A. Freund

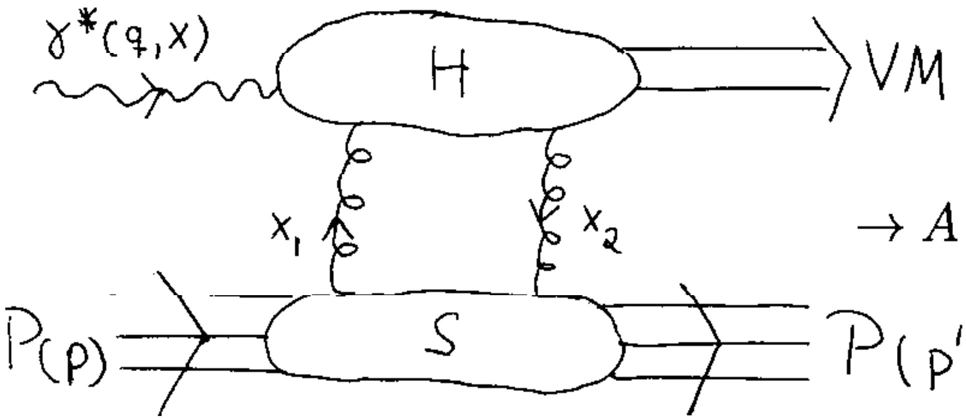
Nondiagonal Parton Distributions (ND-PD) at Small x

- Introduction
- Importance of ND-PD's for Hard Diffractive Scattering
- Proof of absence of singularities from soft gluons in ND PD's
- GLAP-Evolution Equation + ND-Kernels
- Numerical Results of Evolution + Simple connection between D- and ND-distributions at small x
- Conclusions

Motivation: $\gamma^* + P(p) \to P(p') + V, \gamma$, 2 jets etc.

Hard Diffractive Production (**HDP**)

dominated by 2-gluon coupling to $q\bar{q}$-loop.



$\to A = \psi_M \otimes H \otimes S$

Excellent testing ground to measure nondiagonal Gluon distribution:

$$x_2 G(x_1, x_2) =$$
$$-\int_{-\infty}^{\infty} \frac{dy^-}{4\pi} \frac{1}{p^+} e^{-ix_2 p^+ y^-} \langle p' | \, T G_\nu{}^+(0, y^-, \mathbf{0}_T) \, \mathcal{P} \, G^{\nu+}(0) \, | p \rangle$$

Also: The nondiagonal Gluon distribution (Not the diagonal one!) appears in the cross section for HDP.

If we know the nondiagonal Gluon distribution, we can make predictions of the size of the cross section, energy dependences, etc.

II Importance of ND-PD's for HDS

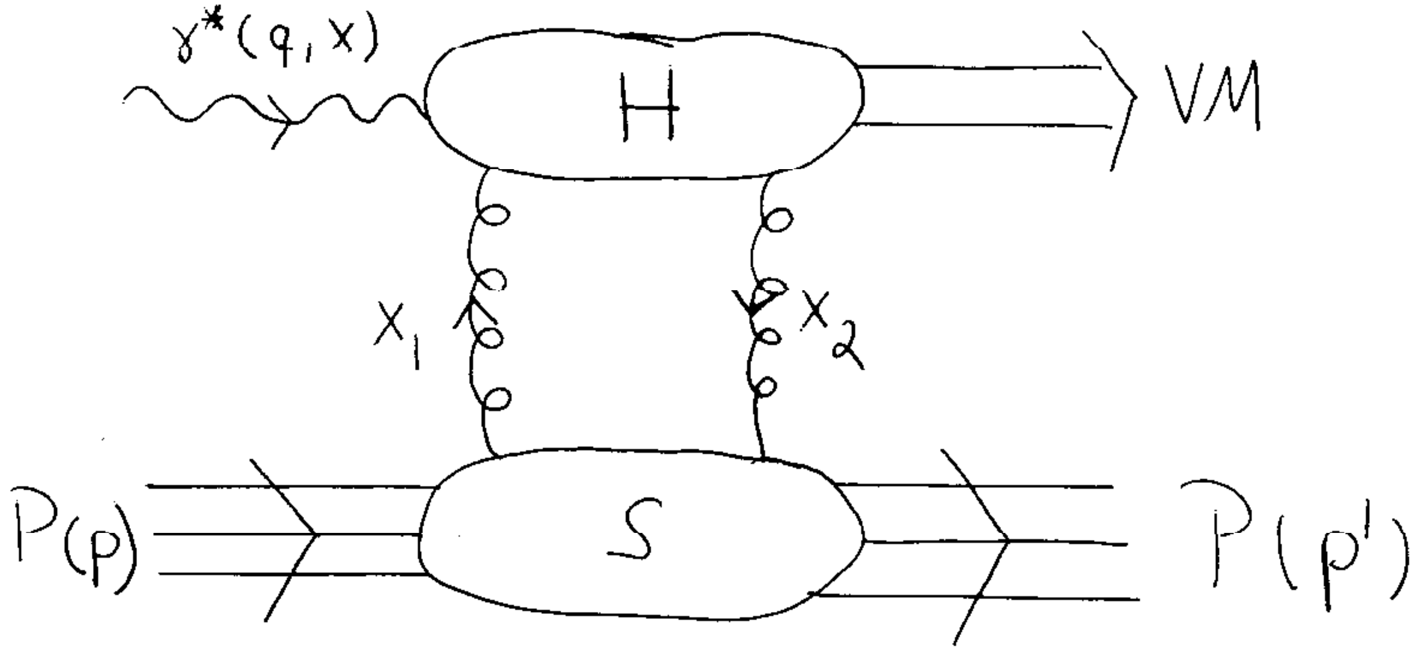
- For leading twist QCD Evol. Eq. have usually been discussed in terms of the imaginary part of the amplitude. **BUT** there exists an Evol. Eq. for the whole amplitude.

<u>**Goal**</u>: For HDP's, the Q^2-evolution at any x is given by a nondiagonal GLAP-type equation with asymmetric DGLAP-type kernels and HDP's can be calculated through discontinuities of parton distributions.

- To express the amplitude in terms of ND-PD's, close integration contour for β over singularities of gluon-nucleon scattering at fixed x_1 and $x_2 = x_1 - x$.

- Causality condition: $x - 1 \le x_1 \le 1$

  but for small $-t > 0$: $x \le x_1 \le 1$.



- For $x \leq x_1 \leq 1$ the amplitude is the sum of terms with s- and u-channel singularities only! $\rightarrow$ Amplitude is expressed through imaginary part of the amplitude for gluon-nucleon scattering. QCD evol. is described by GLAP-type equation + ND-Kernels.

- $x \geq x_1 \geq 0$: Contribution of this region has **no** direct relationship with conventional parton densities, since the integral over β cannot be closed over s- and u-channel discont., only over the gluon "mass". Connection to VM-wavefunction as suggested by Radyushkin'96.

- Dispersion relation over s ($\rightarrow$ Discont. over parton distribution) $\rightarrow$ only subtraction constant in the real part cannot be reconstructed.

- Symmetric contribution (under s $\leftrightarrow$ u) to the disp. rel. is down by an additional power of s and the anti-symmetric piece corresponding to a process with neg. charge parity i.e., electroproduction of π_0, has NO subtraction term at all [increases with energy slower than s] $\rightarrow$ Disp. Rel. gives full discription!

- Small x_i behaviour: Consideration of HD-Feynman diagrams shows that for

$$x_2\, G(x_1, x_2) =$$
$$- \int_{-\infty}^{\infty} \frac{dy^-}{4\pi} \frac{1}{p^+}\, e^{-ix_2 p^+ y^-} \langle p'|\, TG_\nu{}^+(0, y^-, \mathbf{0}_T)\, \mathcal{P}\, G^{\nu+}(0)\; |p\rangle$$

- Leading $\alpha_s \ln x$: $\ln x_i \simeq \ln x \rightarrow G(x_1, x_2) = G(x)$.

- Leading $\alpha_s \ln x_i$ with $x_1 >> x_2$:

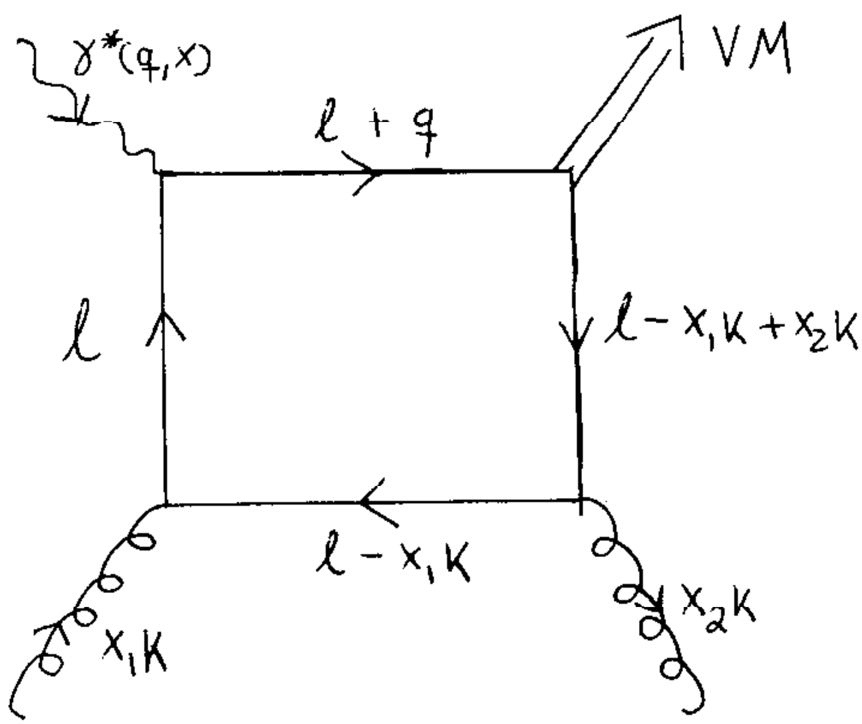
$$x_2 G(x_1, x_2) \simeq x_1 G(x_1)$$

III Proof of no $\ln x_2$ in ND-PD's

Strategy: Show that $x_2 = 0$ is not a leading region!

Step I: Gluon with $x_2 = 0$ corresponds to a soft gluon.

Step II: Use argument by Collins and Sterman that a soft gluon does not give a leading contribution.



To Step I:

- l_- (quark loop) $\simeq$ $m_N x$ $>>$ k_- since k_- $\simeq$ $\frac{(m_{q\bar{q}}^2+M_V^2)m_N}{2qp_N}$

- l_t^2 $<<$ Q^2 due to cut-off by VM-wavefunction, whereas k_t^2 possibly $O(Q^2)$ <u>but</u> $k_\mu T^{\mu\nu} = 0$ or $x_2 p_\mu T^{\mu\nu} + k_{t\mu} T^{\mu\nu} = 0$ and since $x_2 = 0$, $k_t^2 << l_t^2$. $T^{\mu\nu}$ is the scattering amplitude and the Ward identity is very simple since we are dealing with $q\bar{q}$-singlet!

$\rightarrow$ gluon is soft!

To Step II:

Integral for leading region is:

$$\int_{soft\ \ k} d^4k \frac{1}{((l-k)^2+i\epsilon)(k^2+i\epsilon)} f(l-k,p)$$

$$\simeq \int_{soft\ \ k} dk_+ \frac{f(l-k,p)}{((l_+-k_+)l_- - l_t^2+i\epsilon)(2k_+k_- - k_t^2+i\epsilon)}$$

2 possibilities:

- $k_+k_- \geq k_t^2$, No obstructions to contour deformations

  $\rightarrow$ No leading contribution!

- $k_+k_- \ll k_t^2$, No Obstructions to contour deformations

  $\rightarrow$ No leading contribution!

$\rightarrow$ $x_2 = 0$ gives no leading contribution!

IV GLAP-EVOL.EQ. + ND-Kernels

Evolution Equation:

$[g(x_1, x_2) = (x_1 - \Delta)G(x_1, \Delta),\ \Delta = x_1 - x_2]$

$$\frac{dg(x_1, \Delta, Q^2)}{d\ln Q^2} = \int_{x_1}^{1} \frac{dy_1}{y_1} P_{GG}(x_1, y_1) \frac{x_1/y_1 - \Delta/y_1}{1 - \Delta/y_1} g(y_1, \Delta, Q^2) + P_{GQ}(x_1, y_1) \frac{x_1/y_1 - \Delta/y_1}{1 - \Delta/y_1} q(y_1, \Delta, Q^2)$$

In the limit of small x and large Q^2:

$$xG(x) \approx x_2 G(x_1, x_2)$$

Reason: In this limit, main contribution stems from ND-distribution at $\tilde{x}_1, \tilde{x}_2 = \tilde{x}_1 - \Delta$ with $\tilde{x}_1 \gg x_1$. For $\tilde{x}_1, \tilde{x}_2 \gg \Delta$ deviations from the diagonal distributions are small. $\rightarrow$ Main effect of asymmetry from evolution.

<u>Kernels</u>: Calculated using diagramatic method of decay cells

$$\frac{d\phi}{2\pi} dk_\perp^2 dx_1 \frac{\mathcal{M}}{(1-\Delta)} \left(\frac{1-\beta}{k_\perp^2}\right)^2 \frac{1}{1-\beta}$$

$$P_{QQ}(x_1,\Delta) = 2C_F[\frac{1+x_1^2-\Delta(1+x_1)}{(1-\Delta)(1-x_1)} - 2\delta(1-x_1)\int_0^1 \frac{dz}{z} + \frac{3}{2}\delta(1-x_1)]$$

$$P_{QG}(x_1,\Delta) = 2C_F\frac{[x_1^2+(1-x_1)^2-x_1\Delta]}{(1-\Delta)^2}$$

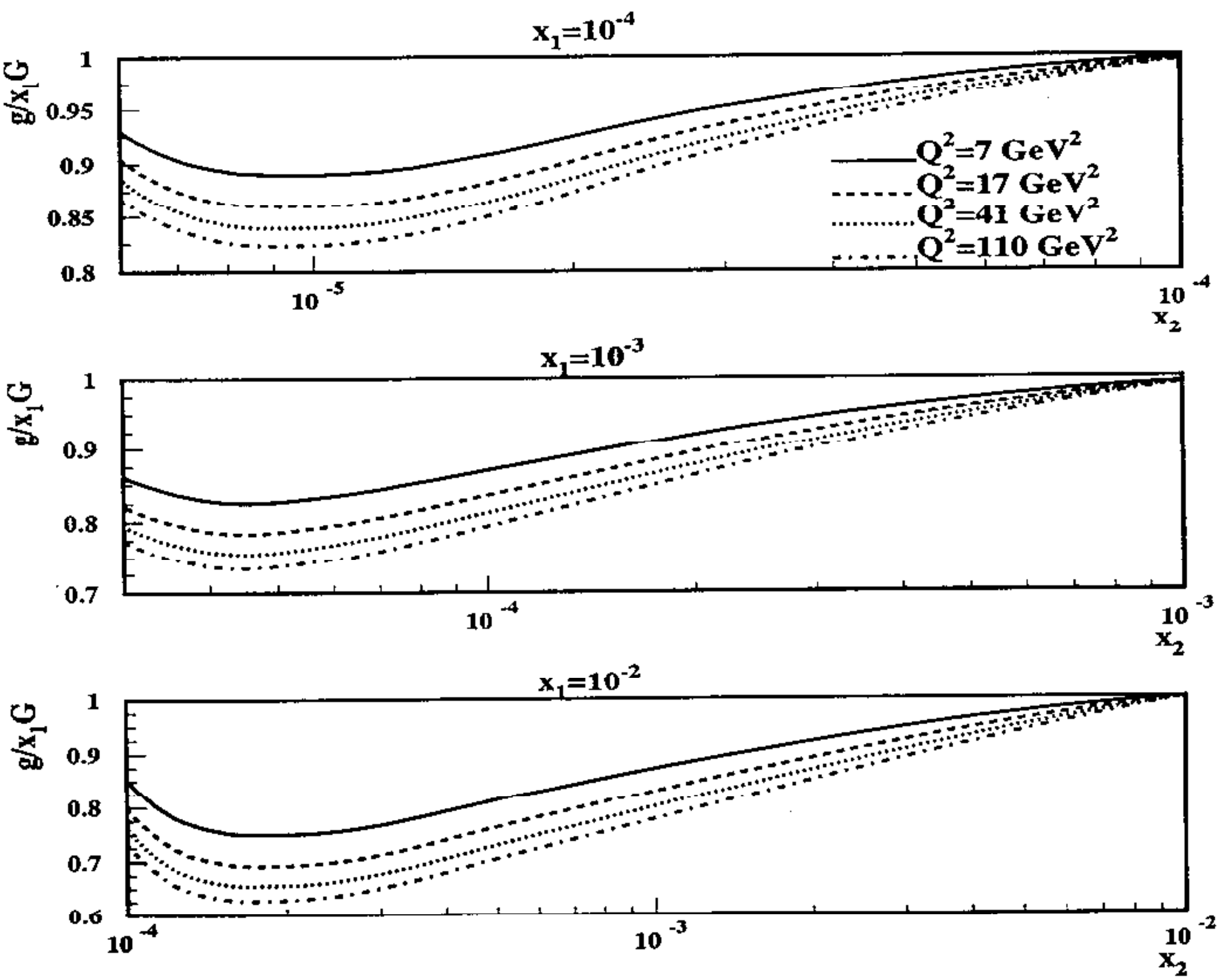
$$P_{GQ}(x_1,\Delta) = 2T_R\frac{[1+(1-x_1)^2-\Delta]}{(x_1-\Delta)}$$

$$P_{GG}(x_1,\Delta) = 4N_c[\frac{x_1(1-x_1)-\frac{1}{2}}{1-\Delta} + \frac{1}{1-x_1} - \frac{1}{2} + \frac{\frac{x_1\Delta}{2}-x_1+1}{x_1-\Delta} - \delta(1-x_1)\left[\frac{\beta_0}{4N_C} - \int_0^1\frac{dz}{z}\right]]$$

$\Delta = 0$ yields the diagonal Kernels!

V Numerical Results of Evolution

- Used modified CTEQ-Package and as initial distribution for diagonal and nondiagonal case the CTEQ4 global fit. $Q_0^2 = 1.6\text{GeV}^2$



Conclusions

- Amplitudes of HDS are given by discontinuities of ND-PD's.
- No $\ln x_2$ terms in ND-PD's.
- $xG(x) \simeq x_2 G(x_1, x_2)$ since the diagonal and nondiagonal case are very similar for small x.
- Numerical study confirms:
  - No $\ln x_2$ and $xG(x) \simeq x_2 G(x_1, x_2)$.
  - Since $G(x_1, x_2)$ enters σ for VMP in DIS, the diagonal approximation used in VMP calculations is valid at the kinematics of HERA within 20%!
- More work is needed, especially NLO calculation to check whether correction is substantial.