sum_s \int dy^- e^{-ixP_A^+ y^-} \\ & \langle P_A, s|\tilde{F}_a^\dagger(0,y^-,\mathbf{0})^{+\nu}\tilde{F}_a(0)_\nu{}^+|P_A, s\rangle \end{aligned}$$

where

$$\tilde{\psi}_j(0,y^-,\mathbf{0}) = \left[\mathcal{P}\exp\left(ig\int_{y^-}^{\infty} dz^- A_c^+(0,z^-,\mathbf{0})\,t_c\right)\right]\psi_j(0,y^-,\mathbf{0})$$

and

$$\tilde{F}_a(0,y^-,\mathbf{0})^{\mu\nu} = \left[\mathcal{P}\exp\left(ig\int_{y^-}^{\infty} dz^- A_c^+(0,z^-,\mathbf{0})\,t_c\right)\right]_{ab} F_b(0,y^-,\mathbf{0})^{\mu\nu}$$

Diffractive Distribution Functions

Quarks:

$$(2\pi)^3 2E'_A \,\frac{d\,f^{\rm diff}_{j/A}(x,\mu)}{d^3\vec{P}'_A} = G^{\rm diff}_{j/A}(p_A, p'_A, x, \mu)$$

$$\equiv \frac{1}{4\pi}\frac{1}{2}\sum_s \int dy^- e^{-ixP_A^+ y^-} \sum_{X,s'} \langle P_A, s|\bar{\tilde{\psi}}_j(0,y^-,\mathbf{0})|P'_A, s'; X\rangle$$

$$\times\gamma^+ \langle P'_A, s'; X|\tilde{\psi}_j(0)|P_A, s\rangle$$

Gluons:

$$(2\pi)^3 2E'_A \,\frac{d\,f^{\rm diff}_{g/A}(x,\mu)}{d^3\vec{P}'_A} = G^{\rm diff}_{g/A}(P_A, P'_A, x, \mu)$$

$$\equiv \frac{1}{2\pi x P_A^+}\frac{1}{2}\sum_s \int dy^- e^{-ixP_A^+ y^-} \sum_{X,s'} \langle P_A, s|\tilde{F}_a(0,y^-,\mathbf{0})^{+\nu}|P'_A, s'; X\rangle$$

$$\times \langle P'_A, s'; X|\tilde{F}_a(0)_\nu{}^+|P_A, s\rangle$$

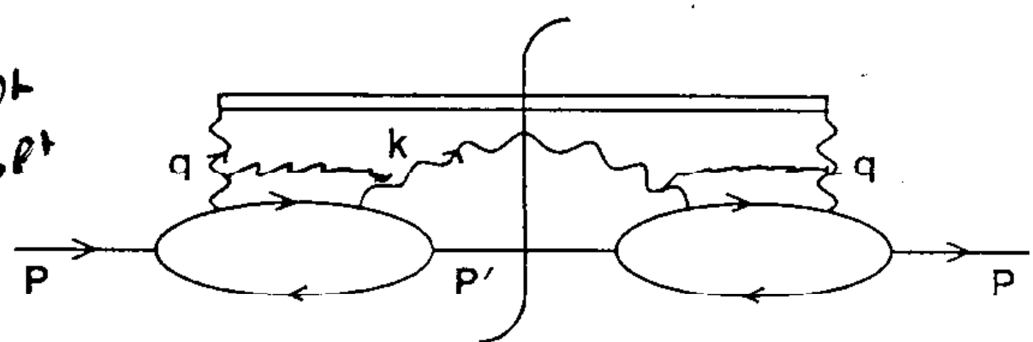
Integrating over the azimuthal angle ϕ, we have

$$\frac{d\,f^{\rm diff}_{a/A}(x,\mu)}{dt\,dz} = \frac{1}{16\pi^2}\,G^{\rm diff}_{a/A}(p,p',x,\mu)$$

Distribution of partons in the Pomeron:

$$f_{a/P}(\xi/z, t, \mu) = \frac{z^{2\alpha(t)}}{\pi\,|\beta_A(t)|^2}\,G^{\rm diff}_{a/A}(P_A, P'_A, x, \mu)$$

$$\mathbf{G_g^{diff}}$$



$q^+ = x\,p^+ = \beta x_{I\!P}\, p^+$

$p'^+ = (1 - x_{I\!P})\, p^+$

We find for the diffractive gluon distribution

$$\frac{df_{g/A}^{\text{diff}}(\beta x_{I\!P}, \mu; x_{I\!P}, t)}{dx_{I\!P}\, dt} \propto (1-\beta)^p \tag{6}$$

for $\beta \to 1$ at moderate values of the scale μ, say 2 GeV, then

$$0 \lesssim p \lesssim 1. \tag{7}$$

$p \approx 0$ corresponds to an effectively massless final state gluon,

$p \approx 1$ corresponds to an effective gluon mass

For the diffractive quark distribution we find,

$$\frac{df_{q/A}^{\text{diff}}(\beta x_{I\!P}, \mu; x_{I\!P}, t)}{dx_{I\!P}\, dt} \propto (1-\beta)^2. \tag{8}$$

Suppose diffractive distribution of gluons is proportional to $(1-\beta)^0$ for β near 1 when the scale μ is not too large

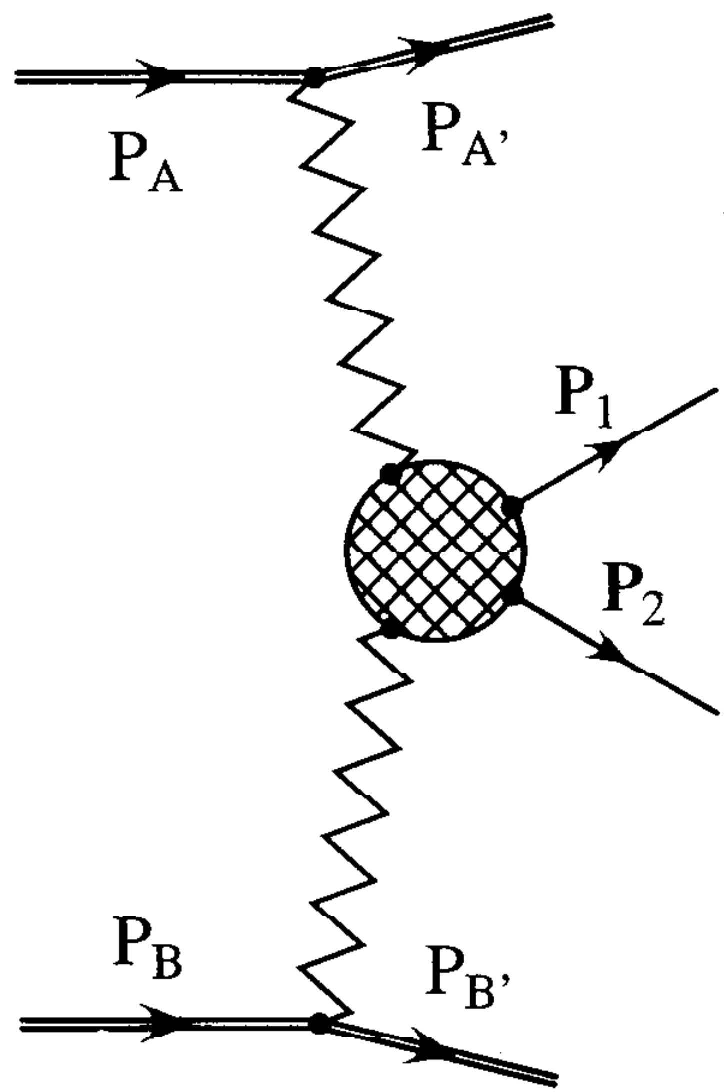
evolution equation for the diffractive parton distributions will give a quark distribution that behaves like

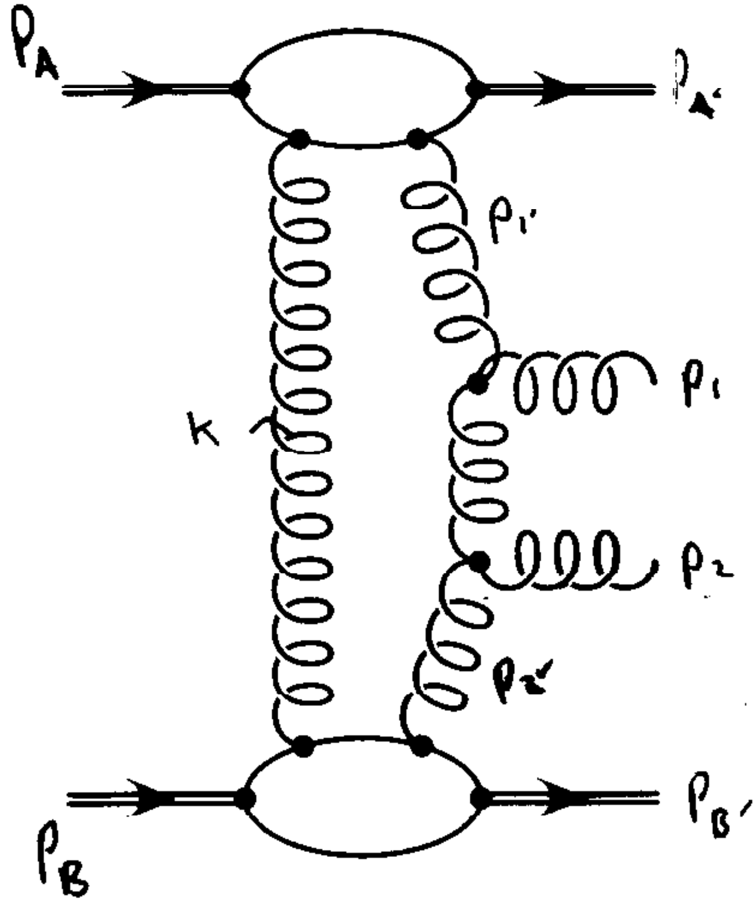
$$\frac{df_{q/A}^{\text{diff}}(\beta x_{I\!P}, \mu; x_{I\!P}, t)}{dx_{I\!P}\, dt} \propto (1-\beta)^1. \tag{9}$$

when the scale μ is large enough that some gluon to quark evolution has occurred, but not so large that effective power p in $(1-\beta)^p$ for the gluon distribution has evolved substantially from $p = 0$

Perhaps seen by data

Double-Pomeron-Exchange
(DPE)





NOPE

The momenta in light-cone components $(+, -; \perp)$ are for the mesons,

$$
\begin{aligned}
P_A &= (\sqrt{\frac{s}{2}}, \frac{M_A^2}{\sqrt{2s}}; \mathbf{0}) \\
P_B &= (\frac{M_B^2}{\sqrt{2s}}, \sqrt{\frac{s}{2}}; \mathbf{0}) \\
P_{A'} &= ((1-x_a)\sqrt{\frac{s}{2}}, \frac{(M_A^2 + \mathbf{Q}_1^2)}{(1-x_a)\sqrt{2s}}; \mathbf{Q}_1) \\
P_{B'} &= (\frac{(M_B^2 + \mathbf{Q}_2^2)}{(1-x_b)\sqrt{2s}}, (1-x_b)\sqrt{\frac{s}{2}}; \mathbf{Q}_2).
\end{aligned}
$$

For the partons,

$$
\begin{aligned}
p_{1'} &= (x_a\sqrt{\frac{s}{2}}, \frac{M^2}{\sqrt{2s}} - \frac{M^2 + Q_1^2}{(1-x_a)\sqrt{2s}}; \mathbf{k} - \mathbf{Q}_1) \\
p_{2'} &= (\frac{M^2}{\sqrt{2s}} - \frac{M^2 + Q_2^2}{(1-x_b)\sqrt{2s}}, x_b\sqrt{\frac{s}{2}}; \mathbf{k} + \mathbf{Q}_2) \\
p_1 &= (ax_a\sqrt{\frac{s}{2}}, bx_b\sqrt{\frac{s}{2}}; E_T\cos\phi, E_T\sin\phi) \\
p_2 &= (bx_a\sqrt{\frac{s}{2}}, ax_b\sqrt{\frac{s}{2}}; -E_T\cos\phi, -E_T\sin\phi)
\end{aligned}
$$

where

$$
\begin{aligned}
a &= \frac{1+\sqrt{1-\kappa}}{2} \\
b &= 1-a
\end{aligned}
$$

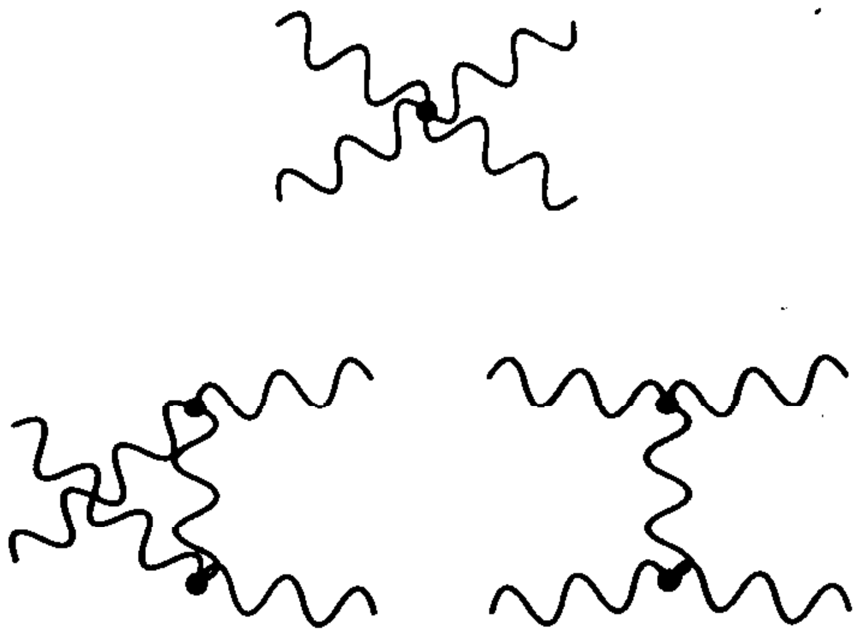
and

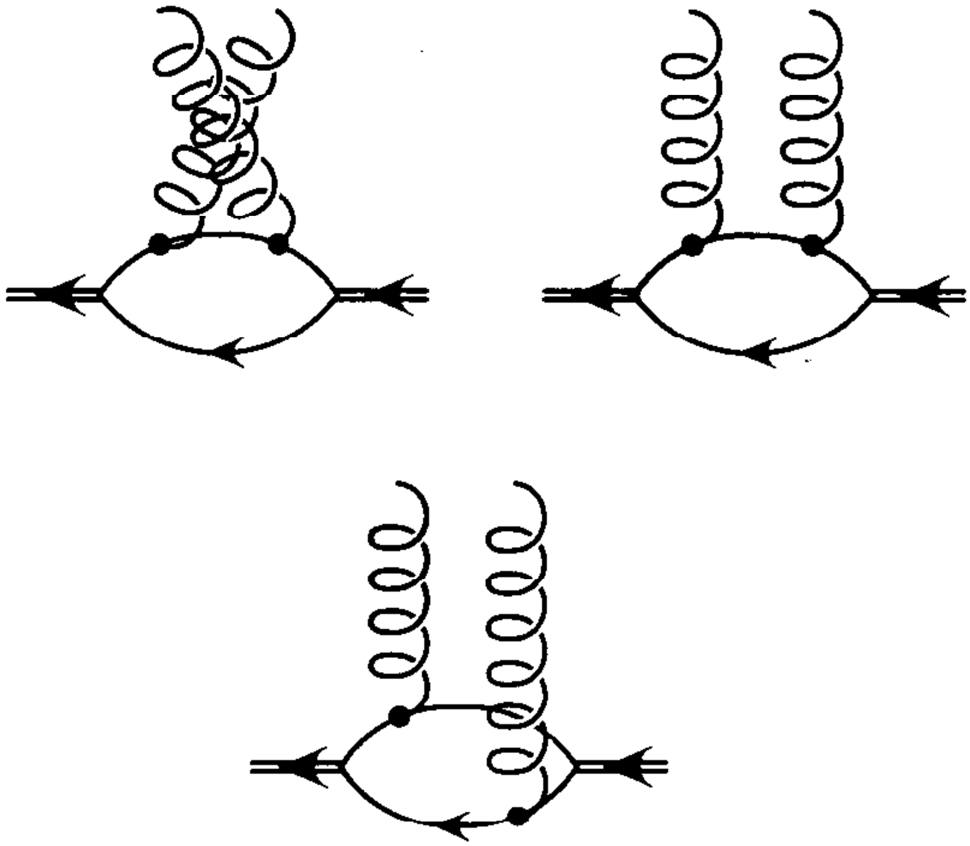
$$
\kappa \equiv \frac{4E_T^2}{x_a x_b s}.
$$

$x_a, x_b \to 0$

$E_T \to \infty$

κ fixed





Amplitude:

$$M = -\frac{4g^4}{x_a x_b}\int \frac{d\mathbf{k}}{(2\pi)^2}\hat{g}_A(\mathbf{k},-\mathbf{Q}_1)\hat{g}_B(\mathbf{k},\mathbf{Q}_2)\epsilon_i(\mathbf{k}-\mathbf{Q}_1)\epsilon_j(\mathbf{k}+\mathbf{Q}_2)A^{ga}(i,j;\tilde{i},\tilde{j})$$

$$\hat{g}_I(\mathbf{k},\mathbf{Q}) \equiv \frac{\delta^{ab}}{\sqrt{N_c}\sqrt{(\mathbf{k})^2}\sqrt{(\mathbf{Q}+\mathbf{k})^2}}\int_0^1 d\alpha\left\{G_I^{ab}(\alpha,\alpha\mathbf{Q}) - G_I^{ab}(\alpha,\mathbf{k}+\alpha\mathbf{Q})\right\}$$

where

$$G_I(\alpha,\mathbf{v}) = -G^2\frac{\delta^{ab}}{2}\frac{\alpha(1-\alpha)}{(2\pi)^2|\mathbf{v}|}\frac{1}{\sqrt{\mathbf{v}^2+4\Lambda^2}}\ln\left[\frac{\sqrt{\mathbf{v}^2+4\Lambda^2}+\mathbf{v}^2}{\sqrt{\mathbf{v}^2+4\Lambda^2}-\mathbf{v}^2}\right]$$

$$\Lambda_I^2 - m^2 - \alpha(1-\alpha)M_I^2$$

and the "polarization" vectors are defined as,

$$\epsilon_i(\mathbf{k}) = \frac{\mathbf{k}_i}{\sqrt{\mathbf{k}^2}}.$$

Quark Jets are Suppressed

For forward scattering of A' and B',

$$|\mathcal{M}(0,0)|^2 = 16\pi \left(\frac{d\sigma(0)}{dt}\right)_{el} \delta_{ij}\delta_{kl}H^{gg}_{ijkl}.$$

where

$$H^{gg}_{ijkl}(\kappa) = \sum_f A^{*ga'}(i,j,\tilde{m},\tilde{n})A^{ga'}(k,l;\tilde{m},\tilde{n})$$

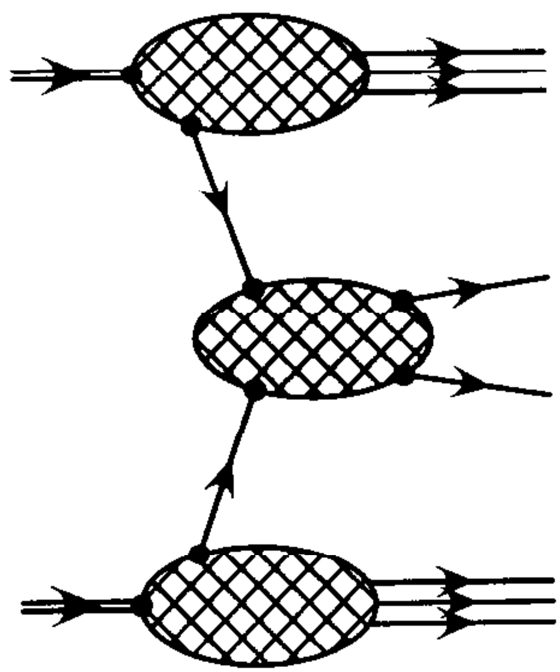
Here $A^{aa'}(i,j,\tilde{i},\tilde{j})$ is the amplitude for incoming parton pair a in polarization state i,j to go to outgoing parton pair a' in polarization state $\tilde{i},\tilde{j}$. In both initial and final states an average over color is done.

By a straightforward perturbative calculation, one can verify that for the quark amplitude, $gg \to q\bar{q}$,

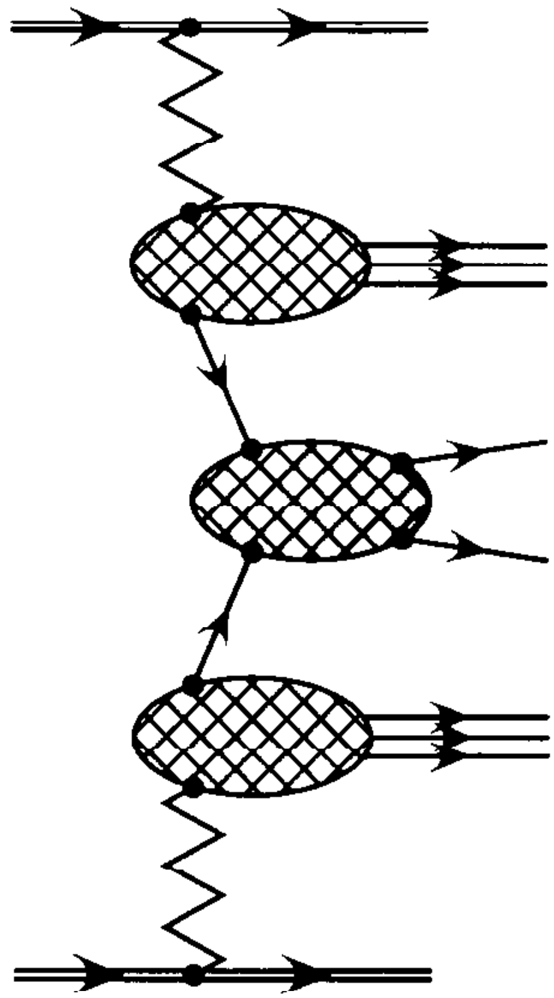
$$A^{gq}(1,1,\lambda_1,\lambda_2) = -A^{gq}(2,2;\lambda_1,\lambda_2)$$

where λ_i is the helicity of quark i. This property implies that the quark amplitude vanishes when A and B are forward scattered. This cancellation does not occur for the gluon amplitude.

Inclusive Two-Jet Production



Factorized Double Pomeron Exchange
(FDPE)



Factorized Two-Jet Cross Sections

Inclusive two-jet production:

$$\frac{d\sigma}{dE_T^2 dy_- dy_+} = \sum_{a,b} x_a f_{a/A}(x_a) x_b f_{b/B}(x_b) \frac{\kappa^2 H^{ab}(\kappa)}{128\pi E_T^4}$$

where,

$$\begin{aligned} x_a &= \frac{E_T}{\sqrt{s}}(e^{y_1} + e^{y_2}) \\ x_b &= \frac{E_T}{\sqrt{s}}(e^{-y_1} + e^{-y_2}) \end{aligned}$$

Factorized Double-Pomeron Exchange (Ingelman-Schlein Model):

$$\begin{aligned} \frac{d\sigma}{dE_T^2 dy_- dy_+} &= \frac{\kappa^2}{128\pi E_T^4}\left(\frac{N}{16\pi}\right)^2 \int dx_{P_A} dx_{P_B} B_A(x_{P_A}) B_B(x_{P_B}) \\ &\quad x_a f_{a/P_A}(x_a/x_{P_A}) x_b f_{b/P_B}(x_b/x_{P_B}) H^{ab}(\kappa) \end{aligned}$$

where,

$$\begin{aligned} B_I(x_{P_I}) &= \int_0^{t_{f_I}} dt |\beta_I(t)|^2 x_{P_I}^{-2\alpha_P(t)} \\ &= x_{P_I}^{-2\alpha} \sigma_0 \int_0^{t_{f_I}} dt e^{(\beta_0 - 2\alpha_0 \ln x_{P_I})t} \end{aligned}$$

with $\alpha_P(t) \equiv \alpha + \alpha_0 t$.

Sudakov suppression factor

$$F = \exp\left(-S(Q_T^2, E_T^2)\right) \qquad (1)$$

where

$$S(Q_T^2, E_T^2) = \frac{3\alpha_S(\sqrt{Q_T^2 + M_T^2})}{4\pi} \ln^2\left(\frac{E_T^2}{4(Q_T^2 + M_T^2)}\right) \qquad (2)$$

(Khoze, Martin, Ryskin hep-ph/9701419)

$\sqrt{s}$	$\sigma_{dijet}^{NDPE}(E_T^{min} = 5.0)$	
	without Sudakov	with Sudakov
(GeV)	(mb)	(mb)
630	0.044	0.024
1800	0.17	0.086
14000	0.65	0.31

Table 1: Nonfactorizing Double Pomeron dijet cross section with and without one loop Sudakov suppression with $M_T^2 = 0.3\text{GeV}^2$ and $E_T^{min} = 5\text{GeV}$.

$\sqrt{s}$	$\sigma_{dijet}^{NDPE}(E_T^{min} = 20.0)$	
	without Sudakov	with Sudakov
(GeV)	(mb)	(mb)
1800	0.00070	0.000070
14000	0.010	0.00080

Table 1: Nonfactorizing Double Pomeron dijet cross section with and without one loop Sudakov suppression with $M_T^2 = 0.3\text{GeV}^2$ and $E_T^{min} = 20\text{GeV}$.