

# Polarized Drell Yan and Double Photon Production to NLO QCD

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based on joint work with L. Gordon and with S. Chang, R. Field and L. Gordon (ARGONNE) (U.FLORIDA)

I present results for the NLO (order  $\alpha_s^2$ ) corrections to 2 polarized cross sections: double prompt photon production and production of a Drell Yan pair in polarized pp collisions. In both cases I wil go through a detailed analysis of the structure of the radiative corrections and I will try to underline the differences between the calculations of the radiative corrections in the polarized case compared to the the unpolarized one. In the case of double photon, I will consider a very special cross section -at large  $p_T$  of the photon pair- and study the dependence of the cross section with NLO evolved structure functions. I will show that the asymmetries are sizeable (L. Gordon and C.C. PRD).

Then I will move to the Drell Yan case. I give complete NLO results for the nonsinglet (order  $\alpha_s^2$ ) cross section for the distributions. (S.Chang, R. D. Field, L. Gordon, C.C., to appear)

Spin physics is a very important avenue for QCD. There are many reasons to believe that the interest in the field will grow even more in the next few years and we have the hope that some of the phenomenological work that many people are doing will be eventually measured (at RHIC and at HERA).

The chapter of the “radiative corrections” is still at its infancy. If we move from DIS to polarized pp colliders, then the story of this corrections may become important. The work, as usual, in this direction, goes through factorization theorems and resummation in order to predict the cross sections with good accuracy.

### *What is known to NLO at pp colliders*

Most of the work has focussed on single and double prompt photon (Gordon and Vogelsang; Gordon and C.C.). Some important features of these calculations have been noticed, especially in the non singlet sector.

The analysis involves the so called “initial state asymmetries”. Basically there is no work to NLO for the “transmitted asymmetries”. Also the study of the transversities at pp colliders is at its beginning and work is in progress (S. Chang et al.).

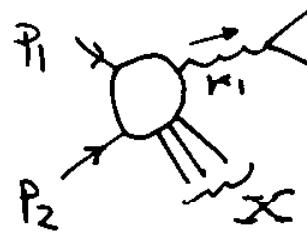
## *Why do we need a hadron colliders*

- At hadron colliders we can definitively set the issue of the gluon polarization, so important after the results of the EMC collaboration.
- We can try to unveil the mystery about the so called transversity distributions (such as  $h_1$ ), some of them which are Leading twist (twist 2) ( Ralston and Soper, Qiu and Sterman, Jaffe and Ji). Drell Yan is the natural process to study these distributions and therefore it is crucial to have complete NLO studies of both the longitudinal and of the transverse asymmetries.
- The calculation are notoriously difficult and extremely time consuming because of the presence of chiral projectors in the initial state.
- Factorization theorems help in “guessing” the structure of the answer, but, as we will see, the structure of the finite contributions to the cross sections are non-trivial.
- In the non singlet sector Drell Yan polarized has a strong resemblance to the corresponding unpolarized.

# The DY mechanism

is intend to study this cross section

$$d\hat{\sigma}^2 \frac{dc}{dtdu}$$



$$\begin{aligned} t_1 &= (p_1 - k_1)^2 \\ u &= (p_1 - k_1)^2 \\ k_1^2 &\equiv \mu^2 \end{aligned}$$

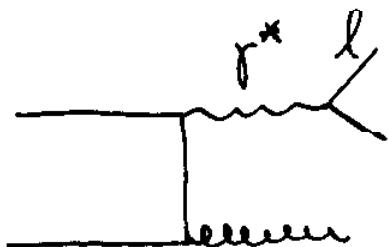
this cross section starts at  $O(\alpha_s)$

Since we are assuming that at least one jet

is in the final state

Originally studied by Ellis Martinelli and Petronzio (NPB 133, 1977)  
in the unpolarized case.

The calculation originally was a study of the  
large  $\tau_T$  behaviour of the process



Born level contribution

since the lepton pair is emitted with a large  $\tau_T$   
(non-zero).

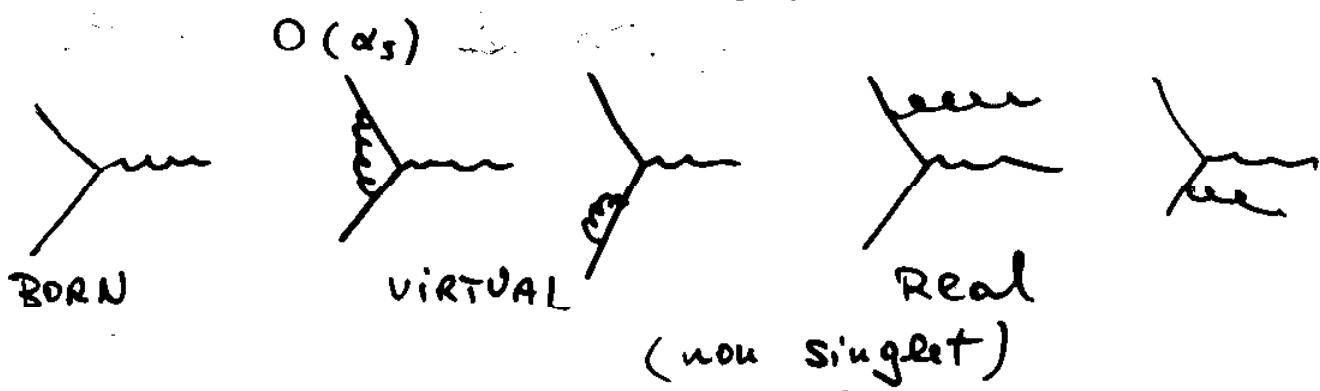
(2)

This cross section is, of course, related to the total cross section  $\frac{d\sigma}{dQ^2} = \sigma_{\text{total}}$

which is only a function of  $x \equiv \frac{Q^2}{s}$

$\sigma_{\text{total}}$  has been studied up to  $O(\alpha_s^2)$  by Van Neerven and collaborators (unpolarized)

and up to  $O(\alpha_s)$  in the polarized case by Ratcliffe (NP B223 (1983)). More recently Gehrman has studied the  $y-t_F$  distribution to  $O(\alpha_s)$ .



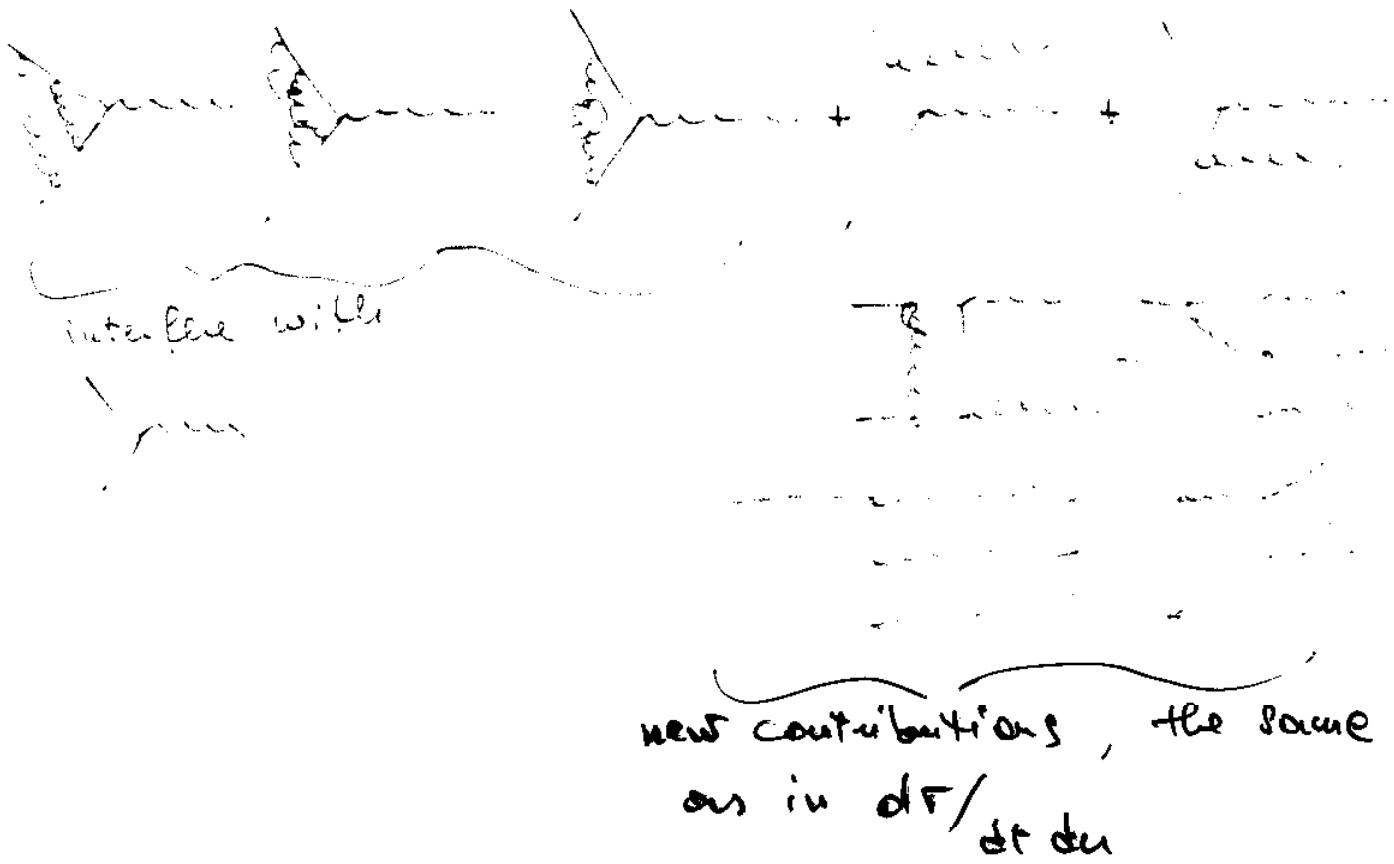
Because computing the contributions is difficult and we get a result involving

$$\ln(1-x), \ln x, \dots$$

the small- $x$  limit also requires resummation (Van Neerven et al.). Work on the resummation of the large  $\ln x$  contributions.

$O(x^2_s)$  corrections to  $d\Gamma/dQ^2$

(3)



therefore, by  $s-t$  integration of our result  
one gets the total  $O(Q^2, s)$  cross section

In principle one does not need to reduce to small  $p_\perp$   
the  $\frac{d\Gamma}{dt ds}$  result to obtain  $G(Q^2, s)$ , but just  
embeds in Dim. Reg.

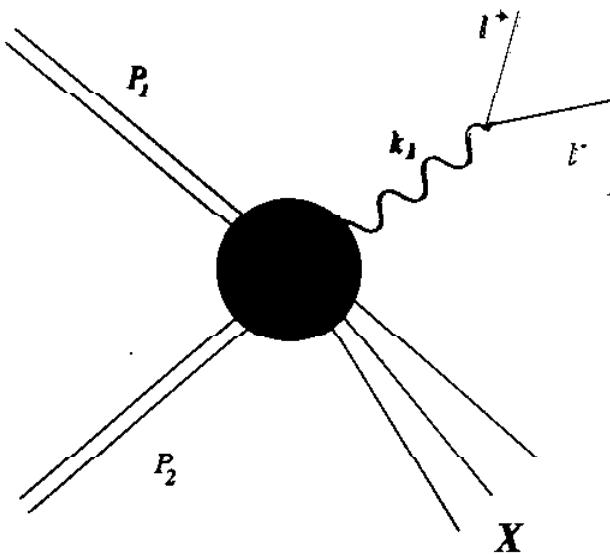


Figure 1: Drell-Yan process

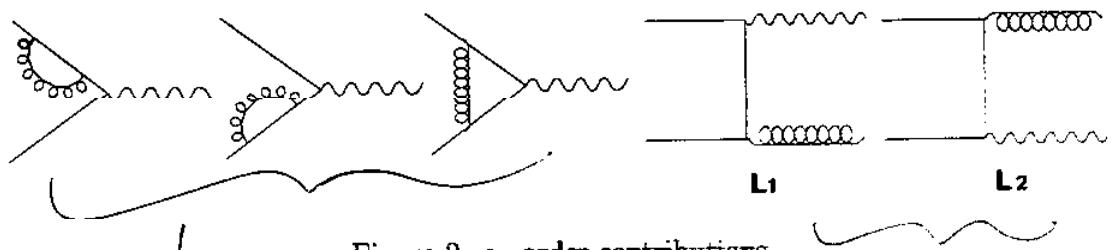


Figure 2:  $\alpha_s$  order contributions

interfering with

$$\frac{d\sigma}{d\alpha^2}$$

Contributing as  
 $L_1 + L_2$

Virtual Corrections up to  $\alpha_s^2$

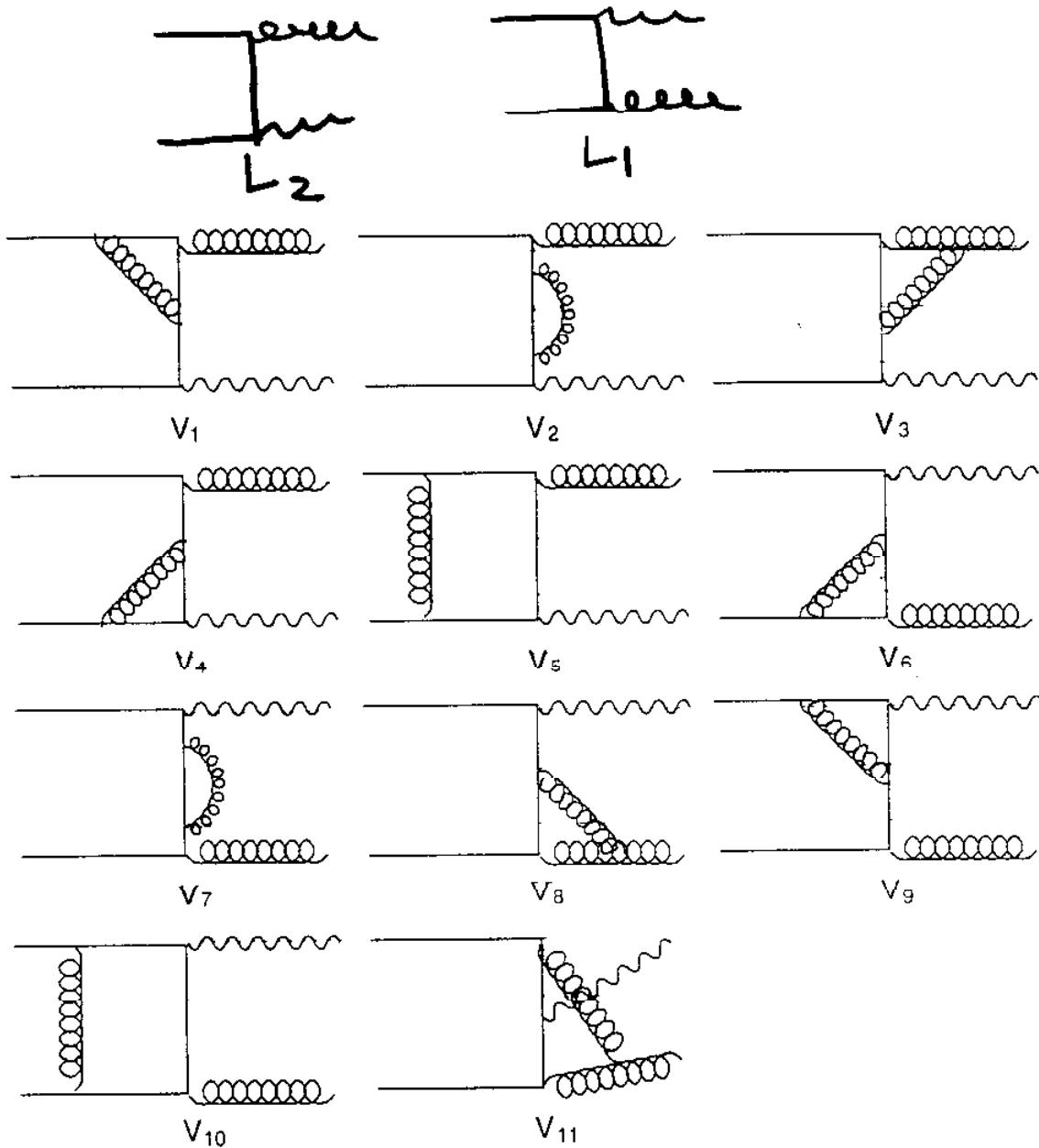
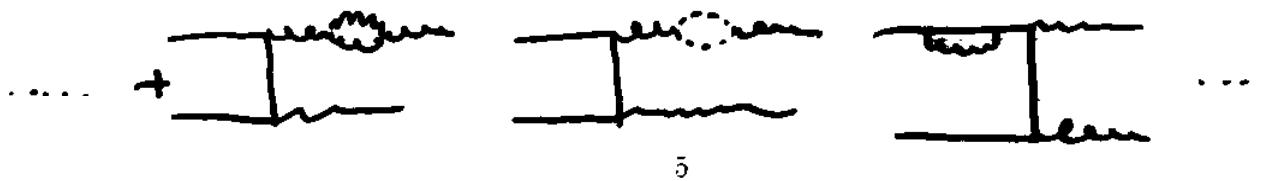


Figure 6: Diagrams which contribute to the process  $q + \bar{q} \rightarrow \gamma^* + G$

+ self energy insertions



## Scheme dependence

As we are going to see

there are subtle issues connected to the Renormalization Scheme dependence ( $\overline{MS}$  and  $\overline{MS}_P$ ) to be considered. Therefore, we are going to calculate again both the unpolarized and the polarized contributions. The EHP result has never been checked before (the virtuals) We are in complete agreement with EHP in the unpolarized limit of our results. The reals have been checked against Gousalres et al. in complete agreement.

We employ Dimensional Reg. to control both the UV and IR singularities and enforce a

t'Hooft-Veltman prescription for  $\gamma_5$

$$\gamma_5 \rightarrow \underbrace{\text{anticommuting}}_{\text{in } D=4} \text{ with } \gamma_\mu$$

in the remaining  $D-4$  dimensions

$\gamma_5$  is commuting

## Virtually

Off-shell Renormalization and tensor reductions

(Methods of Sannan & in QCD)

Renormalization is enforced in the usual way since no  $\gamma_5$  is present. No problem with anomalies since  $\gamma_5$  appears only from the helicity of the initial state.

We use the P.V. reduction procedure

tensor  $\rightarrow$  scalar loop integrals.

We've developed a method - which can be easily implemented in symbolic manipulations - to obtain the scalar expansion of the 1-loop vertex function in the presence of massless states

prescription for massless tadpoles.

example

$$F_C F^2 = \frac{1}{(p+k)^2} - \frac{1}{(p+k)^2} \frac{(p+k)^2}{(p+k)^2} = \frac{1}{(p+k)^2}$$

If we use the PV reduction method

we get generate  $F_C^2$  in loops and find,

They are intrinsically ambiguous.

Usually in DR

$$\bar{B}(0) = 0.$$

This is not correct in general within the  
Feynman - Veltman reduction.

The solution

is to take the  $\bar{B}(0)$  ambiguity into account.

Let's consider the  $\bar{B}(0)$  ambiguity in the Feynman - Veltman reduction.

then we observe that  $\bar{B}(0)$  is intrinsically ambiguous

We interpret  $\bar{B}(0) = \lim_{q^2 \rightarrow 0} \bar{B}(q^2)$

$$\begin{array}{c} \text{---} \\ \text{---} \end{array}$$

UV Renormalized  
(off-shell)

$$\textcircled{1} \quad \xrightarrow{R} \bar{B}(q^2) \rightarrow \bar{B}(0) = \frac{1}{\omega} = \frac{1}{D-4}$$

$$\textcircled{2} \quad \text{while } \rightarrow W \bar{B}(0) = 0$$

This interpretation follows the scheme of the renormalization group expansion of the theory.

A detailed explanation of this can be found in [1].

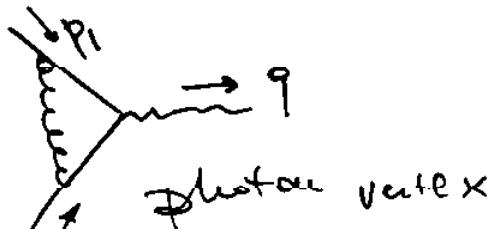
(4)

$$B(c) \rightarrow \bar{B}(c) = f_c$$

$$\omega B(c) \rightarrow c$$

$$\omega B(q^2) \rightarrow -1 + ( )$$

We then obtain all the QCD vertices in a single stroke by symbolic manipulation



$$\begin{aligned}
 V &= \frac{\gamma^\mu}{(t-q^2)^2} \left\{ \frac{2}{\omega} \left[ (L_t - L_{q^2} - 1) q^4 + (L_{q^2} - L_t) q^2 t + t^2 \right] \right. \\
 &\quad + (4 - 3L_{q^2} - L_{q^2}^2 + 2L_t + L_t^2) q^4 + (L_{q^2}^2 - 3L_{q^2} + 3L_t - L_t^2) q^2 t \\
 &\quad \left. + (L_t - 4)t^2 \right\} + \frac{2 \not{p}_1 \not{p}_2}{(t-q^2)^3} \left\{ \frac{2}{\omega} \left[ (L_t - 1 - L_{q^2}) q^4 + (L_{q^2} - L_t) q^2 t \right. \right. \\
 &\quad \left. + t^2 \right] \right. \\
 &\quad + (5 - 5L_{q^2} - L_{q^2}^2 + 3L_t + L_t^2) q^4 \\
 &\quad \left. + (L_{q^2}^2 - 5L_{q^2} + 5L_t - L_t^2) q^2 t \right. \\
 &\quad \left. + (2L_t - 5)t^2 \right\} + \frac{2 \not{p}_2^\mu \not{p}_2}{t-q^2} (L_t - L_{q^2})
 \end{aligned}$$

(5)

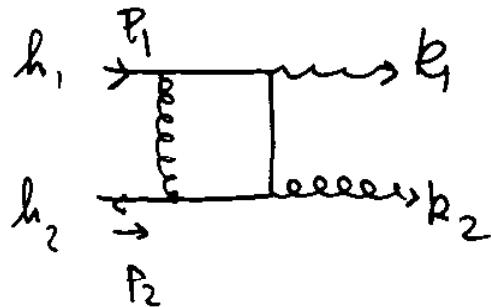
for instance, in the total cross section, we need an expansion up to  $O(\epsilon^2)$  of vertices, boxes, etc.

With our description, this is very easy to obtain.

For jet and GJ diagrams at 1 loop  
we generate immediately the Renormalized  
expressions of the  $T_1$  coefficients  
and implement them in a code.

(6)

Another example

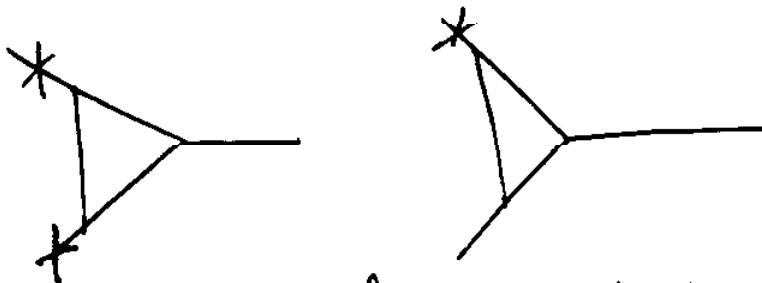


$h_1 \rightarrow$  helicities  
 $h_2$

We generate after loop-integration Rank 1, Rank 2  
 and Rank 3 tensor integrals.

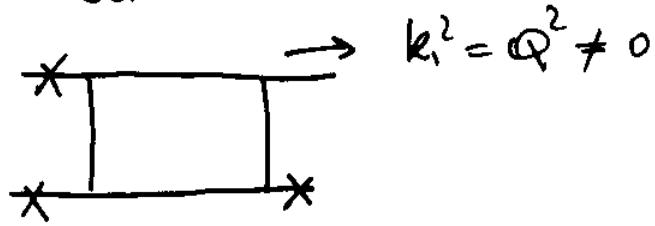
$D^\mu$ ,  $D^{\mu\nu}$ ,  $D^{\mu\nu\rho}$

Using the PV procedure we introduce a huge # of massless tadpoles. Also a large # of triangle type contributions are generated (-f various type)



We use our renormalization prescriptions for  
 $\omega B^{(0)}$ ,  $B^{(0)}$ ,  $B^{(q^2)}$  and the Renormalized expressions  
 of all the vertices  $\Rightarrow$  Renormalized PV coefficients

massive scalar box



$$I_4(s, t, Q^2) = -i(4\pi)^{-n/2} \frac{2}{\epsilon^2} C_F \left\{ \frac{(-Q^2)^{-\epsilon}}{st} F[1, -\epsilon, 1-\epsilon, \frac{-4Q^2}{st}] \right.$$

$$\left. - \frac{(-s)^{-\epsilon}}{st} F[1, -\epsilon, 1-\epsilon, -4/t] - \frac{(-t)^{-\epsilon}}{st} F[1, -\epsilon, 1-\epsilon, -4/s] \right\}$$

(in the Euclidean Region  $s < 0, t > 0$ ) (Van Neerven)

For an analytic continuation:

$$(-s)^{-\epsilon} \rightarrow |s| e^{i\pi\theta(s)}$$

$$\ln(-s) \rightarrow \ln|s| - i\pi\theta(s)$$

For calculations of the total cross section  $I_4$  is not expanded in  $\epsilon$ . Since the integrals for  $\sigma_{\text{total}}$  can be done at finite  $\epsilon$ .

For the distributions instead, we expand up to  $O(1)$

$$I_4(s, t, Q^2) = \frac{i(4\pi)^{-n/2}}{st} C_F \left\{ \frac{2}{\epsilon^2} \left[ -(-Q^2)^{-\epsilon} + (-s)^{-\epsilon} + (t)^{-\epsilon} \right] \right. \\ \left. - 2 \text{Li}_2[1 - Q^2/s] - 2 \text{Li}_2[1 - Q^2/t] - \ln^2 \left( \frac{|s|}{|t|} \right) + \frac{2\pi^2}{3} \right\}$$

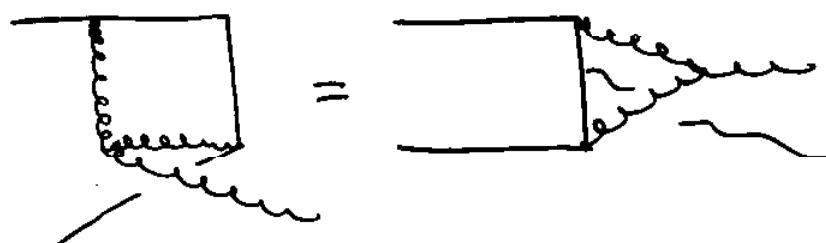
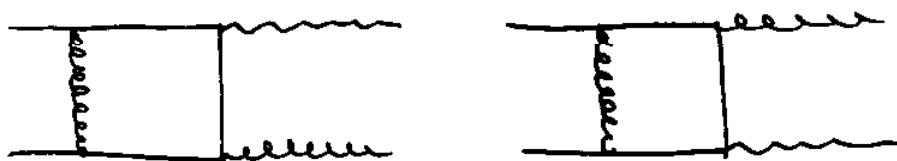
(8)

Direct, exchange etc. require different analytic continuations

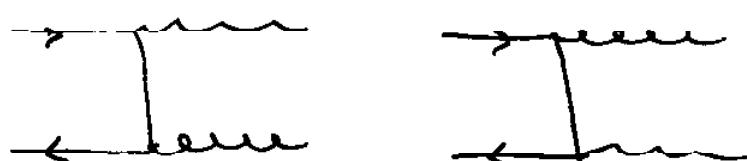
$$\begin{array}{l} Q^2 < 0 \rightarrow Q^2 > 0 \\ s < 0 \quad \quad \quad s > 0 \\ t \geq 0 \rightarrow t < 0 \end{array} \quad \text{physical region of Drell-Yan process}$$

$$\begin{aligned} (-s)^{-\epsilon} &\rightarrow |s|^\epsilon e^{i\pi} \\ (-Q^2)^{-\epsilon} &\rightarrow (Q^2)^\epsilon e^{i\pi} \end{aligned}$$

exchanged diagrams



interference with



Total initial amplitudes  
and helicity violations

the t'Hooft-Veltman-Schweig induces helicity violations for IR-sensitive contributions.

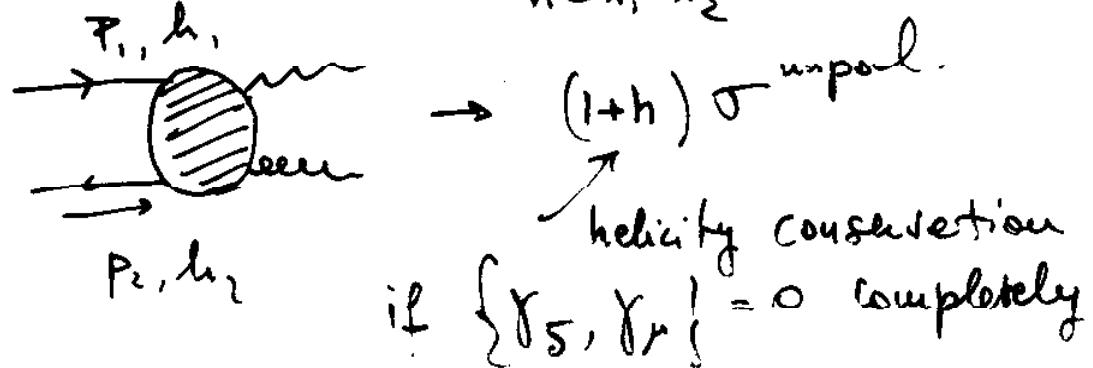
$$u(p,s)\bar{u}(p,s) = \frac{1}{2} \not{p} [1 + \gamma_5 \not{s}] \quad m=0$$

$$v(p,s)\bar{v}(p,s) = \frac{1}{2} \not{p} [1 + \gamma_5 \not{s}]$$

$$\begin{aligned} u(p,h)\bar{u}(p,h) &= \frac{1}{2} \not{p} [1 - \gamma_5 h] \\ v(p,h)\bar{v}(p,h) &= \frac{1}{2} \not{p} [1 + \gamma_5 h] \end{aligned} \quad \left. \right\} \text{"helicity averages"}$$

$$S \cdot p = 0, S^2 = -1$$

$\sigma(h)$  is only a function of the product of  $h_1$  and  $h_2$



helicity + (fermion)  $\rightarrow h_1 = +1$

$$(1+h) = 0$$

helicity + (antifermion)  $\rightarrow h_2 = -1$

therefore for the virtual corrections one would expect

that  $|H(+,+)|^2 = 0 \rightarrow |H(-,-)|^2 = 0$

Now  $\Delta \bar{H}^2 = \frac{1}{2} \left[ |H(+,+)|^2 - |H(+,-)|^2 \right]$

$$\sigma^{unp} = \frac{1}{4} \left\{ |H(+,+)|^2 + |H(+,-)|^2 + |H(-,+)|^2 + |H(-,-)|^2 \right\}$$

$$|H(+,-)|^2 = |H(-,+)|^2$$

whence  $\sigma^{unp} = -\sigma^{pol} = -\Delta \bar{H}^2$

$$\text{Asymmetry} = \sigma^{pol}/\sigma^{unp} = -1$$

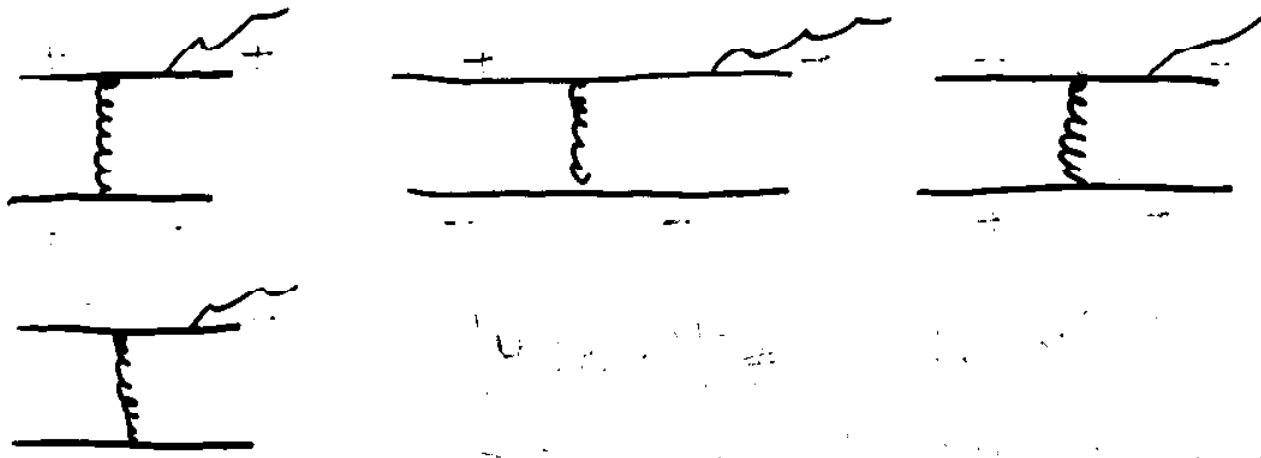
and relations are thus  $\sigma^{unp} \neq \sigma^{pol}$ .

$$\sigma^{unp} = \frac{1}{2} \left( \sigma^{pol} + \sigma^{pol} e^{-i\phi} \right)$$

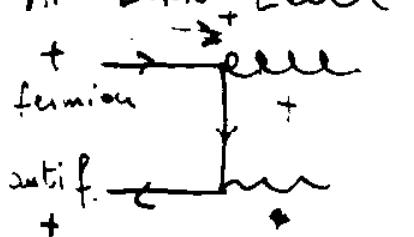
therefore  $|H(+,+)|^2$

"measures" the helicity violating part of the  
|Amplitude|^2

Notice that for scattering contributions (real emissions



At BORN Level



$$\rightarrow |H(+,+)|^2 = 0$$

Now, since there are not singularities  
the helicity violation, in the HVBH scheme  
is  $\epsilon$ -Dependent.

$$\text{Parasitic} = \frac{\mu_0}{\pi} + \frac{I^2}{R_{\text{par}}}$$

(12)

now to  $0 Ce^2$ )

$$|\bar{H}|^2 = \frac{1}{4} \sum_{h_1, h_2} |H(h_1, h_2)|^2$$

$$\Delta \bar{H}^2 = \frac{1}{2} (|H_{++}|^2 - |H_{+-}|^2)$$

$\rightarrow$  is equivalent to with

$$|H(h_1, h_2)|^2 = |H(h)|^2 = |\bar{H}|^2 - h \Delta \bar{H}^2 \quad (\text{CFG})$$

$$= \frac{e_f^2 g_f^2 g_s^2}{N_c t u} \sum_{h_1, h_2} \frac{(1-\epsilon) (2Q^2 s + (1-\epsilon)(t^2 + u^2) - 2\epsilon t u)}{N_c + 1}$$

$$+ \frac{1}{2} [(1+\epsilon)^2 (2Q^2 s + (1+\epsilon)(t^2 + u^2) - 2\epsilon t u) + 2\epsilon^2 t^2 + 2\epsilon^2 u^2]$$

with

$$|\bar{H}|^2 = e_f^2 g_f^2 g_s^2 \frac{1}{N_c t u} \left\{ (1-\epsilon) (2Q^2 s + (1-\epsilon)(t^2 + u^2) - 2\epsilon t u) \right\}$$

$$|H_{++}|^2 = -e_f^2 g_f^2 g_s^2 \frac{1}{N_c t u} \left\{ (1+\epsilon)^2 (2Q^2 s + (1+\epsilon)(t^2 + u^2) - 2\epsilon t u) + 2\epsilon^2 t^2 + 2\epsilon^2 u^2 \right\}$$

$$|H_{+-}|^2 = -e_f^2 g_f^2 g_s^2 \frac{1}{N_c t u} \left\{ (1-\epsilon)^2 (2Q^2 s + (1-\epsilon)(t^2 + u^2) - 2\epsilon t u) + 2\epsilon^2 t^2 + 2\epsilon^2 u^2 \right\}$$

Therefore : since the result is IR safe

$$\epsilon \rightarrow 0$$

then

$$[H_{\text{ext}}] \rightarrow 0 \quad \text{and} \quad [H^{\dagger}] = -[\bar{H}]$$

In the factorization formula for the non-singlet EHP factorize  $|\bar{H}|^2$  up to  $O(\epsilon^2)$

so that we have

$$G_1(Q^2, u, t) = G_1(Q^2, u, t) + \frac{1}{2} \left( \frac{1}{Q^2} + \frac{1}{u^2} \right) G_1(Q^2, u, t) - \frac{1}{2} G_1(Q^2, u, t)$$

$$= G_1(Q^2, u, t) + \frac{1}{2} \left( \frac{1}{Q^2} + \frac{1}{u^2} \right) G_1(Q^2, u, t)$$

$$G_1(Q^2, u, t) = \frac{1}{N_c} \sum_{i=1}^{N_c} \frac{1}{(Q_i^2 + Q^2)^{1/2}} \frac{1}{(u_i^2 + u^2)^{1/2}}$$

$$k_2 T_B(Q^2, u, t) = K T_0(Q^2, u, t) \leftarrow (\text{EHP})$$

$$K = 2\pi \alpha \frac{C_F}{N_c} \frac{(1-\epsilon)}{T[1-\epsilon]} \left( \frac{4\mu^2 \pi}{Q^2} \right)^\epsilon \left( \frac{S Q^2}{u t} \right)^\epsilon$$

$$G_1(Q^2, u, t) = \left( 1 + \epsilon \left( \frac{Q^2}{Q^2} + \frac{u^2}{u^2} \right) + \frac{1}{2} \epsilon^2 \left( \frac{Q^2}{Q^2} + \frac{u^2}{u^2} \right)^2 \right) G_1(Q^2, u, t)$$

EHP

Flavour

Therefore . relativity violating terms absent ( $t \rightarrow \infty$ )

1. Dimensional regularization in the FET scheme

and one more important feature is the automatic cancellation.

The structure of the result for the virtual contributions in EHP (unpolarized non singlet)

$$S \frac{d\sigma}{dt du} = e_F^2 K_2 \frac{\alpha_s}{S} \delta(S+t+u-Q^2) \left\{ \begin{array}{l} \text{BORN LEVEL} \\ \boxed{T_B} \end{array} \right\} \otimes \Theta \left[ \left\{ \frac{C_1}{\epsilon^2} + \frac{C_2}{\epsilon} \right\} \right] + O(\alpha_s) \left\{ \begin{array}{l} \uparrow \\ \text{Complex expression, finite.} \end{array} \right\}$$

in  $\overline{\text{MS}}$

$$\frac{C_1}{\epsilon^2} = \frac{2C_F + N_c}{\epsilon^2}, \quad C_2 = \frac{1}{\epsilon} \left\{ 3C_F - 2C_F \ln S/Q^2 + \frac{11}{6} N_c + N_c \ln S Q^2 / u t - \frac{1}{3} N_F \right\}$$

Double poles and single poles correctly factorize and  $T_B$  is the  $\uparrow$  # of flavours

$O(\epsilon^2)$  Born level 2-to-2 cross section

Virtually in the non-singlet

$$\begin{aligned}
 \frac{s d\sigma^V}{dt du}(s, t, u, h) = & e_F^2 K_2 \frac{\alpha_s}{s} \delta(s+t+u - Q^2) \left\{ \left( (1+h T_B) + h \in A_B \right) \times \right. \\
 & \left[ 1 - \frac{ds}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi/\mu^2}{Q^2} \right)^\epsilon \left( \frac{2C_F + N_c}{\epsilon, \epsilon^2} + \frac{1}{\epsilon} \left( 3N_c - 2C_F \ln \frac{Q^2}{s} + \frac{11}{6} N_c + N_c \ln \frac{3Q^2}{ut} \right. \right. \right. \\
 & \left. \left. \left. - \frac{1}{3} N_F \right) \right) \right] \\
 & \cdot \frac{1}{2\pi} (1+h) \left[ \frac{\pi^2 (4C_F + N_c)}{3tu} \frac{2Q^2 s + t^2 + u^2}{3tu} - 2(2C_F - N_c) \frac{Q^2(t^2 + u^2)}{tu(t+u)} \right. \\
 & - 2C_F \left( \frac{8(2Q^2 s + t^2 + u^2)}{tu(Q^2 - u)(Q^2 - t)} - \frac{Q^4 s(t+u)}{tu(Q^2 - u)(Q^2 - t)} \right. \\
 & \left. \left. - 2 \left( \text{Li}_2 \left( \frac{t}{t-Q^2} \right) + \frac{1}{2} \ln^2 \left( 1 - \frac{Q^2}{t} \right) \right) \left( N_c \frac{2s+u}{t} + 2C_F \frac{s^2 + (s+u)^2}{tu} \right) \right. \\
 & \left. - 2 \left( \text{Li}_2 \left( \frac{u}{u-Q^2} \right) + \frac{1}{2} \ln^2 \left( 1 - \frac{Q^2}{u} \right) \right) \left( N_c \frac{2s+u}{t} + 2C_F \frac{s^2 + (s+u)^2}{tu} \right) \right. \\
 & \left. + 2(2C_F - N_c) \left( \text{Li}_2 \left( -\frac{t+u}{s} \right) \frac{2Q^2 s + u^2 + t^2 + 2s^2}{ut} \right. \right. \\
 & \left. \left. + \left( 2 \ln \left( \frac{s}{Q^2} \right) \frac{Q^4 - (t+u)^2}{(t+u)^2} + \ln^2 \left( \frac{s}{Q^2} \right) \frac{s^2}{tu} \right) \right. \right. \\
 & \left. \left. - \left( \ln |t| \frac{\ln s/Q^2}{Q^2} - \frac{1}{2} \ln |t| \frac{\ln s/Q^2}{Q^2} \right) \frac{s^2 + (s+u)^2}{tu} \right) \right. \\
 & \left. - \left( \ln s/Q^2 \ln u/Q^2 - \frac{1}{2} \ln |u| \frac{\ln s/Q^2}{Q^2} \right) \frac{s^2 + (s+u)^2}{tu} \right) \\
 & \left. + 2 \ln |u| \frac{1}{Q^2} \left( C_F \frac{4Q^2 s - 2su + tu}{(Q^2 - u)^2} + \frac{N_c u}{Q^2 - u} \right) \right. \\
 & \left. + 2 \ln |t| \frac{1}{Q^2} \left( C_F \frac{4Q^2 s - 2st + tu}{(Q^2 - t)^2} + \frac{N_c t}{Q^2 - t} \right) - 2 \ln |t| \frac{\ln |u|}{Q^2} N_c \frac{2Q^2 s + t^2 + u^2}{tu} \right]
 \end{aligned}$$

where we have factorized the lowest order spin-dependent cross section

$$\frac{d\sigma}{dt}(s, t, h) = e_F^2 K_2 \leq [(1+h) T_B(Q^2, u, t) + h e A_B(Q^2, u, t)]$$

if we send  $e \rightarrow 0$  and  $h \rightarrow 0$   $\Rightarrow$  EMP result  
never checked before

left soft:  $(\gamma_{\mu})$  and right soft:  $(\gamma_{\mu})$   $\rightarrow$   $A_B$  contribution

$\rightarrow$   $t = -2$   $E_{\text{miss}}$  contribution  $\approx$  first iteration

$\underline{\underline{h e A_B(Q^2, u, t)}}$  generates an helicity violating contribution

$$\frac{s d\sigma^{++}}{dt} = -e_F^2 K_2 \frac{1}{2\pi s} A_B(s, t) \left\{ 3C_F - 2C_F \ln \frac{s}{Q^2} + \frac{11}{6} N_c + N_c \ln \frac{s Q^2}{ht} - \frac{1}{3} N_F \right\}$$

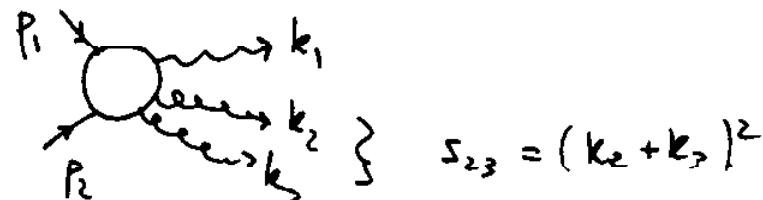
left soft:  $\gamma_{\mu}$  and right soft:  $\gamma_{\mu}$   
and  $\gamma_{\mu}$

We have to look at the real emission diagrams.

# Collinear initial and final state factorization

- distributions obtain in the limit  $\epsilon \rightarrow 0$

$$\frac{1}{\sum_{23,+}} \text{ and } \left( f_{23,+} \right)$$



Bare vs Renormalized structure functions

$$f_j^A(x, M^2) = \int_x^1 \frac{dz}{z} \left\{ \delta_{ij} \delta(z-1) - \frac{\alpha_s}{2\pi} R_{j \rightarrow i}(z, M^2) \right\} f_{j, \text{bare}}^A \left( \frac{x}{z} \right)$$

$$R_{j \rightarrow i} = -\frac{1}{\epsilon} P_{i \rightarrow j}(z) \frac{\Gamma[1-\epsilon]}{\Gamma[1-z\epsilon]} \left( \frac{4\pi \mu^2}{M^2} \right) + C_{j \rightarrow i}(z)$$

$$P_{qq} = C_F \frac{(1+z^2)}{(1-z)_+} - \epsilon(1-z)$$

$$P_{qg} = \frac{1}{1-z} \quad P_{gg} = \frac{1}{(1-z)_+}$$

$$F_{qg} = \frac{1}{1-z} \quad F_{gg} = \frac{1}{(1-z)_+}$$

$$\frac{d\sigma_{ij}}{dt du} = \frac{d\sigma_{ij}}{dt du} - \frac{1}{2\pi} \sum_k \int_0^1 dz R_{i \rightarrow k}(z, M^2) S_{k \rightarrow j}(z, -t, u)$$

$$= \frac{1}{2\pi} \sum_k \int_0^1 dz F_{i \rightarrow k}(z, M^2) S_{k \rightarrow j}(z, -t, u) \delta(z-t-u)$$

IR Sensitive  
Sets of diagrams

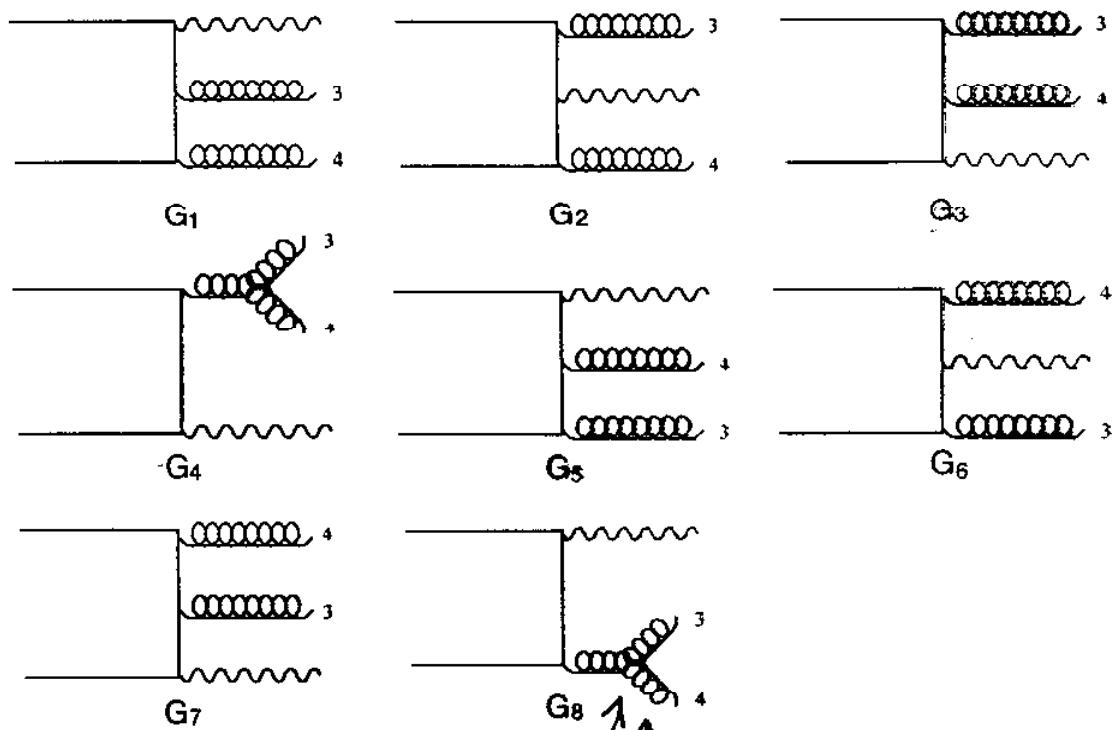
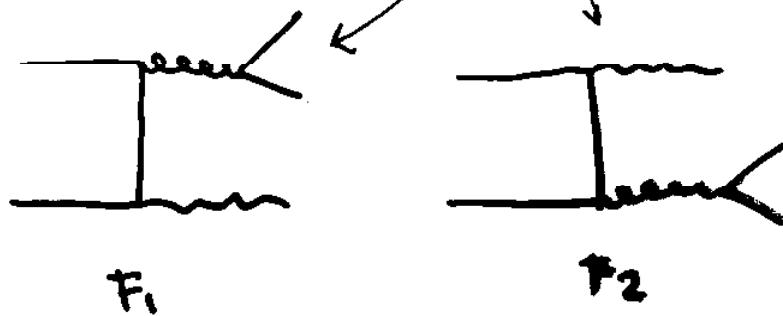


Figure 4: Diagrams which contribute to the process  $q + \bar{q} \rightarrow \gamma^* + G + G$



*included with the C's*

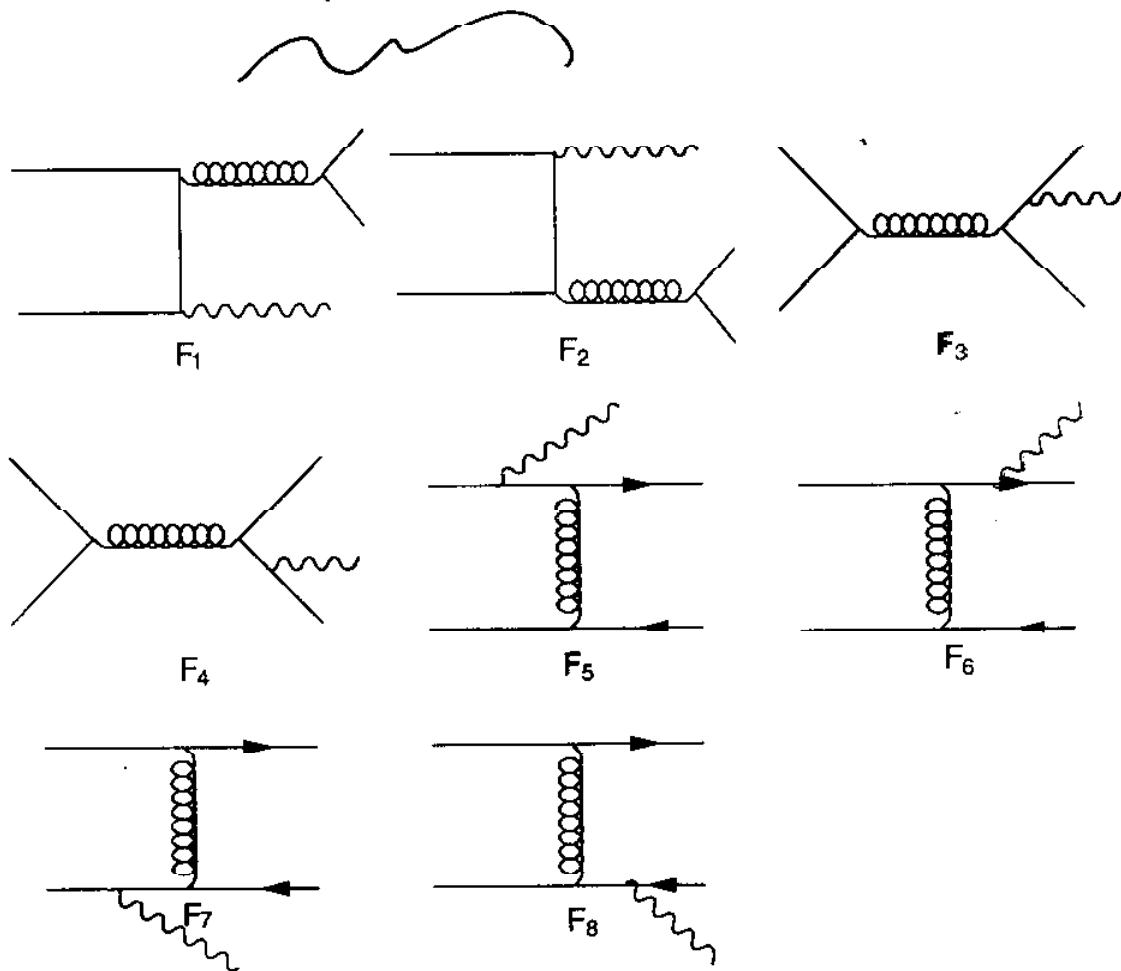


Figure 3: Diagrams which contribute to the process  $q + \bar{q} \rightarrow \gamma^* + q + \bar{q}$

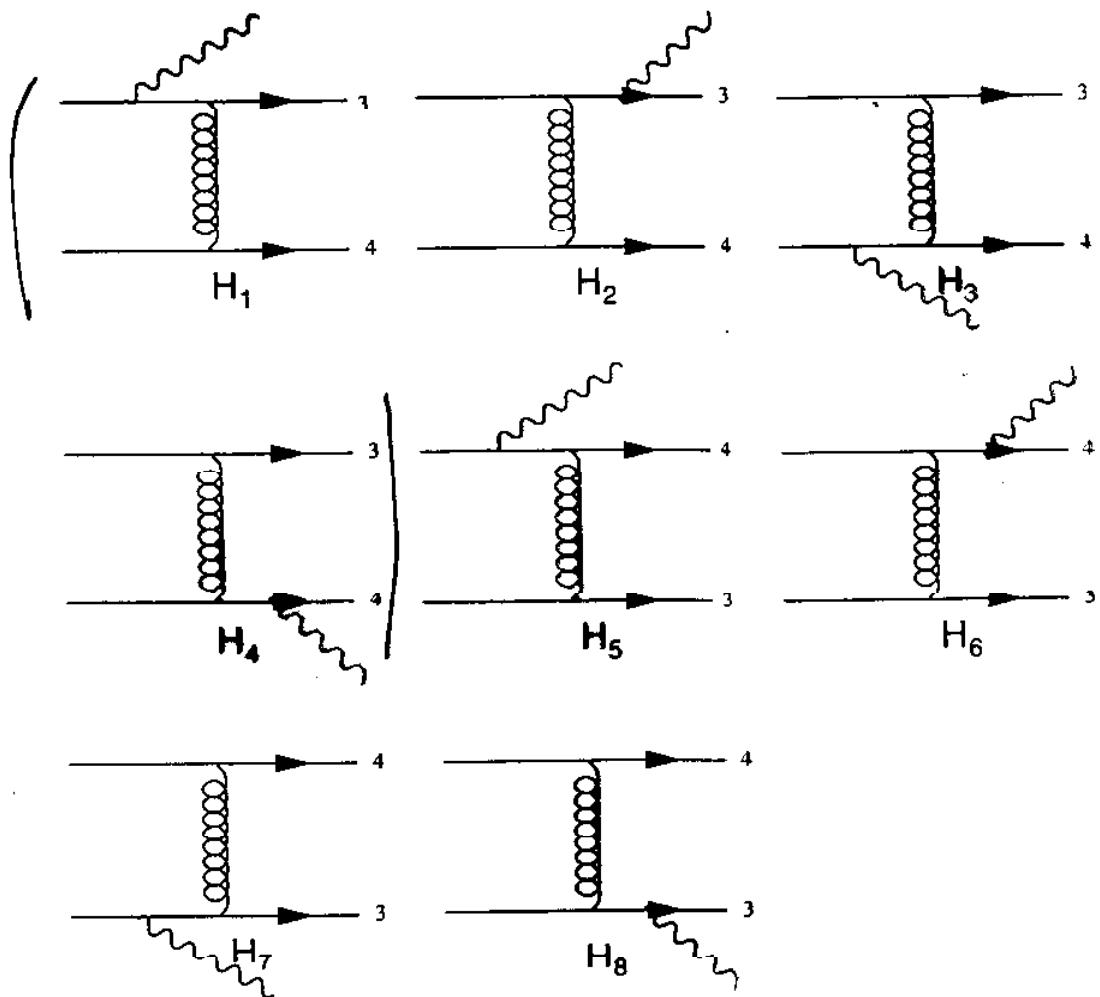


Figure 5: Diagrams which contribute to the process  $q + q \rightarrow \gamma^* + q + q$

Hat-momenta integrations and helicity violation

$$\hat{\gamma}_\mu = \frac{\hat{\gamma}_\mu}{4 \text{Dim}} \bullet \frac{\hat{\gamma}_\mu}{n \cdot 4 \text{Dim.}}$$

$$k^\mu = \hat{k}^\mu + \tilde{k}^\mu$$

non-hat momenta

$$PS_3 = \frac{(4\pi)^{2\epsilon}}{2^8 \pi^4 s \Gamma[1-2\epsilon]} \left\{ \frac{s}{s_{23}(t_u - Q^2 s_{23})} \right\} \times I_{4,2} \underbrace{\quad}_{O(1)}$$

hat-momenta

$$PS_2 = \int d\hat{k}_2 \delta(\hat{k}_2^2) \delta((k_{23} - \hat{k}_2)^2) \hat{k}_2^2$$

We can remove only 3 hat momenta, by choosing a suitable ref. frame, but 1 is left.

$O(\epsilon)$

$\ell^+ + \ell^- + \dots$

interference contributions contain hat-momenta give  $\not{e}$  and  $O(1)$  terms responsible of helicity violation (in the real emission).

the collinear and soft divergences of the two unobserved jets are described by the limit

$$\frac{S_{23}^{1+\epsilon}}{S_{23}^{\epsilon}} = -\frac{1}{\epsilon} \delta(S_{23}) \left\{ 1 - \epsilon \ln A + \gamma_2 \epsilon^2 \ln^2 A \right\} \\ + \left( \frac{1}{S_{23}} \right)_{A+} - \epsilon \left( \frac{\ln S_{23}}{S_{23}} \right)_{A+}$$

Cancellation of the collinear singularities

$$q \quad \overline{q} \quad \overline{q} \quad q_i \quad - q \quad \overline{q} \quad q \\ \overline{q} \quad q \quad \overline{q} \quad \overline{q}^* \quad - \overline{q} \quad \overline{q}^*$$

$$\left| \sum_i Q_i \right|^2 + |F_1 + F_2|^2 \text{ are IR sensitive}$$

polarized and unpolarized are now different

the difference comes in part from hat-momenta contributions

and in part from 3-body contributions  $\sim \frac{1}{(s_2)^{1+\epsilon}} \text{ Trace}^{\otimes}$

3-body contributions

$$① \quad \delta\sigma \sim 4N_f \delta(s_2) \frac{\alpha^4 - Q^2 t + 2t^2 - Q^2 u + 2tu + 2u^2}{3tu}$$

② hat momenta

$$-\frac{t^{1/2}}{m_1} + \frac{t^{1/2}}{m_2} + \dots$$

If we use the  $\overline{MS}$  scheme then

$$\text{Virtual} + |C_1 + \dots + C_{ij}|^2 + |F_1 + F_2|^2$$

$\sim$

$$\sigma_V + \sigma_I \rightarrow \text{IR. corr. and fact.}$$

But there is an overall helicity violation here.

$$(\sigma_V + \sigma_I)_{\text{Fact.}}^{\text{pol}} = (1+h) \sigma_{\text{EMP}}^{\text{unp}} + h H_t$$

However, in the  $\overline{MS}$  Scheme

$$P_{99} = \frac{\Delta P_{99}}{\text{unpol}} = C_F \frac{(1+z^2)}{(1-z)_+} - c(1-z)$$

It has been noted before

Gordon and Helsberg, Fonda and Sorkin

that if

$$\Delta P_{99} = C_F \frac{(1+z^2)}{(1-z)_+} - c(1-z)$$

$\overline{MS}_P$

$$(\sigma_V + \sigma_R)_{\text{Fact}}^{\text{pol}} = (1+h) \sigma_{\text{EMP}}^{\text{unp}}$$

However, we truly need the  $\overline{HS}$  scheme

since the evolution equations are studied in this scheme  
usually

$\Rightarrow$  it is crucial to know the  
velocity violating contribution  
at parton level.. at  $H_t$ .

Question

if we modify  $\Delta P_{gg}$  from  $\overline{HS}$  to  $\overline{HS}_P$

what is the structure of  $\sigma_V + \sigma_R = \sigma_V + \sigma_I + \sigma_{II}$   
 $+ \sigma_{III} - \sigma_{IV}$

$$(\sigma_V + \sigma_R)_{\overline{HS}_P} = (\sigma_V + \sigma_R)_{\overline{HS}} + \text{loop}$$

and how will the velocity of the gluons  
contribute in this modified scheme.

$$\begin{aligned}
S \frac{d\sigma^R}{dt du} = & e_F^2 K_2 \frac{\alpha_S}{S} \left( (1+h) T_B + h e A_B \right) \delta(S_{23}) \frac{\alpha_S \Gamma[1-\epsilon]}{2\pi \Gamma[1-2\epsilon]} \left( \frac{4\pi \mu^2}{Q^2} \right)^\epsilon \textcircled{B} \\
& + \frac{\alpha_S}{2\pi} \left\{ \left( \frac{11N_C - L N_F}{6} \right) \ln \left( \frac{Q^2}{A} \right) + \left( \frac{67}{18} N_C - \frac{5}{9} N_F \right) + N_C \ln \left( \frac{Q^2}{A} \right)^2 \right. \\
& + \left. \left( C_F - \frac{1}{2} N_C \right) \left( \frac{\pi^2}{3} + \ln \left( \frac{A^2 S}{Q^2 u t} \right)^2 \right) \right\} + \frac{\alpha_S}{2\pi} (1+h) T_B \left[ \frac{2N_F - 11N_C}{6 (S_2)_{AT}} \right. \\
& + \left. \left( 2C_F - N_C \right) \left\{ 2 \left( \frac{\ln(S_2/A)}{S_2} \right)_{AT} + \left( \frac{1}{S_2} \right)_{AT} \ln \frac{S^2 A^2}{(u-S_2)(t-S_2)(ut-Q^2 S_2)} \right\} \right. \\
& + \left. \left. + \frac{2C_F}{(S_2)_{AT}} \ln \frac{ut-Q^2 S_2}{(u-S_2)(t-S_2)} + 4C_F \left( \frac{\ln S_2/Q^2}{S_2} \right)_{AT} \right] \right\} \\
& + \frac{1}{2} \left( 2C_F - N_C \right) \left( \frac{1}{S_2} \right)_{AT} \ln \frac{(u-S_2)(t-S_2)}{(ut-Q^2 S_2)} + \frac{Q^2}{2\pi} \\
& + \frac{1}{2} \left( 2C_F - N_C \right) \left( \frac{1}{S_2} \right)_{AT} \ln \frac{(u-S_2)(t-S_2)}{(ut-Q^2 S_2)} + \frac{Q^2}{2\pi} \\
& - \frac{1}{2} \left( 2C_F - N_C \right) \ln \frac{S_2}{ut-Q^2 S_2} \left( \frac{ut-Q^2 S_2}{(u-S_2)(t-S_2)} \right)^2 + \frac{Q^2}{2\pi} \\
& + \frac{1}{2} \left( 2C_F - N_C \right) \ln \frac{S_2}{ut-Q^2 S_2} \left( \frac{(u-S_2)(t-S_2)}{ut-Q^2 S_2} \right)^2 + \frac{Q^2}{2\pi} \\
& + 2C_F \ln \frac{ut-Q^2 S_2}{(u-S_2)(t-S_2)} \cdot \frac{(Q^2 + S^2 + (u^2-t^2)^2 + (Q^2-u^2)^2)}{4(u^2-t^2+2Q^2 S_2)} + \frac{Q^2}{2\pi}
\end{aligned}$$

## Drill Year Summary.

an eight vertex reflected up to order

in the  $\bar{H}^i_S \bar{H}^j_S$  indices,  
clear scheme dependence.

$$\Gamma^{\text{pol}} = (1+h) \Gamma_{\text{EMP}}^{\text{unp}} + h H_t + H_{234} + h H'_{234}$$

$\begin{cases} H_{234} \\ H'_{234} \end{cases}$  Closely related to the EMP result  
for the

↪ IR finite contributions coming from

gg scattering  $\rightarrow$  gg annihilation  $=$  gg → gg

as  $h \rightarrow 0$  we recover for  $H_{234}$  and  $H'_{234}$  the EMP result.

which is the setting of Fig. 10.11

and  $\pi$  scattering (non singlet virtual)

In addition a multiple structure of factorization  
can be considered namely

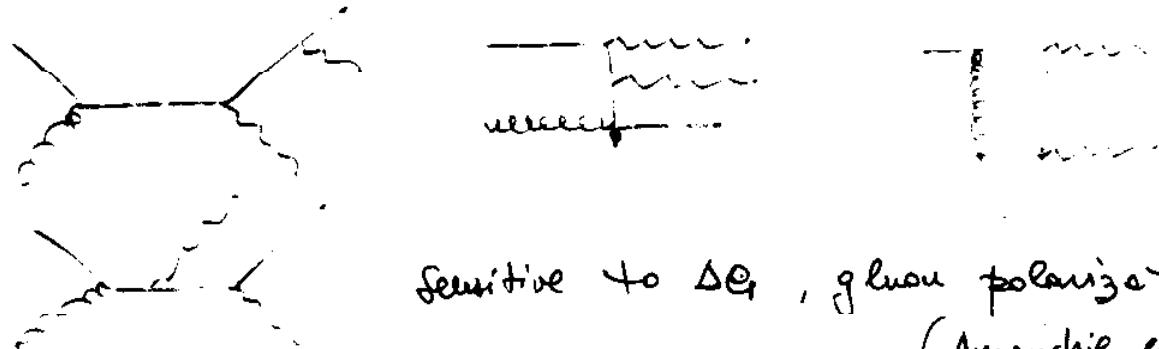
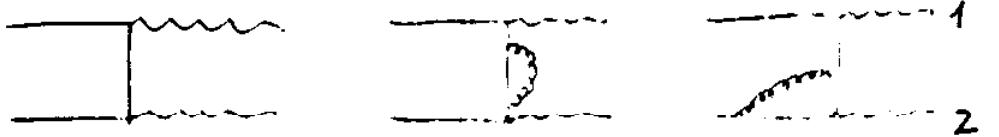
$$T_E^{(2,1,\dots)} = T_E^{(1)} + \sum_{\mu} A_{\mu} \left[ \frac{1}{2} \epsilon_{\mu\nu\lambda} \epsilon_{\rho\sigma\tau} \right] T_E^{(\rho\sigma\tau)}$$

new structures  
due to transverse spin

the longitudinal gg sector of DY  
for longitudinally polarized contributions  
from the initial state is also in progress

Double prompt photon production to NLO

(Gordon and c.c.)



sensitive to  $\Delta Q_s$ , gluon polarization

(Aurenche et al.)  
unpolarized

$$\frac{d\sigma}{dk_t^1 dy^1 dz}$$

↓  
photon 2

$$z = -k_t^1 \cdot k_t^2 / |k_t^1|^2$$

differential in the transverse momentum and rapidity  
of one of the photons

$z \rightarrow$  information about the  $k_t$  of photon 2  
but not its rapidity

studied in the  $\tau_{\gamma\gamma}$  variable of Gordon et al.

$$\tau_{\gamma\gamma} = \frac{E_\gamma^1 E_\gamma^2}{(p_\gamma^1 + p_\gamma^2)^2}$$

lowest order  $\rightarrow z = 1$  back to back  
 $w = 1$

$$\frac{d\sigma}{dvdwdt} (q\bar{q} \rightarrow \gamma\gamma) = \frac{2\pi\alpha_{ew}^2 e_q^4}{3S} \frac{1-2v+2v^2}{v(1-v)} \delta(1-z)\delta(1-w)$$

$$\frac{d\sigma}{dk_t^2 dy dz} = \frac{2\pi k_t^{-1}}{\pi S} \int_{VW}^V \frac{dv}{(1-v)} f_q^A(x_1, M^2) f_{\bar{q}}^B(x_2, M^2) \frac{d\hat{\sigma}}{dvdwdt} + (q\bar{q})$$

There are also fragmentation contributions



$$\frac{d\sigma}{dk_t^2 dy dz} = \frac{2\pi k_t^{-1}}{\pi S} \int_{VW}^V \frac{dv}{(1-v)} f_q^A(x_1, M^2) f_{\bar{q}}^B(x_2, M^2) \Delta f_q^A(x_1, M^2) \Delta f_{\bar{q}}^B(x_2, M^2)$$

$$\Delta f_q^A(x, Q^2) = f_{q,+}^A(x, Q^2) - f_{q,-}^A(x, Q^2)$$

↑ fragmentation scale

$$\Delta f_q^A(x, Q^2) = f_{q,+}^A(x, Q^2) - f_{q,-}^A(x, Q^2)$$

The next quantities are calculated in the C.H. frame,  
of the system - given below.

All the calculation can be done analytically

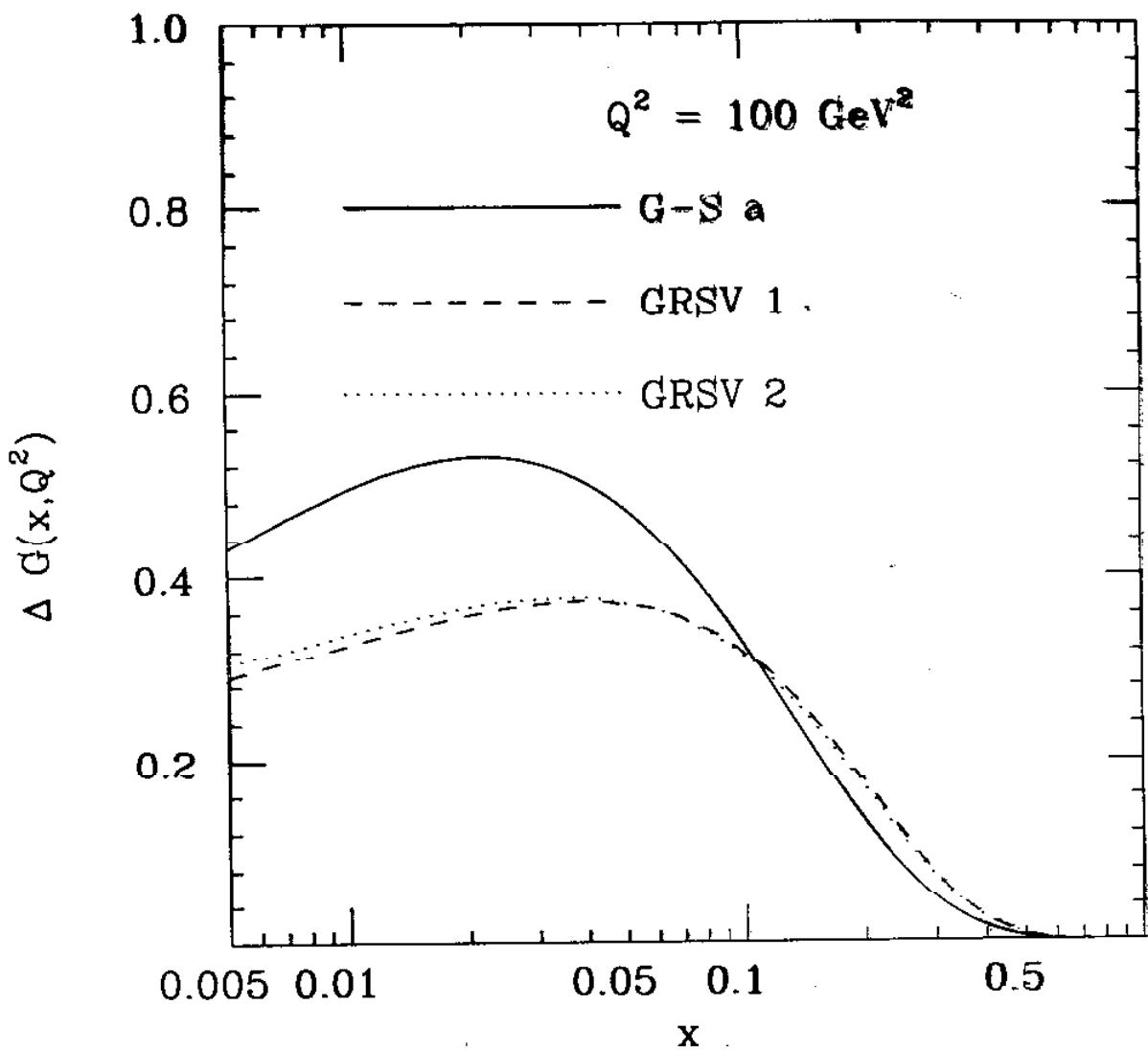
$$\hat{PS}_3 = C \left( \frac{-\epsilon}{1-\epsilon} \right) \int dv dw (1-v)^{-\epsilon} (1-w)^{1-\epsilon} v^{2-2\epsilon} w^{-\epsilon} \\ \times \int_0^{\pi} d\theta_2 dz \delta(z - m \cdot k_2) \sin \theta_2 \int_0^{\pi} d\theta_1 \sin \theta_1$$

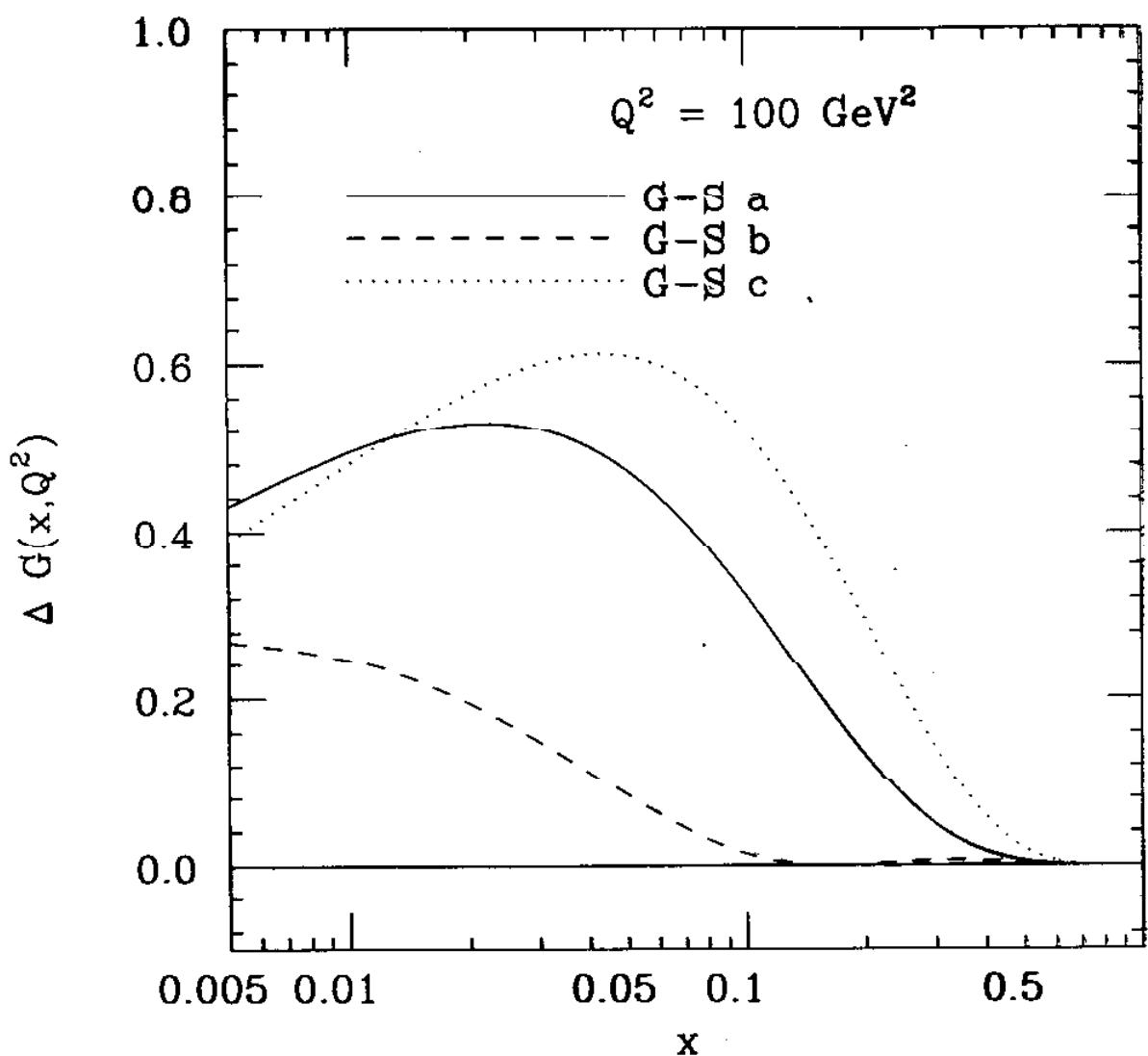
$$m = (sk_3 + t\mu_2 + u\mu_1) / tu$$

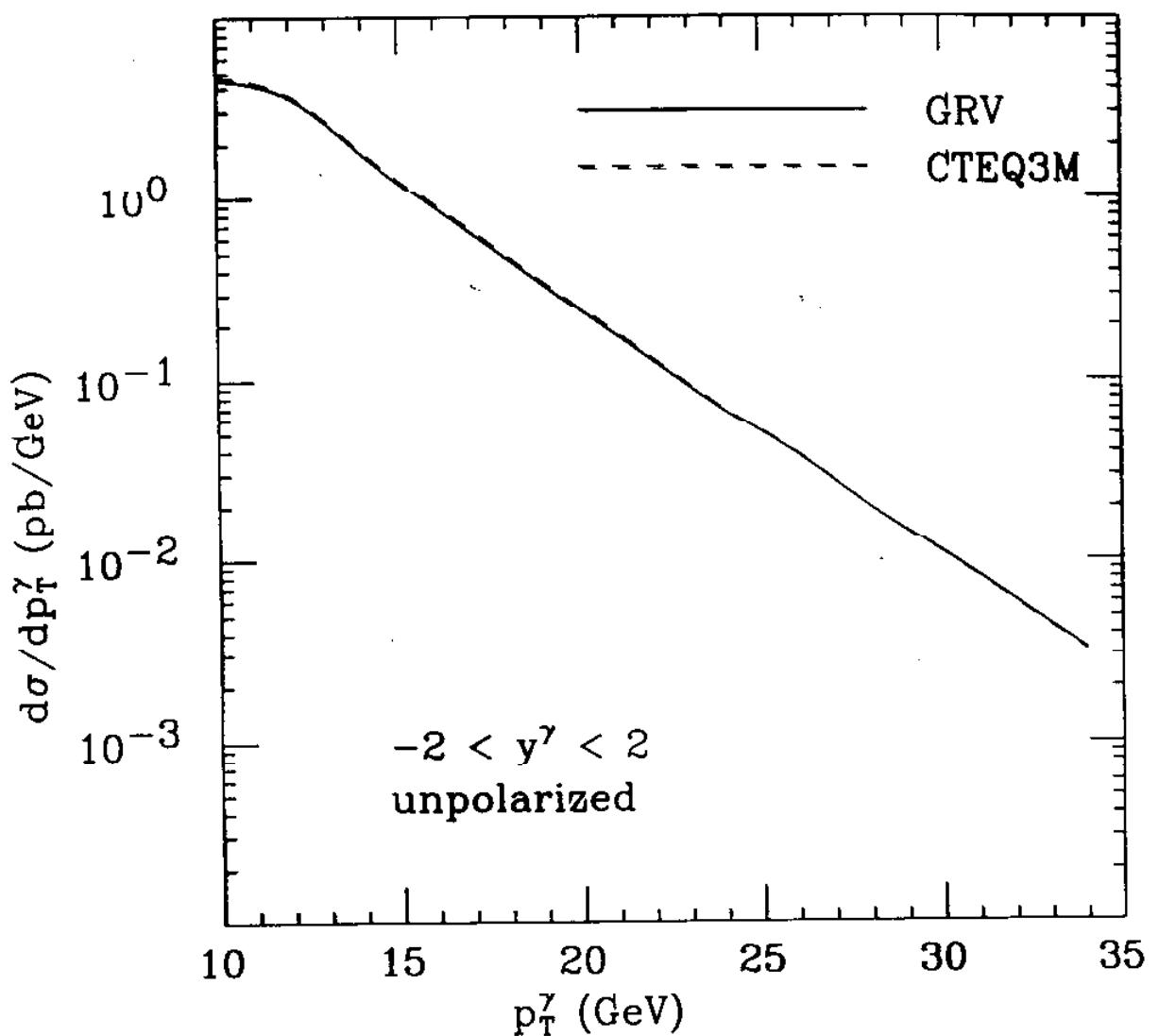
again : in the  $\overline{HS}_p$  scheme

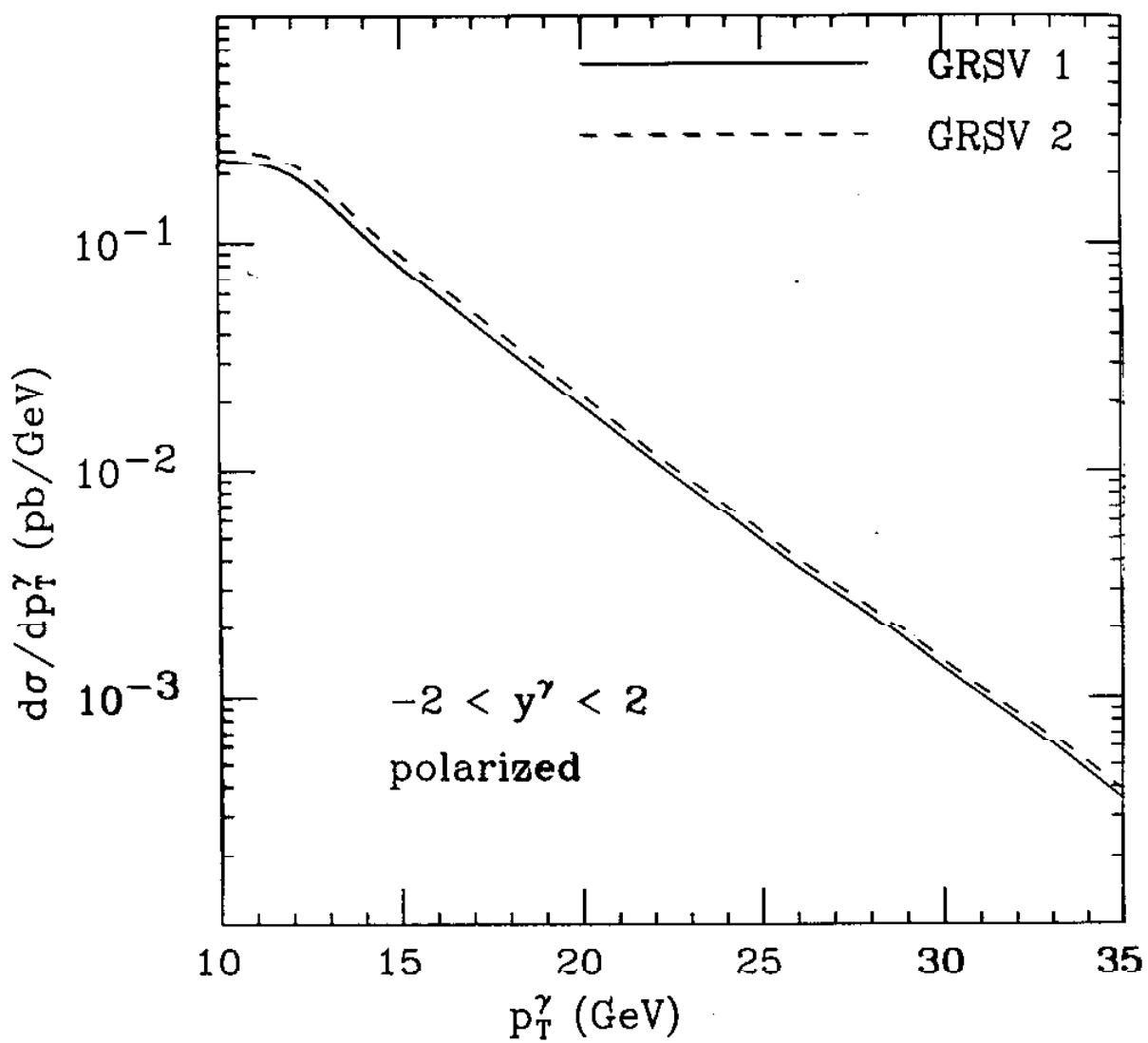
$$\underline{J}^{\text{pol}} = \epsilon^{\text{imp}} \quad \text{and helicity is conserved}$$

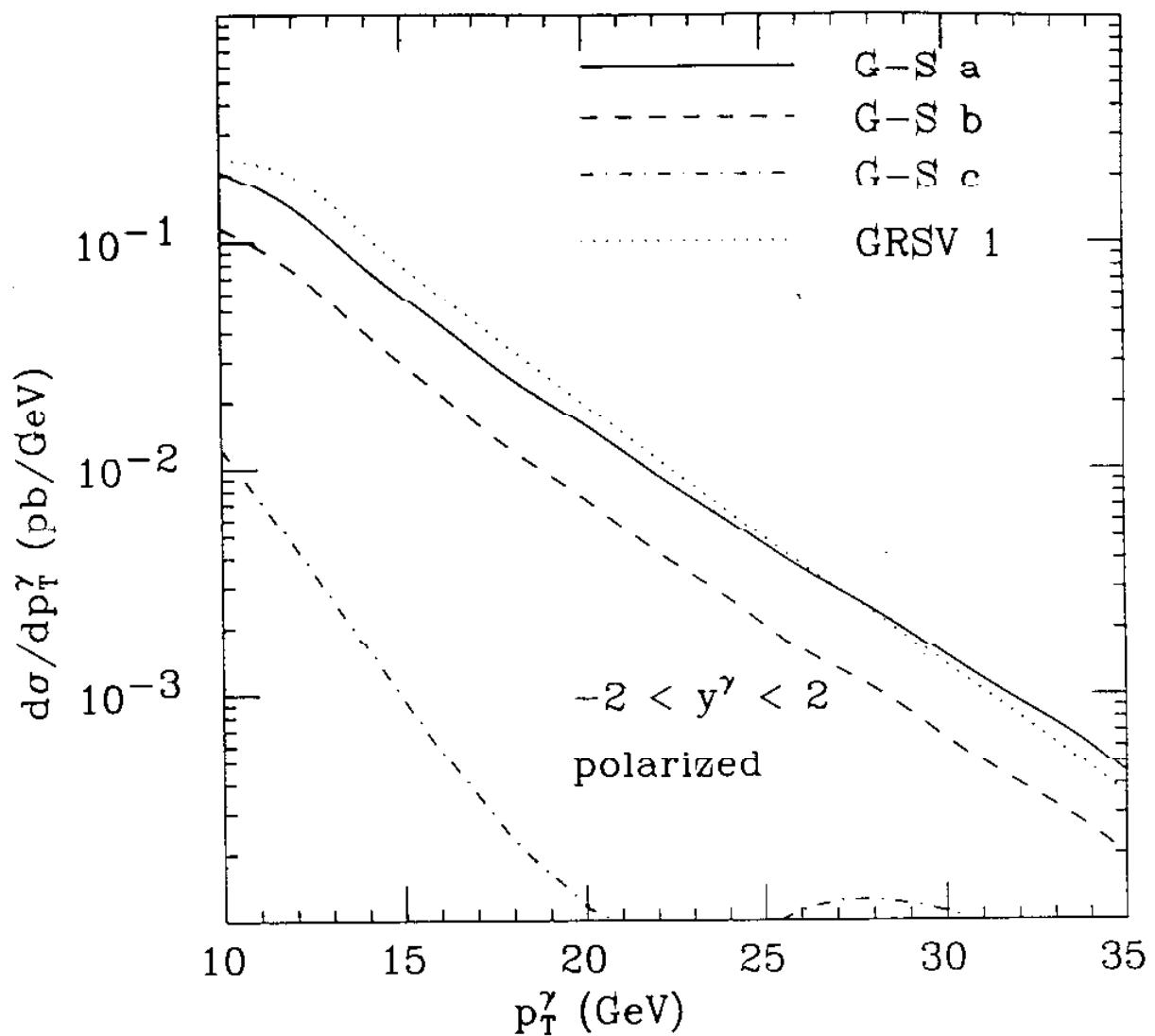
(Arenhoe et al)

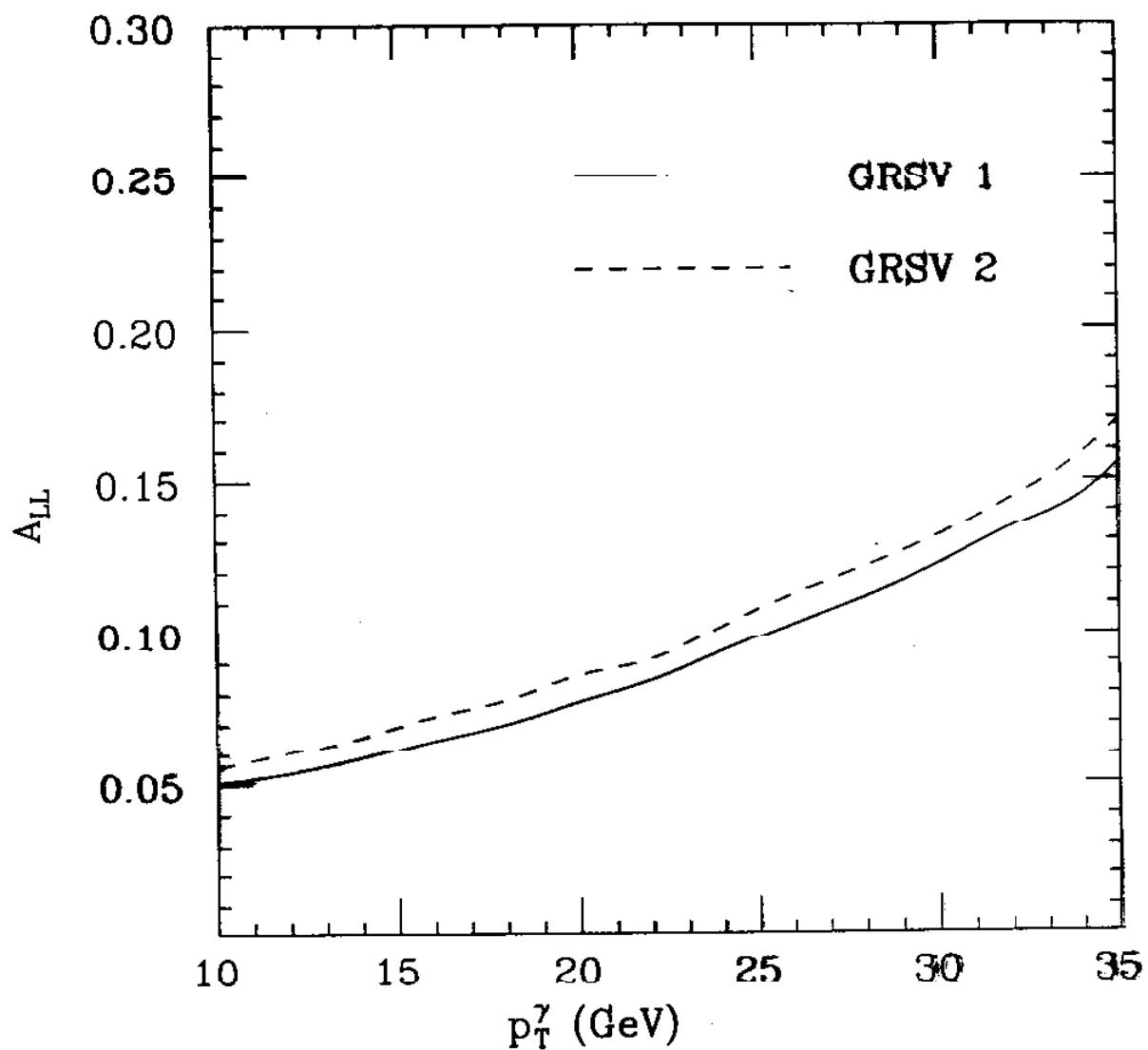


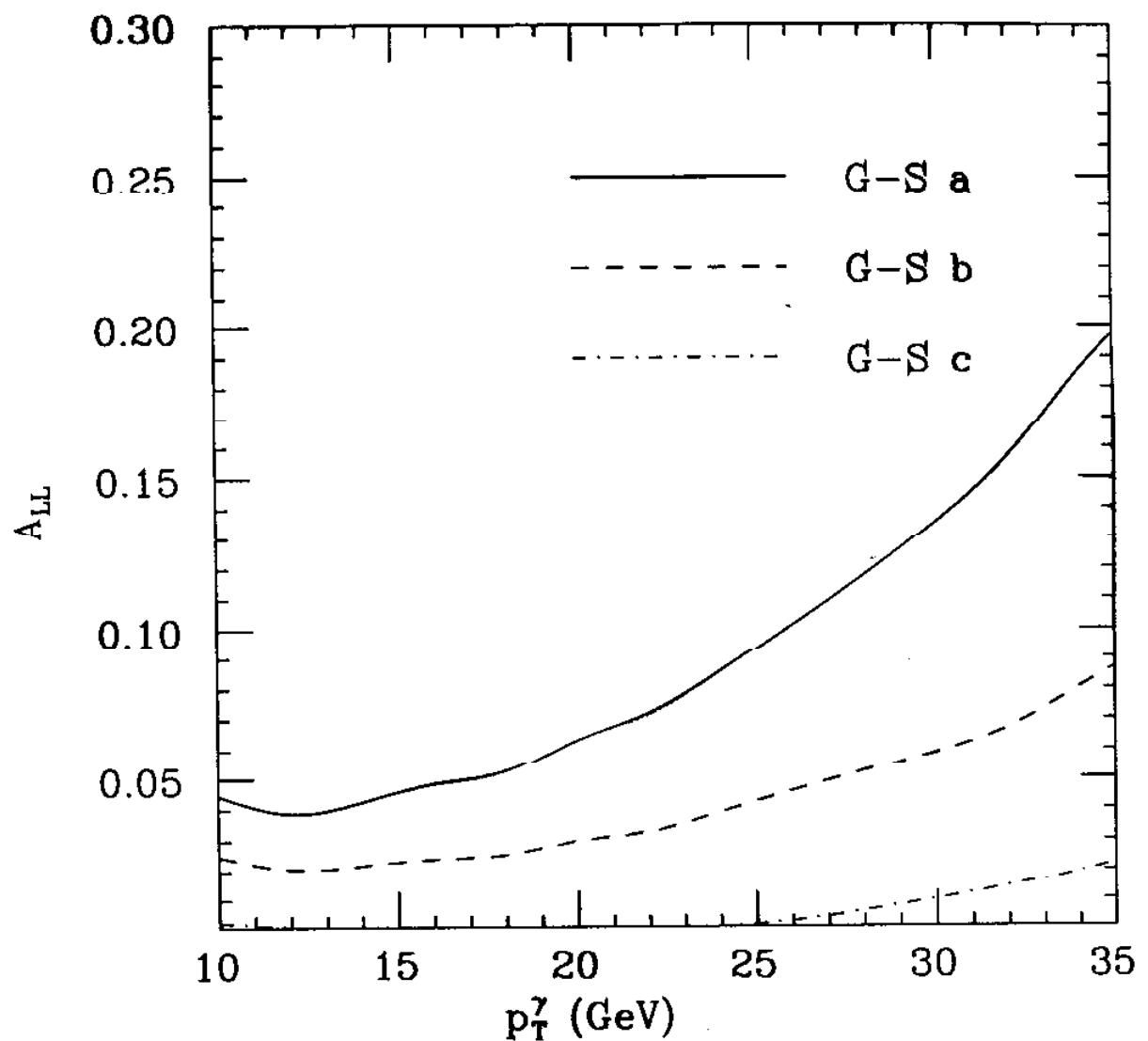


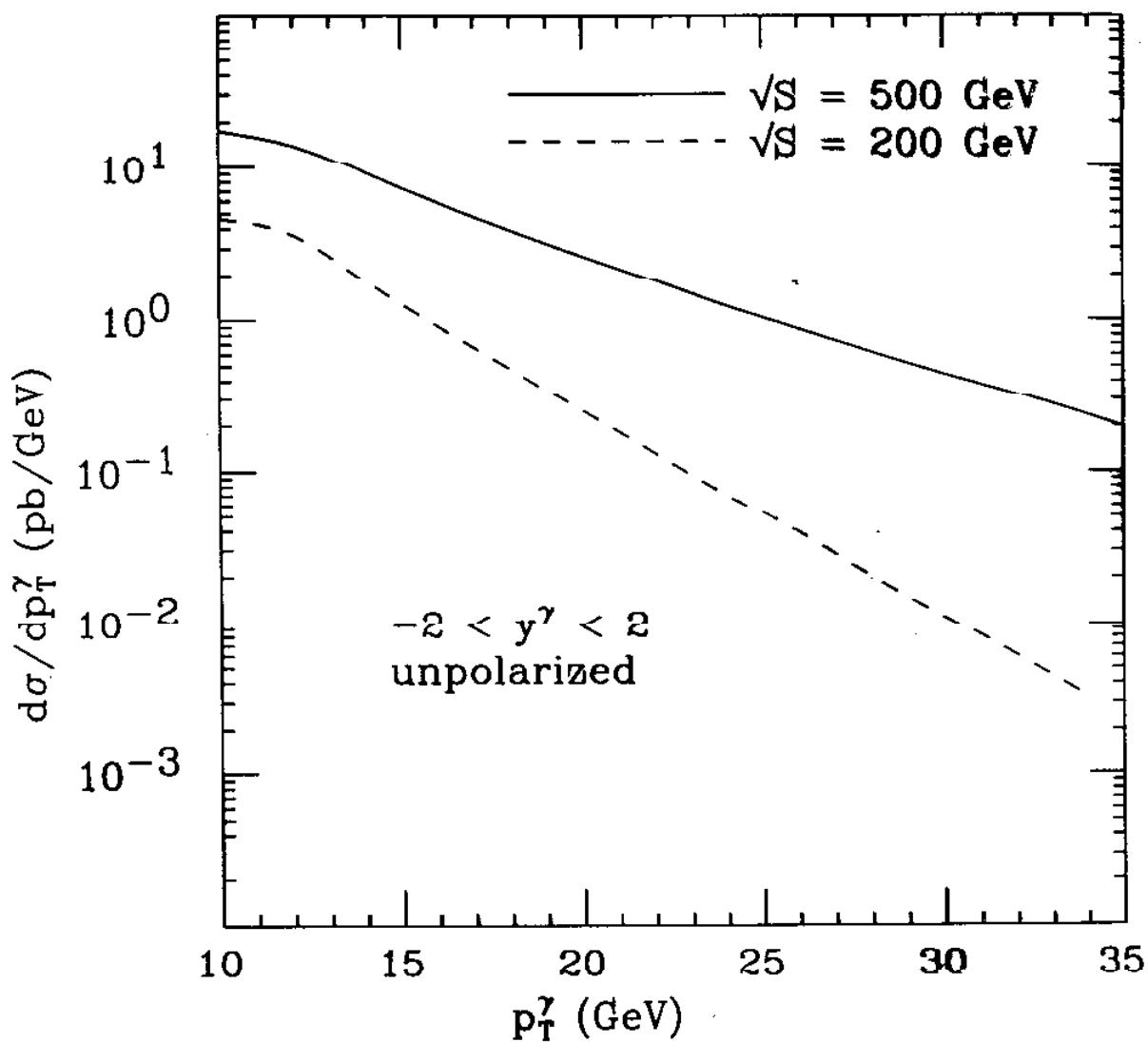


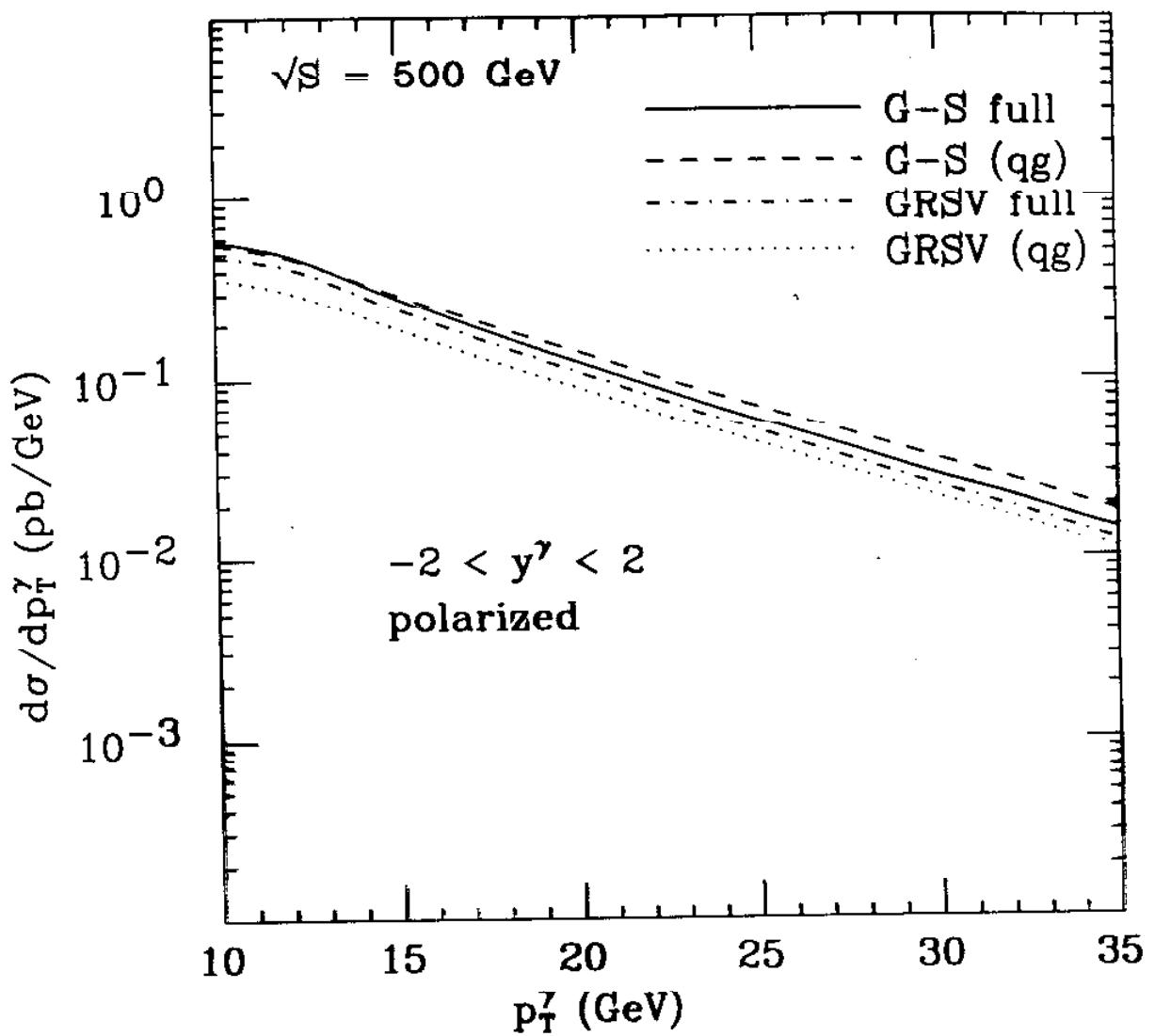


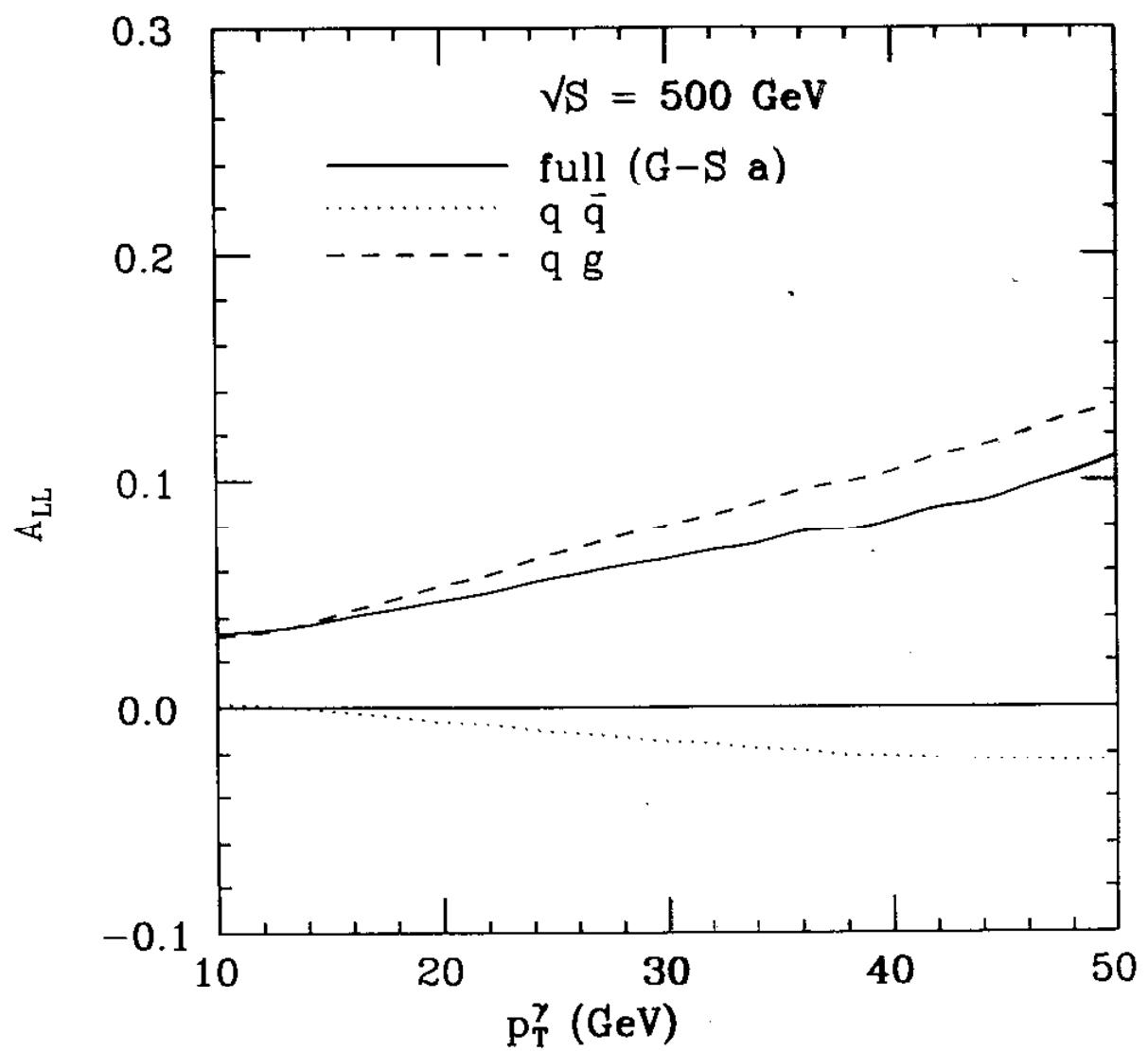


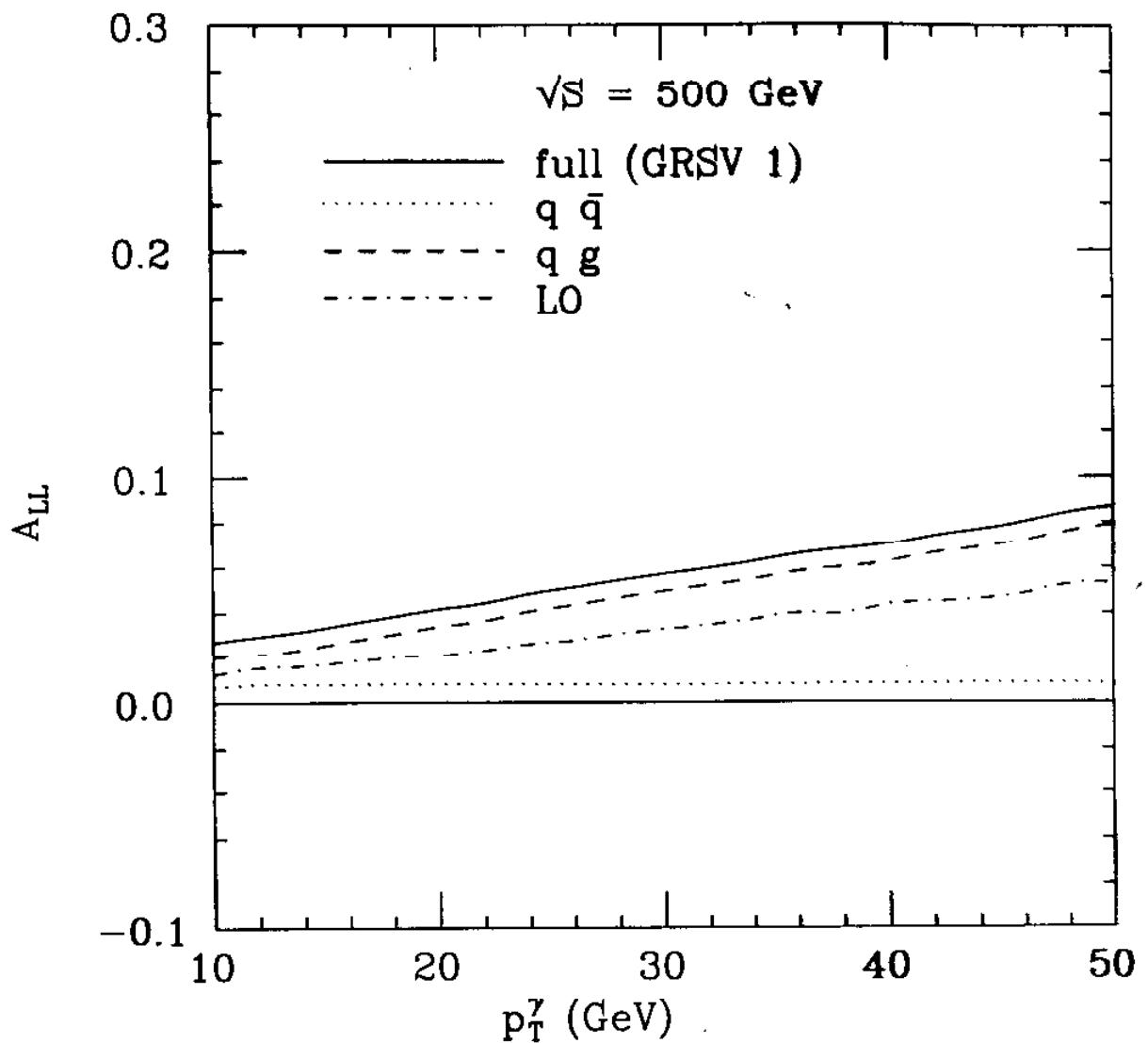


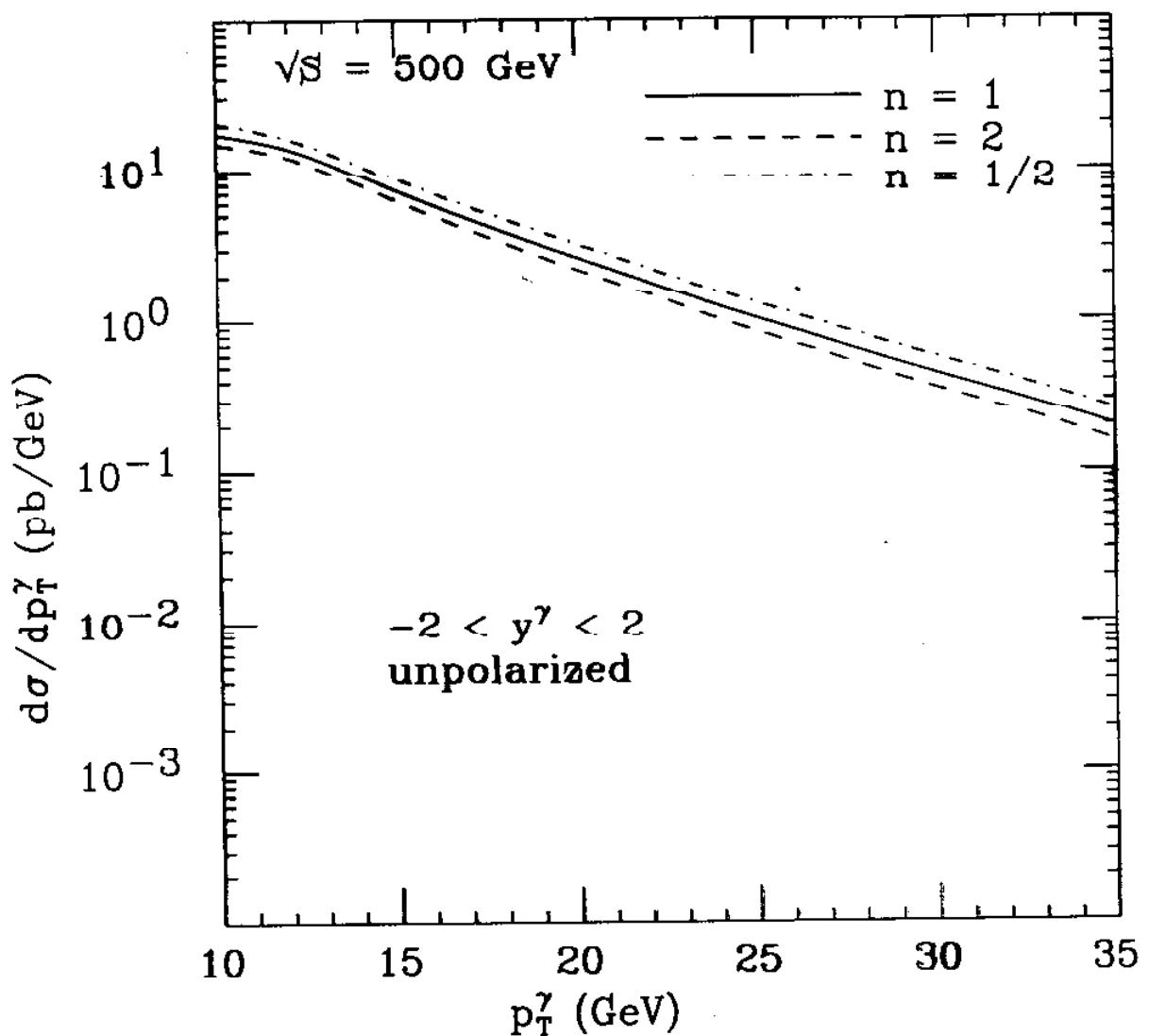


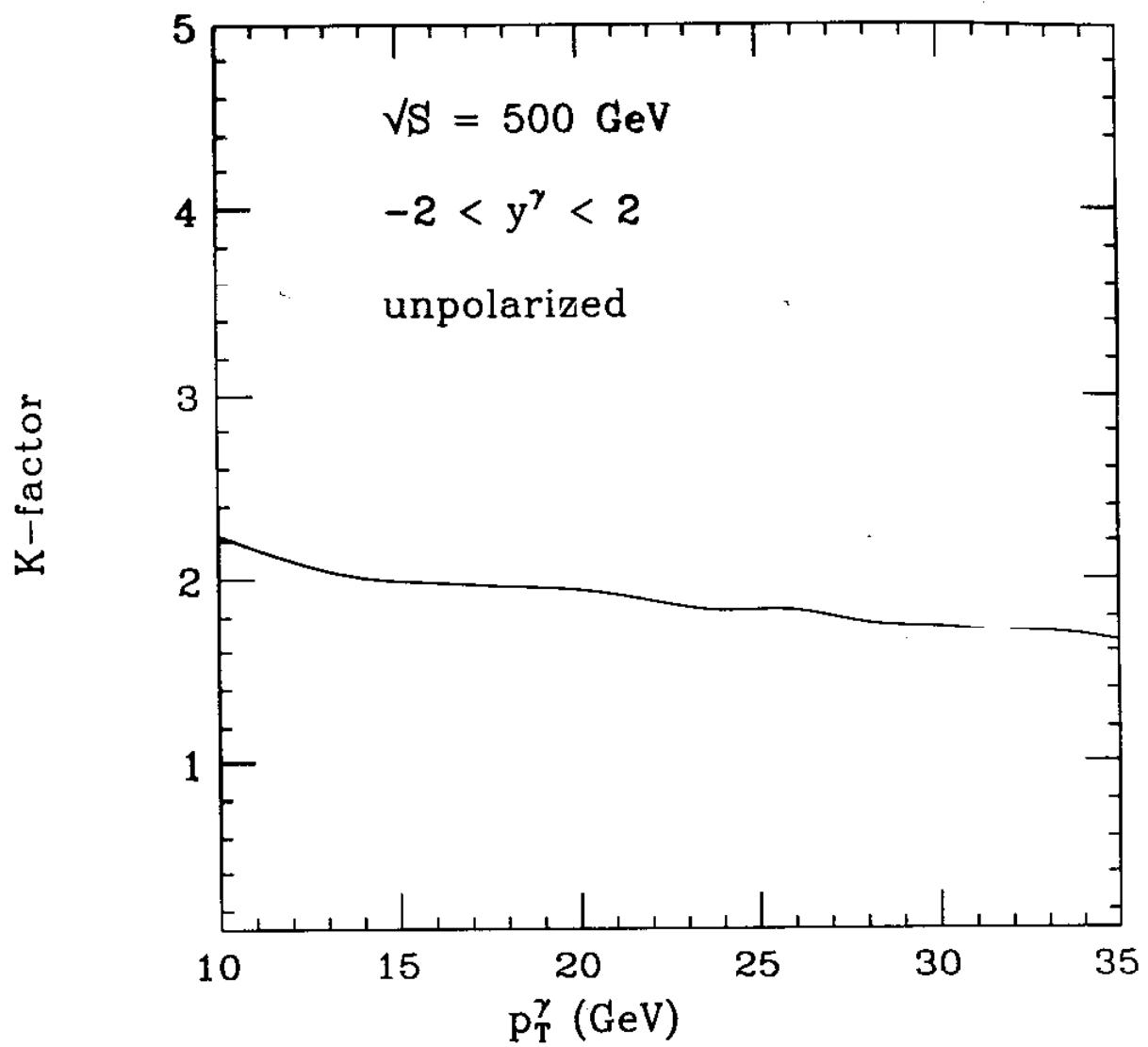












What needs to be done

1. total cross section
2. isolation cuts for  $I^{\pm}$  Muon flight
3. Transresistor

Work in progress Flight

Chang London, Field, c.c.