ASYMMETRIC PARTON DISTRIBUTIONS

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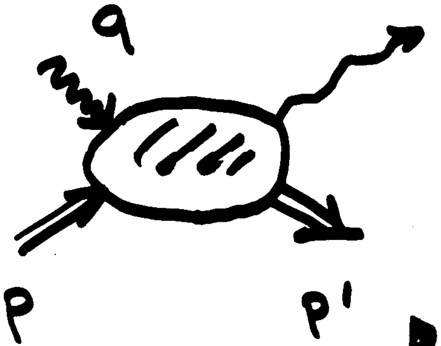
OUTLINE

1. WHERE & WHY THEY ARE NEEDED?
2. WHAT THEY ARE?
3. SAMPLE APPLICATION
4. EVOLUTION EQUATIONS
5. CONCLUSIONS

DEEPLY VIRTUAL COMPTON SCATTERING,

HARD ELASTIC MESON ELECTROPRODUCTION,

etc. EXCLUSIVE PROCESSES.

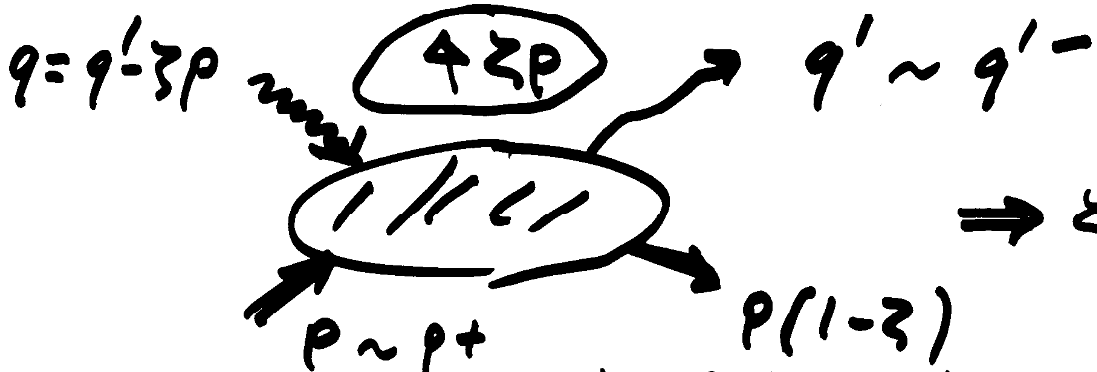


$q^2 = -Q^2$, $\quad q'^2 = 0$, OR $q'^2 = m_M^2 \ll Q^2$

$p^2 \ll Q^2$, $\; p'^2 \ll Q^2$, $|t| \ll Q^2$

Bj LIMIT : $\quad x_{Bj} = \dfrac{Q^2}{2(pq)}$

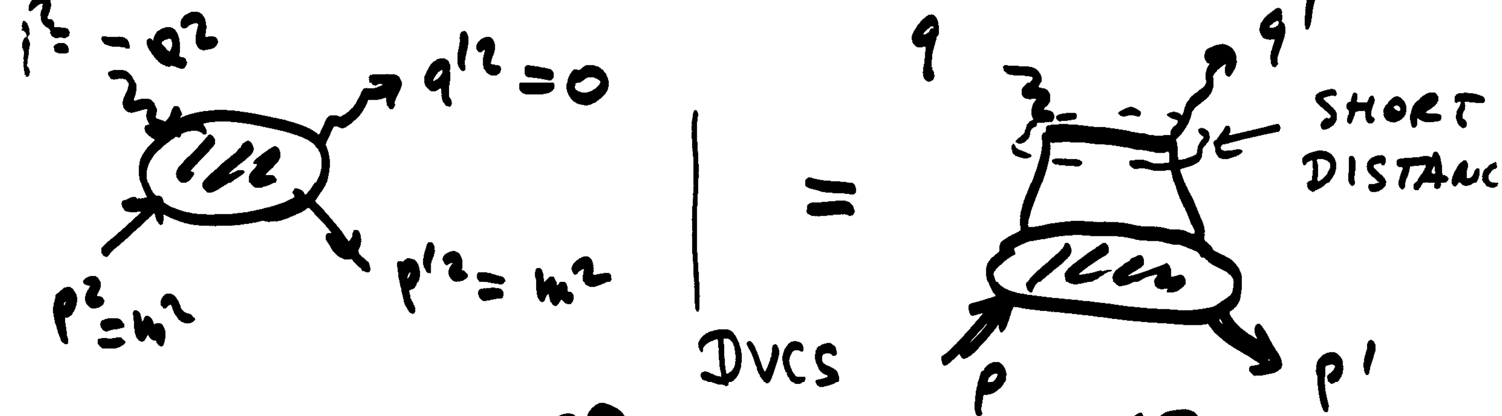
IN LIGHT-CONE DECOMPOSITION:

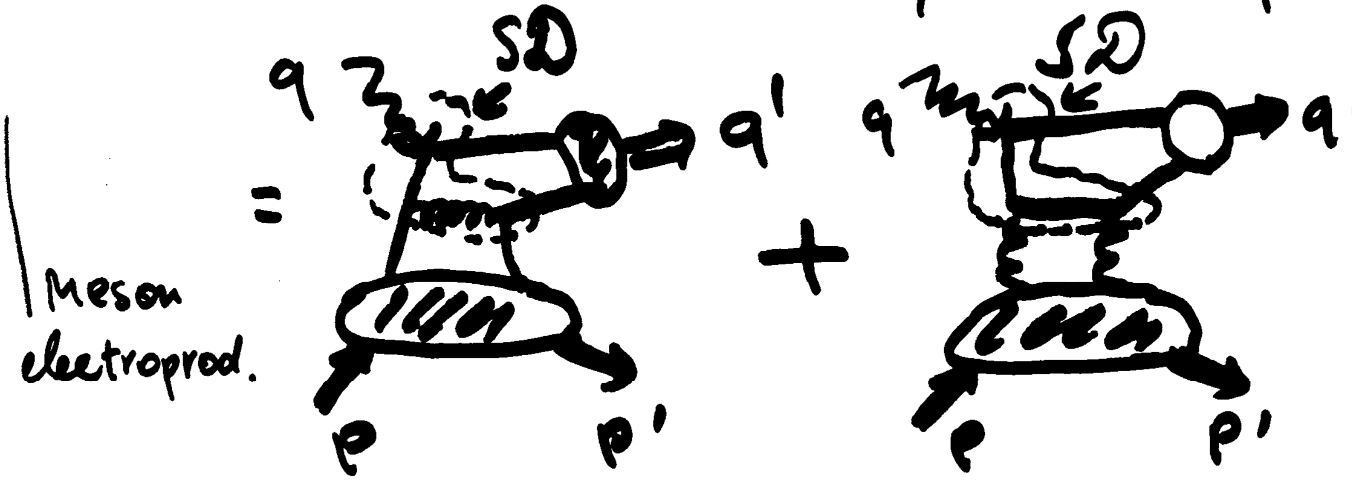


$\Rightarrow \xi = x_{Bj}$

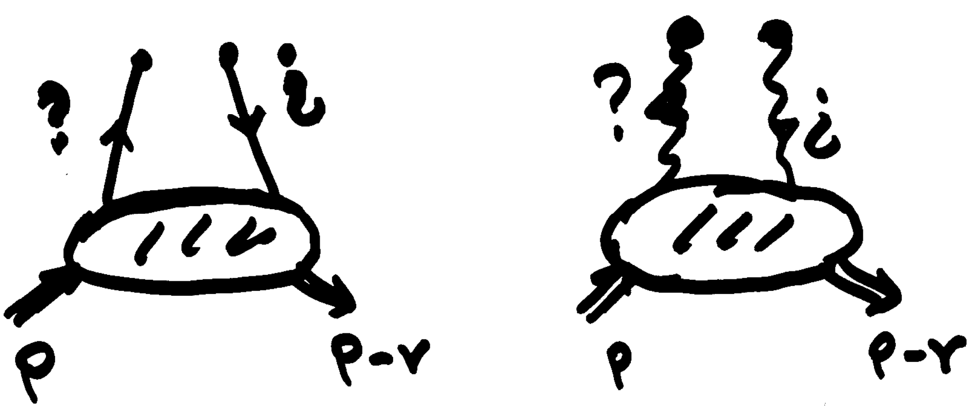
LONGITUDINAL MOMENTUM IN t-chan

FACTORIZATION

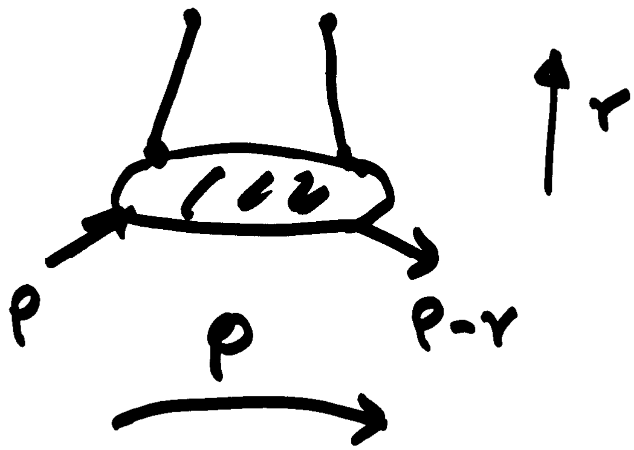




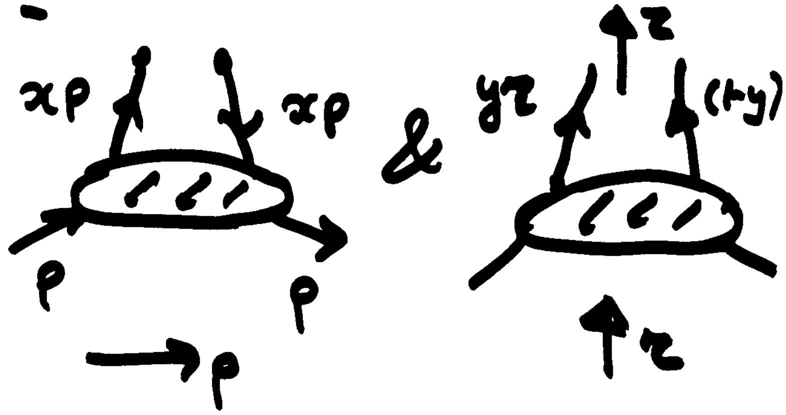
LONG-DISTANCE MATRIX ELEMENTS:



$r = p - p' = \zeta p$



= SUPERPOSITION OF



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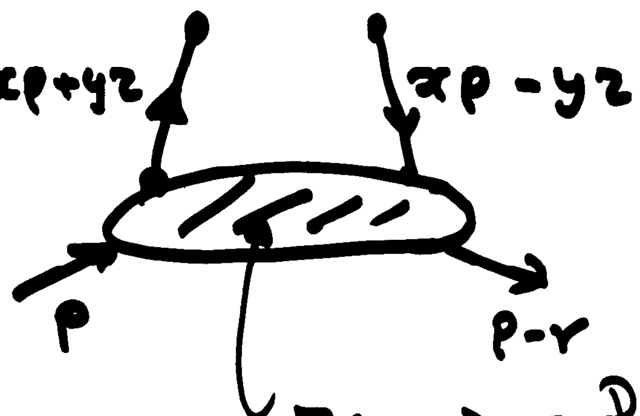


$F(x,y) \equiv$ DOUBLE DISTRIBUTION

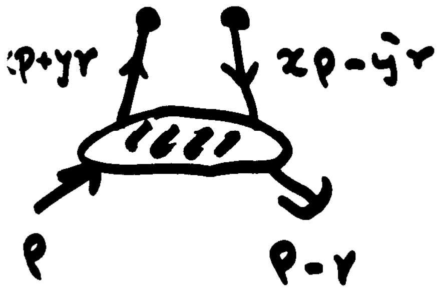
SPECTRAL (PARTON) PROPERTIES:

$$0 \le x \le 1$$

$$0 \le y \le 1$$

$$0 \le x+y \le 1$$

<u>NB</u>: $F(x,y)$ DOES NOT DEPEND ON $r/p = \zeta$



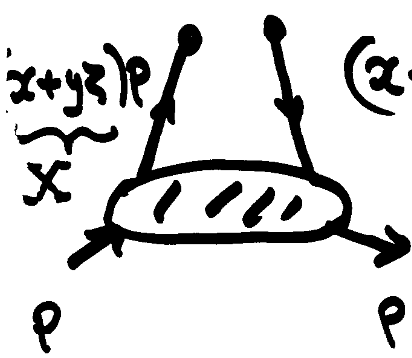
w.r.t. x, $F(x,y)$ WORKS LIKE DF

w.r.t. y, $F(x,y)$ works LIKE DA (WAVE FUNCTION)

IF $(p-p')^2 \equiv t \neq 0 \Rightarrow F(x,y;t)$

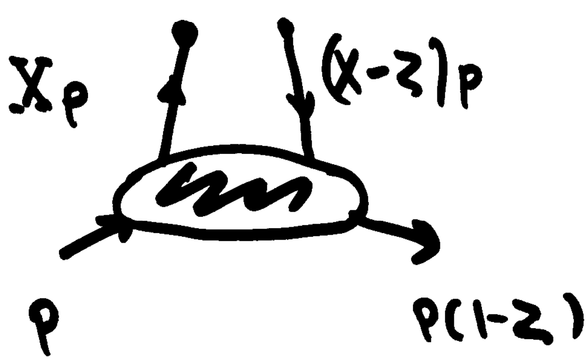
w.r.t. t, $F(x,y;t)$ works LIKE FF

RETURN BACK TO $t=0$, AND USE $r=\zeta p$



$$(x-\bar{y}\zeta)p = (\underbrace{x+y\zeta-\zeta}_{X-\zeta})p$$

$$x+y\zeta = X \rightarrow x = X-y\zeta$$



$$\mathcal{F}_\zeta(X) = \int_0^{\min\{X/\zeta,\ (1-X)/(1-\zeta)\}} F(X-y\zeta, y)\,dy$$

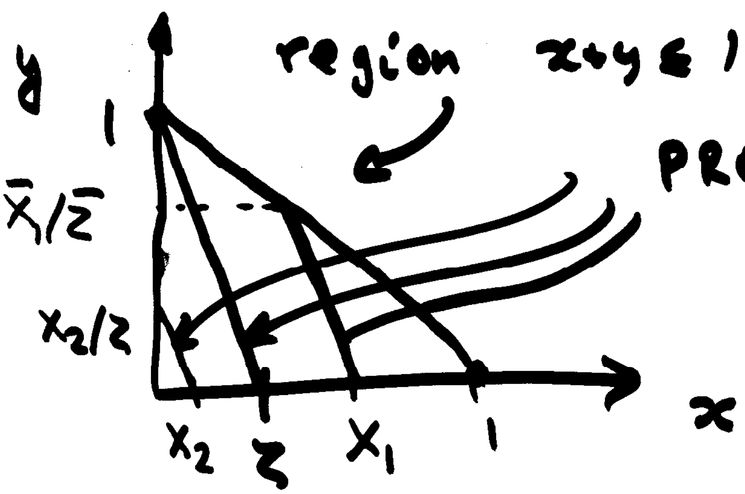
NB: $\mathcal{F}_\zeta(X)$ EXPLICITLY ζ-DEPENDENT:

$\mathcal{F}_\zeta(X)'$ SHAPE CHANGES WITH ζ.

$$\mathcal{F}_\zeta(X) = \theta(X>\zeta)\int_0^{\bar{X}/\zeta} F(X-y\zeta, y)\,dy \; +$$

$$\theta(X<\zeta)\int_0^{X/\zeta} F(X-y\zeta, \zeta)\,dy$$

LIFE ON (x,y)- PLANE:

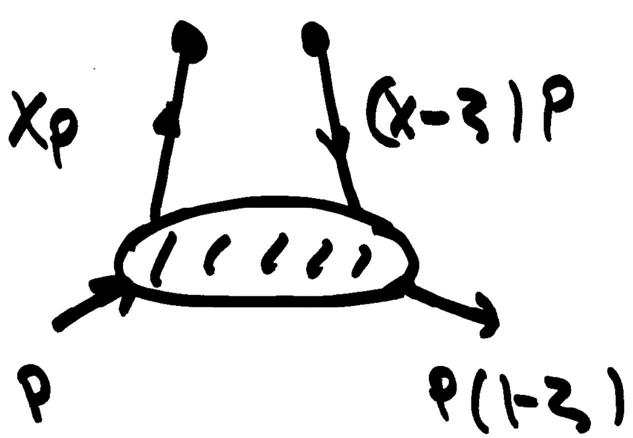


PROFILES OF $F(x,y)$

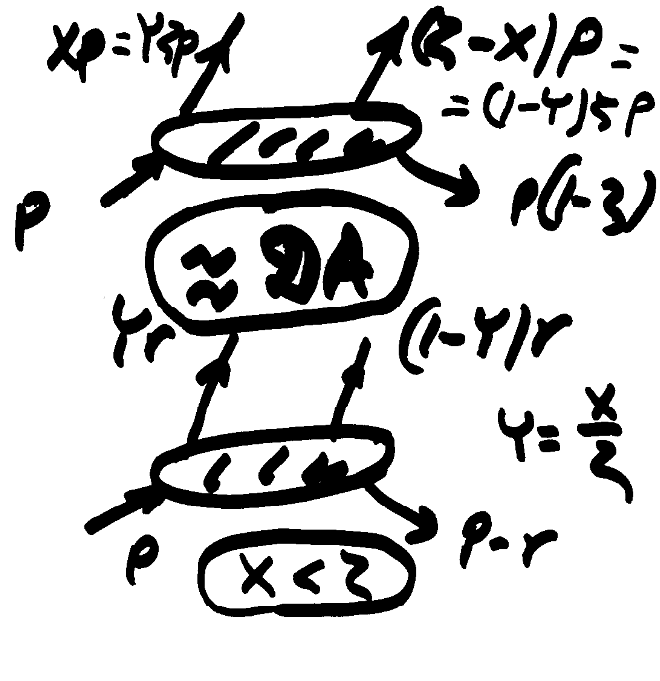
PRODUCING $\mathcal{F}_\zeta(X_2)$,

$\mathcal{F}_\zeta(\zeta)$, $\mathcal{F}_\zeta(X_1)$

WHY TWO COMPONENTS? $(X \lessgtr \zeta)$



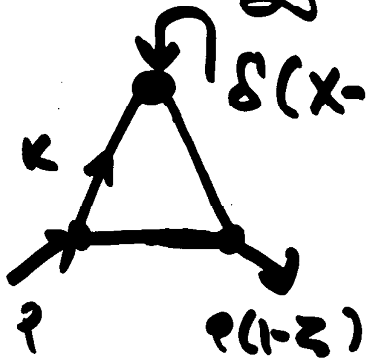
$\boxed{X>\zeta}$ $\approx$ DF



SPECTRAL PROPERTY FOR ASYMMETRIC DF

$$0 \le X = x + y\zeta \Big|_{\zeta \le 1} \le x + y \Big|_{x+y \le 1} \le 1$$

Modelling shape of $\mathcal{F}_\zeta(X)$:



$\delta(X - (kq')/(Pq'))$ 100% SCALAR TRIANG

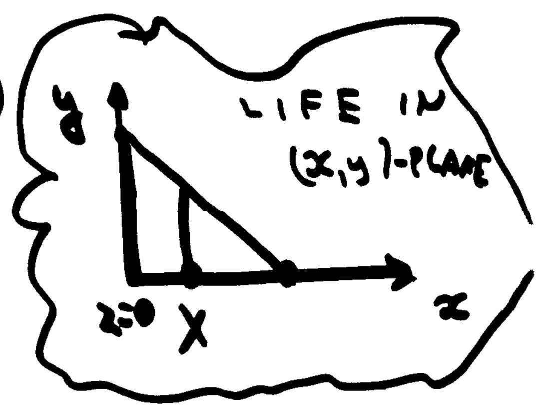
$$\mathcal{F}_\zeta(X) \sim \frac{X}{\zeta}\,\theta(X<\zeta) + \frac{\bar{X}}{\bar{\zeta}}\,\theta(X>\zeta)$$

WHEN $\zeta = 0$, $p' = p \Rightarrow$ REDUCTION FORMULA

$$\mathcal{F}_{\zeta=0}(X) = f(X)$$

OR, IN TERMS OF DD' :

$$\int_0^{1-x} F(x,y)\,dy = f(x)$$



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I. QUARK DISTRIBUTIONS.

$$\langle p', s' \,|\, \bar{\psi}_a(0)\hat{z}E(0,z;A)\psi_a(z) \,|\, p, s\rangle|_{z^2=0} \tag{1.1}$$
$$= \bar{u}(p',s')\hat{z}u(p,s)\int_0^1 \left(e^{-iX(pz)}\mathcal{F}^a_\zeta(X;t) - e^{i(X-\zeta)(pz)}\mathcal{F}^{\bar{a}}_\zeta(X;t)\right) dX$$
$$+\, \bar{u}(p',s')\frac{\hat{z}\hat{r}-\hat{r}\hat{z}}{2M}u(p,s)\int_0^1 \left(e^{-iX(pz)}\mathcal{K}^a_\zeta(X;t) - e^{i(X-\zeta)(pz)}\mathcal{K}^{\bar{a}}_\zeta(X;t)\right) dX,$$

M is the nucleon mass and s, s' specify the nucleon polarization.

Sum rules

$$\int_0^1 \left[\mathcal{F}^a_\zeta(X;t) - \mathcal{F}^{\bar{a}}_\zeta(X;t)\right] dX = F_1^a(t), \tag{1.2}$$

$$\int_0^1 \left[\mathcal{K}^a_\zeta(X;t) - \mathcal{K}^{\bar{a}}_\zeta(X;t)\right] dX = F_2^a(t) \tag{1.3}$$

relate the nonforward distributions $\mathcal{F}^a_\zeta(X;t)$ and $\mathcal{K}^a_\zeta(X;t)$ to the a-flavor components of the Dirac and Pauli form factors, respectively.

When $\zeta \to 0$, the limiting curve for $\mathcal{F}_\zeta(X)$ reproduces $f_a(X)$:

$$\mathcal{F}^a_{\zeta=0}(X) = f_a(X) \;\;;\;\; \mathcal{F}^{\bar{a}}_{\zeta=0}(X) = f_{\bar{a}}(X). \tag{1.4}$$

In the region $X < \zeta$, one can define $Y = X/\zeta$ and treat the function $\mathcal{F}^a_\zeta(X)$ as the distribution amplitude

$$\Psi^a_\zeta(Y) = \mathcal{F}^a_\zeta(Y\zeta) - \mathcal{F}^{\bar{a}}_\zeta(\bar{Y}\zeta). \tag{1.5}$$

Formal definition:

$$\zeta\,\bar{u}(p')\hat{z}u(p)\int_0^1 \left[e^{-iY(rz)}\mathcal{F}^a_\zeta(\zeta Y) - e^{-i(1-Y)(rz)}\mathcal{F}^{\bar{a}}_\zeta(\zeta Y)\right] dY = \zeta\,\bar{u}(p')\hat{z}u(p)\int_0^1 e^{-iY(rz)}\Psi^a_\zeta(Y)\,dY\,, \tag{1.6}$$

see also Robaschik et al.

X. Ji.

Collins, Frankfurt Strikman

$$E(0,z;A) = P\exp\left\{igz^\mu \int_0^1 A_\mu(tz)\,dt\right\}$$

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II. GLUON DISTRIBUTION

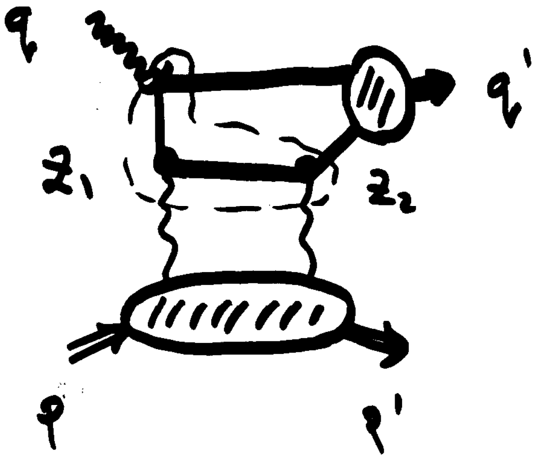
$$\begin{aligned}
&\langle p' \mid z_\mu z_\nu G^a_{\mu\alpha}(0) E^{ab}(0,z;A) G^b_{\alpha\nu}(z) \mid p\rangle|_{z^2=0} \\
&= \bar{u}(p')\hat{z}u(p)\,(z\cdot p)\int_0^1 \frac{1}{2}\left[e^{-iX(pz)} + e^{i(X-\zeta)(pz)}\right]\mathcal{F}^g_\zeta(X;t)\,dX \\
&+ \bar{u}(p')\frac{\hat{z}\hat{r}-\hat{r}\hat{z}}{2M}u(p)(z\cdot p)\int_0^1 \frac{1}{2}\left[e^{-iX(pz)} + e^{i(X-\zeta)(pz)}\right]\mathcal{K}^g_\zeta(X;t)\,dX\,.
\end{aligned} \tag{2.1}$$

In the region $0 < X < \zeta$, $\mathcal{F}^g_\zeta(X;t)$ can be written as

$$\bar{u}(p')\hat{z}u(p)\,(z\cdot r)\int_0^1 e^{-iY(rz)}\,\Psi^g_\zeta(Y;t)\,dY + \text{“}\mathcal{K}\text{” term,} \tag{2.2}$$

with the generalized $Y \leftrightarrow \bar{Y}$ symmetric distribution amplitude $\Psi^g_\zeta(Y;t)$ given by

$$\Psi^g_\zeta(Y;t) = \frac{1}{2}\left(\mathcal{F}^g_\zeta(Y\zeta;t) + \mathcal{F}^g_\zeta(\bar{Y}\zeta;t)\right)\,. \tag{2.3}$$



LIGHT-CONE GAUGE FOR GLUONS: $q'_\rho A^\rho = 0$

$$A_\mu(z,q') = q'^\rho \int_0^\infty G_{\mu\rho}(z+\sigma q')\,e^{-\epsilon\sigma}\,d\sigma$$

$$\Rightarrow \langle p' \mid A^a_\mu(z_1;q')\,A^b_\nu(z;q') \mid p\rangle_{z_1^2,\,z_2^2,\,(z_1-z_2)^2=0} =$$

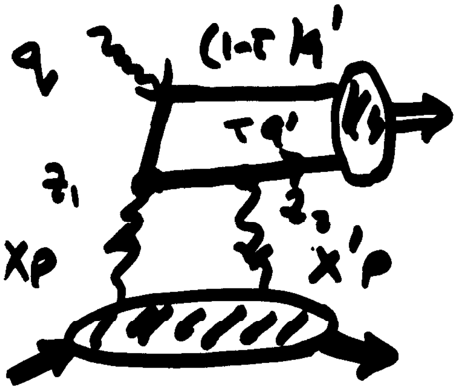
$$\frac{\delta^{ab}}{N_c^2-1}\,\frac{\bar{u}(p')\hat{q}'u(p)}{2(q'p)}\left(-g_{\mu\nu} + \frac{p_\mu q'_\nu + p_\nu q'_\mu}{(p\cdot q')}\right)$$

$$\times\int_0^1\left[e^{-iX(pz_1)+iX(pz_2')} + e^{iX'(pz_1)-iX(pz_2)}\right]\frac{\mathcal{F}^g_\zeta(X)\,dX}{(X-i\epsilon)(X'+i\epsilon)}$$

$$+\ \text{“}\mathcal{K}\text{”}$$

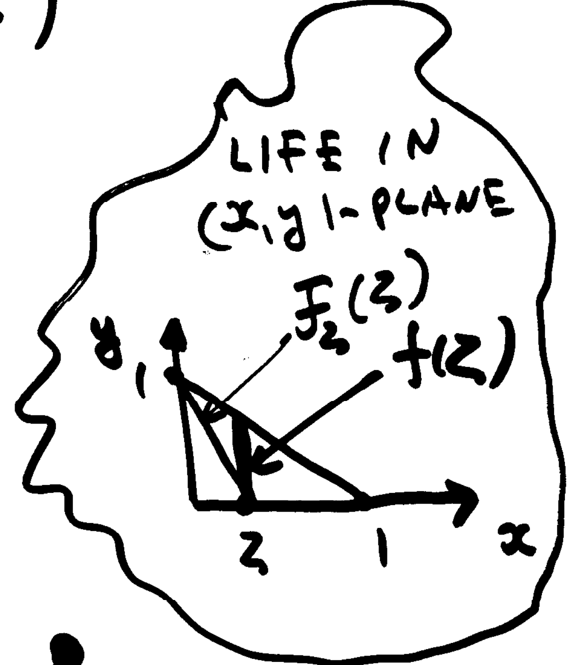
FOR LONGITUDINAL ρ-meson production:

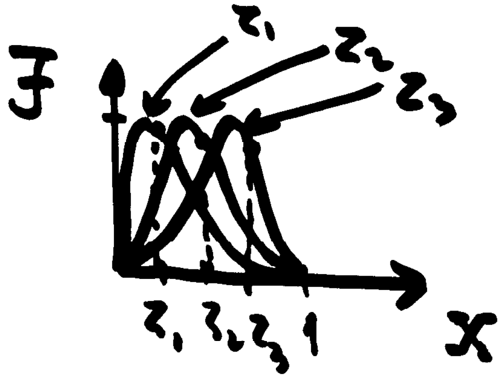
$$T_{LL}(p, q', r) \sim \bar{u}(p')\hat{q}' u(p) \int_0^1 d\tau \, \frac{\varphi_V(\tau)}{\tau} \int_0^1 \frac{\mathcal{F}_\zeta^{g}(x)}{x(x-\zeta+i\epsilon)} \, d$$



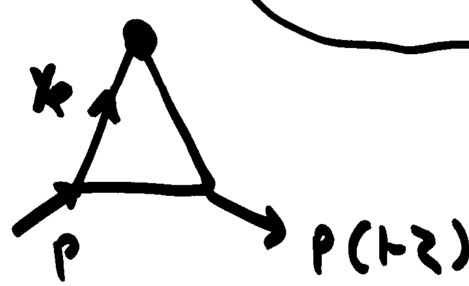
+ (3 others)

$$\mathrm{Im}\, T_{LL} \sim \mathcal{F}_\zeta(\zeta)$$



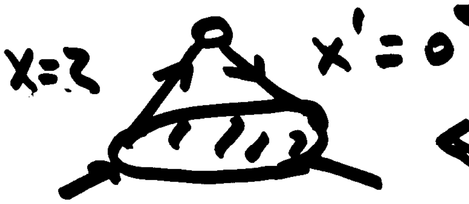


SCALAR TRIANGLE:



$$\left.\begin{aligned} f(\zeta) &= A(1-\zeta) \\ \mathcal{F}_\zeta(\zeta) &= A \end{aligned}\right\} \text{DIFFERENCE } O(\zeta) \text{ FOR SMALL } \zeta$$

NB:

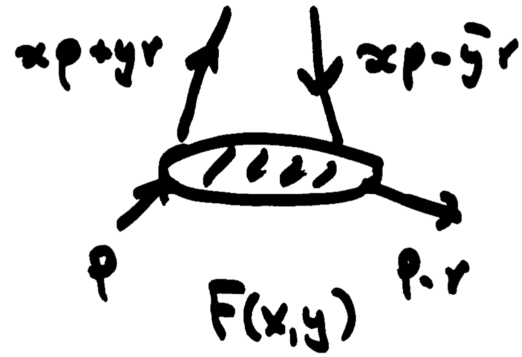


$\leftarrow \mathcal{F}_\zeta(\zeta)$

$\sim f(\zeta)$

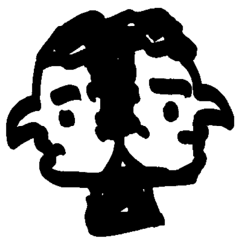
EVOLUTION EQUATIONS.

1. DOUBLE DISTRIBUTIONS

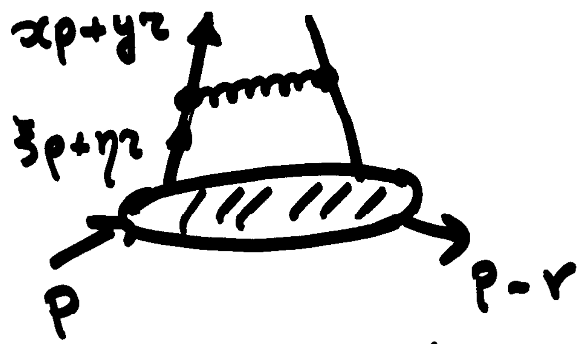


DA: $F(x, y)$

DF DA



$$\mu \frac{d}{d\mu} F(x,y;\mu) = \int_0^1 d\xi \int_0^1 d\eta \, R(x,y;\xi,\eta) F(\xi,\eta;\mu)$$



Remarkable properties of $R(x,y;\xi,\eta)$:

$$(\star) \quad \int_0^{1-x} R(x,y;\xi,\eta)\, dy = \frac{1}{\xi} P\left(\frac{x}{\xi}\right) \leftarrow \text{DGLAP}$$

$$(\star\star) \quad \int_0^{1-y} R(x,y;\xi,\eta)\, dx = V(y,\eta) \leftarrow \text{BL-type}$$

E.g.: $R^{QQ}(x,y;\xi,\eta) = \frac{\alpha_s}{\pi} C_F \frac{1}{\xi}\Big\{\theta\Big(0\le \frac{x}{\xi}\le \min\Big\{\frac{y}{\eta},\frac{\bar y}{\bar \eta}\Big\}\Big)$

$$-\frac{1}{2}\delta\Big(1-\frac{x}{\xi}\Big)\delta\Big(1-\frac{\eta}{y}\Big) +$$

$$+\frac{\theta(0\le x/\xi\le 1)\; x/\xi}{1-x/\xi}\Big[\frac{1}{\eta}\delta\Big(\frac{x}{\xi}-\frac{y}{\eta}\Big)+\frac{1}{\bar\eta}\delta\Big(\frac{x}{\xi}-\frac{\bar y}{\bar \eta}\Big)\Big]$$

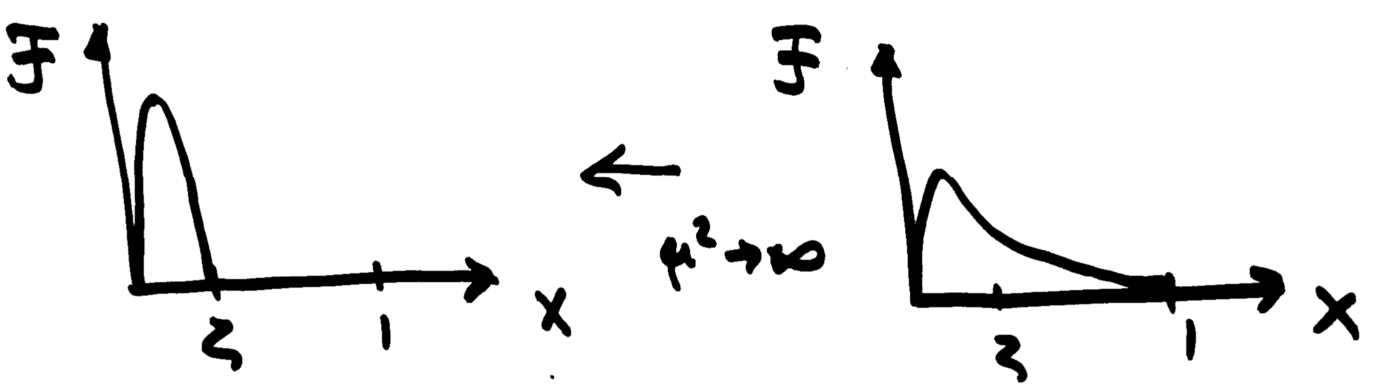
$$-2\,\delta\Big(1-\frac{x}{\xi}\Big)\,\delta(y-\eta)\int_0^1 \frac{z}{1-z}\,dz$$

(HOME ASSIGNMENT: CHECK (★) & (★★)).

FORMAL SOLUTION OF E.E. CAN BE OBTAINED BY COMBINING TAKING x^n-MOMENTS (DGLAP) WITH GEGENBAUER $C_k^{n+3/2}(y-\bar y)$ MOMENTS (BL

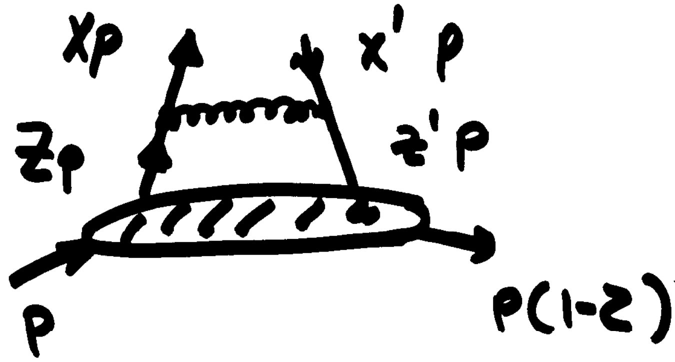
Result: $F(x,y;\mu\to\infty)\sim\delta(x)\,y\bar y$

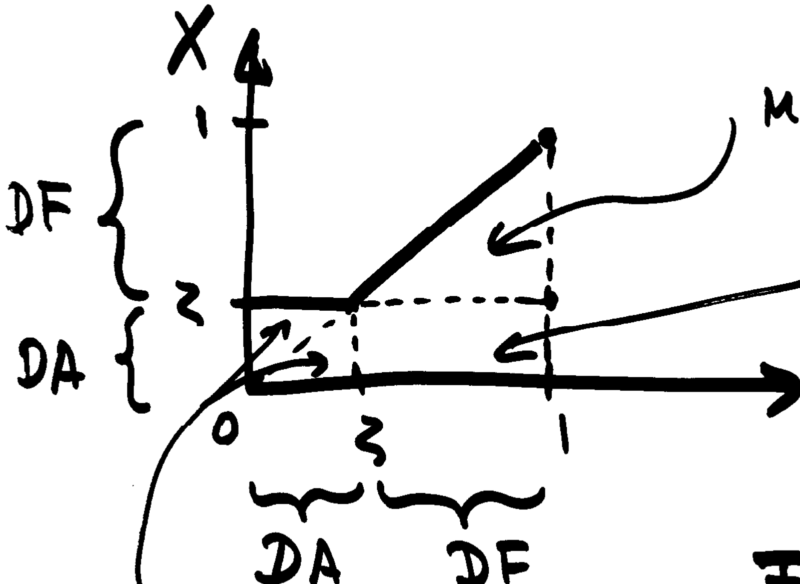
DD → AD ⇒ $\mathcal{F}_\xi(X)\sim\frac{X}{\xi}\Big(1-\frac{X}{\xi}\Big)$



EVOLUTION EQUATION FOR ASYMMETRIC DF:

$$\mu\frac{d}{d\mu}\mathcal{F}_\zeta^{a}(X;\mu)=\int_0^1\sum_b W_\zeta^{ab}(X,Z)\,\mathcal{F}_\zeta^{b}(Z;\mu)\,dZ$$



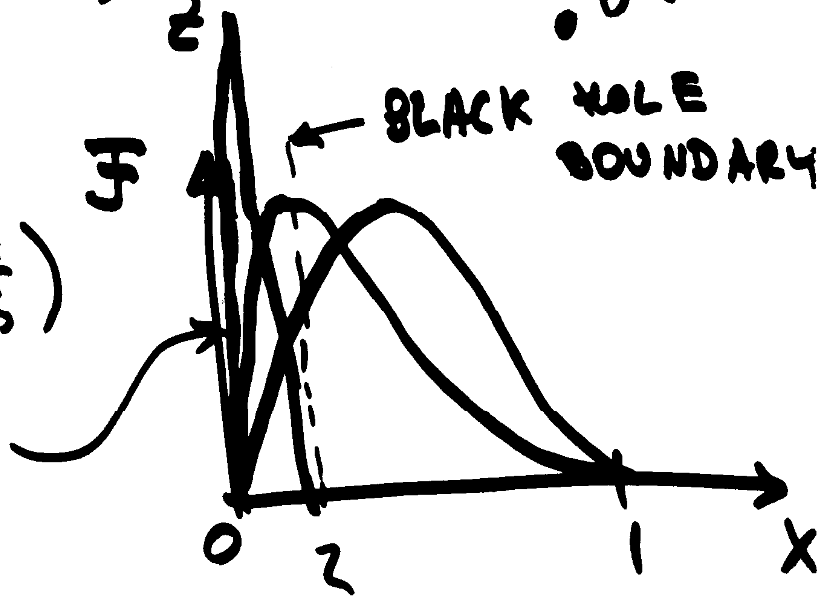


MODIFIED DGLAP

DF → DA transition

BL-type $\frac{1}{\zeta}V\left(\frac{X}{\zeta},\frac{Z}{\zeta}\right)\cdot\theta(X<Z)$

BL-type $\frac{1}{\zeta}V\left(\frac{X}{\zeta},\frac{Z}{\zeta}\right)$



equivalent of $\delta(x)$ for $f(x)$ $\Leftarrow$ $\frac{X}{Z}\left(1-\frac{X}{Z}\right)$

IV. BL-TYPE EVOLUTION KERNELS

When $\zeta = 1$, evolution is governed by the BL-type kernels:

$$W^{ab}_{\zeta=1}(X,Z) = V^{ab}(X,Z). \tag{4.1}$$

Explicit form:

$$V^{QQ}(X,Z) = \frac{\alpha_s}{\pi} C_F \left\{ \left[\frac{X}{Z}\left(1 + \frac{1}{Z-X}\right) \theta(X<Z) \right]_+ + \{X \to \bar{X}, Z \to \bar{Z}\} \right\} \tag{4.2}$$

$$V^{Qg}(X,Z) = \frac{\alpha_s}{\pi} N_f \left\{ \frac{X}{Z}\left[2(2-X) + \frac{1-X}{Z}\right] \theta(X<Z) - \{X \to \bar{X}, Z \to \bar{Z}\} \right\}, \tag{4.3}$$

$$V^{gQ}(X,Z) = \frac{\alpha_s}{\pi} C_F \left\{ \left(2 - \frac{X^2}{Z}\right) \theta(X<Z) + \frac{(1-X)^2}{1-Z}\, \theta(X>Z) \right\}, \tag{4.4}$$

$$\begin{aligned} V^{gg}(X,Z) = \frac{\alpha_s}{\pi} N_c \Bigg\{ & 2\frac{X^2}{Z}\left(3 - 2X + \frac{1-X}{Z}\right) + \frac{1}{Z-X}\left(\frac{X}{Z}\right)^2 \\ & + \delta(X-Z)\left[\frac{\beta_0}{2N_c} - \int_0^1 \frac{dz}{1-z}\right] \Bigg\} \theta(X<Z) + \{X \to \bar{X}, Z \to \bar{Z}\}. \end{aligned} \tag{4.5}$$

BL-type kernels appear as a part of the asymmetric kernel $W^{ab}_\zeta(X,Z)$ even in the general $\zeta \neq 1, 0$ case when both X and Z are smaller than ζ.

Denote $L^{ab}_\zeta(X,Z) \equiv W^{ab}_\zeta(X,Z)|_{0 \leq \{X,Z\} \leq \zeta}$. Then:

$$\begin{aligned} L^{QQ}_\zeta(X,Z) &= \frac{1}{\zeta} V^{QQ}(X/\zeta, Z/\zeta) \;;\; L^{gQ}_\zeta(X,Z) = V^{gQ}(X/\zeta, Z/\zeta) \;; \\ L^{Qg}_\zeta(X,Z) &= \frac{1}{\zeta^2} V^{Qg}(X/\zeta, Z/\zeta) \;;\; L^{gg}_\zeta(X,Z) = \frac{1}{\zeta} V^{gg}(X/\zeta, Z/\zeta). \end{aligned} \tag{4.6}$$

BL-type kernels also govern the evolution in the region corresponding to transitions from a fraction Z which is larger than ζ to a fraction X which is smaller than ζ.

V. REGION $Z \geq \zeta,\ X \geq \zeta$

Auxiliary kernels:

$$M_\zeta^{ab}(X,Z)|_{\zeta\leq X\leq Z\leq 1} = \frac{Z-X}{ZZ'}\int_0^1 B_{ab}\left(\bar{w}\,(1-X/Z)\,,\ w\,(1-X'/Z')\right)dw\,, \tag{5.1}$$

where $X' \equiv X-\zeta$ and $Z' \equiv Z-\zeta = v/(1-X'/Z')$ are the "returning" partners of the original fractions X, Z. Since $Z-X = Z'-X'$, the kernels $M_\zeta^{ab}(X,Z)$ are symmetric with respect to the interchange of X, Z with X', Z'.

Introducing the notation $P_\zeta^{ab}(X,Z) \equiv W_\zeta^{ab}(X,Z)|_{\zeta\leq X\leq Z\leq 1}$ we have

$$P_\zeta^{QQ}(X,Z) = \frac{\alpha_s}{\pi}C_F\left\{\frac{1}{Z-X}\left[1+\frac{XX'}{ZZ'}\right] - \delta(X-Z)\int_0^1\frac{1+z^2}{1-z}\,dz\right\} \to \frac{1}{Z}P_{QQ}(X/Z)\,, \tag{5.2}$$

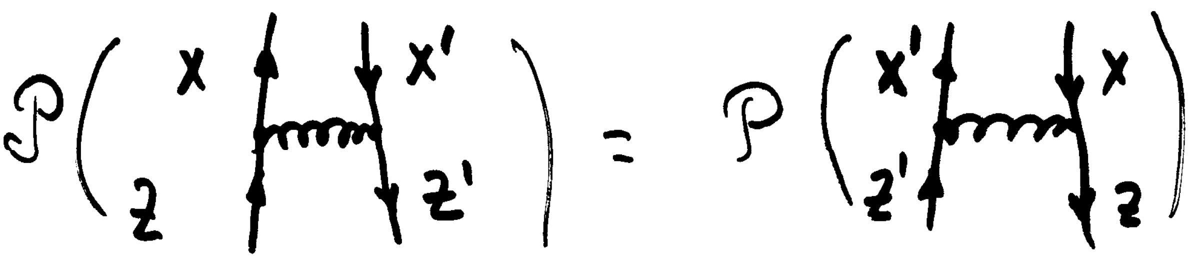
$$P_\zeta^{Qg}(X,Z) = \frac{\alpha_s}{\pi}N_f\,\frac{1}{ZZ'}\left\{\left(1-\frac{X}{Z}\right)\left(1-\frac{X'}{Z'}\right)+\frac{XX'}{ZZ'}\right\} \to \frac{1}{Z^2}P_{Qg}(X/Z)\,, \tag{5.3}$$

$$P_\zeta^{gQ}(X,Z) = \frac{\alpha_s}{\pi}C_F\left\{\left(1-\frac{X}{Z}\right)\left(1-\frac{X'}{Z'}\right)+1\right\} \to \frac{X}{Z}P_{gQ}(X/Z)\,, \tag{5.4}$$

$$P_\zeta^{gg}(X,Z) = \frac{\alpha_s}{\pi}N_c\left\{2\left[1+\frac{XX'}{ZZ'}\right]\frac{Z-X}{ZZ'}+\frac{1}{Z-X}\left[\left(\frac{X}{Z}\right)^2+\left(\frac{X'}{Z'}\right)^2\right]\right.$$
$$\left. +\,\delta(X-Z)\left[\frac{\beta_0}{2N_c}-2\int_0^1\frac{dz}{1-z}\right]\right\} \to \frac{X}{Z^2}P_{gg}(X/Z)\,. \tag{5.5}$$

They also have a symmetric form. The arrows indicate how the asymmetric kernels $P_\zeta^{ab}(X,Z)$ are related to the GLAPD kernels in the $\zeta = 0$ limit when $Z = Z'$ and $X = X'$.

SYMMETRY:



(FORMALLY, THOUGH → IN OUR CASE $X > X'$, $Z > Z'$)

#1 CALCULATION OF ASYMMETRIC EVOLUTION KERNELS:

L. GRIBOV, E. LEVIN & M. RYSKIN

Phys. Reports #100 (198)

(also Robaschik et al. (VPA); X. Ji ; Frankfurt et al.)

CONCLUSIONS

1) DVCS (& OTHER HARD ELECTROPRODUCTION PROCESSES
INTRODUCE NEW TYPE OF DISTRIBUTIONS, HYBRIDS OF DF'S, DA'S & FF'S.

2) FOR SMALL ζ ($= x_{Bj}$)

$$\mathcal{F}_\zeta(x, t\to 0) \simeq f(x)$$

→ SAME DISTRIBUTION APPEARS IN INCLUSIVE & EXCLUSIVE PROCESSES

3) NEW TYPE OF SPIN-FLIP DISTRIBUTIONS (ANALOGOUS TO $F_2(t)$ FORM FACTOR)

4) POSSIBILITY TO GET INFORMATION ABOUT PARTON HELICITY SENSITIVE DISTRIBUTIONS IN UNPOLARIZED EXPERIMENTS LIKE $\gamma^* p \to \pi p'$,

NON-DIAGONAL DISTRIBUTIONS $\langle n | \ldots | p \rangle$,

etc.