NLO Analysis of SLAC E154 Data

Piotr Zyla
Temple University

On behalf of the SLAC E154 Collaboration

DIS 97
Chicago, Illinois, USA
April 14-18, 1997

- **Motivation**
- **Formalism**
- **Experimental and Theoretical Uncertainties**
- **Results and Interpretation**
- **Summary**

Motivation

- **E154 $g_1^n(x)$ large and strongly x dependent at low $x \leq 0.1$ at $Q^2 = 5$ GeV^2**
  What does it mean?

- **Evolution based on g_1/F_1 = (constant in Q^2) assumption**
  - **consistent within errors with experimental data**
  - **incompatible with pQCD**
  - **no associated systematic error**

- **Low x extrapolation for the sum rules**
  - **Regge-like, $\sim$constant in x**
  - **Power-like, $\sim x^{-0.9\pm0.2}$**

  model dependent.

- **Reasonable kinematic range and precision of polarized data**

- **Known NLO spin-dependent anomalous dimensions**

  R. Mertig and W. L. van Neerven (1996), W. Vogelsang (1996)

- **Careful study of statistical and systematic uncertainties**

- **New E154 data on g_1^n**

- **Global analysis of x and Q^2 dependence**

- **Allows spin decomposition of structure functions onto valence and sea quarks and gluons**

- **Standard in unpolarized DIS data analyses**

Factorization Theorem

Flavor Decomposition of g_1

$$g_1^p(x, Q^2) = \frac{1}{2} \sum_q^{N_f} e_q^2 \ [\, C_q \otimes \Delta q + \ \frac{1}{N_f} C_G \otimes \Delta G \,]$$

and for the neutron and deuteron

$$\begin{aligned} g_1^n &= g_1^p(u \leftrightarrow d) \\ g_1^d &= \frac{1}{2}(g_1^n + g_1^p)(1 - \frac{3}{2}\omega_D), \quad \omega_D = 0.05 \pm 0.01 \end{aligned}$$

Convolution $\otimes$ is defined as

$$g_1(x) \equiv (C \otimes q)(x) = \int_x^1 \frac{dy}{y} C\left(\frac{x}{y}\right) q(y)$$

Mellin transform

$$f(n) = \int_0^1 dx \, x^{n-1} \, f(x)$$

$$g_1^p(n,Q^2) = \frac{1}{2}\sum_q^{N_f} e_q^2 \;[\,C_q\,\Delta q + \frac{1}{N_f} C_G\,\Delta G\,]$$

* $C_{q,G}$ – **perturbative coefficients**
* $\Delta q = \delta q + \delta\bar{q}$ – **non-perturbative input**
* **Factorization scheme dependent in NLO**

- **Common Schemes**
  * $\overline{\mathrm{MS}}$ – **gluons do not contribute to the integral of** g_1, $C_G^{(1)}(n=1)=0$
  * **Adler-Bardeen** – **include axial anomaly contribution,** $C_G^{(1)}(n=1)=-N_f$

$$\int_0^1 dx\; g_1^p(x) = \frac{1}{2}\left[\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right] - \frac{\alpha_s}{6\pi}\Delta G$$

- **Consequence: ambiguity in definition of the "quark spin" contribution**

$$\Delta\Sigma(\mathrm{AB}) = \Delta\Sigma(\overline{\mathrm{MS}}) + \frac{N_f}{2\pi}\alpha_s\,\Delta G$$

- **Product $\alpha_s(Q^2)\,\Delta G(Q^2)$ is independent of Q^2 to $O(\alpha_s^2)$ J. Kodaira (1980)**
  $\Rightarrow \Delta\Sigma$ is defined up to a constant
  $\Rightarrow$ Asymptotically $\Delta G \sim 1/\alpha_s \to \infty$

- **Angular momentum sum rule revised:**
  X. Ji (1996)

$$S_p = \frac{1}{2} = \left(\frac{1}{2}\Delta\Sigma + \langle L_z^q\rangle\right) + \left(\Delta G + \langle L_z^G\rangle\right)$$

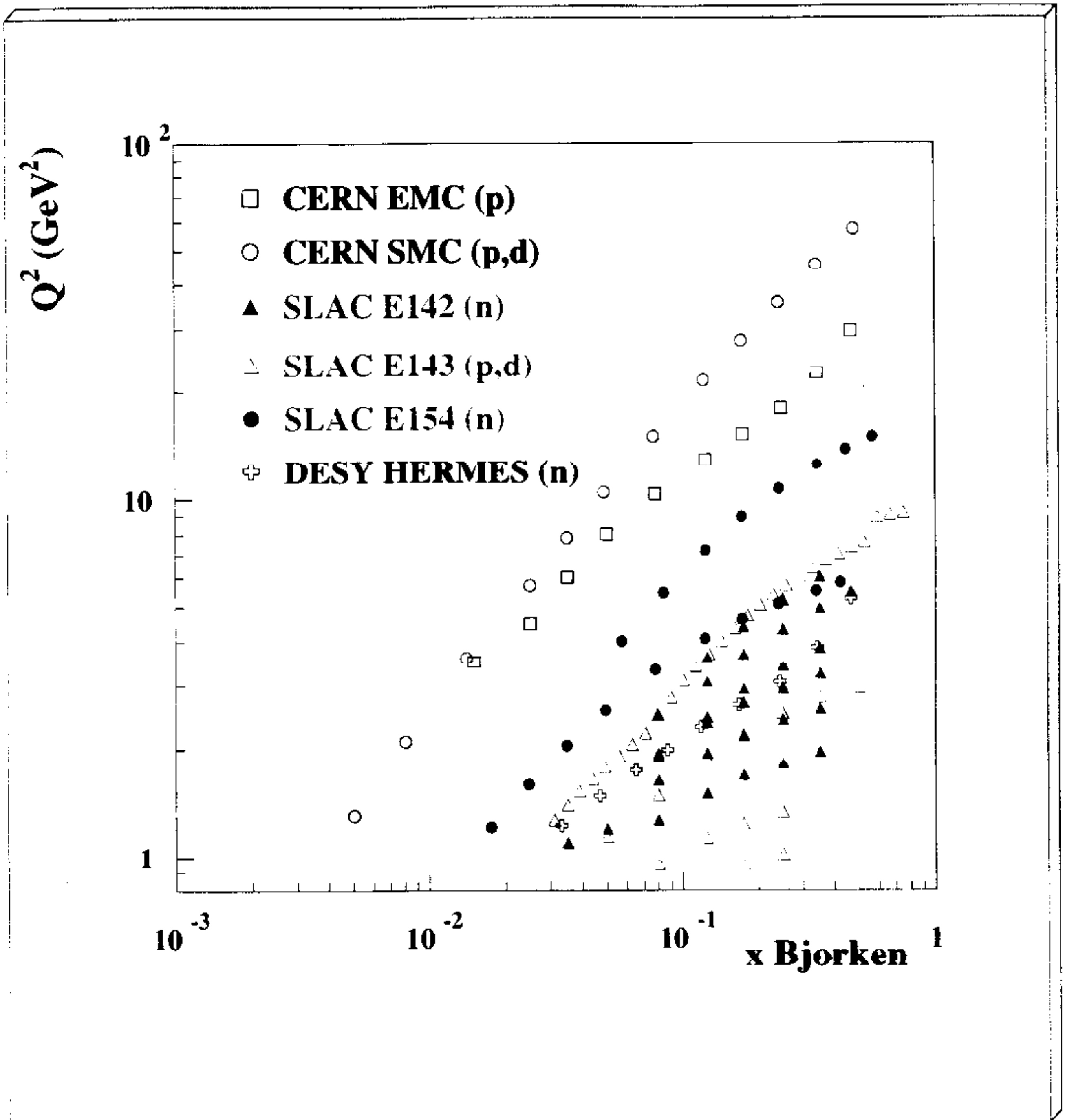
Fitting Procedure

- **Start with parametrized parton distributions $\Delta q(x, Q_0^2)$ at low Q_0^2**
- **Evolve up to experimental Q^2 using NLO DGLAP equations**
- **Compute g_1 at experimental (x, Q^2) and compare with data. Contributions to χ^2 weighted by statistical error**
- **Constrain 8 free parameters to give the best fit**

Notice that we do not assume

$$\begin{aligned}\Delta q_3 &= g_A = 1.2601 \pm 0.0025 \\ \Delta q_8 &= 0.579 \pm 0.025\end{aligned}$$

Kinematic coverage



Initial Distributions

at $Q_0^2 = 0.34\mathrm{GeV}^2$

- **Assume the x dependence**

$$\Delta f(x, Q_0^2) = A_f\, x^{\alpha_f}\, f(x, Q_0^2)\, (1-x)^{\beta_f}$$

where

* $\Delta f = \Delta u_V, \Delta d_V, \Delta \bar{Q}, \Delta G$
* $f(x, Q_0^2)$ – **unpolarized distributions of GRV95**

- **Positivity constraint**

$$|\Delta f(x)| \;\leq\; f(x) \qquad \Rightarrow \alpha_f \geq 0,\;\; \beta_f \geq 0$$

- **Helicity retention property**

$$\frac{\Delta f(x)}{f(x)} \xrightarrow{x\to 1} 1 \quad \Rightarrow \beta_f = 0$$

The fits are compatible with this assumption

- **Assume sea isospin symmetry**

$$\Delta\bar{u} = \Delta\bar{d}$$

- **Then**

$$g_1^{\mathrm{sea}} \sim \frac{\Delta\bar{u} + \Delta\bar{d}}{2} + \frac{1}{5}\Delta\bar{s}$$

  - **DIS does not probe light and strange sea separately**
  - **Sensitivity to the difference only via evolution, beyond present experimental precision**

Thus parametrize

$$\Delta\bar{Q} \equiv \frac{\Delta\bar{u} + \Delta\bar{d}}{2} + \frac{1}{5}\Delta\bar{s}$$

- **Assume** $\Delta s(x, Q_0^2)$

$$\Delta s = \Delta \bar{s} = \lambda_s \frac{\Delta \bar{u} + \Delta \bar{d}}{2} = \frac{\lambda_s}{1 + \lambda_s/5} \Delta \bar{Q}$$

with parameter $\lambda_s = 0 - 1$

- **Initial distributions**

$$\begin{aligned}
\Delta u_V(x, Q_0^2) &= A_u\, x^{\alpha_u}\, u_V(x, Q_0^2) \\
\Delta d_V(x, Q_0^2) &= A_d\, x^{\alpha_d}\, d_V(x, Q_0^2) \\
\Delta \bar{Q}(x, Q_0^2) &= A_Q\, x^{\alpha_Q}\, \bar{Q}(x, Q_0^2) \\
\Delta G(x, Q_0^2) &= A_G\, x^{\alpha_G}\, G(x, Q_0^2)
\end{aligned}$$

specified in terms of 8 free parameters.

Unpolarized distributions of GRV95

$$
\begin{aligned}
u_V(x, Q_o^2) &= 0.988\, x^{-0.457}\, (1 + 1.58\, \sqrt{x} + 2.58\, x + 18.1\, x^{3/2})\, (1-x)^{3.380} \\
d_V(x, Q_o^2) &= 0.182\, x^{-0.684}\, (1 + 2.51\, \sqrt{x} + 25.0\, x + 11.4\, x^{3/2})\, (1-x)^{4.113} \\
\bar{Q}(x, Q_o^2) &= 0.654\, x^{-0.70}\, (1 + 2.65\, x)\, (1-x)^{8.33} \\
G(x, Q_o^2) &= 26.20\, x^{0.9}\, (1-x)^{4.0}
\end{aligned}
$$

NLO Evolution ($\overline{\mathrm{MS}}$)

- **DGLAP equations in n-moments space**

$$Q^2 \frac{d}{dQ^2} \Delta q^\eta_{NS} = \frac{\alpha_s(Q^2)}{2\pi} \gamma^\eta_{NS} \cdot \Delta q^\eta_{NS}, \qquad \eta = \pm 1,$$

$$Q^2 \frac{d}{dQ^2} \begin{pmatrix} \Delta\Sigma \\ \Delta G \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \begin{pmatrix} \gamma_{qq} & \gamma_{qG} \\ \gamma_{Gq} & \gamma_{GG} \end{pmatrix} \cdot \begin{pmatrix} \Delta\Sigma \\ \Delta G \end{pmatrix}$$

with the NS distributions defined as

$$\left.\begin{array}{lcl} \Delta u_V & = & \delta u - \delta \bar{u} \\ \Delta d_V & = & \delta d - \delta \bar{d} \end{array}\right\} \eta = 1$$

$$\left.\begin{array}{lcl} \Delta q_3 & = & \Delta u - \Delta d \\ \Delta q_8 & = & \Delta u - \Delta d - 2\Delta s \end{array}\right\} \eta = -1$$

and the singlet

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

- **Fixed flavor scheme, N_f=3**

- **Heavy flavors contribution included in running of two-loop α_s**

$$\alpha_s(m_q^2, f) = \alpha_s(m_q^2, f+1)$$

- **Quark masses: m$_c$=1.5** GeV**, m$_b$=4.5** GeV

- **For consistency with the unpolarized fit**

$$\alpha_s(M_Z^2) = 0.109 \quad \Rightarrow \quad \alpha_s(5\,\mathrm{GeV}^2) = 0.237$$

- **Thus**

$$\Lambda_{f=3,4,5} = 248,\ 200,\ 131\,\mathrm{MeV}$$

Experimental and Theoretical Uncertainties

- Statistical Errors
  - Added in quadrature contributions from experimental points
- Systematic Errors
  - Usually dominated by normalization errors (beam and target polarizations, dilution factor) - largely correlated from point to point
  - Assume 100% correlation within experiment
  - Add contributions from any given experiment linearly

- **Theoretical Uncertainties**

  * α_s **from fixed target DIS experiments,**

$$\alpha_s(M_Z^2) = 0.108 - 0.116,$$

    has the biggest effect (renormalization and factorization scale dependence)
  * **Variation of current quark masses - small effect**

$$m_c = 1 - 2 \text{ GeV} \qquad m_b = 4 - 5 \text{ GeV}$$

  * **Initial strange sea distribution - small effect**

$$\lambda_s = 0 - 1$$

  * **The** x **dependence of initial distributions and starting** Q_0^2 **scale: fit with MRS96 initial unpolarized distributions at** $Q_0^2 = 1 \text{ GeV}^2$**. Results consistent within statistical errors**
  * **Factorization scheme dependence:** $\overline{\text{MS}}$ **and AB schemes results consistent within errors**
  * **Possible higher twist effects neglected. Expected to drop as 1/W**^{2} **and W**$^2 > 4 \text{ GeV}^2$ **for all experimental data with majority W**$^2 > 8 \text{ GeV}^2$

NLO Results

- **Very good fits,**

  χ^2 **= 146 for 168 points** ($\overline{\rm MS}$)

  148 (AB)

- **Low x behavior of valence distributions at the initial Q_0^2 scale consistent with Regge prediction**

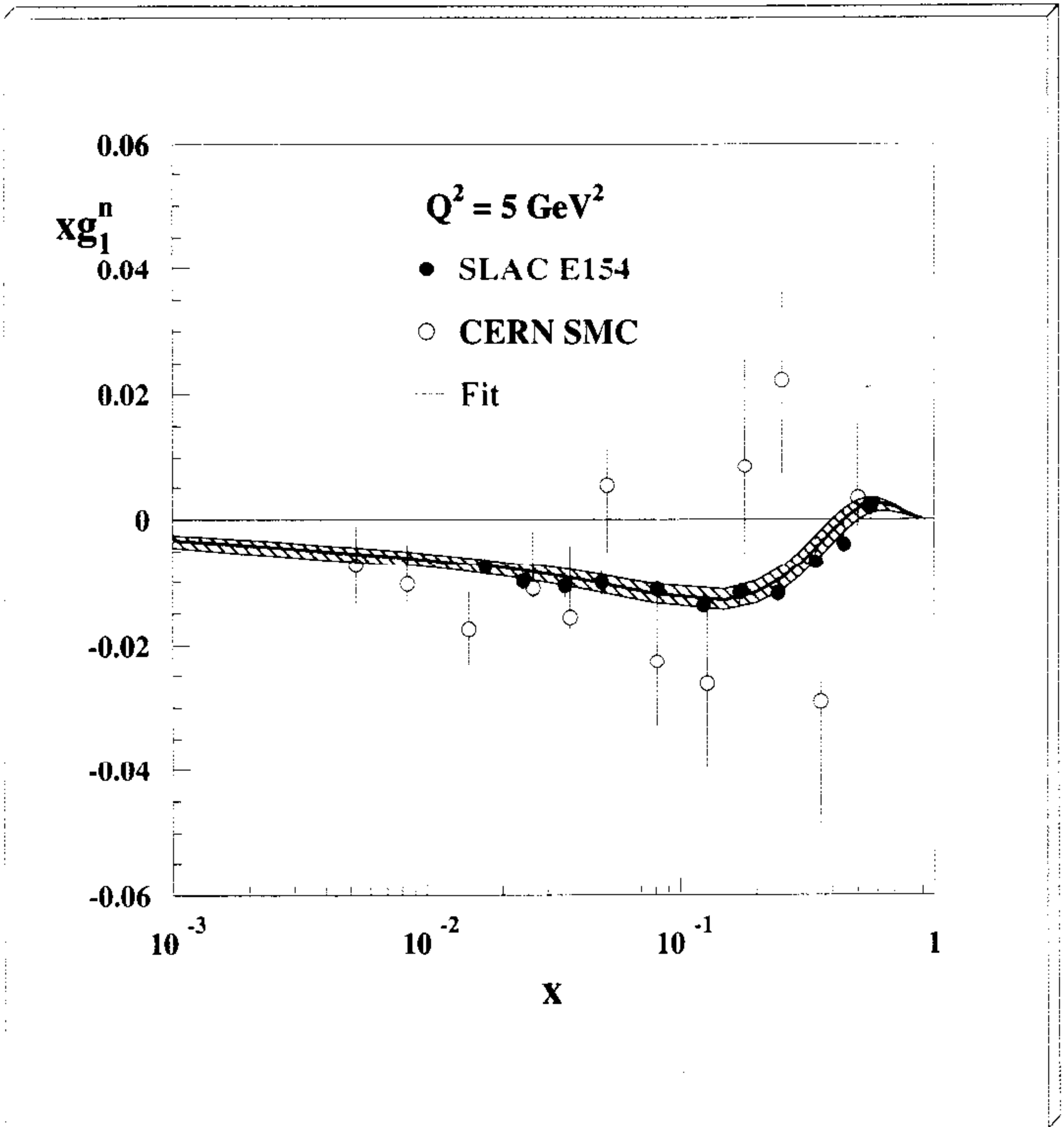
- **First moments of valence distributions well determined**

- **First moments of sea quarks and gluons only qualitatively constrained**

- **Contribution of systematic errors comparable to the statistical**

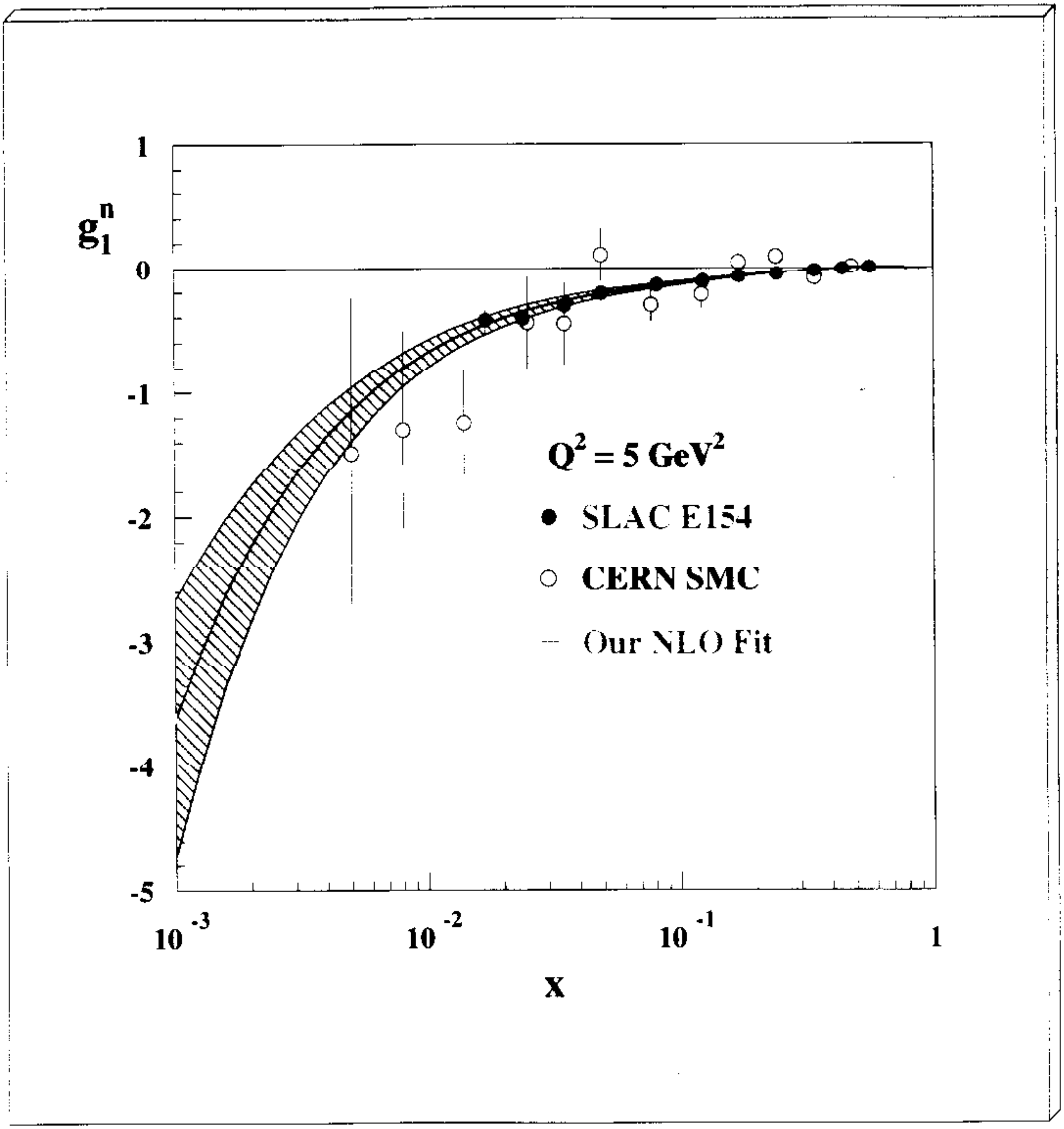
- **Theoretical uncertainty quite large. Could be reduced by simultaneous analysis of unpolarized and polarized DIS data with α_s as one of the parameters**

Neutron



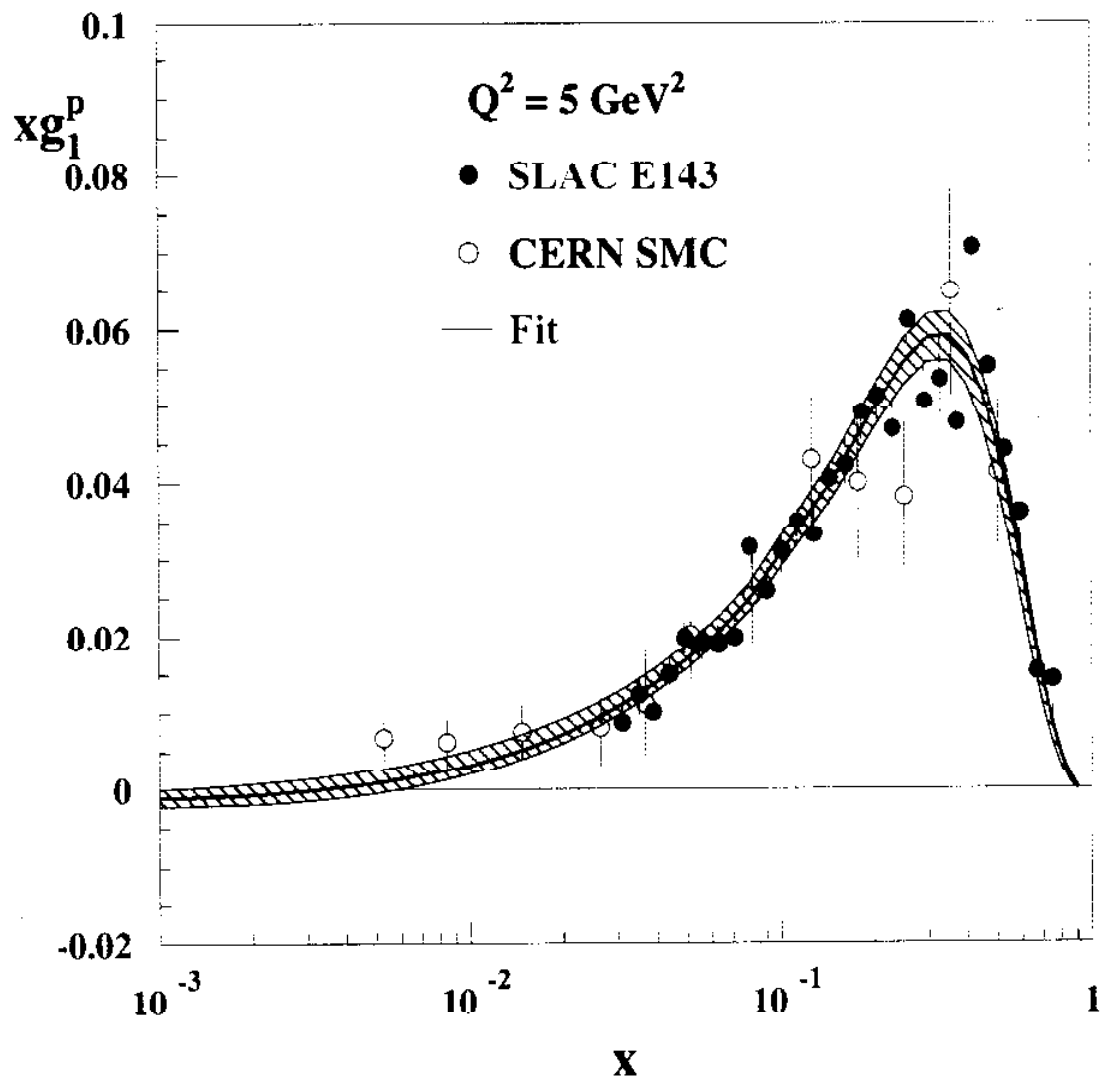
0.06
xg_1^n
0.04
0.02
0
-0.02
-0.04
-0.06
$Q^2 = 5\ GeV^2$
SLAC E154
CERN SMC
Fit
10^{-3}
10^{-2}
10^{-1}
1
x

Neutron



g_1^n
1
0
-1
-2
-3
-4
-5
$Q^2 = 5\ GeV^2$
SLAC E154
CERN SMC
Our NLO Fit
10^{-3}
10^{-2}
10^{-1}
1
x

Proton



$Q^2 = 5\ GeV^2$
SLAC E143
CERN SMC
Fit
xg_1^p
0.1
0.08
0.06
0.04
0.02
0
-0.02
10^{-3}
10^{-2}
10^{-1}
1
x

Fitted Values of Parameters ($\overline{\rm MS}$)

	Value	Stat.	Syst.	Theory
A_u	0.99	$^{+0.08}_{-0.08}$	$^{+0.04}_{-0.05}$	$^{+0.97}_{-0.11}$
A_d	−0.78	$^{+0.14}_{-0.20}$	$^{+0.05}_{-0.05}$	$^{+0.05}_{-1.28}$
A_Q	−0.02	$^{+0.03}_{-0.06}$	$^{+0.01}_{-0.02}$	$^{+0.01}_{-0.35}$
A_G	1.6	$^{+1.1}_{-0.9}$	$^{+0.6}_{-0.6}$	$^{+0.2}_{-1.3}$
α_u	0.63	$^{+0.06}_{-0.07}$	$^{+0.04}_{-0.05}$	$^{+0.36}_{-0.06}$
α_d	0.28	$^{+0.15}_{-0.11}$	$^{+0.05}_{-0.03}$	$^{+0.75}_{-0.03}$
α_Q	0.04	$^{+0.29}_{-0.03}$	$^{+0.12}_{-0.03}$	$^{+0.55}_{-0.01}$
α_G	0.8	$^{+0.4}_{-0.5}$	$^{+0.3}_{-0.3}$	$^{+0.1}_{-0.6}$

	Value	Stat.	Syst.	Theory
Δu_V	0.69	$^{+0.03}_{-0.02}$	$^{+0.05}_{-0.04}$	$^{+0.14}_{-0.01}$
Δd_V	−0.40	$^{+0.03}_{-0.04}$	$^{+0.03}_{-0.03}$	$^{+0.07}_{-0.00}$
$\Delta \bar{Q}$	−0.02	$^{+0.01}_{-0.02}$	$^{+0.01}_{-0.01}$	$^{+0.00}_{-0.03}$
ΔG	1.8	$^{+0.6}_{-0.7}$	$^{+0.4}_{-0.5}$	$^{+0.1}_{-0.6}$
Δq_3	1.09	$^{+0.03}_{-0.02}$	$^{+0.05}_{-0.05}$	$^{+0.06}_{-0.01}$
Δq_8	0.30	$^{+0.06}_{-0.05}$	$^{+0.05}_{-0.05}$	$^{+0.23}_{-0.01}$
$\Delta \Sigma$	0.20	$^{+0.05}_{-0.06}$	$^{+0.04}_{-0.05}$	$^{+0.01}_{-0.01}$
Γ_1^p	0.112	$^{+0.006}_{-0.006}$	$^{+0.008}_{-0.008}$	$^{+0.009}_{-0.001}$
Γ_1^n	−0.056	$^{+0.006}_{-0.007}$	$^{+0.005}_{-0.006}$	$^{+0.002}_{-0.001}$
Γ_1^d	0.026	$^{+0.005}_{-0.006}$	$^{+0.005}_{-0.006}$	$^{+0.005}_{-0.001}$
Γ_1^{p-n}	0.168	$^{+0.005}_{-0.004}$	$^{+0.008}_{-0.007}$	$^{+0.007}_{-0.001}$

- **Notice that $\Delta q_3 = 1.09^{+0.09}_{-0.06}$ is $2\,\sigma$ lower than $g_A = 1.2601$. Higher order corrections could be ~5% at 5 GeV^2 bringing Δq_3 to a better agreement**

- **NLO evolution of data points**

$$g_1^{exp}(x_i, \langle Q^2 \rangle) = g_1^{exp}(x_i, Q_i^2) - \Delta g_1^{fit}(x_i, Q_i^2, \langle Q^2 \rangle)$$

where

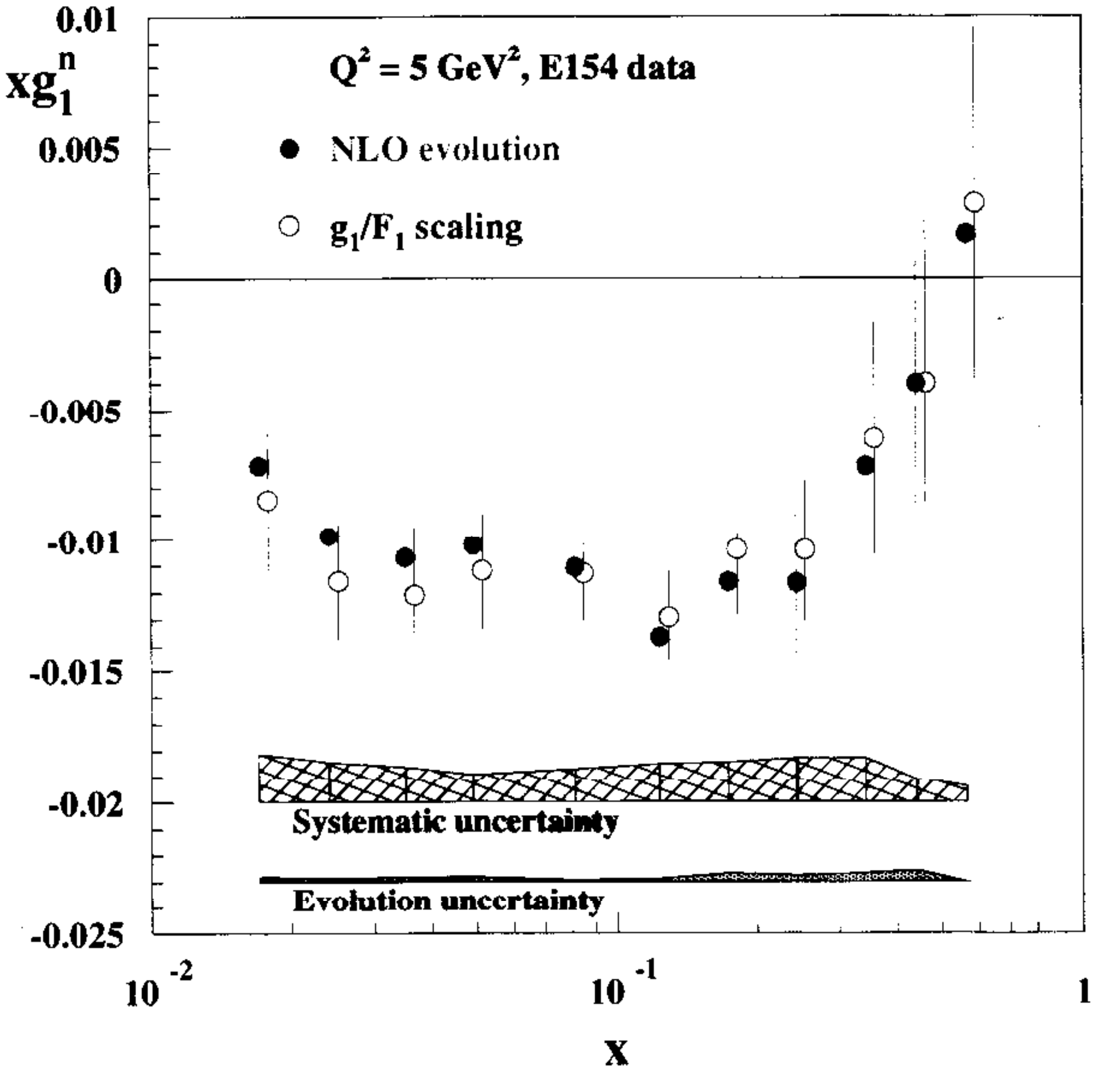
$$\Delta g_1^{fit}(x_i, Q_i^2, \langle Q^2 \rangle) = g_1^{fit}(x_i, Q_i^2) - g_1^{fit}(x_i, \langle Q^2 \rangle)$$

and

$$\sigma^2(g_1^{exp}(x_i, Q_i^2)) = \sigma^2(g_1^{exp})_{stat} + \sigma^2(g_1^{exp})_{syst} + \sigma^2(g_1)_{evol}$$

- **Difference between NLO and g_1/F_1=const evolutions is on the order of present day experimental precision**

Q^2 evolution



- **The integrals over measured range less sensitive to the type of evolution if computed at the average $\langle Q^2 \rangle$ of the experiment. For E154 at 5 GeV^2**

$$\int_{0.014}^{0.7} dx\, g_1^n(x) = -0.035 \pm 0.003 \pm 0.005 \pm 0.001 \text{ NLO}$$

$$= -0.036 \pm 0.004 \pm 0.005 \qquad g_1^n / F_1^n$$

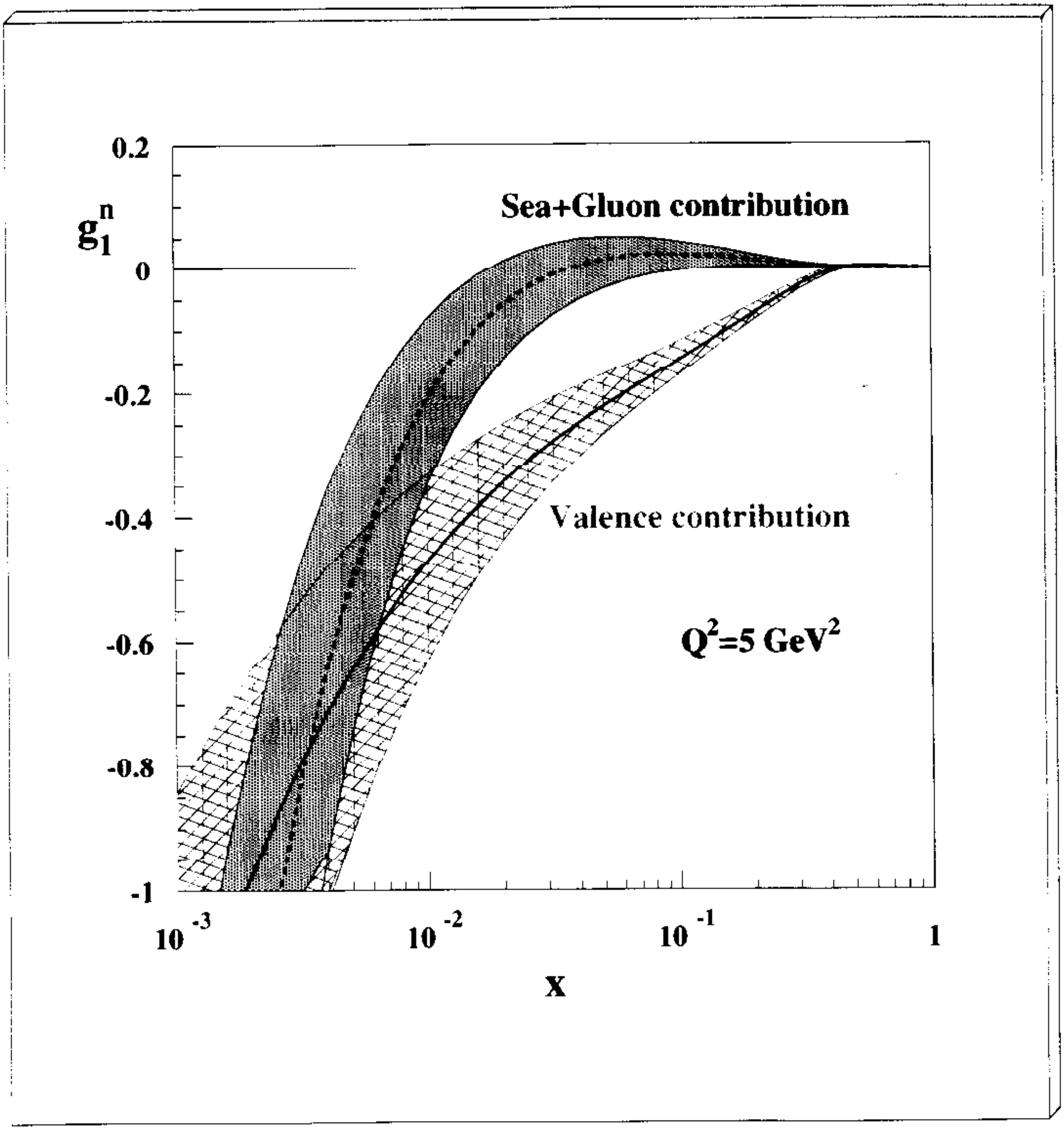
- **Strong x behavior of**

$$g_1^n \sim x^{-0.8}$$

for $x \leq 0.1$ at $Q^2 = 5$ GeV^2 largely due to the sea and gluon contributions

- **Negative value of g_1^p at lower x and higher Q^2 would provide direct evidence of polarized sea is on the order of present day experimental precision**

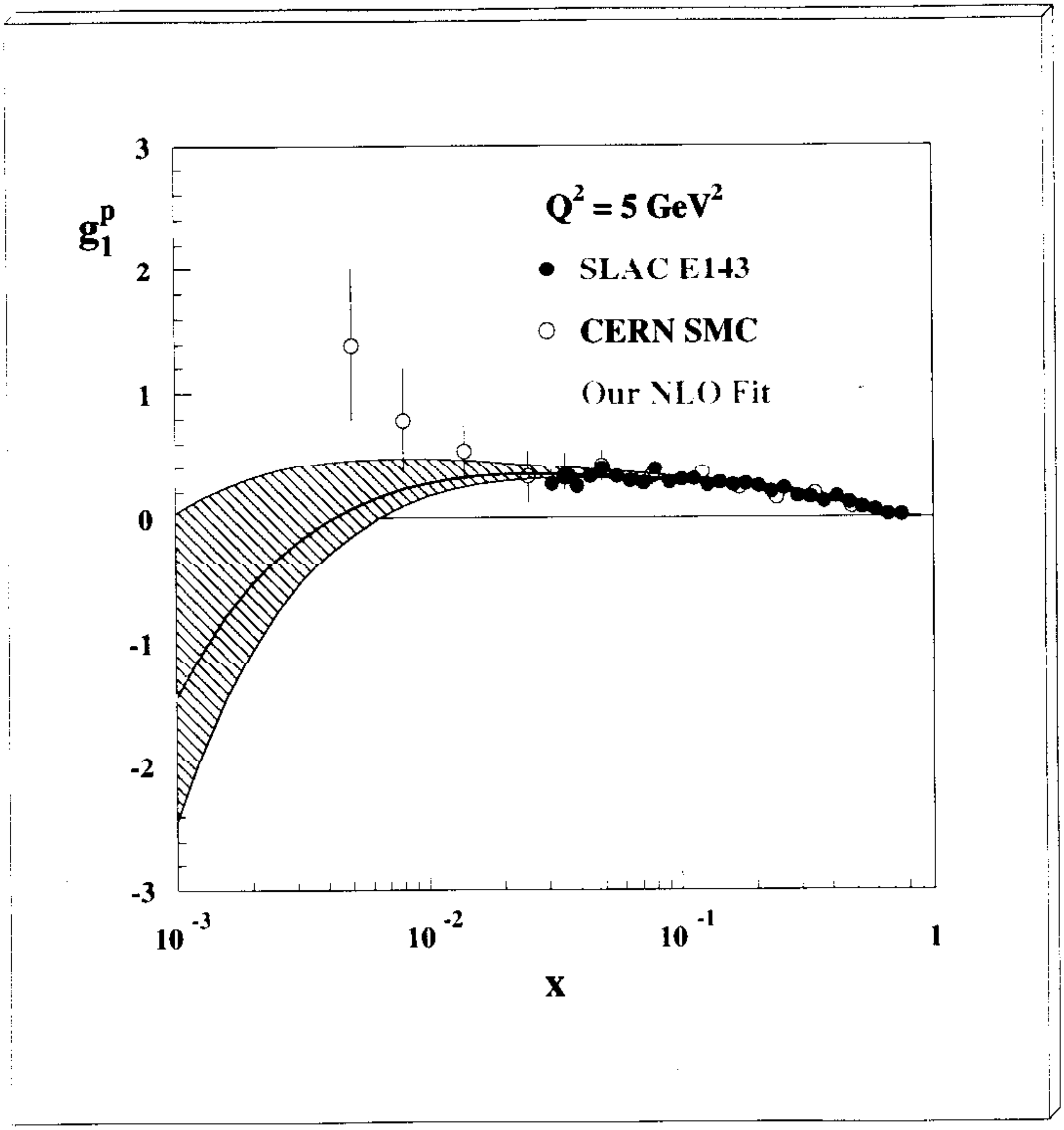
Sea and Valence



ERROR: timeout
OFFENDING COMMAND: timeout

STACK:

Proton



- **First moment of $g_1^n(x)$**
  - **Using E154(n) + $\overline{\mathrm{MS}}$ fit for low and high x extrapolations**

$$\Gamma_1^n = -0.058 \pm 0.004 \pm 0.007 \pm 0.007$$

where the errors are ±stat. ±syst. ±extrap.

- **Bjorken sum rule $\Gamma_1^{p-n} = \int_0^1 dx\,(g_1^p(x) - g_1^n(x))$**
  - **Using E143(p) + E154(n) + $\overline{\mathrm{MS}}$ extrapolations**

$$\Gamma_1^{p-n} = 0.171 \pm 0.004 \pm 0.010 \pm 0.006$$

  - **Prediction up to $O(\alpha_s^3)$,**

$$\Gamma_1^{p-n}(\text{theory}) = 0.186$$

Summary

- **The first moments of polarized valence distributions well established**

- **Gluon and sea quarks distributions only qualitatively constrained**

- **Q^2 dependence of g_1/F_1 is sizable in comparison to present experimental uncertainties**

- **Low x behavior of the non-singlet distributions understood**

  * **"Regge-like" at low Q_0^2**
  * **Divergent at $Q^2 > 1\ \mathrm{GeV}^2$ due to evolution**
  * **Powers well constrained**

- **Bjorken sum rule confirmed within one σ**