DVCS

DEEPLY VIRTUAL COMPTON SCATTERING :

THEORY OF NONFORWARD PARTON DISTRIBUTION

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OUTLINE

1. DVCS & PROTON SPIN
2. KINEMATICS
3. BUILDING pQCD PARTON PICTURE
4. ASYMMETRIC/NONFORWARD DISTRIBUTIONS & DOUBLE DISTRIBUTIONS
5. DVCS AMPLITUDE AT LOWEST ORDER
6. EVOLUTION OF NONFORWARD DISTRIBUTION FUNCTIONS

3 DVCS & PROTON SPIN (X. Ji)

ANGULAR MOMENTUM IN QCD

$$J^{\mu\nu} = \int d^3\vec{x}\; M^{0\mu\nu}(x) \quad ; \quad M^{\mu\nu\alpha} = T^{\mu\alpha}x^{\nu} - T^{\mu\nu}x^{\alpha}$$

WITH $T^{\mu\nu}$ BEING ENERGY-MOMENTUM TENSOR

$$T^{\mu\nu} = T^{\mu\nu}_{q} + T^{\mu\nu}_{g} = \frac{i}{4}\bar{\psi}\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi + \left(\frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F\right.$$

QUARK & GLUON CONTRIBUTION TO PROTON SPIN CAN BE OBTAINED FROM FORM FACTORS OF ENERGY-MOMENTUM TENSOR AT $t=0$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right]$$

$$J_q + J_g = \frac{1}{2}$$

CHARGE-LIKE

WHERE

$$\langle p'|\, T^{\mu\nu}_{q,g}\, |p\rangle = \bar{u}(p')\Big[A_{q,g}(t)\,\gamma^{\{\mu}p^{\nu\}} +$$

ANOM. MAG. MOM. TYPE

$$+ B_{q,g}(t)\, p^{\{\mu}\sigma^{\nu\}\alpha}\frac{r_\alpha}{2M} + C_{q,g}(t)(r^\mu r^\nu - g^{\mu\nu}t)$$

$$+ D_{q,g}(t)\, g^{\mu\nu}M\Big]u(p)$$

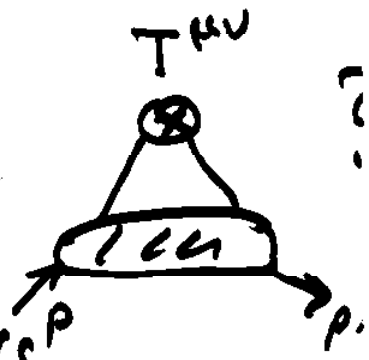
$$\boxed{r = p - p' \;;\; t \equiv r^2 = (p-p')^2}$$

4] HENCE, ONE CAN GET $J_{q,g}$ MEASURING $t=0$ LIMIT OF ENERGY-MOMENTUM TENSOR FORM FACTORS.

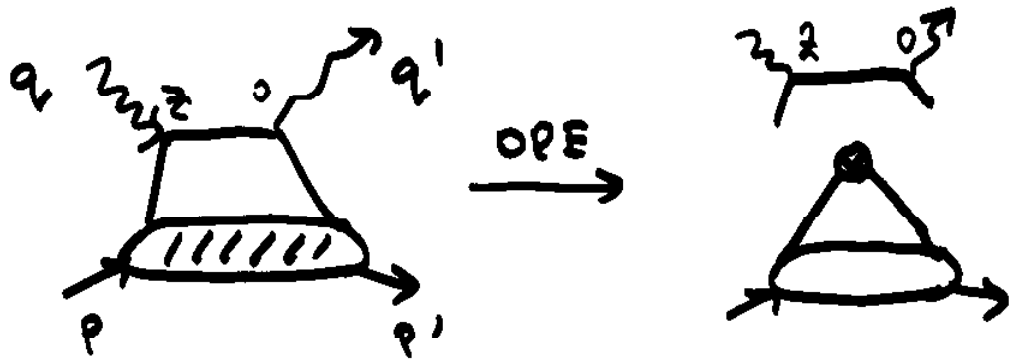
TWO COMPONENTS: CHARGE-LIKE $A_{q,g}$ (NON-SPIN-FLIP)

AND SPIN-FLIP $B_{q,g}$.

PROBLEM: HOW TO ARRANGE



X. JI'S SUGGESTION: USE DVCS



CONTAINS $\bar{\Psi}(0)\,\hat{z}\,\psi(z) =$

$$z_\alpha\Big[\underbrace{\bar{\Psi}\gamma_\alpha\psi}_{\int F(x)\,dx} + z_\beta\,\underbrace{\bar{\Psi}\gamma_\alpha D_\beta\psi}_{\int xF(x)\,dx\ \ T_{\alpha\beta}} + \ldots\Big]$$

SO, THE IDEA IS TO MEASURE

STRUCTURE FUNCTIONS $\mathcal{F}(x,t)$, $\mathcal{K}(x,t$
NON-SPIN-FLIP SPIN-FLI

OF DVCS AND GET A_q, B_q

FROM

$$\int x \underbrace{\mathcal{F}(x, t\to 0)}_{A_q} dx + \int x \underbrace{\mathcal{K}(x, t\to 0)}_{B_q} dx$$

(LIKE MEASURING # OF VALENCE QUARKS FROM $N_a = \int_0^1 \left(f_a(x) - f_{\bar{a}}(x)\right) dx$)

WHETHER THIS IS FEASIBLE OR NOT,

DVCS IS INTERESTING ON ITS OWN

(HYBRID DISTRIBUTIONS, NEW EXAMPLE OF SCALING, etc.) (ROBASCHIK et al. ~10 years ago)

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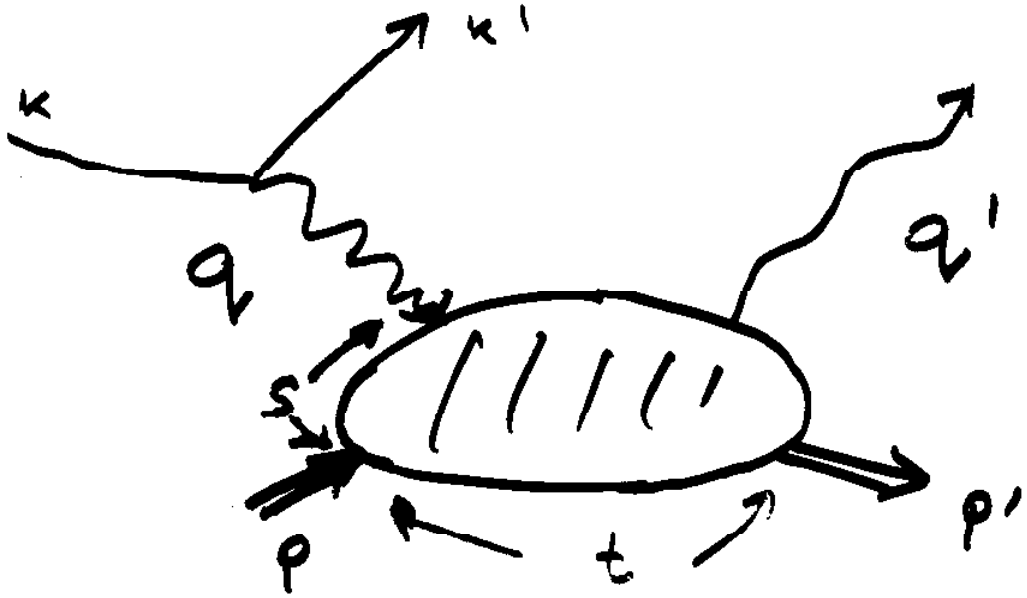


KINEMATICS:

(LARGE Q^2, FIXED x, SMALL t)

$$p^2 = p'^2 = m_H^2$$

$$q^2 = -Q^2 \quad , \quad Q^2 \gg m_H^2$$

$$q'^2 = 0 \qquad \text{REAL PHOTON}$$

$$s = (q+p)^2 = m_H^2 + 2(q\cdot p) - Q^2 \gg m_H^2$$

$$\frac{Q^2}{2(q\cdot p)} = x_{Bj} \quad - \text{ fixed}$$

$$t = (p-p')^2 \quad ; \quad |t| \ll Q^2$$

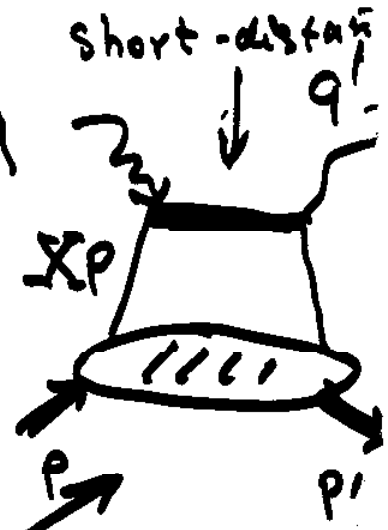
7)

$Q^2 \to \infty$

x_{Bj} fixed

t small

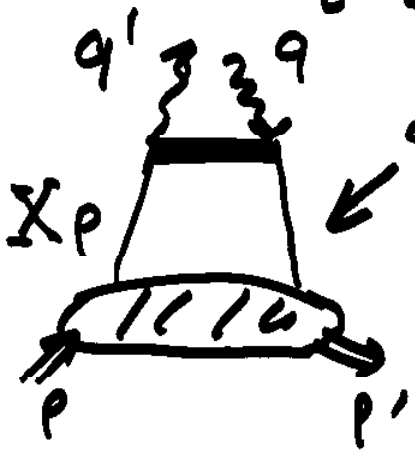
$=$



(s-channel handbag)

SCALING

$+$

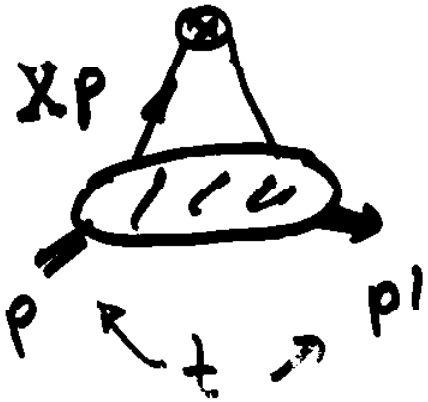


(u-channel handbag)

$+ \; O(1/Q^2)$

LONG-DISTANCE FACTOR

NONFORWARD DISTRIBUTION



$$\equiv \mathcal{F}_{\xi}(X; t, \mu)$$

ξ — x_{Bj}

X — LIGHT-CONE FRACTION

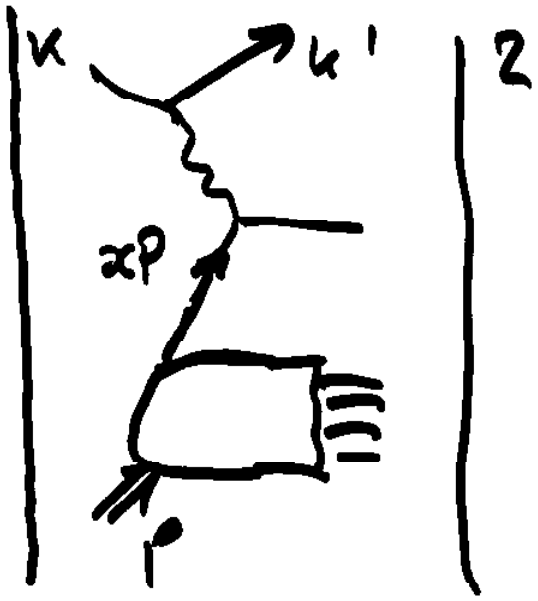
t — $(p-p')^2$

μ — FACTORIZATION SCALE

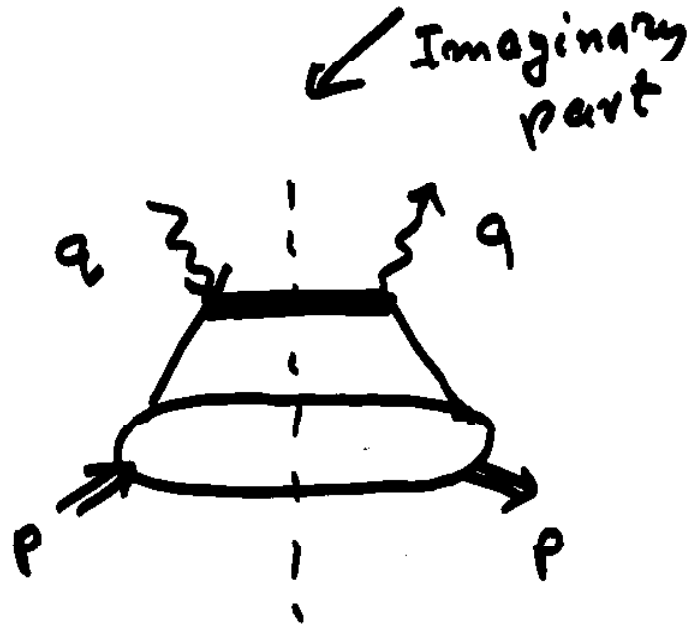
REMINDER FROM pQCD PARTONOLOGY

1) DISTRIBUTION FUNCTIONS

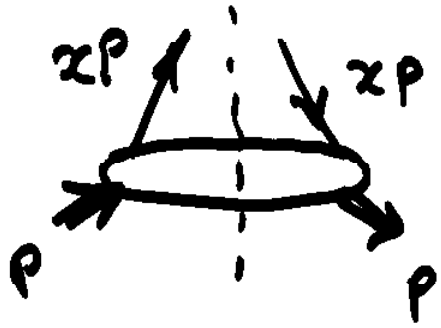
TAKE DIS:



$\Rightarrow$



VIRTUAL FORWARD COMPTON AMPLITUDE



$f(x) \equiv$ PROBABILITY TO FIND PARTON WITH MOMENTUM xP

<u>NB</u>: AT THIS STAGE $x \neq x_B$;;
x VARIES FROM 0 TO 1

PARTON CONVOLUTION FORMULA

$$\sigma^{hadron}(q,P) = \int_0^1 \sigma^{parton}(q, xP) f(x)\, dx$$

IN LOWEST ORDER: $\sigma^{parton} \sim \delta(x - x_{Bj})$

$\Rightarrow \quad x = x_{Bj}$

COORDINATE REPRESENTATION/
OPERATOR DEFINITION:

$$\langle p,s' | \bar{\psi}_a(0)\, \hat{z}\, E(0,z;A)\, \psi_a(z) | p,s \rangle \Big|_{z^2=0} =$$

$$= \bar{u}(p,s')\hat{z}\, u(p,s) \int_0^1 \Big[e^{-ix(pz)} \underbrace{f_a(x)}_{\text{QUARK}} - e^{ix(pz)} \underbrace{f_{\bar{a}}(x)}_{\text{ANTIQUARK}} \Big] dx$$

$a = u, d, s, \ldots$

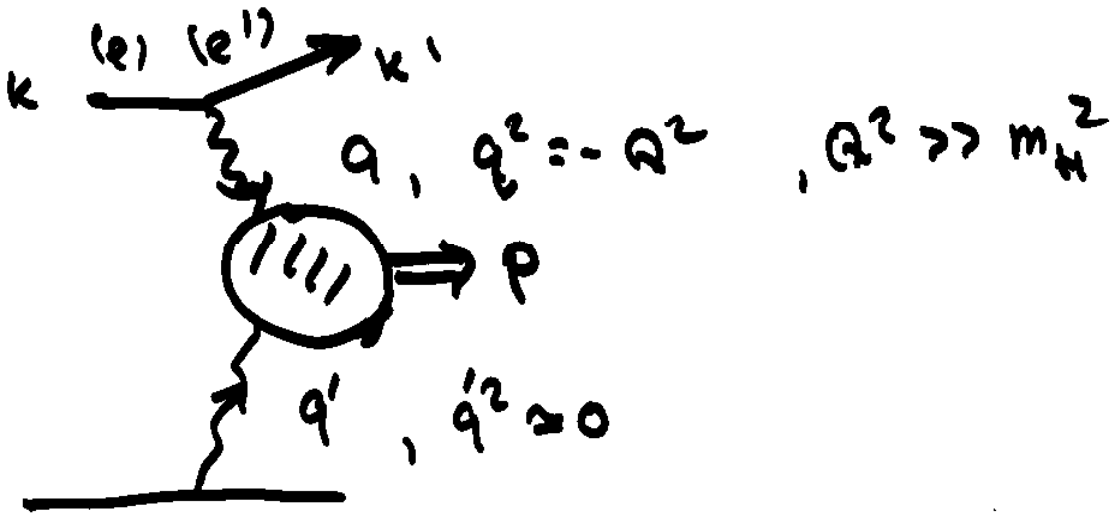
$\hat{z} = z_\mu \gamma^\mu$ (HAT CONVENTION, $\hat{z} = \not{z}$)

$E(0,z;A) = P \exp\left\{ ig \int_0^1 z_\mu A^\mu(tz)\, dt \right\}$

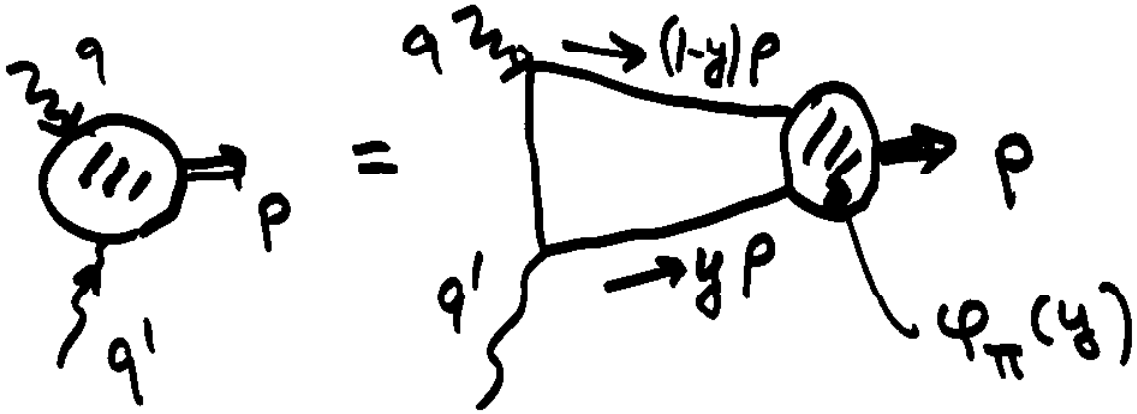
$\bar{u}(p,s')$, $u(p,s)$ = PROTON DIRAC SPINORS

2) DISTRIBUTION AMPLITUDES.

TAKE $\gamma\gamma^* \to \pi^0$ PROCESS



IN LOWEST ORDER



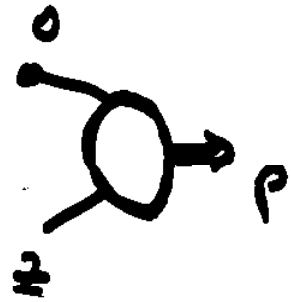
$$\sigma^{\gamma\gamma^* \to \pi}(q, q', p) = \int_0^1 \sigma^{\gamma\gamma^* \to q\bar{q}}(q, q'; yp, \bar{y}p) \cdot \varphi_\pi(y)\, dy$$

($\bar{y} \equiv 1-y$

"BAR" CONVENTION)

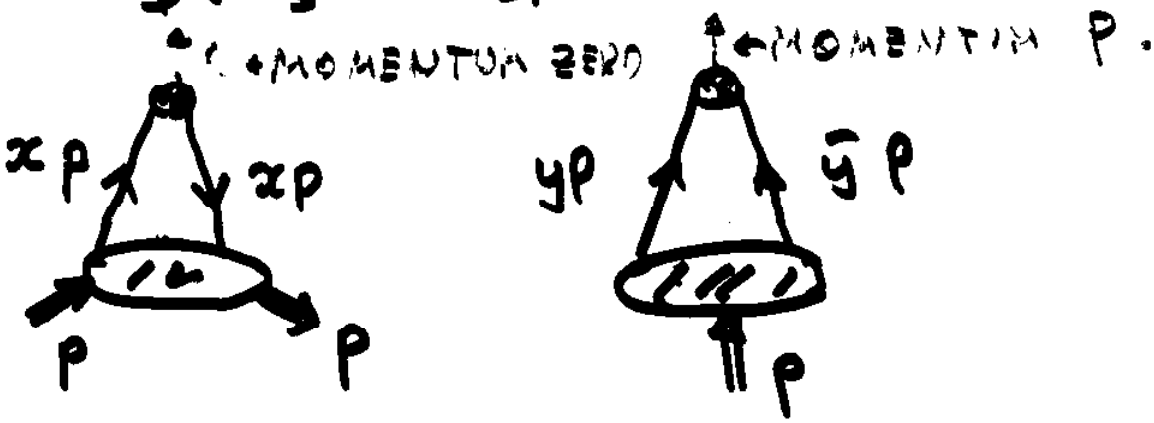
$\varphi_\pi(y) \equiv$ PROBABILITY AMPLITUDE TO FIND PION IN $q\bar{q}$-STATE WITH MOMENTUM P SHARED IN FRACTIONS yP AND $\bar{y}P$. $(y+\bar{y}=1)$

OPERATOR DEFINITION:

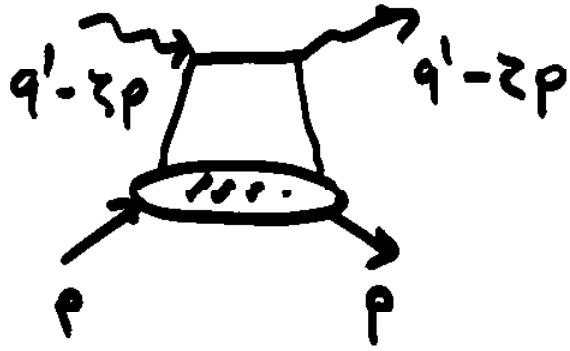


$$\langle 0|\bar{\psi}_d(0)\gamma_5 \hat{z}\, E(0,z;A)\,\psi_u(z)|P(\pi)\rangle = i(zP)\int_0^1 \varphi_\pi(y)\, e^{-iy(zP)}\, dy$$

DF'S VS. DA'S:



12 FORWARD COMPTON AMPLITUDE VS NONFORWARD



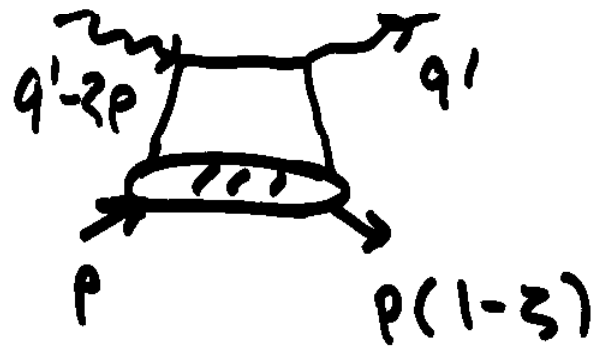
$q'^2 = 0$; TAKE ALSO $p^2 =$

$$q = q' - \zeta p$$

$$\Rightarrow \quad q^2 = -2\zeta(q'p) = -2\zeta(qp)$$

i.e. $\zeta = x_B$;

TAKE DVCS FOR $t = 0$



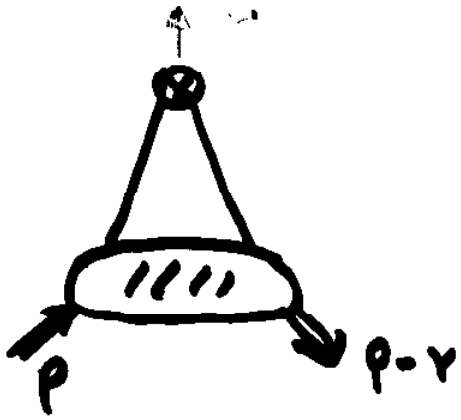
ALL KINEMATICS SATISFIED:

$$q^2 = (q' - \zeta p)^2 = -2\zeta(qp)$$

$$\Rightarrow \zeta = x_B;$$

$$q'^2 = 0. \quad p'^2 \ll Q^2$$

CRUCIAL FACT: MOMENTUM TRANSFER $r \equiv p - p'$ HAS LONGITUDINAL COMPONENT $r = \zeta p$

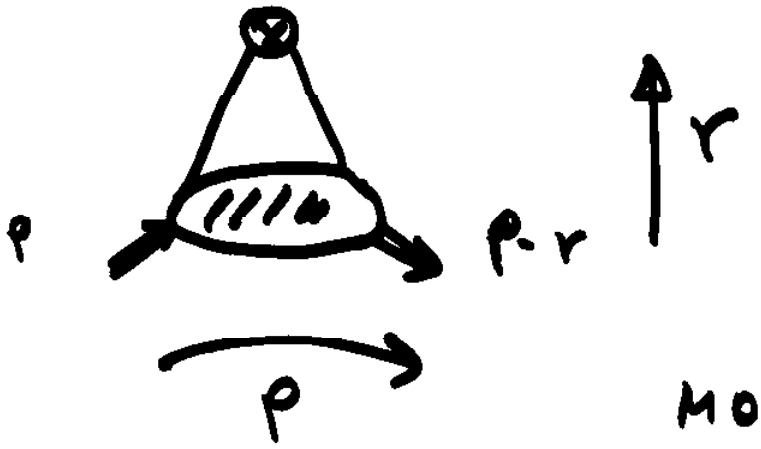


$\langle p-r | \mathcal{O}(0,z) | p \rangle$

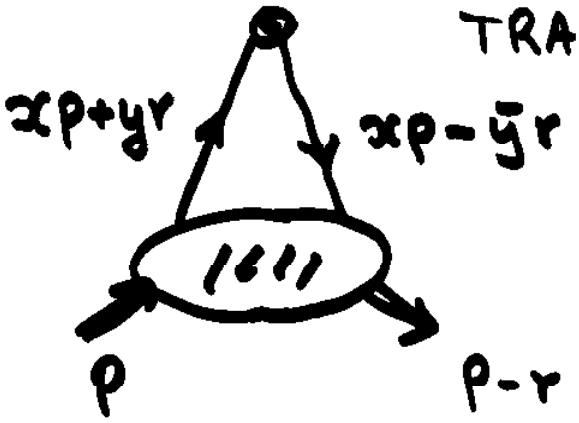
ASYMMETRIC DISTRIBUTIO FUNCTION [t=0]

(if $t \neq 0 \Rightarrow$ NONFORWARD D.F.)

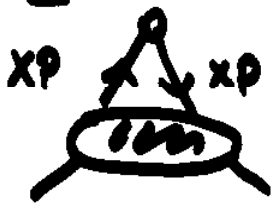
<u>NB</u>: MOMENTUM FLOWS IN TWO CHANNELS:



$\Rightarrow$ PARTON "+" MOMENTUM COMES BOTH FROM <u>INITIAL</u> HADRON MOMENTUM p AND FROM MOMENTUM TRANSFER r



= SUPERPOSITION OF



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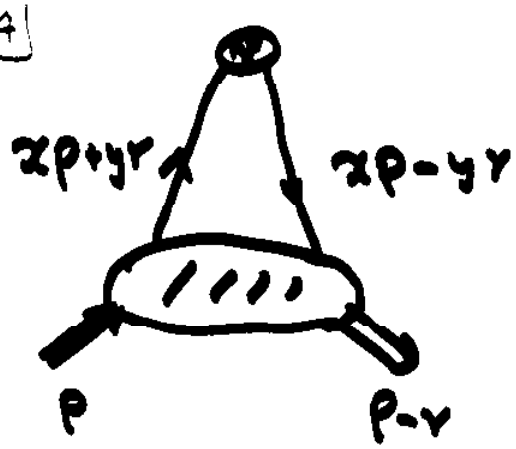

DOUBLE DISTRIBUTION

$$F(x,y)$$

SPECTRAL PROPERTIES

$$0 \le x \le 1 \; ; \; 0 \le y \le 1$$

$$0 \le x+y \le 1$$

<u>NB</u>: *) WITH RESPECT TO x, $F(x,y)$ WORKS LIKE DISTRIBUTION FUNCTIO

**) WITH RESPECT TO y, $F(x,y)$ WORKS LIKE DISTRIBUTION AMPLITU (WAVE FUNCTION)

OPERATOR DEFINITION:

$$\langle p-r|\bar{\psi}_a(0)\,\hat{z}\,E(0,z;A)\,\psi_a(z)|p\rangle =$$

$$= \bar{u}(p-r)\,\hat{z}\,u(p)\int_0^1\int_0^1\Big[e^{-ix(pz)-iy(rz)}F_a(x,y)$$

$$- e^{ix(pz)-iy(rz)}F_{\bar{a}}(x,y)\Big]\theta(x+y\le 1)\,dx\,dy$$

<u>NB</u>: IF $r=0 \rightarrow$ USUAL DF

19 REDUCTION FORMULA:

$$\int_0^{1-x} F_a(x,y)\,dy = f_a(x)$$

IMPORTANT FEATURE OF DOUBLE DISTRIBUTIONS $F(x,y)$:

THEY ARE UNIVERSAL, i.e. DO NOT DEPEND ON RATIO $(rz)/(pz) \equiv \zeta$.

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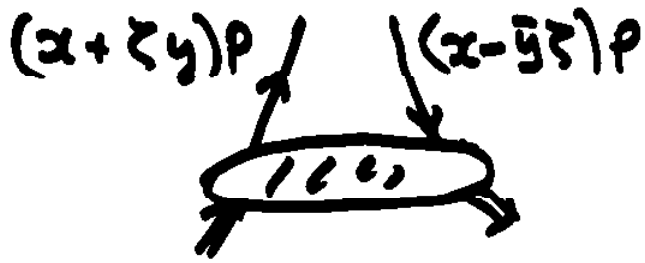
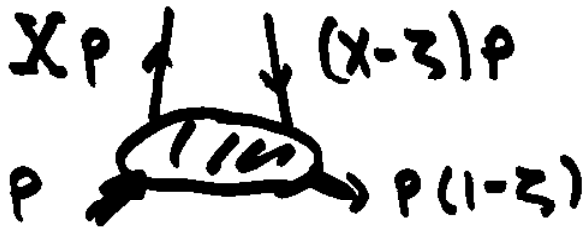


SINCE $x - \bar{y}\zeta = x - \zeta + y\zeta = (x+y\zeta) - \zeta$

x & APPEAR ONLY IN COMBINATION

$X \equiv (x+y\zeta) \rightarrow$ ASYMMETRIC D.F.



i.e. y- DEPENDENCE CAN BE INTEGRATED OUT

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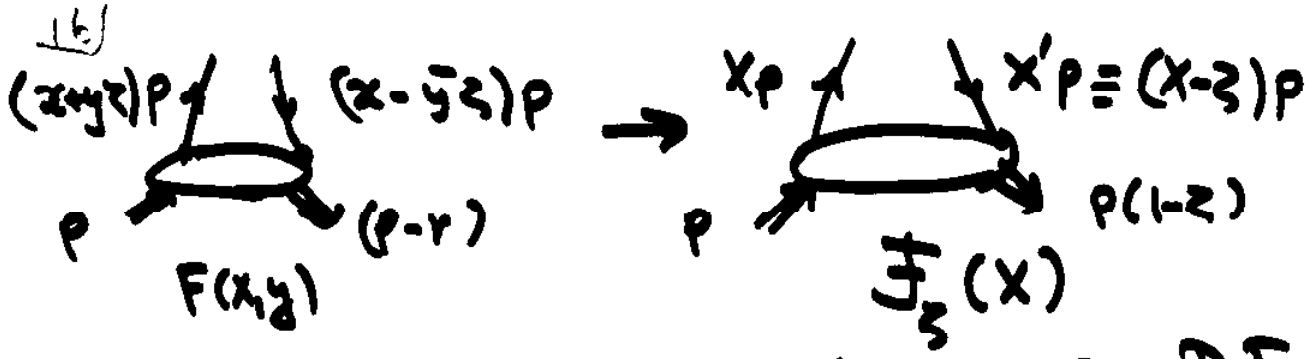
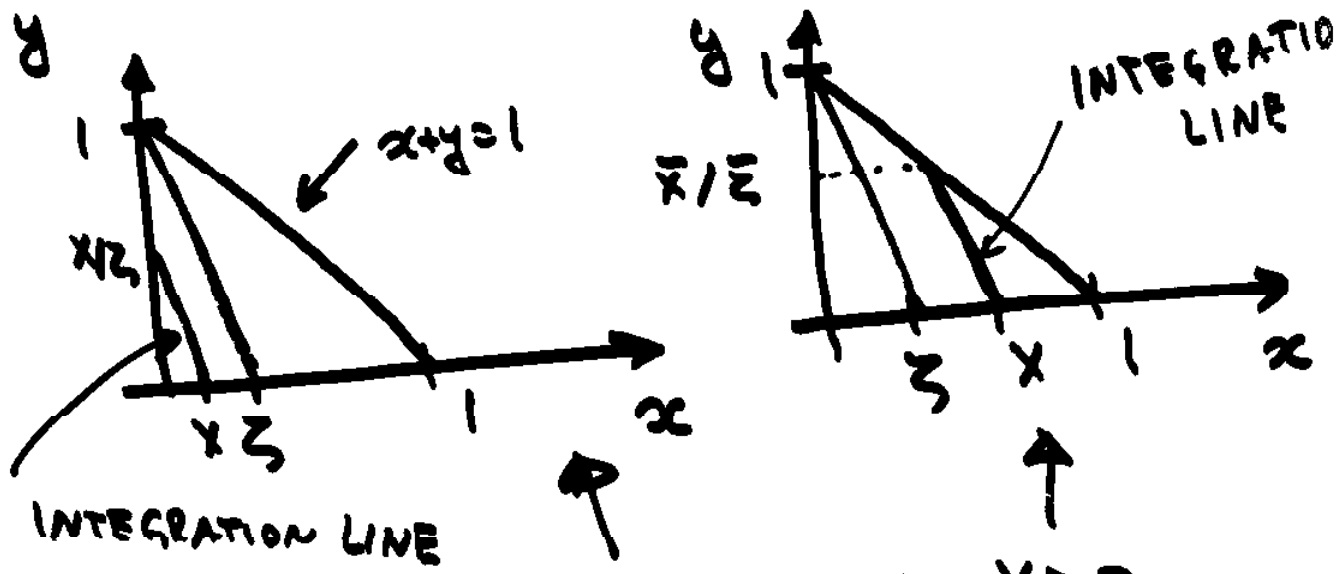


DOUBLE DISTRIBUTION ASYMMETRIC D.F.

$$\mathcal{F}_\xi(X) = \theta(X>\xi)\int_0^{\bar{X}/\bar{\xi}} F(X-y\xi, y)\,dy$$

$$+\,\theta(X<\xi)\int_0^{X/\xi} F(X-y\xi, y)\,dy$$

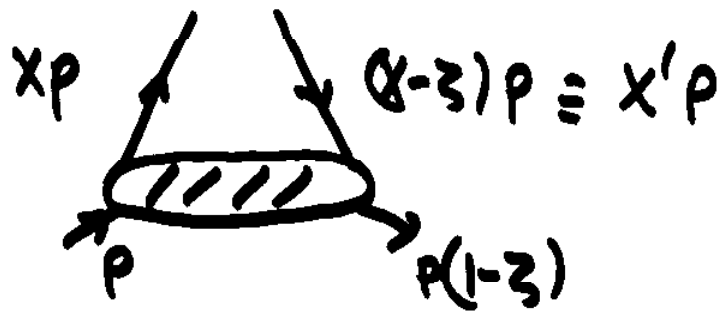


TWO REGIONS: $X<\xi$ AND $X>\xi$

<u>NB</u>: $X' \equiv X-\xi < 0$ IF $X<\xi$

(4) TWO COMPONENTS OF $F_\zeta(x)$

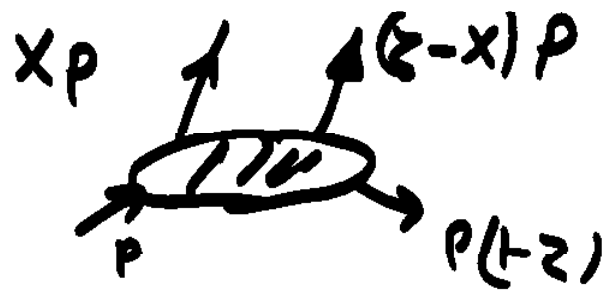
1) $x > \zeta$



$x, x' > 0$ $\quad F_\zeta(x)$ IS SIMILAR FOR USUAL DF, BUT IN ASYMMETRIC CASE

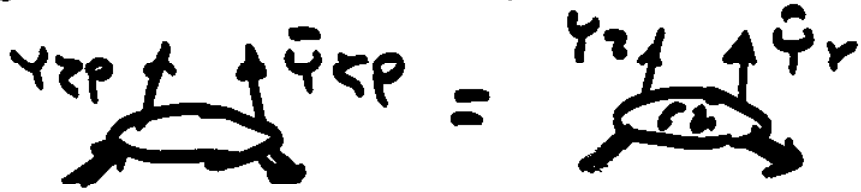
2) $0 < x < \zeta \rightarrow x' = x - \zeta < 0$

$\Rightarrow$ SECOND PARTON GOES <u>OUT</u> OF HADRONIC BLOB



TOTAL t-channel MOMENTUM $\zeta p \equiv r$ IS SHARED AS xp AND $(\zeta - x)p$

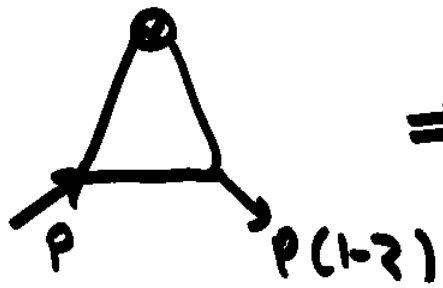
DEFINE $x = y\zeta$, $0 \le y \le 1$, THEN $(\zeta - x) = \bar{y}\zeta$



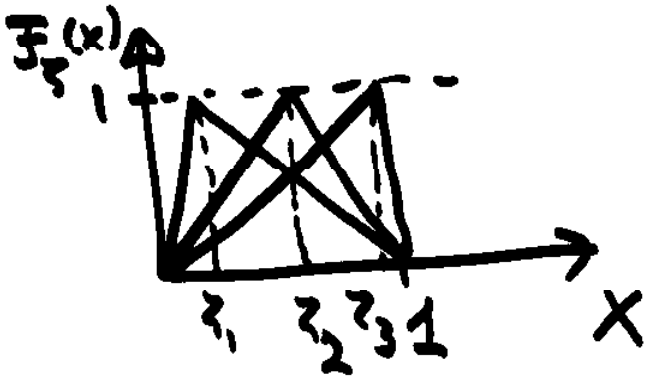
$\rightarrow$ DA-LIKE FUNCTION $\Psi_\zeta($

IMPORTANT FEATURE: $\mathcal{F}_\zeta(x)$ CHANGE X-SHAPE WHEN $\zeta \equiv$ (LONGITUDINAL MOMENTUM ASYMMETRY PARAMETER) CHANGES.

e.g. FOR SCALAR TRIANGLE



$$\Rightarrow \mathcal{F}_\zeta(x) \sim \frac{x}{\zeta}\,\theta(x<\zeta) + \frac{\bar{x}}{\bar{\zeta}}\,\theta(x>$$



REDUCTION FORMULA:

$$\mathcal{F}^a_{\zeta=0}(x) = f_a(x)$$

USUAL DF

<u>NB</u>: TAKING DOUBLE DISTRIBUTION $F(x,y)$, ONE GETS $\mathcal{F}_\zeta(x)$ FOR ALL ζ, i.e., THE WHOLE FAMILY OF ASYMMETRIC DF'S.

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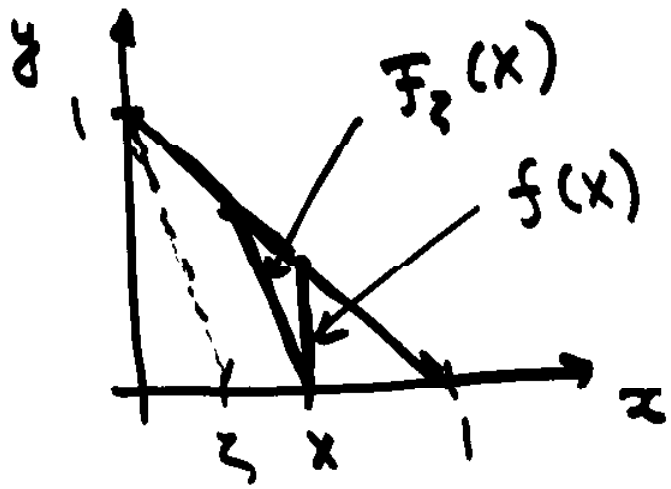


GETTING $\mathcal{F}_\zeta(x)$ AND $f(x)$ FROM $F(x,y)$

FOR SMALL ζ, $\mathcal{F}_\zeta(x)$ IS EXPECTED TO DIFFER FROM $f(x)$ BY $O(\zeta)$.

OPERATOR DEFINITION OF $\mathcal{F}_\zeta(x,t)$

$$\langle p',s'|\bar{\psi}_a(0)\,\hat{z}\,E(0,z;A)\,\psi_a(z)|p,s\rangle\Big|_{z^2=0}$$

$$= \bar{u}(p',s')\,\hat{z}\,u(p,s)\int_0^1 dx\left[e^{-ix(pz)}\mathcal{F}_\zeta^a(x,t) - e^{ix'(pz)}\mathcal{F}_\zeta^{\bar{a}}(x\right.$$

$$\underbrace{+\,\bar{u}(p',s')\,\frac{\hat{z}\hat{r}-\hat{r}\hat{z}}{2M}\,u(p,s)\int_0^1 dx\left[e^{-ix(pz)}\mathcal{K}_\zeta^a(x,t) - e^{ix'(pz)}\mathcal{K}_\zeta^{\bar{a}}(x,t\right.}_{\text{SPIN-FLIP TERM}}$$

$$\int_0^1\left[\mathcal{F}_\zeta^a(x,t)-\mathcal{F}_\zeta^{\bar{a}}(x,t)\right]dx = F_1^a(t) \leftarrow \text{DIRAC F.F.}$$

$$\int_0^1\left[\mathcal{K}_\zeta^a(x,t)-\mathcal{K}_\zeta^{\bar{a}}(x,t)\right]dx = F_2^a(t) \leftarrow \text{PAULI F.F}$$

20/ $\langle p', s' | \bar{\Psi}_a(0)\, \gamma_5 \hat{z}\; E(0,z;A)\, \Psi_a(z) | p, s \rangle \Big|_{z^2=0}$

$$= \bar{u}(p',s')\, \gamma_5 \hat{z}\, u(p,s) \int_0^1 dx \Big[e^{-iX(pz)}\, \mathcal{G}_z^a(x,t)$$

$$+ e^{iX'(pz)}\, \mathcal{G}_z^{\bar{a}}(x,t) \Big] +$$

$$+ (rz)\, \bar{u}(p',s')\, \gamma_5 u(p,s) \int_0^1 dx \Big[e^{-iX(pz)}\, \mathcal{P}_z^a(x,t)$$

$$+ e^{iX'(pz)}\, \mathcal{P}_z^{\bar{a}}(x,t) \Big]$$

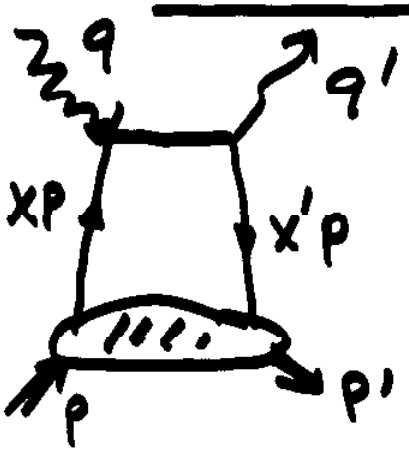
REDUCTION FORMULAS:

$$\mathcal{G}_0^a(x, t=0) = g_1^a(x)$$ SPIN-DEPENDEN DF.

$$\int_0^1 dx \left[\mathcal{G}_z^a(x,t) + \mathcal{G}_z^{\bar{a}}(x,t) \right] = G_A(t)$$ AXIAL F.F.

$$\int dx \left[\mathcal{P}_z^a(x,t) + \mathcal{P}_z^{\bar{a}}(x,t) \right] = G_P(t)$$ PSEUDOSCALAR

DVCS AMPLITUDE IN LOWEST ORDER



$$T^{\mu\nu}(p,q,q') = \frac{1}{2(pq')} \sum_a e_a^2 \Big[\Big(-g^{\mu\nu} + \frac{1}{p\cdot q'}(p^\mu q'^\nu + p^\nu q'^\mu)\Big)$$

$$\cdot \Big\{ \bar{u}(p')\hat{q}' u(p)\, T_F^a(\zeta) + \frac{1}{2M}\bar{u}(p')(\hat{q}'\hat{r} - \hat{r}\hat{q}')u(p)\, T_K^a(\zeta) \Big\}$$

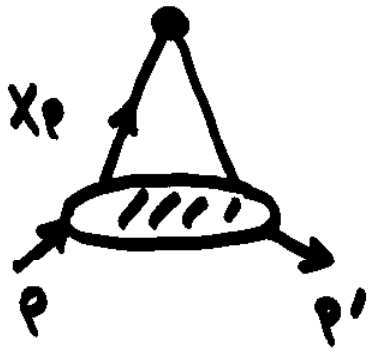
$$+ i\epsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q'_\beta}{(pq')} \Big\{ \bar{u}(p')\gamma_5 \hat{q}' u(p)\, T_G^a(\zeta) +$$

$$\frac{(q'r)}{2M}\bar{u}(p')\gamma_5 u(p)\, T_P^a(\zeta) \Big\}$$

WHERE

$$T_F^a(\zeta) = -\int_0^1 \Big[\frac{1}{x-\zeta+i\epsilon} + \frac{1}{x-i\epsilon} \Big] \big(F_2^a(x,t) + F_2^{\bar{a}}(x,t) \big)$$

etc..



SINCE NUCLEON IS LOWEST BOUND STATE, $\mathcal{F}_\zeta(x,t)$ IS REAL $\rightarrow$ REAL & IM. PARTS OF TOTAL $T^{\mu\nu}$ ARE GIVEN BY THOSE OF SD-SUBPROCESS

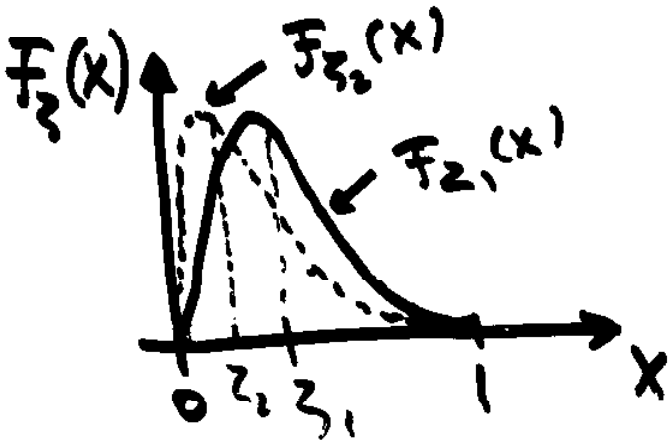
e.g.,

$$T_F^a(\zeta) = -\int_0^1 \left[\frac{1}{x-\zeta+i\epsilon} + \frac{1}{x-i\epsilon}\right]\left(\mathcal{F}_\zeta^a(x,t) + \mathcal{F}_\zeta^{\bar{a}}(x,t)\right)$$

$$\rightarrow \quad \frac{1}{\pi}\,\mathrm{Im}\,T_F^a(\zeta) = \mathcal{F}_\zeta^a(\zeta;t) + \mathcal{F}_\zeta^{\bar{a}}(\zeta;t)$$

$$\& \quad \mathrm{Re}\,T_F^a(\zeta) = -P\int_0^1 \left(\mathcal{F}_\zeta^a(x,t) + \mathcal{F}_\zeta^{\bar{a}}(x,t)\right)\frac{dx}{x-\zeta}$$

<u>NB</u>: $\mathrm{Im}\,T \sim \mathcal{F}_\zeta(\zeta,t)$

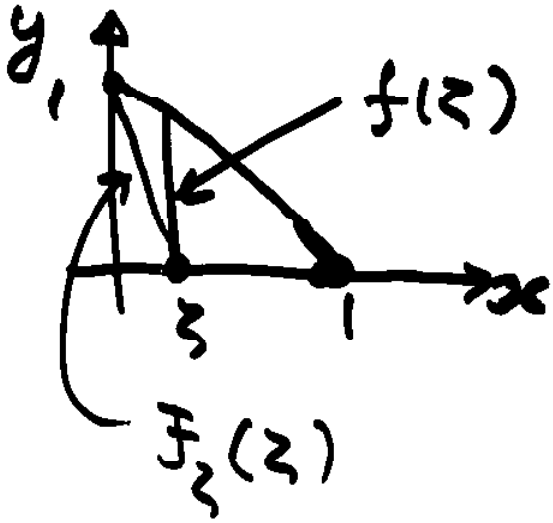


ζ APPEARS TWICE!

MOMENTUM ASYMMETRY PARAMETER

FRACTION x, FOR WHICH $\mathrm{Im}\,T$ APPEARS

4) RELATION BETWEEN $\mathcal{F}_\zeta(\zeta, t=0)$ AND $f(\zeta)$

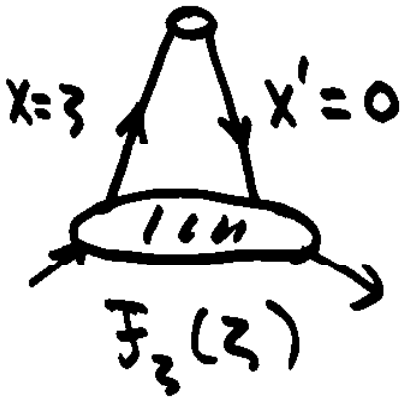


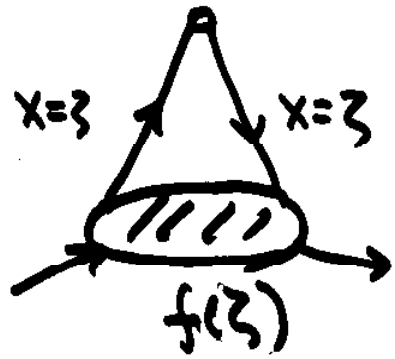
F (x, y) - PLANE

SCALAR TRIANGLE

$$\left.\begin{aligned} f(\zeta) &= A(1-\zeta) \\ \mathcal{F}_\zeta(\zeta) &= A \end{aligned}\right\}$$ DIFFERENCE $O(\zeta)$ FOR SMALL ζ.

NB: WHEN $X=\zeta$, $X' = X-\zeta = 0$





ASYMPTOTIC SOLUTIONS OF EVOLUTION EQUATIONS

NONSINGLET CASE / DOUBLE DISTRIBUTION

$$\mu \frac{d}{d\mu} F(x,y;\mu) = \int_0^1 d\xi \int_0^1 d\eta \, R(x,y;\xi,\eta;\alpha_s) F(\xi,\eta;\mu)$$

COMBINE METHODS USED TO GET FORMAL SOLUTIONS OF DGLAP & BL EQUATIONS.

→ TAKE MOMENTS w.r.t. x

$$F_n(y;\mu) \equiv \int_0^1 x^n F(x,y;\mu)\, dx$$

$$\mu \frac{d}{d\mu} F_n(y;\mu) = \int_0^1 R_n(y,\eta;\alpha_s) F_n(\eta,\mu)\, d\eta$$

← BL-type equation

EXPLICIT FORM:

$$R(x,y;\xi,\eta;\alpha_s) = \frac{\alpha_s}{\pi} C_F \frac{1}{\xi} \Big\{ \theta\Big(0 \le \frac{x}{\xi} \le \min\Big\{\frac{y}{\eta}, \frac{\bar{y}}{\bar{\eta}}\Big\}\Big) - \frac{1}{2}\delta\Big(1-\frac{x}{\xi}\Big)\delta(1$$

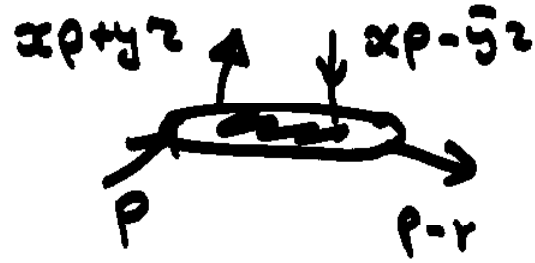
$$+ \frac{\theta(0 \le x/\xi < 1)\, x/\xi}{1 - x/\xi} \Big[\frac{1}{\eta}\delta\Big(\frac{x}{\xi} - \frac{y}{\eta}\Big) + \frac{1}{\bar{\eta}}\delta\Big(\frac{x}{\xi} - \frac{\bar{y}}{\bar{\eta}}\Big)\Big] - 2\delta\Big(1-\frac{x}{\xi}\Big)\delta(y-\eta) \int_0^1 \frac{z}{1-z}$$

$$R_n(y,\eta;\alpha_s) = \frac{\alpha_s}{\pi} C_F \Big\{ \Big[\Big(\frac{y}{\eta}\Big)^{n+1} \Big[\frac{1}{n+1} + \frac{1}{\eta - y} \Big] - \delta(y-\eta) \int_0^1 \frac{z}{1-z}\, dz \Big] + (y \to \bar{y}, \eta \to \bar{\eta}$$

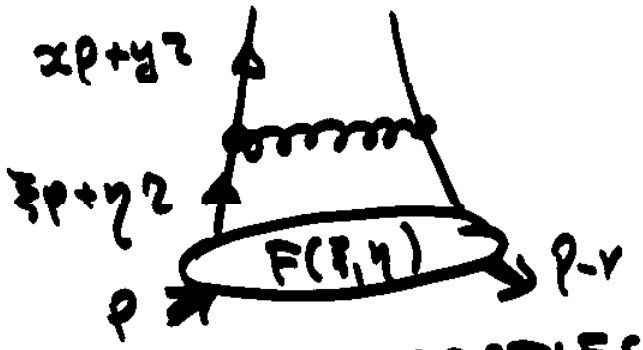
25) EVOLUTION OF NONFORWARD & DOUBLE DISTRIBUTIONS

DD'S $\quad F(x,y)$

DF $\quad$ DA



$$\mu \frac{d}{d\mu} F(x,y;\mu) = \int_0^1 d\xi \int_0^1 R(x,y;\xi,\eta) \cdot F(\xi,\eta;\mu)\, d\eta$$



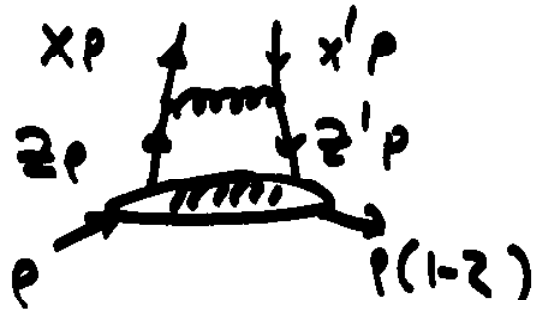
REMARKABLE PROPERTIES OF $R(x,y;\xi,\eta)$:

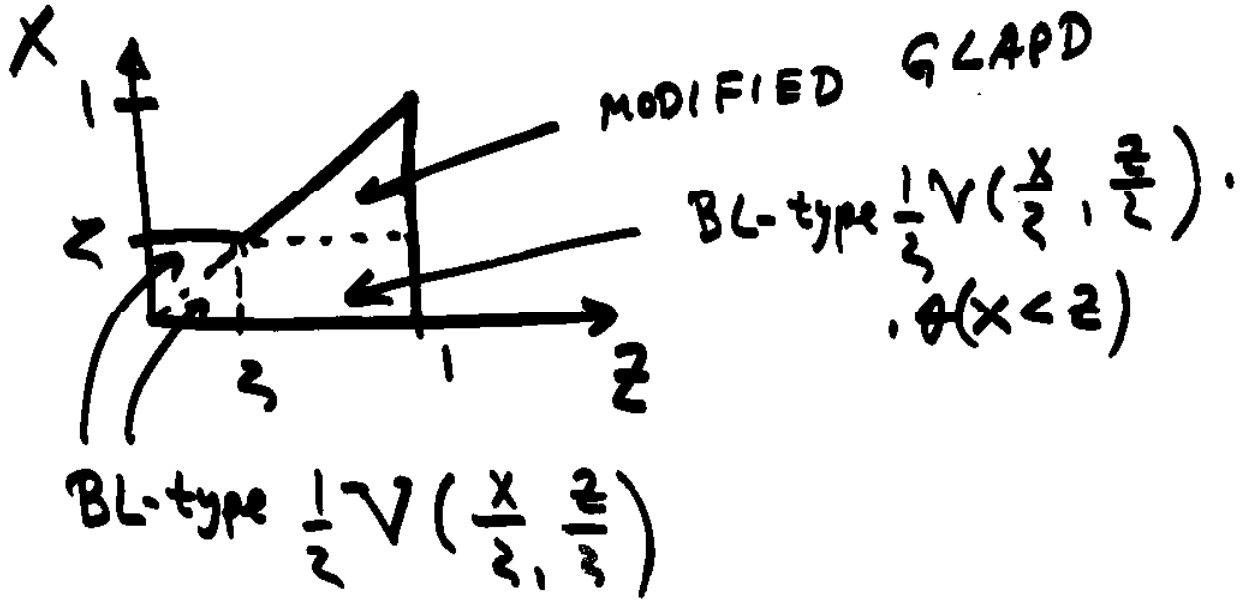
$$\int_0^{1-x} R(x,y;\xi,\eta)\, dy = \frac{1}{\xi} P\left(\frac{x}{\xi}\right) \leftarrow \text{GLAPD}$$

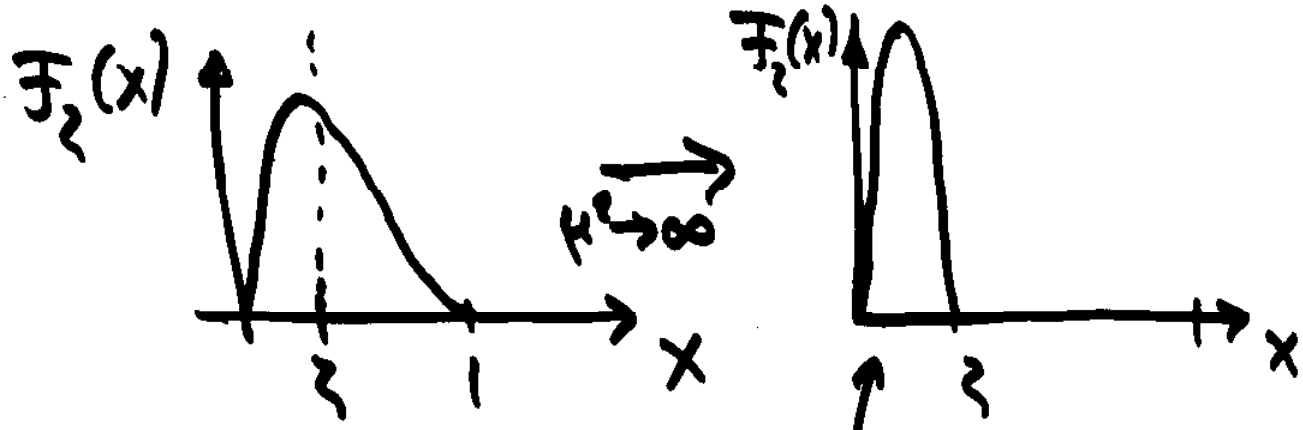
$$\int_0^{1-y} R(x,y;\xi,\eta)\, dx = V(y,\eta) \leftarrow \text{BL-type}$$

NONFORWARD / ASYMMETRIC DF'S :

$$\mu \frac{d}{d\mu} \mathcal{F}^a_\xi(x;t,\mu) = \int_0^1 \sum_b W^{ab}_\xi(x,z)\, \mathcal{F}^b_\xi(z;t,\mu)\, dz$$







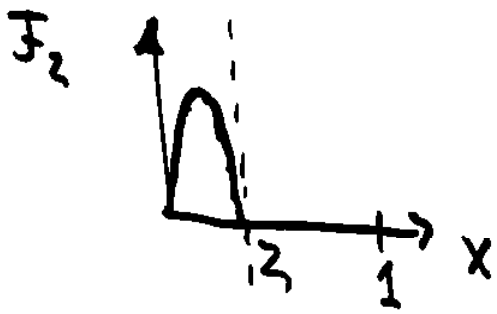
ANALOGOUS TO $\delta(x)$ LIMIT FOR USUAL DF'S $\longrightarrow \frac{x}{\xi^2}\left(1-\frac{x}{\xi}\right) \sim \text{YY}\ \frac{1}{\xi}$

27 ASYMPTOTIC SHAPE

$$F(x, y; \mu \to \infty) \sim \delta(x)\, y\bar{y}$$

FOR ASYMMETRIC DF:

$$\mathcal{F}_\zeta(x; \mu \to \infty) \sim \frac{x}{\zeta^2}\left(1 - \frac{x}{\zeta}\right)$$



28

OTHER APPROACHES.

OFF-FORWARD PARTON DISTRIBUTIONS (X. Ji)

$$\int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{\psi}(-\lambda \tfrac{n}{2}) \hat{n}\, \psi(\lambda \tfrac{n}{2}) | p \rangle =$$

$$= H(x, \xi; t)\, \bar{u}(p') \hat{n}\, u(p)$$

$$+ E(x, \xi; t)\, \bar{u}(p') \frac{\hat{n}\hat{r} - \hat{r}\hat{n}}{2M} u(p)$$

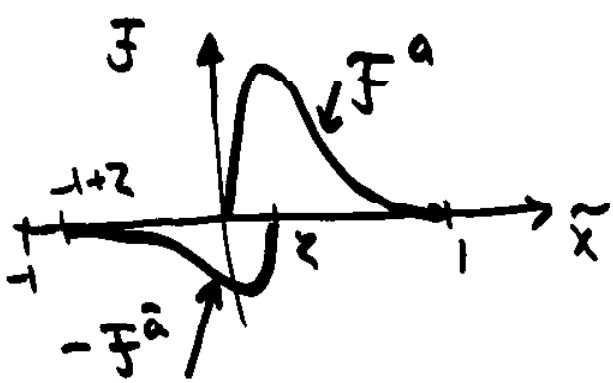
i.e., FOURIER TRANSFORMATION IN OUR (QZ)

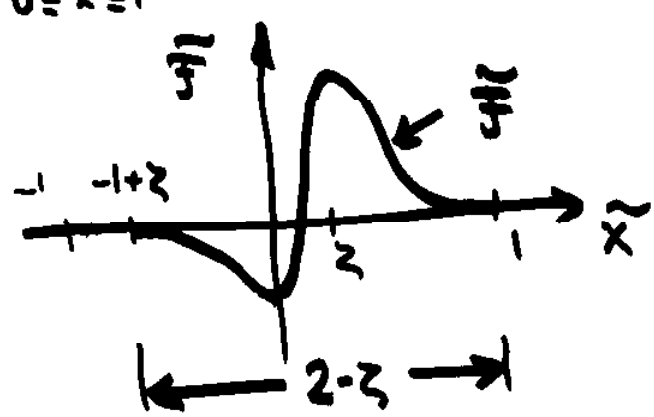
$$\langle p' | \bar{\psi}_a(0) \hat{z} \psi_a(z) | p \rangle = \bar{u}(p') \hat{z}\, u(p) \int_0^1 \Big[e^{-iX(pz)} \mathcal{F}_\zeta^a(X, t)$$

$$- e^{i(X-\zeta)(pz)} \mathcal{F}_\zeta^{\bar{a}}(X, t) \Big] dX + \ldots$$

$$= \bar{u}(p') \hat{z}\, u(p) \int_{-1+\zeta}^{1} d\tilde{X}\, e^{-i\tilde{X}(pz)}\, \tilde{\mathcal{F}}_\zeta(\tilde{X}, t)$$

$$\tilde{\mathcal{F}}_\zeta(\tilde{X}, t) = \mathcal{F}_\zeta^a(\tilde{X}, t)\Big|_{0 \le \tilde{X} \le 1} - \mathcal{F}_\zeta^{\bar{a}}(\zeta - \tilde{X})\Big|_{-1+\zeta \le \tilde{X} \le \zeta}$$





24) JI'S VARIABLES: (ALSO ROBASCHIK et al.)

$$\tilde{x} = \frac{\tilde{X} - \xi/2}{1-\xi/2} \quad ; \quad \xi = \frac{\zeta}{1-\zeta/2} \; ; \; 0 \le \xi \le 2$$

INVERSE RELATIONS:

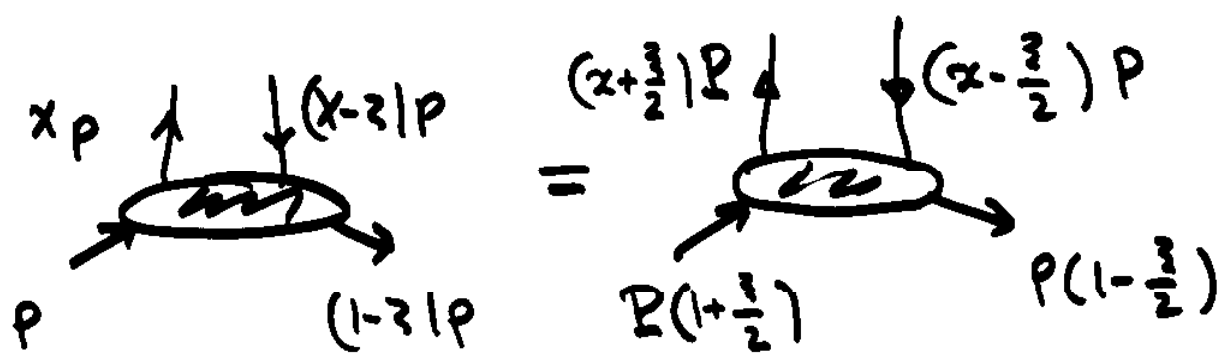
$$\zeta = \frac{\xi}{1+\xi/2} \quad , \quad \tilde{X} = \frac{\tilde{x} + \xi/2}{1+\xi/2} \quad , \quad \tilde{X} - \zeta = \frac{\tilde{x} - \xi/2}{1+\xi/2}$$

AVERAGE MOMENTUM

$$P = \tfrac{1}{2}(p + p') \rightarrow \tfrac{1}{2}\left(p + (1-\zeta)p\right) = \left(1 - \tfrac{\zeta}{2}\right)p = \frac{p}{1+\zeta/2}$$

$$\Rightarrow \quad \tilde{X}\, p = \frac{\tilde{x} + \xi/2}{1+\xi/2}\, p = \left(\tilde{x} + \tfrac{\xi}{2}\right) P$$

PARTON PICTURE:



$$\tilde{F}_\zeta(\tilde{X}, t) = (1+\xi/2)\, H(\tilde{x}, \xi; t)$$

NONFORWARD OFF-FORWARD