B. ERMOLAEV 1

Small-x-behaviour of g_1 and g_2

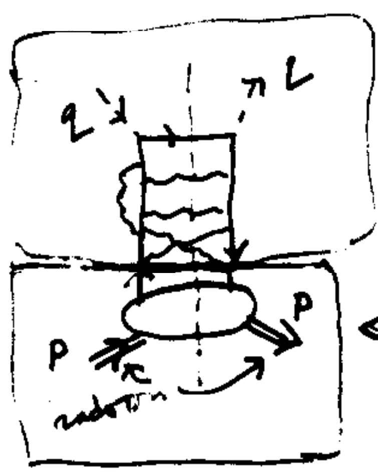
Spin-dependent part of the hadronic tensor is

$$W^{(A)}_{\mu\nu} = i\varepsilon_{\mu\nu\alpha\beta}\frac{Mq_\alpha}{pq}\left\{ S_\beta\, g_1(x,Q^2) + \left(S_\beta - P_\beta \frac{Sq}{pq}\right) g_2(x,Q^2)\right\}$$

$x = Q^2/2pq$, $Q^2 = -q^2 > 0$. $x \ll 1 \Rightarrow$

$$W^{(A)}_{\mu\nu} = i\varepsilon_{\mu\nu\alpha\beta}\frac{Mq_\alpha}{pq}\left\{ S^{\parallel}_\beta \cdot g_1 + S^{\perp}_\beta (g_1+g_2)\right\},$$

$S^{\parallel} \in \{p,q\}$, $S^{\perp} \perp \{p,q\}$.

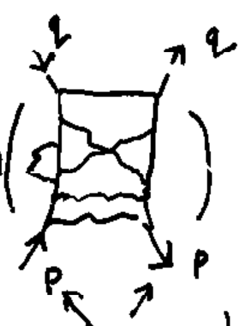


Pert QCD realm, depends on x explicitly

Altarelli-Ball-Forte, Moch-..., Roberts-R., Dyakonov et al

non-Pert QCD realm $\Leftrightarrow$ fits/models

Small-x-behavior of g_1, g_2 essentially is stipulated by, i.e. can be predicted by PQCD:

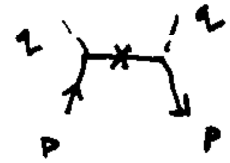
Im() $= i\varepsilon_{\mu\nu\alpha\beta}\frac{mq_\alpha}{pq}\left\{ S^{\parallel}_\beta \cdot g^{\parallel} + S^{\perp}_\beta\, g^{\perp}\right\},$

$g^{\parallel} = g_1$, $g^{\perp} = g_1 + g_2$.

 $\leftrightarrow$ flavour nonsinglet

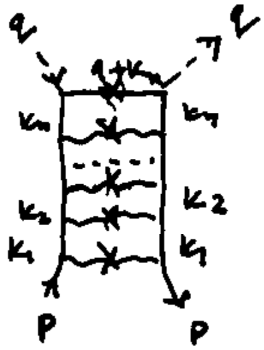
 $\leftrightarrow$ flavour singlet

Born approximation:



$$g^\perp = g^\parallel = \frac{e_q^2}{2}\,\delta(1-x) \Rightarrow g_2^{Born} = 0.$$

Beyond Born approximation, Leading (Double) Logarithmic Approximation



$$\sim \int \frac{dk_1 \ldots dk_n\, \delta((p-k_1)^2) \prod \delta((k_i-k_{i-1})^2)\, \delta((q+k_n)^2)}{(k_1^2-m^2)^2 (k_2^2-m^2)^2 \cdots (k_n^2-m^2)^2} \cdot N^{(n)}_{\mu\nu},$$

$$N^{(n)}_{\mu\nu} = i\,\varepsilon_{\mu\nu\alpha\beta}\, m\, q_\alpha \left\{ \overset{g_1}{N^{(n)}_\parallel} S^\parallel_\beta + \overset{g_1+g_2}{N^{(n)}_\perp} S^\perp_\beta \right\}.$$

$\underline{n=1}$ $\quad N^{(1)}_\perp = N^{(1)}_\parallel \Rightarrow g_2^{(1)} = 0$

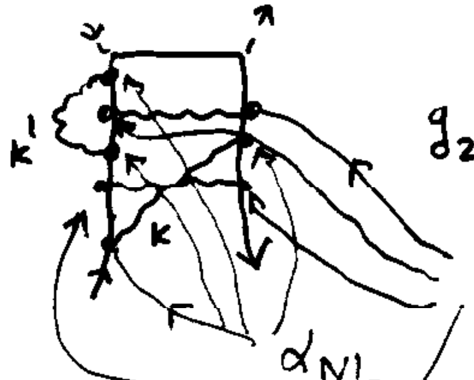
It coincides with results by Altarelli-Lampe-Nason-Ridolfi, Mertig-van Neerven

$n=2$: $\quad N^\perp = -N^\parallel$

$n=3,4,\ldots$ $\quad N^\perp = -(n-1)\,N^\parallel \Rightarrow$

$$g_2^{NS} = -\frac{\partial g_1^{NS}}{\partial \ln \alpha_s}$$

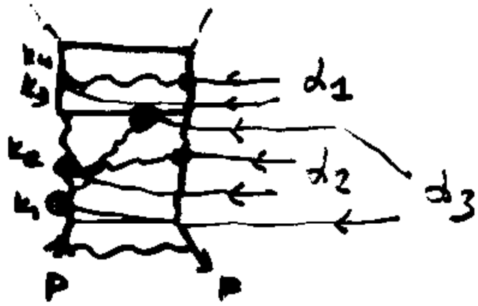
Nonladder graph contribution:



Erm-Troyan

$$g_2^{NS} = -\left.\frac{\partial g_1^{NS}(\alpha_L, \alpha_{NL})}{\partial \ln \alpha_L}\right|_{\alpha_L = \alpha_{NL} = \alpha}$$

Flavour singlet:



$$N^{\perp} = -\left[(n_q - 1) + \frac{1}{2}(n_g - 1) \right] N^{\parallel}$$

Together with nonladder graphs: Erm-Troyan

$$g_2^S = -\left(\frac{\partial g_1^S}{\partial \ln \alpha_1} + \frac{1}{2} \frac{\partial g_1^S}{\partial \ln \alpha_2} \right) g_1^S(\alpha_1, \alpha_2, \alpha_3) \Big|_{\alpha_1 = \alpha_2 = \alpha_3 = \alpha_s}$$

Therefore, we can use the above expressions only if the analytical expression for g_1 is known.

At small x in LLA Bartels-Manayenkov-Erm-Ryskin

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} \frac{\omega}{\omega - \sqrt{\omega^2 - \frac{2\alpha_s C_F}{\pi}\left(1 + \frac{\alpha_s}{\pi} \frac{1}{N\omega^2}\right)}}$$

$$\cdot \exp\left\{ \frac{1}{2} \ln \frac{Q^2}{m^2} \cdot \left[\omega - \sqrt{\omega^2 - \frac{2\alpha_s C_F}{\pi}\left(1 + \frac{\alpha_s}{\pi N} \frac{1}{\omega^2}\right)} \right] \right\}$$

$$\Rightarrow \quad g_1, g_2 \underset{x \to 0}{\sim} \left(\frac{1}{x}\right)^{b_{NS}}$$

$$b_{NS} = \left(\frac{2\alpha_s C_F}{\pi} \right)^{1/2} \left(1 + \frac{1}{2N^2} \right).$$

$$g_1^S = \left(\sum_q e_q^2\right) \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} e^{\lambda y} \frac{z(z^2+\omega z + b_3 - b_4)}{z^2(\omega+z)^2 - 4b_1 b_2 - (b_3-b_4)^2}$$

$$y = \ln\frac{Q^2}{\mu^2}, \quad 2z^2 = \omega^2 - 2(b_3+b_4) + \sqrt{\omega^4 - 4\omega^2(b_3+b_4) + 16 b_3 b_4 - 8 b_1 b_2},$$

$$b_1 = b_{qq} = \frac{\alpha_s C_F}{\pi} - \frac{\alpha_s^2}{\pi^2}\frac{C_F N}{\omega^2},$$ Erm - Troyan

$$b_2 = b_{qg} = -\frac{\alpha_s}{2\pi} + \frac{\alpha_s^2}{\pi^2}\frac{N}{2}\frac{1}{\omega^2},$$

$$b_3 = b_{gq} = \frac{\alpha_s C_F}{2\pi} + \frac{\alpha_s^2}{\pi^2}\frac{C_F}{2N}\frac{1}{\omega^2},$$

$$b_4 = b_{gg} = \frac{2\alpha_s N}{\pi} - \frac{\alpha_s^2}{\pi^2} 2N^2 \frac{1}{\omega^2}.$$

Therefore,

$$g_1, g_2 \underset{x\to 0}{\sim} \left(\frac{1}{x}\right)^{\omega_0}$$

where ω_0 is solution of

$$\omega_0^4 - 4(b_3+b_4)\omega^2 + 16 b_3 b_4 - 8 b_1 b_2 = 0.$$

if nonladder graphs are neglected,

$$\omega_0 = \left(\frac{\alpha_s}{2\pi}\right)^{1/2}\left[8N + C_F + 2\sqrt{(4N-C_F)^2 - 2C_F}\right]^{1/2}$$

$N=3$, $C_F = \frac{4}{3}$. $N \gg C_F \Rightarrow$ gluon contribution dominates like in the Pomeron case (unpolarized DIS)

Pure gluon ladder + nonladder contributions

yields

$$\omega_g = \omega_0^{\text{Ladder}}\left(1 - \frac{1}{16}\right)$$

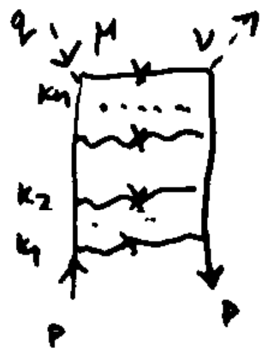
↑ does not depend on $C_F, N, \ldots$

In pure quark (non-singlet) case ← opposite signs

$$\omega_q = \omega_q^{\text{Ladder}}\left(1 + \frac{1}{2N^2}\right)$$

General case: $\omega_0 = \left(\frac{\alpha_s N}{2\pi}\right)^{1/2} \cdot 3.45$ Bartels-Ermolaev-Ryskin

$S^{\perp}$-dynamics VS $S^{\parallel}$-dynamics:



$$\sim \int \frac{dk_1 \ldots dk_n \;\prod \delta(\;)}{(k_1^2-m^2)^2 \ldots (k_n^2-m^2)^2} \cdot N_{\mu\nu}$$

$$N_{\mu\nu}^{(n)} = m\,\mathrm{Tr}\left(\gamma_5 \hat{S} \cdot \gamma_{\alpha_1}\hat{k}_1 \cdots \gamma_{\alpha_n}\hat{k}_n \cdot \gamma_\nu \hat{q} \gamma_\mu \cdot \hat{k}_n \gamma_{\alpha_n} \cdots \hat{k}_1 \gamma\right.$$

$$\gamma_\nu \hat{q} \gamma_\mu = i\,\varepsilon_{\mu\nu\lambda\rho}\, q_\lambda \cdot \gamma_\rho \gamma_5 + \underbrace{\mathrm{symm}(\mu,\nu)}_{\text{does not contrib.}}$$

$$N_{\mu\nu}^{(n)} \sim m\,(-1)^n q_\lambda\, i\,\varepsilon_{\mu\nu\lambda\rho}\,\mathrm{Tr}\left(\gamma_\rho \cdot \hat{k}_n \ldots \hat{k}_2 \hat{k}_1 \hat{S} \hat{k}_1 \hat{k}_2 \ldots \hat{k}_n\right)$$

$$\equiv m\, i\,\varepsilon_{\mu\nu\lambda\rho}\, q_\lambda \cdot N_\rho^{(n)};$$

$$N_\rho^{(n)} = (-1)^n\,\mathrm{Tr}\left(\gamma_\rho \cdot \hat{k}_n \ldots \hat{k}_1 \hat{S} \hat{k}_1 \ldots \hat{k}_n\right).$$

$n = 1 \qquad N_\rho = 4\left(k_1^2 S_\rho - 2k_{1\rho} \cdot k_1 S\right) \approx$

$k_1^2 \approx k_{1T}^2$ in LLA $\qquad \approx 4\left(k_{1T}^2 S_\rho - 2k_{1\rho} \cdot k_1 S\right)$

$$S^{\parallel}: \; N_\rho^{\parallel} = 4k_{1T}^2 S_\rho^{\parallel}$$

$S^\perp$: $\quad N_\rho^\perp = 4\left(k_{1\perp}^2 S_\rho^\perp - 2\underbrace{k_{1\perp\rho}\, k_{1\perp}}_{=\frac{1}{2}k_{1\perp}^2} S_\perp \right) \Rightarrow N_\rho^\perp = 0$

when integrated over the azimuthal angle

$n=2$

$$\frac{N_\rho}{4} = k_1^2 k_2^2 S_\rho - 2\overbrace{k_1 S \cdot k_{1\rho}}^{k_{1\perp}^2/2} k_2^2 - 2\overbrace{k_2 S \cdot k_{2\rho}}^{k_{2\perp}^2/2} k_1^2$$
$$+ 2k_1 S \cdot k_{2\rho} \cdot 2k_1 k_2$$

$S^\parallel$: $\quad k_{1\perp}^2 k_{2\perp}^2 S_\rho^\parallel$

$S^\perp$: $\quad k_{1\perp}^2 k_{2\perp}^2 S_\rho^\perp - k_{1\perp}^2 \cdot k_{2\perp}^2 \cdot S_\rho^\perp - k_{1\perp}^2 k_{2\perp}^2 \cdot S_\rho^\perp$

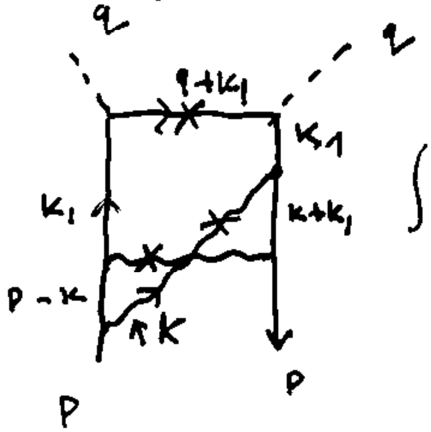
$\delta((k_1-k_2)^2) \longrightarrow \quad + \underbrace{2k_1^+ S^\perp \cdot k_{2\rho} \cdot (k_1^2 + k_2^2)}$ $=$

does not contribute in LLA

$= \; - N_\rho^\parallel$

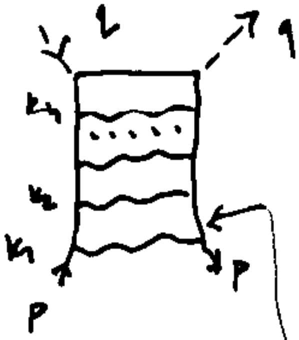
$\forall n$: $\quad \frac{N_\rho^\perp}{4} = k_1^2 \ldots k_n^2 S_\rho - 2\overbrace{k_1 S \cdot k_{1\rho}}^{k_{1\perp}^2/2} \cdot k_2^2 \ldots k_n^2 -$
$$- 2\underbrace{k_2 S \cdot k_{2\rho}}_{k_{2\perp}^2/2} k_1^2 k_3^2 \ldots k_n^2 - \ldots$$
$$- 2\underbrace{k_n S \cdot k_{n\rho}}_{k_{n\perp}^2/2} k_1^2 \ldots k_{n-1}^2$$

$\Rightarrow \quad N^\perp = -(n-1)\, N^\parallel$



$$\int \frac{dk_1\, dk}{(k_1^2 - m^2)^2\; k^2} \; \frac{N_{\mu\nu} \prod \delta(\;)}{(k+k_1)^2 (p-k)}$$

Beyond DLA. g_1^{NS} : DL + SL



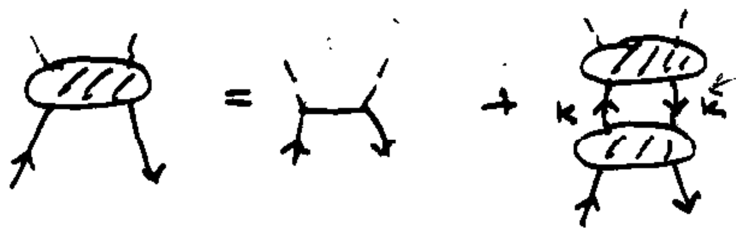
DGLAP $\quad k_{1\perp} < k_{2\perp} < \dots < k_{n\perp}$ } true for $x \sim 1$ but false for $x \ll 1$

Gribov, Gorshkov, Frolov, Lipatov 196.

k_T-ordering results into neglecting e.g. $\alpha_s \ln^2 x$ compared to $\alpha_s \ln Q^2 \ln x$ which is not correct when $x \ll 1$.

DGLAP

min $k_\perp \equiv k_{1\perp}$



Solution of IREE and DGLAP:

DGLAP
$$\tilde{g}_1^{NS} = \frac{e_q^2}{2} \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} e^{\frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\omega}+\frac{1}{2}\right) \ln \frac{Q^2}{Q_0^2}}$$

IREE
$$g_1^{NS} = \frac{e_q^2 \cdot 2\pi}{\alpha_s C_F} \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} \omega F_0 \, e^{\left(\frac{1}{\omega}+\frac{1}{2}\right)\omega F_0 \ln \frac{Q^2}{\mu^2}}$$

Ermolaev + Troyan

where $C_F = \frac{N^2-1}{2N}$,

$$F_0 = 4\pi^2 \frac{\omega - \left[\omega^2 - \overbrace{\frac{\alpha_s C_F}{2\pi}}^{\equiv a^2}\left(1+\frac{\omega}{2}\right)\right]^{1/2}}{1+\omega/2}.$$

DGLAP eqs were obtained for $x \sim 1 \leftrightarrow \omega \sim 1$;

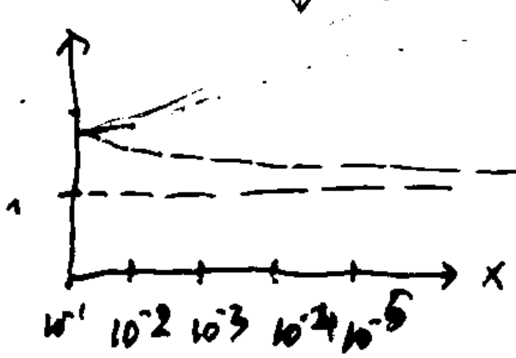
when $\omega \sim 1$ one can expand F_0 into series in $\frac{a}{\omega^k}$, $k = 1, 2$

$$\frac{F_0}{4\pi^2} = \frac{\alpha_s C_F}{4\pi\omega} \Rightarrow$$

$$g_1 = \frac{i}{2}\int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)\left(\frac{1}{\omega}+\cdots\right)\omega\, e^{\frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\omega}+\frac{1}{2}\right)\ln\frac{Q^2}{\mu^2}}$$

corresponds to DGLAP with 1 loop anom. dimension.

$$r(x) = g_1^{SL+DL}/g_1^{DL}.$$



could be anticipated

The reason for the increase is that SL-contributio change the power:

$$\text{DL:} \quad g_1 \underset{x\to 0}{\sim} \left(\frac{1}{x}\right)^{b} \left(\frac{Q^2}{\mu^2}\right)^{b/2}$$ Bartels-Erm-Manayenkov-Ryskin

$$\text{SL+DL:} \quad g_1 \sim \left(\frac{1}{x}\right)^{b'} \left(\frac{Q^2}{\mu^2}\right)^{b'/2}$$ Erm-Troyan

$$b' = b\left[\left(1+\frac{b^2}{16}\right)^{1/2} + \frac{b}{4}\right]$$

when $\alpha_s \approx 0.18$, $b \approx 0.4$, $b' = 0.4(1+0.1)$

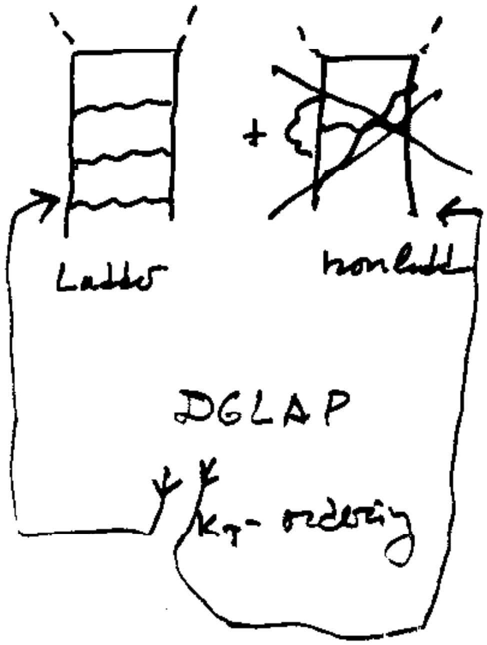
Therefore, taking SL into account brings a new effect (though well-known in the Regge calculations) compared to the DGLAP: SL-corrections change the power of the asymptotic power-like behaviour whereas

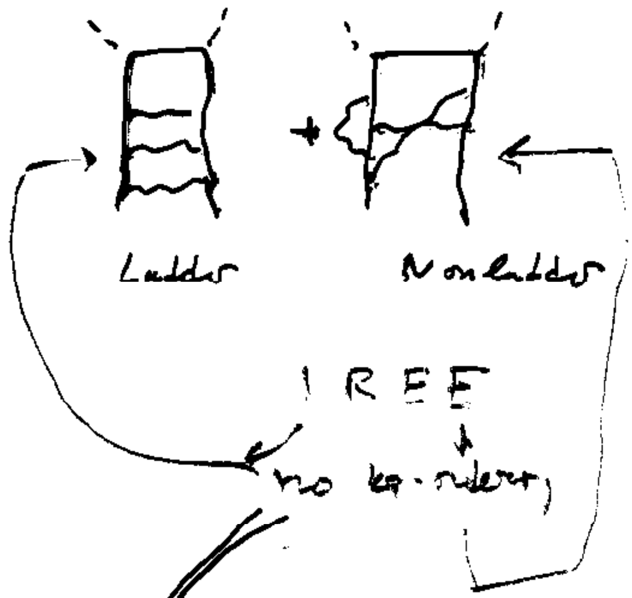
$$g_1^{NS} \underset{x\to 0}{\sim} \exp\left(\frac{2\alpha_s C_F}{\pi} \ln\frac{1}{x} \ln\frac{Q^2}{\mu^2}\right)^{1/2}$$

without SL, ...

IREE vs DGLAP:

actually both approaches sum the same graphs!





neglects the fact that g_1 has the negative signature

takes into account the running QCD effects

takes into account the negative signature effect

$\Downarrow$

the phase space much greater than in DGLAP.

Conclusion

1. In the double-log approximation g_2 small-x asymptotics is identical to the asymptotics of g_1, singular at $x=0$.

2. Taking into account single log contributions for g_1 changes the value of g_1 at fixed, though small, values of x and also changes the exponent in $\left(\frac{1}{x}\right)^{b}$ expression for its asymptotics. So, strictly speaking, it makes the range of value of $LLA = \text{zero}$

3. Taking into account, besides $\sim \ln Q^2$, also independent on Q^2, $\sim \ln^k x$ - contributions in DLA and SLA drastically change the small-x asymptotics of g_1, g_2 making them power-like.

4. Although one should increase accuracy of IREE there is 1st phen. observation $\left(\begin{smallmatrix}\text{Soffer,}\\ \text{Teryaev}\end{smallmatrix}\right)$ of $\left(\frac{1}{x}\right)^{\ell}$ be.