DIS '97

CHICAGO

On the Twist-2 and Twist-3 Contributions to Polarized Structure Functions and New Sum Rules

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DESY

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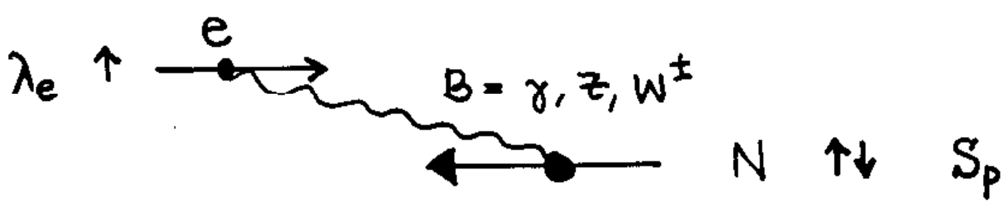
PHYS. LETT. B381 (1996) 296

NUCL. PHYS. B (1997), IN PRINT.

1. INTRODUCTION



N $|N\rangle = |q_i\uparrow, G\uparrow, \bar{q}_i\uparrow\rangle$

(up to) 5 STRUCTURE FUNCTIONS ON BORN LEVEL.

$g_k^i\big|_{k=1}^{5}$; $i = |\gamma|^2, |\gamma Z|, |Z|^2, |W^+|^2, |W^-|^2$.

BORN LEVEL : $g_1 \propto \Delta q + \Delta\bar{q}$

$g_5 \propto \Delta q - \Delta\bar{q}$

$\longrightarrow$ 3 FURTHER STRUCTURE FUNCTIONS

THERE SHOULD 3 LINEAR OPERATORS EXIST WHICH YIELD : (TWIST 2)

$g_2 = A_1(\underline{g_1}, g_5)$ WANDZURA-WILCZEK OPERAT 1977

$g_3 = A_2(g_1, g_5)$ NEW

$g_4 = A_3(g_1, \underline{g_5})$. DICUS - OPERATOR 1972

$\longrightarrow$

$\exists$ TWIST 3 CONTRIBUTIONS.

TECHNIQUES USED SO FAR:

(COLLINEAR) PARTON MODEL	KAUR 1977 BARTELSKI 1979 ANSELMINO, GAMBINO, KALINOWSKI 1994 ANSELMINO, EFREMOV, LEADER 1995 PHYS.REP.
ONLY LONG. POL.	VOGELSANG, WEBER 1991 LAMPE 1992 MATHEWS, RAVINDRAN 1992 DE FLORIAN, SASSOT 1995
LIGHT CONE CURRENT ALGEBRA	DICUS 1972 JOSHIPURA, ROY 1977
• OPERATOR PRODUCT EXPANSION	AHMED, ROSS 1976 JI 1993 RAVISHANKAR 1992
• <u>COVARIANT</u> PARTON MODEL	NASH 1971 JACKSON, ROBERTS, ROSS 1989 ROBERTS, ROSS 1996

STILL DISCREPANCIES.

$\longrightarrow$ TWIST 2 !

TWIST 3.

2. BASIC NOTATION

HADRONIC TENSOR:

$$W^{ab}_{\mu\nu} = \frac{1}{4\pi}\int d^4x\, e^{iqx} \langle pS|[J^a_\mu(x), J^b_\nu(0)]|pS\rangle$$

$$J^a_\mu(x) = \sum_{ff'} U_{ff'}\, \bar{q}_{f'}(x)\, \gamma_\mu (g^a_V + g^a_A \gamma_5)\, q_f(x)$$

$$\begin{aligned}
W^{ab}_{\mu\nu} = {} & \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F^i_1(x,Q^2) + \frac{\hat{p}_\mu \hat{p}_\nu}{p\cdot q} F^i_2(x,Q^2) \\
& - i\varepsilon_{\mu\nu\lambda\sigma} \frac{q_\lambda p_\sigma}{2p\cdot q} F^i_3(x,Q^2) \\
& + i\varepsilon_{\mu\nu\lambda\sigma} \frac{q_\lambda S_\sigma}{pq} g^i_1(x,Q^2) \\
& + i\varepsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda (p{\cdot}q S^\sigma - S{\cdot}q\, p^\sigma)}{(p\cdot q)^2} g^i_2(x,Q^2) \\
& + \left[\frac{\hat{p}_\mu \hat{S}_\nu + \hat{p}_\nu \hat{S}_\mu}{2} - S\cdot q \frac{\hat{p}_\mu \hat{p}_\nu}{p\cdot q}\right] \frac{1}{p\cdot q} g^i_3(x,Q^2) \\
& + S\cdot q \frac{\hat{p}_\mu \hat{p}_\nu}{(p\cdot q)^2} g^i_4(x,Q^2) + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \frac{S\cdot q}{p\cdot q} g^i_5(x,Q
\end{aligned}$$

$$\hat{p}_\mu = p_\mu - \frac{p\cdot q}{q^2} q_\mu \qquad \hat{S}_\mu = S_\mu - \frac{S\cdot q}{q^2} q_\mu$$

- LORENTZ & TIME REVERSAL INVARIANCE
- NO SF'S WHICH BREAK CURRENT CONSERVATION, I.E $\propto m_q^2 \ldots$

$$
\begin{aligned}
W_{\mu\nu} \;=\;& (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})F_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q}F_2(x,Q^2) - i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda P^\sigma}{2P\cdot q}F_3(x,Q^2) \\
& + i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda S^\sigma}{P\cdot q}g_1(x,Q^2) + i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda(P\cdot q S^\sigma - S\cdot q P^\sigma)}{(P\cdot q)^2}g_2(x,Q^2) \\
& + [\frac{\hat{P}_\mu \hat{S}_\nu + \hat{S}_\mu \hat{P}_\nu}{2} - S\cdot q\frac{\hat{P}_\mu \hat{P}_\nu}{(P\cdot q)}]\frac{g_3(x,Q^2)}{P\cdot q} \\
& + S\cdot q\frac{\hat{P}_\mu \hat{P}_\nu}{(P\cdot q)^2}g_4(x,Q^2) + (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})\frac{(S\cdot q)}{P\cdot q}g_5(x,Q^2), \qquad (8)
\end{aligned}
$$

with

$$
\hat{P}_\mu = P_\mu - \frac{P\cdot q}{q^2}q_\mu, \qquad \hat{S}_\mu = S_\mu - \frac{S\cdot q}{q^2}q_\mu. \qquad (9)
$$

Note: X. Ji: $\hat{S}_\rho = S_\rho - \frac{S\cdot q}{P\cdot q}P_\rho$

our notation	AEL	[illegible]	Lampe	Dicus	Wray
g_1	g_1	g_1	g_1	$F_3 + F_4$	$2(F_1 + F_2)$
g_2	g_2	g_2	g_2	$-F_4$	$-2F_2$
g_3	$-g_3$	g_5	$(g_4 - g_5)/2$	$2F_6$	$4F_4$
g_4	$g_4 - g_3$	$g_4 + g_5$	g_4	$2F_6 + F_7$	$2(F_6 + 2F_4)$
g_5	$-g_5$	$-g_3$	g_3	$F_7/(2x) - F_6$	$F_7/(2x) - 2F_5$
	J+R	Frankfurt	Ravishankar	Nash	BC.
g_1	$G_1 + G_2$	g_1	g_1	$\bar{F}_1$	$M\nu \mathrm{Im}\, G_1/\pi$
g_2	$-G_2$	g_2	g_2	$\bar{F}_2/(2x)$	$-\nu^2 \mathrm{Im}\, G_2/(M\pi)$
g_3	$2G_3$	$b_1 + b_2$	$(A_2 - A_3)/2$	$-2\bar{F}_3$	
g_4	$2G_4 + G_5$	$a_2 + b_1 + b_2$	A_2		
g_5	$-G_4$	a_1	A_1		

Table 1: A comparison of different conventions to denote the polarized nucleon structure functions.

$$
\begin{aligned}
\frac{d^2\sigma(\lambda,\pm S_L)}{dx\,dy} &= 2\pi S\frac{\alpha^2}{Q^4}\sum_i C_i\eta_i(Q^2)\left\{y^2 2xF_1^i + 2\left(1-y-\frac{xyM^2}{S}\right)F_2^i - 2\lambda y\left(1-\frac{y}{2}\right)xF_3^i\right.\\
&\pm \left[-2\lambda y\left(2-y-\frac{2xyM^2}{S}\right)xg_1^i + 8\lambda\frac{yx^2M^2}{S}g_2^i + \frac{4xM^2}{S}\left(1-y-\frac{xyM^2}{S}\right)g_3^i\right.\\
&- \left.\left. 2\left(1+\frac{2xM^2}{S}\right)\left(1-y-\frac{xyM^2}{S}\right)g_4 - 2xy^2\left(1+\frac{2xM^2}{S}\right)g_5^i\right]\right\}. \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
\frac{d^3\sigma(\lambda,\pm S_T)}{dx\,dy\,d\phi} &= S\frac{\alpha^2}{Q^4}\sum_i C_i\eta_i(Q^2)\left\{y^2 2xF_1^i + 2\left(1-y-\frac{xyM^2}{S}\right)F_2^i - 2\lambda y\left(1-\frac{y}{2}\right)xF_3^i\right.\\
&\pm 2\sqrt{\frac{M^2}{S}}\sqrt{xy\left[1-y-\frac{xyM^2}{S}\right]}\cos(\alpha-\phi)\left[-2\lambda yxg_1^i - 4\lambda g_2^i\right.\\
&- \left.\left. \frac{1}{y}\left(2-y-\frac{2xyM^2}{S}\right)g_3^i + \frac{2}{y}\left(1-y-\frac{xyM^2}{S}\right)g_4 + y^2 2xg_5^i\right]\right\}. \qquad (12)
\end{aligned}
$$

3.1. The Forward Compton Amplitude

The forward Compton amplitude $T_{\mu\nu}$ is related to the hadronic tensor by

$$W^i_{\mu\nu} = \frac{1}{2\pi}\mathrm{Im}\ T^i_{\mu\nu}, \tag{13}$$

with

$$T^i_{\mu\nu} = i\int d^4x e^{iqx}\langle PS \mid (TJ^{i_1 \dagger}_\mu(x)J^{i_2}_\nu(0))|PS\rangle. \tag{14}$$

It can be represented in terms of the amplitudes T^i_k and A^i_k as

$$\begin{aligned} T^i_{\mu\nu} &= (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})T^i_1(q^2,\nu) + \frac{\hat{P}_\mu \hat{P}_\nu}{M^2}T^i_2(q^2,\nu) - i\varepsilon_{\mu\nu\lambda\sigma}\frac{q_\lambda P\sigma}{2M^2}T^i_3(q^2,\nu) \\ &+ i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda S^\sigma}{M^2}A^i_1(q^2,\nu) + i\varepsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda(P\cdot qS^\sigma - S\cdot qP^\sigma)}{M^4}A^i_2(q^2,\nu) \\ &+ [\frac{\hat{P}_\mu\hat{S}_\nu + \hat{S}_\mu\hat{P}_\nu}{2} - S\cdot q\frac{\hat{P}_\mu\hat{P}_\nu}{(P\cdot q)}]\frac{A^i_3(q^2,\nu)}{M^2} \\ &+ S\cdot q\frac{\hat{P}_\mu\hat{P}_\nu}{M^4}A^i_4(q^2,\nu) + (-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})\frac{S\cdot q}{M^2}A^i_5(q^2,\nu), \end{aligned} \tag{15}$$

where $\nu = P.q$. The structure functions $g_i(x,Q^2)$ and amplitudes $A_i(q^2,\nu)$ are related by

$$\begin{aligned} g_{1,3,5}(x,Q^2) &= \frac{1}{2\pi}\frac{\nu}{M^2}\mathrm{Im}A_{1,3,5}(q^2,\nu), \\ g_{2,4}(x,Q^2) &= \frac{1}{2\pi}\frac{\nu^2}{M^4}\mathrm{Im}A_{2,4}(q^2,\nu). \end{aligned} \tag{16}$$

Subsequently we will consider the polarized part of $T^i_{\mu\nu}$ only.

NC :

For neutral current interactions the current operators obey

$$J^{\gamma,Z\dagger}_\mu = J^{\gamma,Z}_\mu. \tag{17}$$

Therefore, the crossing relation for the amplitude for $q \to -q, P \to P$ reads

$$T^i_{\mu\nu}(q^2,-\nu) = T^i_{\nu\mu}(q^2,\nu). \tag{18}$$

The corresponding relations for the amplitudes $A^{\mathrm{NC}}_i(q^2,\nu)$ are

$$\begin{aligned} A^{\mathrm{NC}}_{1,3}(q^2,-\nu) &= A^{\mathrm{NC}}_{1,3}(q^2,\nu), \\ A^{\mathrm{NC}}_{2,4,5}(q^2,-\nu) &= -A^{\mathrm{NC}}_{2,4,5}(q^2,\nu). \end{aligned} \tag{19}$$

Furthermore, the amplitudes obey the following forward dispersion relations :

$$\begin{aligned} A^{\mathrm{NC}}_{1,3}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^{\infty} d\nu'\frac{\nu'}{\nu'^2 - \nu^2}\mathrm{Im}\ A^{\mathrm{NC}}_{1,3}(q^2,\nu'), \\ A^{\mathrm{NC}}_{2,4,5}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^{\infty} d\nu'\frac{\nu}{\nu'^2 - \nu^2}\mathrm{Im}\ A^{\mathrm{NC}}_{2,4,5}(q^2,\nu'). \end{aligned} \tag{20}$$

3.2. THE OPERATOR PRODUCT EXPANSION

$$\hat{T}^{i}_{\mu\nu} = T(J^{i\dagger}_{\mu}(x)J^{i}_{\nu}(0)).$$

Near the light cone one obtains for neutral currents

$$\begin{aligned}\hat{T}^{NC}_{\mu\nu} &= \bar{q}(x)\gamma_{\mu}(g_{V_1} + g_{A_1}\gamma_5)S(x)\gamma_{\nu}(g_{V_2} + g_{A_2}\gamma_5)P^{+}q(0)\\ &+ \bar{q}(0)\gamma_{\nu}(g_{V_2} + g_{A_2}\gamma_5)S(-x)\gamma_{\mu}(g_{V_1} + g_{A_1}\gamma_5)P^{+}q(x),\end{aligned}$$

and for the charged current combinations

$P_+ = 1\!\!1$, $P_- = \tau_3$

$$\hat{T}^{\pm}_{\mu\nu} = \hat{T}^{W^-}_{\mu\nu} \pm \hat{T}^{W^+}_{\mu\nu},$$

$$\begin{aligned}\hat{T}^{\pm}_{\mu\nu} &= \bar{q}(x)\gamma_{\mu}(g_{V_1} + g_{A_1}\gamma_5)S(x)\gamma_{\nu}(g_{V_2} + g_{A_2}\gamma_5)P^{\pm}q(0)\\ &\pm \bar{q}(0)\gamma_{\nu}(g_{V_2} + g_{A_2}\gamma_5)S(-x)\gamma_{\mu}(g_{V_1} + g_{A_1}\gamma_5)P^{\pm}q(x).\end{aligned}$$

$S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta}$

$$\begin{aligned}\hat{T}^{\pm}_{\mu\nu,spin} &= \frac{4ix^{\alpha}}{(2\pi)^2(x^2 - i0)^2}\Big\{-i(g_{V_1}g_{V_2} + g_{A_1}g_{A_2})\varepsilon_{\mu\alpha\nu\beta}\rho^{\pm\beta}_{-}\\ &\qquad - (g_{V_1}g_{A_2} + g_{A_1}g_{V_2})S_{\mu\alpha\nu\beta}\rho^{\pm\beta}_{+}\Big\}\end{aligned} \tag{43}$$

is obtained. Here we used the abbreviations

$$\begin{aligned}\rho^{\pm\beta}_{+} &= \sum_{n\ \mathrm{even}}\frac{1}{n!}x_{\mu_1}\ldots x_{\mu_n}\bar{q}(0)\gamma^{\beta}\gamma_5 D^{\mu_1}\ldots D^{\mu_n}P^{\pm}q(0),\\ \rho^{\pm\beta}_{-} &= \sum_{n\ \mathrm{odd}}\frac{1}{n!}x_{\mu_1}\ldots x_{\mu_n}\bar{q}(0)\gamma^{\beta}\gamma_5 D^{\mu_1}\ldots D^{\mu_n}P^{\pm}q(0),\end{aligned}$$

for which it is convenient to define (cf. e.g. [25])

$$\rho^{\pm\beta}_{\pm} \equiv \sum_{n\ \mathrm{even/odd}}\frac{1}{n!}ix_{\mu_1}\ldots ix_{\mu_n}\Theta^{\pm\beta\{\mu_1\ldots\mu_n\}} \tag{44}$$

for later analysis. The Fourier transforms of eqs. (42, 43) read

$$
\begin{aligned}
\widehat{T}^{+}_{\mu\nu,spin} &= i\int d^4x e^{iqx}\hat{T}^{+}_{\mu\nu,spin} \\
&= \frac{1}{\pi^2}\int d^4x e^{iqx}\frac{x^\alpha}{(x^2-i0)}\left\{i(g_{V_1}g_{V_2}+g_{A_1}g_{A_2})\varepsilon_{\mu\alpha\nu\beta}\rho_+^{+\beta}+(g_{V_1}g_{A_2}+g_{A_1}g_{V_2})S_{\mu\alpha\nu\beta}\rho_-^{+\beta}\right\} \\
&= -i(g_{V_1}g_{V_2}+g_{A_1}g_{A_2})\varepsilon_{\mu\alpha\nu\beta}q^\alpha\sum_{n\ \mathrm{even}}q^{\mu_1}\ldots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n+1}\Theta^{+\beta\{\mu_1\ldots\mu_n\}} \\
&+ (g_{V_1}g_{A_2}+g_{A_1}g_{V_2})[-g_{\mu\nu}\sum_{n\ \mathrm{even}}q^{\mu_1}\ldots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n}\Theta^{+\mu_1\{\mu_2\ldots\mu_n\}} \\
&+ \sum_{n\ \mathrm{even}}q^{\mu_1}\ldots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n+1}(\Theta^{+\mu\{\nu\mu_1\ldots\mu_n\}}+\Theta^{+\nu\{\mu\mu_1\ldots\mu_n\}})]\,,
\end{aligned}
\tag{45}
$$

and

$$
\begin{aligned}
\widehat{T}^{-}_{\mu\nu,spin} &= i\int d^4x e^{iqx}\hat{T}^{-}_{\mu\nu,spin} \\
&= \frac{1}{\pi^2}\int d^4x e^{iqx}\frac{x^\alpha}{(x^2-i0)}\left\{i(g_{V_1}g_{V_2}+g_{A_1}g_{A_2})\varepsilon_{\mu\alpha\nu\beta}\rho_-^{-\beta}+(g_{V_1}g_{A_2}+g_{A_1}g_{V_2})S_{\mu\alpha\nu\beta}\rho_+^{-\beta}\right\} \\
&= -i(g_{V_1}g_{V_2}+g_{A_1}g_{A_2})\varepsilon_{\mu\alpha\nu\beta}q^\alpha\sum_{n\ \mathrm{odd}}q^{\mu_1}\ldots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n+1}\Theta^{-\beta\{\mu_1\ldots\mu_n\}} \\
&+ (g_{V_1}g_{A_2}+g_{A_1}g_{V_2})[-g_{\mu\nu}\sum_{n\ \mathrm{odd}}q^{\mu_1}\ldots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n}\Theta^{-\mu_1\{\mu_2\ldots\mu_n\}} \\
&+ \sum_{n\ \mathrm{odd}}q^{\mu_1}\ldots q^{\mu_n}\left(\frac{2}{Q^2}\right)^{n+1}(\Theta^{-\mu\{\nu\mu_1\ldots\mu_n\}}+\Theta^{-\nu\{\mu\mu_1\ldots\mu_n\}})]\,.
\end{aligned}
\tag{46}
$$

The operators $\Theta^{\pm\beta\{\mu_1\ldots\mu_n\}}$ can be decomposed into a symmetric and a remainder part, Θ_S and Θ_R, respectively

$$
\Theta^{\pm\beta\{\mu_1\ldots\mu_n\}}=\Theta_S^{\pm\beta\{\mu_1\ldots\mu_n\}}+\Theta_R^{\pm\beta\{\mu_1]\ldots\mu_n\}}, \tag{47}
$$

where

$$
\begin{aligned}
\Theta_S^{\pm\beta\{\mu_1\ldots\mu_n\}} &= \frac{1}{n+1}\left[\Theta^{\pm\beta\{\mu_1\ldots\mu_n\}}+\Theta^{\pm\mu_1\{\beta\ldots\mu_n\}}+\ldots+\Theta^{\pm\mu_n\{\mu_1\ldots\beta\}}\right], \qquad (48)\\
\Theta_R^{\pm\beta\{\mu_1]\ldots\mu_n\}} &= \frac{1}{n+1}\left[\Theta^{\pm\beta\{\mu_1\ldots\mu_n\}}-\Theta^{\pm\mu_1\{\beta\ldots\mu_n\}}+\Theta^{\pm\beta\{\mu_1\mu_2\ldots\mu_n\}}-\Theta^{\pm\mu_2\{\mu_1\beta\ldots\mu_n\}}+\ldots\right].
\end{aligned}
$$

The nucleon matrix elements of these operators are

$$
\begin{aligned}
\langle PS|\Theta_S^{\pm\beta\{\mu_1\ldots\mu_n\}}|PS\rangle &= \frac{a_n^\pm}{n+1}\left[S^\beta P^{\mu_1}P^{\mu_2}\ldots P^{\mu_n}+S^{\mu_1}P^\beta P^{\mu_2}\ldots P^{\mu_n}+\ldots-\mathrm{traces}\right], \qquad (49)\\
\langle PS|\Theta_R^{\pm\beta\{\mu_1]\ldots\mu_n\}}|PS\rangle &= \frac{d_n^\pm}{n+1}\left[\left(S^\beta P^{\mu_1}-S^{\mu_1}P^\beta\right)P^{\mu_2}\ldots P^{\mu_n}\right.\\
&\quad+\left(S^\beta P^{\mu_2}-S^{\mu_2}P^\beta\right)P^{\mu_1}P^{\mu_3}\ldots P^{\mu_n}\\
&\quad\left.+\ldots+\left(S^\beta P^{\mu_n}-S^{\mu_n}P^\beta\right)P^{\mu_1}P^{\mu_2}\ldots P^{\mu_{n-1}}-\mathrm{traces}\right]. \qquad (50)
\end{aligned}
$$

$$
\langle PS|\Theta_R^{\pm\beta\{\mu_1]\ldots\mu_n\}}|PS\rangle \approx \frac{d_n^\pm}{n+1}\left[\left(S^\beta P^{\mu_1}-S^{\mu_1}P^\beta\right)P^{\mu_2}\ldots P^{\mu_n}-\mathrm{traces}\right]. \tag{51}
$$

AHMED & ROSS 1975 : NOT APPLICABLE
FOR a

For the charged current interactions it is suitable to study the linear combination of amplitudes

CC:
$$T_{\mu\mu}^{\pm}(q^2,\nu) = T_{\mu\mu}^{W^-}(q^2,\nu) \pm T_{\mu\mu}^{W^+}(q^2,\nu). \tag{21}$$

Due to the transformation

$$J_\mu^{W^{\pm}\dagger} = J_\mu^{W^{\mp}}, \tag{22}$$

the following crossing relations hold :

$$T^{\pm}(q^2,-\nu) = \pm T^{\pm}(q^2,\nu). \tag{23}$$

Correspondingly, one obtains for the combination of the amplitudes

$$A_i^{\pm} = A_i^{W^-} \pm A_i^{W^+} \tag{24}$$

the relations

$$\begin{aligned} A_{1,3}^{\pm}(q^2,-\nu) &= \pm A_{1,3}^{\pm}(q^2,\nu), \\ A_{2,4,5}^{\pm}(q^2,-\nu) &= \mp A_{2,4,5}^{\pm}(q^2,\nu). \end{aligned} \tag{25}$$

The respective dispersion relations for $A_i^+(q^2,\nu)$ and $A_i^-(q^2,\nu)$ are:

$$\begin{aligned} A_{1,3}^{+}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^{\infty} d\nu' \frac{\nu'}{\nu'^2-\nu^2}\mathrm{Im}A_{1,3}^{+}(q^2,\nu'), \\ A_{2,4,5}^{+}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^{\infty} d\nu' \frac{\nu}{\nu'^2-\nu^2}\mathrm{Im}A_{2,4,5}^{+}(q^2,\nu'), \\ A_{1,3}^{-}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^{\infty} d\nu' \frac{\nu'}{\nu(\nu'^2-\nu^2)}\mathrm{Im}\left[\nu' A_{1,3}^{-}(q^2,\nu')\right], \\ A_{2,4}^{-}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^{\infty} d\nu' \frac{\nu'}{\nu^2(\nu'^2-\nu^2)}\mathrm{Im}\left[\nu'^2 A_{2,4}^{-}(q^2,\nu')\right], \\ A_{5}^{-}(q^2,\nu) &= \frac{2}{\pi}\int_{Q^2/2}^{\infty} d\nu' \frac{\nu'}{\nu'^2-\nu^2}\mathrm{Im}\ A_{5}^{-}(q^2,\nu'). \end{aligned} \tag{26}$$

For the case of charged current interactions we introduce the structure function combinations

$$g_i^{\pm}(x,Q^2) = g_i^{W^-}(x,Q^2) \pm g_i^{W^+}(x,Q^2). \tag{27}$$

The integral representations of the amplitudes A_i^{NC} and $A_i^{\pm}$ can be finally expressed by the moments of the corresponding structure functions as

$$\begin{aligned} A_{1,3}^{NC,+}(q^2,\nu) &= \frac{4M^2}{\nu}\sum_{n=0,2\ldots}\frac{1}{x^{n+1}}\int_0^1 dy y^n g_{1,3}^{NC,+}(y,Q^2), \\ A_{2,4}^{NC,+}(q^2,\nu) &= \frac{4M^4}{\nu^2}\sum_{n=2,4\ldots}\frac{1}{x^{n+1}}\int_0^1 dy y^n g_{2,4}^{NC,+}(y,Q^2), \\ A_{5}^{NC,+}(q^2,\nu) &= \frac{4M^2}{\nu}\sum_{n=1,3\ldots}\frac{1}{x^{n+1}}\int_0^1 dy y^n g_{5}^{NC,+}(y,Q^2), \end{aligned} \tag{28}$$

and

$$\begin{aligned} A_{1,3}^{-}(q^2,\nu) &= \frac{4M^2}{\nu}\sum_{n=-1,1\ldots}\frac{1}{x^{n+1}}\int_0^1 dy y^n g_{1,3}^{-}(y,Q^2), \\ A_{2,4}^{-}(q^2,\nu) &= \frac{4M^4}{\nu^2}\sum_{n=-1,1\ldots}\frac{1}{x^{n+1}}\int_0^1 dy y^n g_{2,4}^{-}(y,Q^2), \\ A_{5}^{-}(q^2,\nu) &= \frac{4M^2}{\nu}\sum_{n=0,2\ldots}\frac{1}{x^{n+1}}\int_0^1 dy y^n g_{5}^{-}(y,Q^2), \end{aligned} \tag{29}$$

$$
\begin{aligned}
T^{+}_{\mu\nu,spin} &= -i(g_{V_1}g_{V_2}+g_{A_1}g_{A_2})\frac{\varepsilon_{\mu\alpha\nu\beta}q^{\alpha}}{\nu}\sum_{n\ \mathrm{even}}\frac{1}{x^{n+1}}\left[\frac{a^{+}_{n}+nd^{+}_{n}}{n+1}S^{\beta}+\frac{n(a^{+}_{n}-d^{+}_{n})}{n+1}\frac{(S.q)}{\nu}P^{\beta}\right] \\
&+ (g_{V_1}g_{A_2}+g_{A_1}g_{V_2})\Bigg\{-g_{\mu\nu}\frac{(S.q)}{\nu}\sum_{n\ \mathrm{odd}}\frac{a^{+}_{n}}{x^{n+1}} \\
&+ \frac{2}{\nu}\sum_{n\ \mathrm{odd}}\frac{1}{x^{n}}\left[\frac{2a^{+}_{n}+(n-1)d^{+}_{n}}{n+1}\left(\frac{S^{\mu}P^{\nu}+P^{\mu}S^{\nu}}{2}-\frac{P^{\mu}P^{\nu}}{\nu}(S.q)\right)\right. \\
&+ \left.\left. a^{+}_{n}\frac{P^{\mu}P^{\nu}}{\nu}(S.q)\right]\right\}, \qquad (52)
\end{aligned}
$$

$$
\begin{aligned}
T^{-}_{\mu\nu,spin} &= -i(g_{V_1}g_{V_2}+g_{A_1}g_{A_2})\frac{\varepsilon_{\mu\alpha\nu\beta}q^{\alpha}}{\nu}\sum_{n\ \mathrm{odd}}\frac{1}{x^{n+1}}\left[\frac{a^{-}_{n}+nd^{-}_{n}}{n+1}S^{\beta}+\frac{n(a^{-}_{n}-d^{-}_{n})}{n+1}\frac{(S.q)}{\nu}P^{\beta}\right] \\
&+ (g_{V_1}g_{A_2}+g_{A_1}g_{V_2})\Bigg\{-g_{\mu\nu}\frac{(S.q)}{\nu}\sum_{n\ \mathrm{even}}\frac{a^{-}_{n}}{x^{n+1}} \\
&+ \frac{2}{\nu}\sum_{n\ \mathrm{even}}\frac{1}{x^{n}}\left[\frac{2a^{-}_{n}+(n-1)d^{-}_{n}}{n+1}\left(\frac{S^{\mu}P^{\nu}+P^{\mu}S^{\nu}}{2}-\frac{P^{\mu}P^{\nu}}{\nu}(S.q)\right)\right. \\
&+ \left.\left. a^{-}_{n}\frac{P^{\mu}P^{\nu}}{\nu}(S.q)\right]\right\}. \qquad (53)
\end{aligned}
$$

Here we arranged the structure as in eq. (15) according to the contributions to the different amplitudes.

$$\int_0^1 dx x^n g_5^{NC,+}(x,Q^2) = \sum_q \frac{g_V^q g_A^q a_n^{+q}}{2}, \quad n = 1,3 \ldots \tag{60}$$

and

$$\int_0^1 dx x^n g_1^-(x,Q^2) = \sum_q \frac{((g_V^q)^2 + (g_A^q)^2) a_n^{-q}}{4}, \quad n = 1,3 \ldots \tag{61}$$

$$\int_0^1 dx x^n g_2^-(x,Q^2) = \sum_q \frac{((g_V^q)^2 + (g_A^q)^2) n (d_n^{-q} - a_n^{-q})}{4(n+1)}, \quad n = 1,3 \ldots \tag{62}$$

$$\int_0^1 dx x^n g_3^-(x,Q^2) = \sum_q \frac{g_V^q g_A^q (2a_{n+1}^{-q} + n d_{n+1}^{-q})}{(n+2)}, \quad n = 1,3 \ldots \tag{63}$$

$$\int_0^1 dx x^n g_4^-(x,Q^2) = \sum_q g_V^q g_A^q a_{n+1}^{-q}, \quad n = 1,3 \ldots \tag{64}$$

$$\int_0^1 dx x^n g_5^-(x,Q^2) = \sum_q \frac{g_V^q g_A^q a_n^{-q}}{2}, \quad n = 0,2 \ldots . \tag{65}$$

3.3 Relations between the Moments of Structure Functions

From eqs. (52) and (53) one derives the following representations for the amplitudes $A_i^{NC}(q^2,\nu)$ and $A_i^{\pm}(q^2,\nu)$:

$$
\begin{aligned}
A_1^{NC,+}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ \mathrm{even}} \frac{((g_V^q)^2+(g_A^q)^2)a_n^{+q}}{x^{n+1}} \\
A_2^{NC,+}(q^2,\nu) &= \frac{M^4}{\nu^2}\sum_{n\ \mathrm{even}} \frac{((g_V^q)^2+(g_A^q)^2)n(d_n^{+q}-a_n^{+q})}{x^{n+1}(n+1)} \\
A_3^{NC,+}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ \mathrm{odd}} \frac{4g_V^q g_A^q(2a_n^{+q}+(n-1)d_n^{+q})}{x^n(n+1)} \\
A_4^{NC,+}(q^2,\nu) &= \frac{M^4}{\nu^2}\sum_{n\ \mathrm{odd}} \frac{4g_V^q g_A^q a_n^{+q}}{x^n} \\
A_5^{NC,+}(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ \mathrm{odd}} \frac{2g_V^q g_A^q a_n^{+q}}{x^{n+1}},
\end{aligned}
\tag{54}
$$

and

$$
\begin{aligned}
A_1(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ \mathrm{odd}} \frac{((g_V^q)^2+(g_A^q)^2)a_n^{-q}}{x^{n+1}} \\
A_2^-(q^2,\nu) &= \frac{M^4}{\nu^2}\sum_{n\ \mathrm{odd}} \frac{((g_V^q)^2+(g_A^q)^2)n(d_n^{-q}-a_n^{-q})}{x^{n+1}(n+1)} \\
A_3^-(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ \mathrm{even}} \frac{4g_V^q g_A^q(2a_n^{-q}+(n-1)d_n^{-q})}{x^n(n+1)} \\
A_4^-(q^2,\nu) &= \frac{M^4}{\nu^2}\sum_{n\ \mathrm{even}} \frac{4g_V^q g_A^q a_n^{-q}}{x^n} \\
A_5^-(q^2,\nu) &= \frac{M^2}{\nu}\sum_{n\ \mathrm{even}} \frac{2g_V^q g_A^q a_n^{-q}}{x^{n+1}}.
\end{aligned}
\tag{55}
$$

On the other hand, the representations eq. (28) and (29) are valid, from which the following relations between the operator matrix elements $a_n^{\pm,q}$ and $d_n^{\pm,q}$ and the moments of the structure functions $g_i^{\mathrm{NC},\pm}(x,Q^2)$ are obtained[4] :

$$\int_0^1 dx x^n g_1^{NC,+}(x,Q^2) = \sum_q \frac{((g_V^q)^2+(g_A^q)^2)a_n^{+q}}{4}, \quad n=0,2\ \ldots \tag{56}$$

$$\int_0^1 dx x^n g_2^{NC,+}(x,Q^2) = \sum_q \frac{((g_V^q)^2+(g_A^q)^2)n(d_n^{+q}-a_n^{+q})}{4(n+1)}, \quad n=2,4\ \ldots \tag{57}$$

$$\int_0^1 dx x^n g_3^{NC,+}(x,Q^2) = \sum_q \frac{g_V^q g_A^q(2a_{n+1}^{+q}+nd_{n+1}^{+q})}{(n+2)}, \quad n=0,2\ \ldots \tag{58}$$

$$\int_0^1 dx x^n g_4^{NC,+}(x,Q^2) = \sum_q g_V^q g_A^q a_{n+1}^{+q}, \quad n=2,4\ \ldots \tag{59}$$

[4]In a previous publication [22] the twist 3 terms in $g_3^{NC}(x,Q^2)$ were missed. This was caused using the representation (51) instead of the exact one, (50). Eq. (17) of ref. [22] is corrected by eq. (58).

$\Longrightarrow$ CONSIDER TWIST 2 FIRST.

CHOOSE A BASIS: g_1, g_5 $\qquad d_n^{\pm} \underset{\text{DF}}{=} 0.$

NC:
$$g_1^i = \frac{1}{2} \sum_q \alpha_i^q \, (\Delta q + \Delta \bar{q})$$
$$g_5^i = \frac{1}{2} \sum_q \beta_i^q \, (\Delta q - \Delta \bar{q})$$

CC:
$$g_1^{-(+)} = \sum_q \left[\Delta q_{u(d)} + \Delta \bar{q}_{d(u)}\right]$$
$$g_5^{-(+)} = -\sum_q \left[\Delta q_{u(d)} - \Delta \bar{q}_{d(u)}\right]$$

ONE OBTAINS: $N \in \mathbb{N} \to z \in \mathbb{C}$; INVERSE MELLIN TRANSFORM

TWIST 2: $g_2^i(x) = -g_1^i(x) + \int_x^1 \frac{dy}{y} g_1^i(y)$ WANDZURA-WILCZEK 1977

$g_4^i(x) = 2x \, g_5^i(x)$ DIC.. 197..

TWIST 2: $g_3^i(x) = 4x \int_x^1 \frac{dy}{y} g_5^i(y)$ NEW.

$\longrightarrow$ NUMERICAL RESULTS.

SUM RULES: (neglecting d_n^{q})
$$\int_0^1 dx \, x^n \left[g_3^k(x,Q^2) - \frac{2}{2+n} g_4^k(x,Q^2)\right] = 0$$
$$\int_0^1 dx \, g_3^k(x,Q^2) = \int_0^1 dx \, g_4^k(x,Q^2).$$

CAN WE FIND THESE RESULTS ALSO IN A PARTONIC APPROACH ?

COLLINEAR PARTON MODEL:

IMF :

$$\overset{P}{\Longrightarrow} \;\to\; \underset{p \approx x P}{\longrightarrow}$$

NR APPROXIMATION

$\sim$ LONGITUDINAL

$\vec{S} \sim p$ ($\approx$ OK)

$S \to S_\perp$ $\quad S \cdot k_\perp$ RELEVANT !

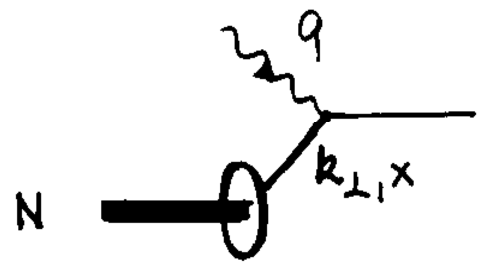
BETTER USE: COMPLETE DIRAC EQ.

COVARIANT PARTON MODEL

LANDSHOFF, POLKINGHORNE, NASH ....
early 70ies.

- JACKSON, ROBERTS, ROSS '89
- ROBERTS, ROSS '96

4. COVARIANT PARTON MODEL



(APPROACH FOR TWIST 2).

$$W_{\mu\nu,ab}(q,p,S) = \sum_{\lambda,q_i} \int d^4k \; f_\lambda^{q_i}(p,k,S)\, w^{q_i}_{\mu\nu,\lambda,ab}(k,q) \cdot \delta\left[(k+q)^2 - m^2\right]$$

$$\begin{aligned} w^{q_i,\text{spin}}_{\mu\nu,\lambda,ab}(k,q) = \lambda \Big\{ 2i\,\varepsilon_{\mu\alpha\nu\beta} \Big[& g^{q_i}_{Aa} g^{q_i}_{Ab} k_\alpha n_\beta \\ & + (g^{q_i}_{Aa} g^{q_i}_{Ab} + g^{q_i}_{Va} g^{q_i}_{Vb})\, q_\alpha n_\beta \\ & + g^{q_i}_{Va} g^{q_i}_{Ab} \left[2k_\mu n_\nu - n.q\, g_{\mu\nu}\right] \\ & + g^{q_i}_{Aa} g^{q_i}_{Vb} \left[2n_\mu k_\nu - n.q\, g_{\mu\nu}\right] \Big\} \end{aligned}$$

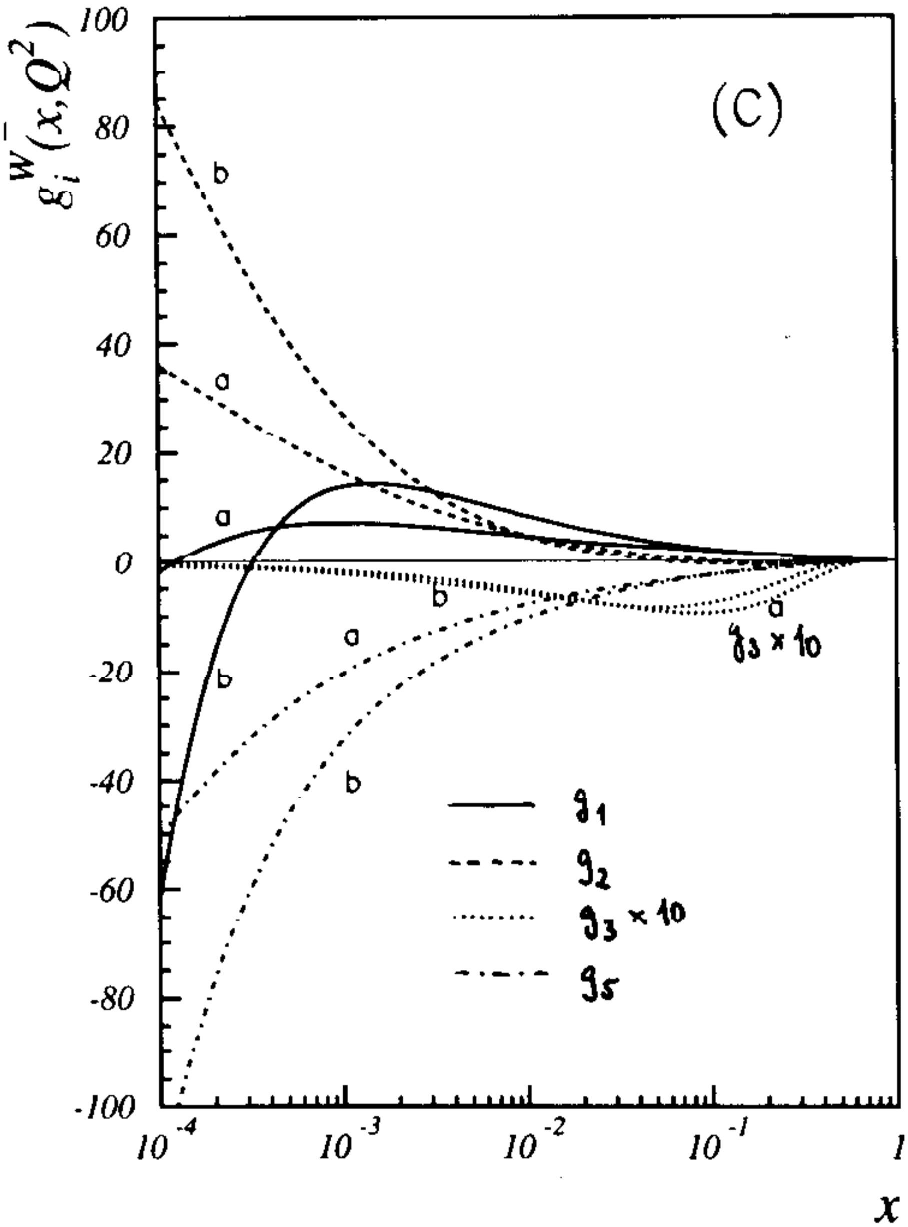
$$n_\sigma = \frac{m\; p.k}{\sqrt{(pk)k^2 - M^2 k^4}} \left(k_\sigma - \frac{k^2}{p.k} p_\sigma\right)$$

$$\Delta f(p.k, S.k, k^2) = -\frac{S.k}{M^2} \hat{f}(p.k, k^2)$$

$$W^{\parallel j}_{\mu\nu} = i\,\varepsilon_{\mu\nu\alpha\beta} \frac{q_\alpha p_\beta}{\nu} \underline{g_1^j(x)} + \frac{p_\mu p_\nu}{\nu} \underline{g_4^j(x)} - g_{\mu\nu} \underline{g_5^j(x)}$$

$$W^{\perp j}_{\mu\nu} = i\,\varepsilon_{\mu\alpha\nu\beta} \frac{q_\alpha S^\perp_\beta}{\nu} \left[\underline{g_1^j(x)} + \underline{g_2^j(x)}\right] + \frac{p_\mu S^\perp_\nu + p_\nu S^\perp_\mu}{2\nu} \underline{g_3^j(x)}$$

$$k = xp + \frac{k^2 + k_\perp^2 - x^2 M^2}{2x\nu} (q + xp) + k_\perp$$



(C)
$g_i^{W^-}(x,Q^2)$
100
80
60
40
20
0
-20
-40
-60
-80
-100
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x
a
b
$g_3 \times 10$
g_1
g_2
$g_3 \times 10$
g_5

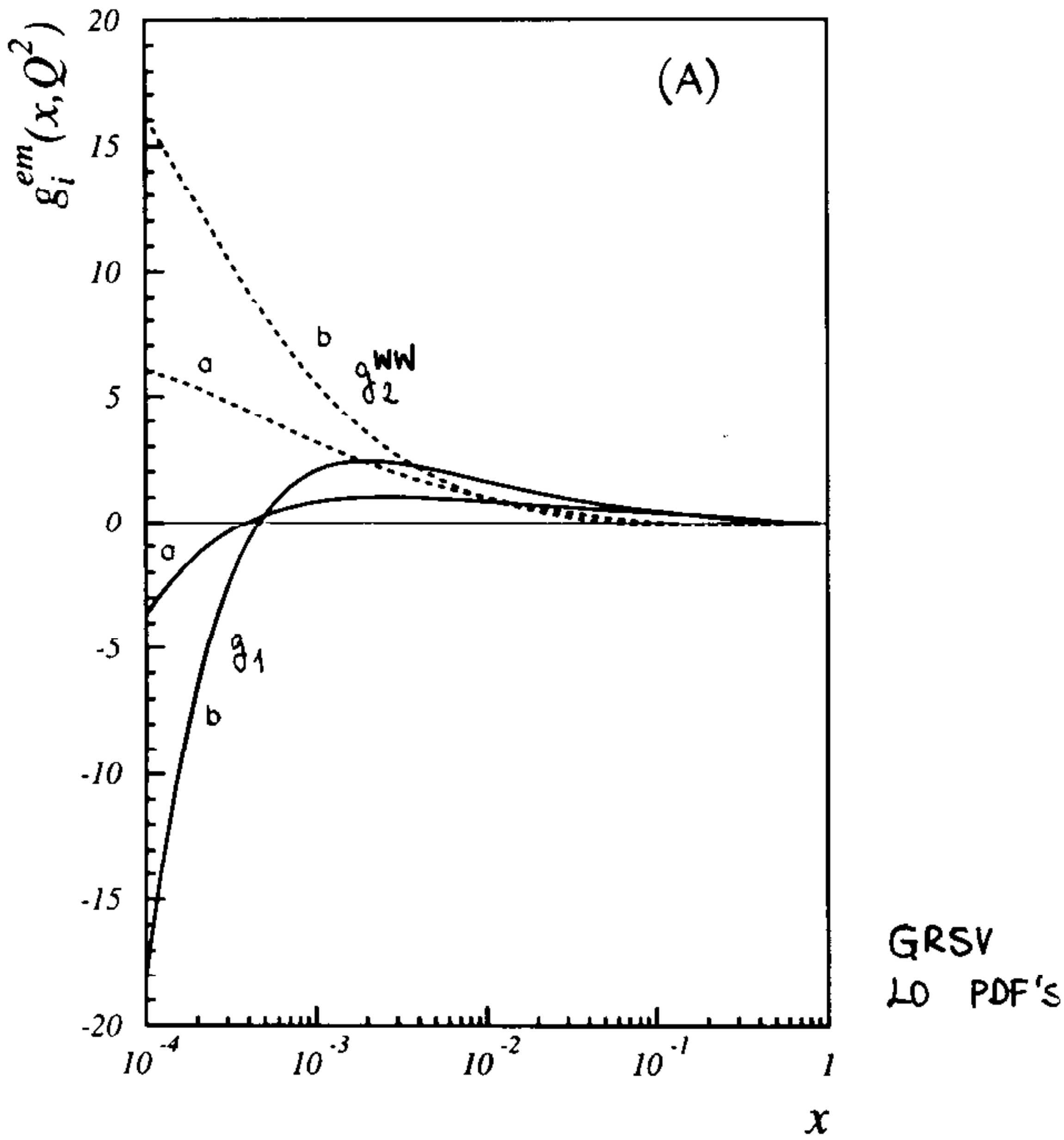


Figure 1: x and Q^2 dependence of the structure functions $g_i^h(x,Q^2)\Big|_{i=1}^{5}$ using the parton parametrization [21] (LO, STD). The lines correspond to $Q^2 = 10\,\mathrm{GeV}^2$ (a) and $Q^2 = 10^4\,\mathrm{GeV}^2$ (b). The structure functions for photon exchange, g_i^{em} (A), γZ interference, $g_i^{\gamma Z}$ (B), and W^- exchange in charged current lN scattering, $g_i^{W^-}$ (C), are compared separately. Full lines: g_1, dashed lines: g_2, dotted lines: $10 \times g_3$, dash-dotted line: g_5. The structure function g_4 can be obtained by the Dicus relation $g_4 = 2xg_5$ directly.

ONE OBTAINS FOR $m \to 0$:

& IN THE BJORKEN LIMIT : $M^2/Q^2 \to 0$, $M^2/\nu \to 0$
$Q^2/\nu = \text{const.}$

$$\boxed{\begin{aligned}
g_1^j(x) &= \frac{\pi x M^2}{8} \sum_q \alpha_q^j \int_x^1 dy\,(2x-y)\,\hat{h}_q(y) \\
g_2^j(x) &= \frac{\pi x M^2}{8} \sum_q \alpha_q^j \int_x^1 dy\,(2y-3x)\,\hat{h}_q(y) \\
g_3^j(x) &= \frac{\pi x^2 M^2}{2} \sum_q \beta_q^j \int_x^1 dy\,(y-x)\,\hat{h}_q(y) \\
g_4^j(x) &= \frac{\pi x^2 M^2}{4} \sum_q \beta_q^j \int_x^1 dy\,(2x-y)\,\hat{h}_q(y) \\
g_5^j(x) &= \frac{\pi x M^2}{8} \sum_q \beta_q^j \int_x^1 dy\,(2x-y)\,\hat{h}_q(y)
\end{aligned}}$$

$$g_2^j(x) = -\,g_1^j(x) + \int_x^1 \frac{dy}{y}\, g_1^j(y)$$

$$g_4^j(x) = 2x\, g_5^j(x)$$

$$g_3^j(x) = 4x \int_x^1 \frac{dy}{y}\, g_5^j(x)$$

$\surd_{m_q \to 0}$

$$\hat{h}_q(y) = \int dk^2\, \hat{f}_q(y, k^2)$$

$$y = x + k_\perp^2/(xM^2)$$

RELATIONS FOR THE TWIST 2 CONTRIBUTIONS

Dicus

$$W^{\parallel}_{\mu\nu} = i\epsilon_{\mu\nu\alpha\beta}\frac{q_\alpha P_\beta}{\nu} g_1(x) + \frac{P_\mu P_\nu}{\nu} g_4(x) - g_{\mu\nu} g_5(x)$$

Wandzura – Wilczek | **this paper, eq. (71)**

$$W^{\perp}_{\mu\nu} = i\epsilon_{\mu\nu\alpha\beta}\frac{q_\alpha S^{\perp}_\beta}{\nu}[g_1(x) + g_2(x)] + \frac{P_\mu S^{\perp}_\nu + P_\nu S^{\perp}_\mu}{2\nu} g_3(x)$$

$\overbrace{\Delta q \quad + \quad \Delta\bar{q}}$ | $\overbrace{\Delta q \quad - \quad \Delta\bar{q}}$

Figure 1 : Relations between the twist 2 contributions of the polarized structure functions

Table 2 : A comparison of different structure function relations (twist 2) derived in the literature with the results obtained in the local operator product expansion in section 5. The signs in the last column mark agreement or disagreement.

	sum rule	ref.	$m_q = 0$	
1	$g_4 = 2x g_5$	[11, 10, 16, 23]	+	Dicus' relation
2	$12x[(g_1+g_2)^{ep} - (g_1+g_2)^{en}] = g_3^{\nu n} - g_3^{\nu p}$	[11, 12, 9]	−	
3	$12x[g_2^{ep} - g_2^{en}] = (g_3 - 2g_4)^{\nu n} - (g_3 - 2g_4)^{\nu p}$		−	
4	$12x(g_1^{ep} - g_1^{en}) = g_4^{\nu n} - g_4^{\nu p}$	[12]	+	wrong
5	$3\int_0^1 dx(g_1^{ep} - g_1^{en}) - \int_0^1 dx(g_1^{\nu p} + g_1^{\nu n}) = -\frac{1}{6} g_A^8$		−	
6	$6\int_0^1 dx(g_2^{ep} - g_2^{en}) - \int_0^1 dx(g_1^{\nu p} - g_1^{\nu n}) = -g_A^3$		+	
7	$12\int_0^1 dx(g_2^{ep} - g_2^{en}) - \int_0^1 \frac{dx}{x}(g_4^{\nu p} - g_4^{\nu n}) = -2g_A$		+	
8	$12x[g_1^{ep} - g_1^{en}] = g_3^{\nu n} - g_3^{\nu p}$		−	
9	$\int_0^1 dx[(g_1+g_2)^{\bar{\nu} p} - (g_1+g_2)^{\nu p}] = g_A^3$	[17]	+	Nash
10	$\int_0^1 \frac{dx}{x}[g_3^{\bar{\nu} p} + g_3^{\nu p}] = -g_A^{8}$		+	
11	$\int_0^1 dx\, g_2^{\gamma} = 0$	[24]		BC

	sum rule	ref.	$m_q = 0$	
12	$\int_0^1 dx x (g_1 + 2g_2)^{\nu p - \bar{\nu} p} = 0$	[13]	+	Joshipura, Roy
13	$\int_0^1 dx (g_3 - 2x g_5)^{\nu p + \bar{\nu} p} = 0$		+	
14	$\int_0^1 dx (g_4 - g_3)^{\nu p + \bar{\nu} p} = 0$		+	
15	$\int_0^1 dx (g_5^{\nu p} - g_5^{\nu n}) = g_A$	[9]	+	Bartelski
16	$\int_0^1 dx \frac{[(g_4 - g_3)^{\nu p} - (g_4 - g_3)^{\nu n}]}{x} = 0$		−	
17	$\int_0^1 dx \frac{(g_3^{\nu p} - g_3^{\nu n})}{x} = 2g_A$		−	
18	$g_4 - g_3 = 2x g_5$	[15]	−	
19	$\int_0^1 dx x^n (\frac{n-7}{n+1} g_4 + 2g_3) = 0$	[16]	−	Ravishankar
20	$g_3 = 2x g_5$	[6]	−	
21	$g_3 = g_4$		−	
22	$g_2^{\gamma} = g_2^{\gamma Z} = 0$		−	
23	$g_1^{W^\pm} = -2g_2^{W^\pm}$		−	

	sum rule	ref.	$m_q = 0$
24	$\int_0^1 dx (g_3 - g_4)^{(\nu+\bar{\nu}),\gamma,Z} = 0$	[23]	
25	$\int_0^1 dx\, g_2^{\nu+\bar{\nu}} = 0$		
26	$\int_0^1 dx\, x\, [g_1 + 2g_2]^{W^- - W^+} = 0$		+
27	$\int_0^1 dx (g_3 - 2x g_5)^{(\nu+\bar{\nu}),\gamma,Z} = 0$		+
28	$\int_0^1 dx \frac{g_3^{\nu p} - g_3^{\nu n}}{x} = 4 g_A$	this paper	+
29	$24x[(g_1 + g_2)^{ep} - (g_1 + g_2)^{en}] = g_3^{\nu n} - g_3^{\nu p}$		+
30	$24x[g_2^{ep} - g_2^{en}] = (g_3 - 2g_4)^{\nu n} - (g_3 - 2g_4)^{\nu p}$		+
31	$\int_0^1 dx (g_1^{ep} + g_1^{en}) - \frac{2}{9} \int_0^1 dx (g_1^{\nu p} + g_1^{\nu n}) = \frac{1}{18} g_A^8$		+

Schäfer et al.

A RELATION BETWEEN THE TWIST-3 TERMS

EXPRESS:

$$g_2^{III}(x,Q^2) = \underbrace{g_2(x,Q^2)}_{=0} + \overbrace{g_1(x,Q^2) - \int_x^1 \frac{dy}{y}\, g_1(y,Q^2)}^{WW}$$

$$g_3^{III}(x,Q^2) = \underbrace{g_3(x,Q^2)}_{=0} - \underbrace{4x\int_x^1 \frac{dy}{y}\, g_5(y,Q^2)}_{BK}$$

$$\int_0^1 dx\, x^n \left[4 g_5^- - \frac{n+1}{x} g_3^-\right] = \sum_q (n-1) d_n^{q-} \qquad n = 2,4 \ldots$$

$$\int_0^1 dx\, x^n \left[g_1^\gamma + (n+1) g_2^\gamma\right] = \sum_q \frac{n e_q^2}{4} d_n^{q+}$$

$$12\left[x\, g_2^{III}(x,Q^2) - \int_x^1 dy\, g_2^{III}(y,Q^2)\right]^{\gamma p - \gamma n} = g_3^{III,\nu n - \nu p}(x,Q^2)$$

relation for the TWIST 3 valence part!

$d_n^+ \neq d_n^-$ in general.

REMARK ON A RELATION BY : Efremov, Leader, Teryaev.

$$\int_0^1 dx\ x\ \left(g_1^V(x) + 2g_2^V(x)\right) = 0$$

What is $g_{1,2}^V$ FOR THE em. – INTERACTION ?

$\longrightarrow$ CONSIDER : CC $\longrightarrow$ DEFINE 'VALENCE SENSITIVITY'.

$$\int_0^1 dx\ x^n \left[g_1^- + 2g_2^- \right] = \sum_q \frac{\left[(g_V^q)^2 + (g_A^q)^2\right]\left[n d_n^{-q} - (n-1) a_n^{-q}\right]}{4(n+1)}$$

$n = 1, 3, \ldots$

$g_A \to 0$, only g_V.

$$\int_0^1 dx\ x\ \left(g_1^{Vq}(x,Q^2) + 2g_2^{Vq}(x,Q^2)\right) = \frac{e_q^2}{8}\, d_1^{Vq}.$$

ONE ALSO HAS:

$$\langle PS|\,\bar{q}\,(\gamma_\beta\gamma_5 D^\mu - \gamma_\mu\gamma_5 D^\beta)\,q\,|PS\rangle - d_1^{Vq}\,(S^\beta P^\mu - S^\mu P^\beta)$$

$$= m_q \langle PS|\,\bar{q}\, i\gamma_5\sigma^{\beta\mu} q\,|PS\rangle \xrightarrow[m_q \to 0]{} 0. \quad \text{iff finite.}$$

$$\langle PS|\bar{q}\, i\gamma_5\sigma^{\beta\mu} q\,|PS\rangle = \frac{\delta q}{M}\,(P^\beta S^\mu - P^\mu S^\beta)$$

$$\delta q = \int_0^1 dx \left[h_1^q(x) - h_1^{\bar{q}}(x)\right]$$

$\curvearrowright d_1^{Vq} = 0.$

OPE : $\quad \int_0^1 dx\ x\ \left[g_1^V + 2g_2^V\right] = 0$

5. CONCLUSIONS

1) WE HAVE DERIVED A CONSISTENT PICTURE FOR POLARIZED STRUCTURE FUNCTIONS IN LO QCD INCLUDING WEAK CURRENTS.

2) THE RESULTS FOUND IN OPE AND THE <u>COVARIANT</u> PARTON MODEL ARE FULLY CONSISTENT FOR TWIST 2.

3) THE 'NAIVE' PARTON MODEL DOES NOT DESCRIBE THE SPIN PROPERTIES CORRECTLY.

4) A NEW INTEGRAL RELATION WAS FOUND:

$$g_3^i(x) = 4x \int_0^1 dy \, \frac{1}{y} \, g_5^i(y) .$$

IT COMPLETES THE SET OF LO LINEAR OPERATOF DESCRIBING THE STRUCTURE FUNCTIONS IN TERM OF g_1^i AND g_5^i.

5) NEW ASSOCIATED SUM RULES ARE DERIVED.

6) TWIST 2 LIGHT QUARK MASS CORRECTIONS WERE CALCULATED.

7) ASIDE OF g_2^i, g_3^i RECEIVES TWIST 3 CONTRIBUTIONS.

8) AN INTEGRAL RELATION FOR THE VALENCE PART OF TWIST 3 CONTRIBUTIONS TO SF'S WAS DERIVED:

$$\underline{g_3^{III,\,\nu n-\nu p}(x,Q^2) = 12\left[x\, g_2^{III}(x,Q^2) - \int_x^1 dy\, g_2^{III}(y,Q^2)\right]^{\gamma p-\gamma n}}$$

9) THE (RECENT) ELT SUM RULE

$$\int_0^1 dx\, x(g_1^v + 2 g_2^v) = 0$$

COULD BE DERIVED USING THE OPE.