Evolution Kernels for Light-Ray Operators Twist-2 and Twist-3

I Amplitudes as Fouriertransforms of matrix elements

II Light-Ray Operators, Anomalous Di

III Anomalous Dim. of Twist-2 operators

IV Evolution Eq. for Amplitudes

V Twist-3

11.4.97
for "DIS 97"

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- Th. Braunschweig, B. Geyer, D. R
  Annalen d. Physik (Leipzig) 44 (1987) 403-411
- D. Müller, D.R., B. Geyer, F.M. Dittes
  J. Hořejši
  Fortschr. d. Physik 42 (1994) 101-141
- J. Blümlein, B. Geyer, D.R.

X. Ji 1996 / 1997

A. Radyushkin 1996 / 1997

Elements of Light-Ray Operators

- Parton Distribution Function

$$q(z,\mu^2) = \int_{-\infty}^{+\infty} \frac{d\tilde{x}p\kappa 2}{2\pi(\tilde{x}p)^2} e^{2i(\kappa\tilde{x}p)z} \langle p|\bar{\psi}(-\kappa\tilde{x})\gamma\tilde{x}\,\psi(\kappa\tilde{x})|p\rangle$$

κ: parameter. $\tilde{x}$ light-like vector, $\tilde{x}^2=0$

- Meson Wave Function

$$\phi(t,\mu^2) = \int_{-\infty}^{+\infty} \frac{d\tilde{x}p\kappa}{2\pi\tilde{x}p} e^{i\kappa\tilde{x}pt} \langle 0|\bar{\psi}(-\kappa\tilde{x})\gamma\tilde{x}\,\psi(\kappa\tilde{x})|p\rangle$$

Generalization I

$$q(t,\tau,\mu^2) = \int \frac{d\tilde{x}p_+\kappa}{\tilde{x}p_+} e^{i\kappa\tilde{x}p_+t} \langle p_2|\bar{\psi}(-\kappa\tilde{x})\gamma\tilde{x}\,\psi(\kappa\tilde{x})|p_1\rangle$$

$$\tau = \frac{\tilde{x}p_-}{\tilde{x}p_+}$$

support: $|t|\le 1$, $|\tau|\le 1$

$p_+ = p_2 + p_1 \qquad p_- = p_2 - p_1$

special cases:

$$q(z,\mu^2) = \lim_{\tau\to 0} q(z,\tau,\mu^2)$$

$$\phi(t,\mu^2) = \lim_{\tau\to -1} q(t,\tau,\mu^2)$$

$$F(z_+, z_-, \mu^2) = \int_{-\infty}^{+\infty} \frac{d\kappa \tilde{x} p_+}{2\pi \tilde{x} p_+} \int_{-\infty}^{+\infty} \frac{d\kappa \tilde{x} p_-}{2\pi} e^{i\kappa \tilde{x} p_+ z_+ + i \kappa \tilde{x} p_- z_-}$$

$$\cdot \langle p_2 | \bar{\psi}(-\kappa \tilde{x}) \gamma \tilde{x} \psi(\kappa \tilde{x}) | p_1 \rangle$$

connection:

$$q(t, \tau, \mu^2) = \int dz_- \, F(t - \tau z_-, z_-, \mu^2)$$

$$q^a(z, Q^2) = \lim_{\tau \to 0} q^a(z, \tau, Q^2) \quad \text{quark distribution function (formally!).} \tag{2.13}$$

In short we add some further properties of the distribution amplitude (2.10):

- Because of the conjugation properties of the light-ray operators $(O^a(\kappa_-; \tilde{n}))^+ = O^a(\kappa_-; \tilde{n})$ it satisfies

$$(q^a(t, \tau, \mu^2))^* = q^a(t, -\tau, \mu^2). \tag{2.14}$$

  This means that this amplitude is real for $P_1 = P_2$.

- The normalization of this distribution amplitude follows from the definition (2.10) and (2.5),

$$\int dt\, q^a(t, \tau, \mu^2) = (\tilde{n}P_+)^{-1} \tilde{n}^\nu \langle P_2 | J_\nu^a(0) | P_1 \rangle |_{\tilde{n}P_- = \tau \tilde{n}P_+}, \tag{2.15}$$

  where $J_\nu^a(0) = :\bar{\psi}(0) \gamma_\nu \lambda^a \psi(0):$ is a current with the flavour content a.

At last we give a representation of the generalized distribution amplitude with the help of a "spectral" function which is helpful for later calculations. Taking into account the ideas of the general Jost-Lehmann representation for T-amplitudes of two local operators (our operators (2.5) are in fact bilocal if we use the light-cone gauge in QCD) then the spectral functions have a finite support with respect to the new distribution variables z_+ and z_-. In this way the matrix elements of light-ray operators can be expressed by "spectral" functions $f^a(z_+, z_-, \mu^2)$ as follows

$$\frac{1}{\tilde{n}P_+} \langle P_2 | O^a(\kappa_-; \tilde{n}) | P_1 \rangle = \iint dz_+ \, dz_- \, e^{-i\kappa_-(\tilde{n}P_-)z_- - i\kappa_-(\tilde{n}P_+)z_+} f^a(z_+, z_-, \mu^2). \tag{2.16}$$

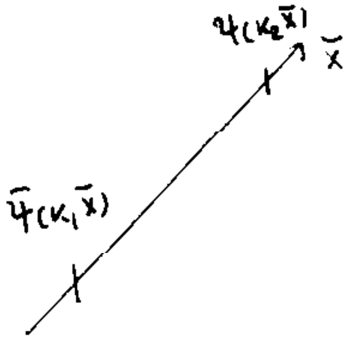
If we insert now this equation (2.16) into the definition of the distribution function (2.10) we obtain a new representation

$$\begin{aligned} q^a(t, \tau, Q^2) &= \iint dz_+ \, dz_- \int \frac{d\kappa_- |\tilde{n}P_+|}{2\pi} e^{i\kappa_-(\tilde{n}P_+)(t - z_+ - \tau z_-)} f^a(z_+, z_-, Q^2) \\ &= \int dz_- f^a(z_+ = t - \tau z_-, z_-, Q^2), \end{aligned} \tag{2.17}$$

which shows in which way the mathematically independent distribution variables z_+ and z_- turn over to the more physical variables t and τ.

- $O^q(\kappa_1,\kappa_2) = \bar{\psi}(\kappa_1\tilde{x})\,\gamma\tilde{x}\,\psi(\kappa_2\tilde{x})$

this operator lives on the light-cone



$\tilde{x}$ light-ray , $\tilde{x}^2=0$

κ_i parameters

"light-ray operators"
"nonlocal operators"
"string operators"

$\Rightarrow$ basic QFT heoretic object !

$O^q(\kappa_1,\kappa_2)$ is a generalization of local operators

$$O^q = \bar{\psi}(\kappa_1\tilde{x})\,\gamma\tilde{x}\,\psi(\kappa_2\tilde{x}) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} \frac{\kappa_1^{n_1}}{n_1!}\,\frac{\kappa_2^{n_2}}{n_2!} \cdot \underbrace{\left(\frac{\partial}{\partial\kappa_1'}\right)^{n_1}\bar{\psi}(\kappa_1'\tilde{x})\,\gamma\tilde{x}\left(\frac{\partial}{\partial\kappa_2'}\right)^{n_2}\psi(\kappa_2'\tilde{x})\Big|_{\kappa_i'=0}}_{O_{n_1,n_2}}$$

$$O_{n_1,n_2} = (\tilde{x}\partial_x)^{n_1}\,\bar{\psi}(x)\,\gamma\tilde{x}\,(\tilde{x}\partial_x)^{n_2}\,\psi(x)\Big|_{x=0}$$

axial gauge!

$d\mu^2$

analogously

$$\mu^2 \frac{d}{d\mu^2} O_{n_1 n_2} = \sum_{n_1' n_2'} \gamma_{n_1 n_2 n_1' n_2'} O_{n_1' n_2'}$$

indeed simpler: scale invariance
translations, ..

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all order proof:

$$\mu^2 \frac{d}{d\mu^2} O^q(\kappa_1, \kappa_2) = \int dw_1\, dw_2\, \hat{\gamma}(w_1, w_2)\, O^q(\kappa_1', \kappa_2')$$

$$w_1 = \frac{\kappa_+' - \kappa_+}{\kappa_-}, \quad w_2 = \frac{\kappa_-'}{\kappa_-} \qquad \kappa_\pm = \frac{\kappa_2 \pm \kappa_1}{2}$$

support:

$\hat{\gamma}(w_1, w_2) \neq 0$ for $|w_1| \leq 1$ $|w_2| \leq 1$
$|w_1 \pm w_2| \leq 1$

Relations to <u>local anomalous dimensions</u>

$$\mu^2 \frac{d}{d\mu^2} \hat{O}^q(\kappa_+, \kappa_-) = \int dw_1\, dw_2\, \hat{\gamma}(w_1, w_2)\, \hat{O}^q(w_1\kappa_- + \kappa_+, w_2$$

$$\downarrow \; (\partial_{\kappa_+})^{n_+} (\partial_{\kappa_-})^{n_-}$$

$$\mu^2 \frac{d}{d\mu^2} \hat{O}^q_{n_+, n_-} = \sum_{l=0}^{n_-} \overbrace{\binom{n_-}{l} \int dw_1 dw_2\, \hat{\gamma}(w_1, w_2)\, w_1^l\, w_2}^{\gamma_{n_+ + l,\, n_- - l}}$$

$$\cdot\, \hat{O}^q_{n_+ + l,\, n_- - l}$$

A spin-independent case

$$O^q(\kappa_1,\kappa_2) = \frac{i}{2}\left(\bar\psi(\kappa_1\tilde x)\gamma\tilde x\,\psi(\kappa_2\tilde x) - \bar\psi(\kappa_2\tilde x)\gamma\tilde x\,\psi(\kappa_1\tilde x)\right)$$

$$O^G(\kappa_1,\kappa_2) = \tilde x^\mu F_{\mu\nu}(\kappa_1\tilde x)\,\tilde x^{\mu'} F_{\mu'\nu}(\kappa_2\tilde x)$$

$$\mu^2\frac{d}{d\mu^2}O^i(\kappa_1,\kappa_2) = \frac{\alpha_s}{2\pi}\int_0^1 d\alpha_1\,d\alpha_2\,\theta(1-\alpha_1-\alpha_2)$$

$$K^{ij}(\alpha_1,\alpha_2,\kappa_1,\kappa_2)\,O^j(\kappa_1(1-\alpha_1)+\kappa_2\alpha_1,\kappa_1\alpha_2+\kappa_2(1-$$

for $\kappa_1=1,\ \kappa_2=0$ Th. Braunschweig, B. Geyer, D. Robasch. Annalen d. Physik (Leipzig) 44 (1987) 407

$$K^{qq} = C_F\left(1-\delta(\alpha_1)-\delta(\alpha_2)+\delta(\alpha_1)\left[\frac{1}{\alpha_2}\right]_+ + \delta(\alpha_2)\left[\frac{1}{\alpha_1}\right]_+\right.$$
$$\left.+\frac{3}{2}\delta(\alpha_1)\,\delta(\alpha_2)\right)$$

$$K^{Gq} = 2C_F\left(1+\frac{1}{2}\delta(\alpha_1)\delta(\alpha_2) - \delta(1-\alpha_1-\alpha_2)\right)$$

$$K^{qG} = N_F\left(1-\alpha_1-\alpha_2+4\alpha_1\alpha_2\right)$$

$$K^{GG} = C_A\Big[4(1-\alpha_1-\alpha_2)+3\alpha_1\alpha_2+\left(\frac{11}{6}-\frac{2}{6}\frac{N_F}{C_A}\right)\delta(\alpha_1)\delta$$
$$+\delta(\alpha_1)\left(\left[\frac{1}{\alpha_2}\right]_+ -2-\alpha_2\right)+\delta(\alpha_2)\left(\left[\frac{1}{\alpha_1}\right]_+ -2-\alpha_1\right.$$

I. Balitsky V. Braun 1989

where

$$\hat{\gamma}^{AB}(\underline{w}) = -\frac{g^2}{8\pi^2}\,\theta(1 + w_1 - w_2)\,\theta(w_2)\,\theta(1 - w_1 - w_2)\,\gamma_0^{AB},$$

with

$$\begin{aligned}
\gamma_0^{qq}(\underline{w}) &= C_N\left\{\frac{1}{2} - \left[1 - \frac{3}{4}\,\delta(1 - w_2) - \frac{1}{(1 - w_2)_+}\right]\right.\\
&\qquad \left.\times\,[\delta(1 + w_1 - w_2) + \delta(1 - w_1 - w_2)]\right\},\\
\gamma_0^{Gq}(\underline{w}) &= C_N\{1 + \delta(w_1)\,\delta(1 - w_2)\},\\
\gamma_0^{qG}(\underline{w}) &= T\{1 - w_1{}^2 - w_2(1 - w_2)\},\\
\gamma_0^{GG}(\underline{w}) &= C_A\left\{\frac{3}{2}\,(1 - w_1{}^2 + w_2{}^2) - w_2\right.\\
&\qquad + \left[\frac{w_2{}^2}{(1 - w_2)_+} + \left(\frac{11}{12} - \frac{1}{3}\,\frac{T}{C_A}\right)\delta(1 - w_2)\right]\\
&\qquad \left.\times\,[\delta(1 + w_1 - w_2) + \delta(1 - w_1 - w_2)]\right\}.
\end{aligned}$$

($C_N = 4/3$, $C_A = 3$ and $T = 1/2$ are the well known group invariants c
In eqs. (2.16) the following definition for the $(\ldots)_+$-description is used

$$\frac{1}{(z - z')_+} \overset{\text{def}}{=} \frac{1}{z - z'} - \delta(z - z')\cdot\int^{z}\frac{\mathrm{d}y}{z - y},$$

to shorten the notation.

$$O_5^q(\kappa_1,\kappa_2) = \frac{1}{2}\left(\bar{\psi}(\kappa_1\tilde{x})\gamma_5\gamma\tilde{x}\,\psi(\kappa_2\tilde{x}) + \bar{\psi}(\kappa_2\tilde{x})\gamma_5\gamma\tilde{x}\,\psi(\kappa_1\tilde{x})\right)$$

$$O_5^G(\kappa_1,\kappa_2) = \frac{1}{2}\left(\tilde{x}^\mu F_{\mu\nu}(\kappa_1\tilde{x})\,\tilde{x}^\rho F^{D}_{\rho\nu}(\kappa_2\tilde{x}) - (\kappa_1 \leftrightarrow \kappa_2)\right)$$

analogously

$$K^{qq} = C_F\Big(1 - \delta(\alpha_1) - \delta(\alpha_2) + \delta(\alpha_1)\left[\frac{1}{\alpha_2}\right]_+ + \delta(\alpha_2)\left[\frac{1}{\alpha_1}\right]_+ + \frac{3}{2}\delta(\alpha_1)\delta(\alpha_2)\Big)$$

$$K^{qG} = 2C_F\left(1 - \frac{1}{2}\delta(\alpha_1)\delta(\alpha_2)\right)$$

$$K^{qG} = -N_F(1-\alpha_1-\alpha_2)$$

$$K^{GG} = C_A\Big[4(1-\alpha_1-\alpha_2) + \left(\frac{11}{6} - \frac{2}{6}\frac{N_F}{C_A}\right)\delta(\alpha_1)\delta(\alpha_2 + \delta(\alpha_1)\left(\left[\frac{1}{\alpha_2}\right]_+ - 2 + \alpha_2\right) + \delta(\alpha_2)\left(\left[\frac{1}{\alpha_1}\right]_+ - 2 +$$

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use : (a) RG Equation for Operators

(b) Definition of Amplitudes with matrix elements of the operators.

- $$\mu^2 \frac{d}{d\mu^2} \langle p_2 | O^q(\kappa_1, \kappa_2) | p_1 \rangle = \int dw_1\, dw_2\, \hat{\gamma}(w_1, w_2) \langle p_2 | O^q(\kappa_1', \kappa_2') | p_1 \rangle$$

- $$\left. \frac{\langle p_2 | O^q(-\kappa, \kappa) | p_1 \rangle}{\tilde{x} p_+} \right|_{\tau = \frac{\tilde{x} p_-}{\tilde{x} p_+}} = \int dt\, e^{-i\kappa_- \tilde{x} p_+ t}\, q(t, \tau, \mu^2)$$

$$\Rightarrow \quad \mu^2 \frac{d}{d\mu^2} q(t, \tau, \mu^2) = \int_{-1}^{+1} \frac{dt'}{|\tau|}\, \gamma_E\left(\frac{t}{\tau}, \frac{t'}{\tau}\right) q(t', \tau, \mu^2)$$

$$\gamma_E(T, T') = \int dw_2\, \hat{\gamma}(T' w_2 - T, w_2)$$

Generalized Evolution Equation

Generalized Kernel

Ditter, Geyer, Hořejši, Müller, Robaschik

Phys. Lett. 209 B 325 (1988)

Fortschr. Physik 42 101–141 (1994)

forward direction).

$$q_1 + p_1 \to q_2 + p_2$$

Photon Hadron Photon Hadron

kinematics: $q = \frac{q_1+q_2}{2}$ $\quad p_+ = p_1 + p_2$, $p_- = p_2 - p_1 = q_1 - q_2$

Scaling variables:

$$\nu \Rightarrow p_+ q \to \infty \qquad Q^2 = -q^2 \to \infty$$

$$\xi = \frac{Q^2}{\nu} \text{ fix}$$

$$\eta = \frac{p_- q}{p_+ q} = \frac{q_1^2 - q_2^2}{2\nu} \text{ fix. } \quad \frac{q_1^2}{\nu} = -(\xi - \eta) \quad \frac{q_2^2}{\nu} = -(\xi + \eta)$$

Representation of the Scattering Amplitude

$$T_\mu^\mu = i \int d^4x \, e^{iqx} \langle p_2 | T j_\mu(\tfrac{x}{2}) j^\mu(-\tfrac{x}{2}) | p_1 \rangle$$

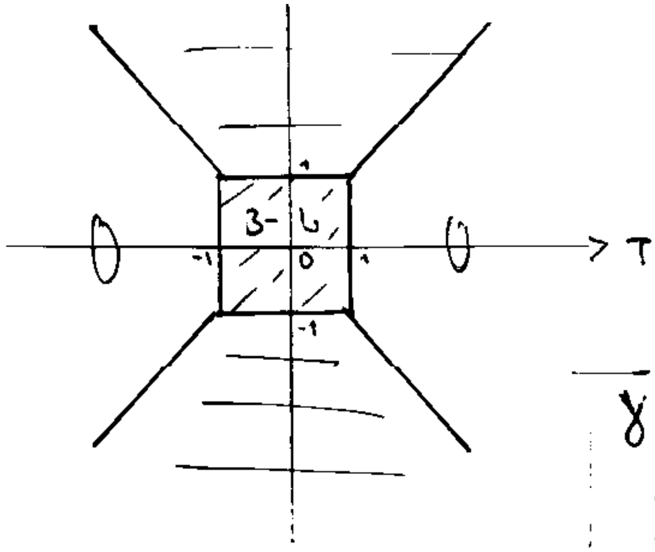
$$= 2 \int_{-1}^{+1} dt \underbrace{\left(\frac{1}{\xi + t} - \frac{1}{\xi - t} \right)}_{\text{perturbative part (coeff. function)}} \underbrace{e_a}_{\text{flavour}} \underbrace{q^a(t, \eta, \mu^2 = Q^2)}_{\text{nonperturbat. distribution function}}$$

- representation as convolution
- <u>one</u> distribution function
- new evolution eq. for distribution function

Fortschr. d. Physik 1994

Ji 1996 Radyushkin 1996

K8



Extended Brodsky - Lepage Kernel

$$\gamma_E(T,T') = \int dw_2\, \hat{\gamma}(T'w_2 - T, w_2)$$

Special cases:

- $\tau \to 0 \quad \sim P_1 = P_2$

$$Q^2 \frac{d}{dQ^2} q^a(z, Q^2) = \int_{-1}^{+1} \frac{dz'}{|z'|} P\left(\frac{z}{z'}, \alpha_s(Q^2)\right) q^a(z', Q^2)$$

$$|z'|^{-1} P\left(\frac{z}{z'}\right) = \lim_{\tau \to 0} \frac{1}{|\tau|}\left[\hat{\gamma}\left(\frac{z}{\tau}, \frac{z'}{\tau}\right)\right]_+$$

Altarelli Parisi

- $\tau \to -1 \quad P_2 \to 0$ (!)

$$Q^2 \frac{d}{dQ^2} \phi^a(x, Q^2) = \int_0^1 dy\, V_{BL}(x, y, \alpha_s)\, \phi^a(y, Q^2)$$

$$V_{BL}(x, y, \alpha_s) = \left[\hat{\gamma}(2x-1, 2y-1)\right]_+ \quad 0 \le x, y \le 1$$

Brodsky - Lepage

reconstruction!

$$\gamma_E^{qq}(T,T') = \frac{\alpha_S}{2\pi} C_F \left\{ \mathrm{sign}(1-T')\, \theta\left(\frac{1-T}{1-T'}\right) \theta\left(1-\frac{1-T}{1-T'}\right) \right.$$

$$\left. \cdot \frac{1-T}{1-T'} \left(1 - \frac{2}{T-T'}\right)_+ + \begin{matrix} T \to -T \\ T' \to -T' \end{matrix} \right\}$$

$$Q^2 \frac{d}{dQ^2} q^a(z, Q^2) = \int_{-1}^{1} \frac{dz'}{|z'|} P\left(\frac{z}{z'}; \alpha_s(Q^2)\right) q^a(z', Q^2), \tag{4.12}$$

$$|z'|^{-1} P\left(\frac{z}{z'}\right) = \lim_{\tau \to 0} \frac{1}{|2\tau|} \left[\gamma\left(\frac{z}{\tau}, \frac{z'}{\tau}\right)\right]_+, \quad \text{Altarelli-Parisi-(Lipatov) kernel.} \tag{4.13}$$

- Evolution equation for hadron wave functions:

This case can be obtained as a formal limit too [compare the remarks after Eq. (2.12)].

$$\Phi^a(x = (1+t)/2, Q^2) = \lim_{\tau \to -1} q^a(t, \tau, Q^2),$$

$$Q^2 \frac{d}{dQ^2} \Phi^a(x, Q^2) = \int_0^1 dy\, V_{BL}(x, y; \alpha_s(Q^2))\, \Phi^a(y, Q^2), \tag{4.14}$$

$$V_{BL}(x, y) = [\gamma(2x-1, 2y-1)]_+ |_{0 \leqq x, y \leqq 1}, \quad \text{Brodsky-Lepage kernel.} \tag{4.15}$$

Here, the variables $x = (1+t)/2$, $y = (1+t')/2$ are restricted to $0 \leqq x$, $y \leqq 1$. Note that both kernels (4.13) and (4.15) are calculated separately in QCD. From the above considerations it is obvious, however, that they have a common origin, namely the anomalous dimension $\gamma(w_+, w_-)$ of the light-ray operators being hidden within the corresponding amplitudes. We return to this point in section V.

Let us add a remark concerning the distribution amplitude introduced for the non-forward virtual Compton scattering. This distribution amplitude appearing in (3.14) or (3.23) satisfies in the flavour nonsinglet case the evolution equation

$$Q^2 \frac{\partial}{\partial Q^2} q^a(t, \eta, Q^2) = \int_{-1}^{1} \frac{dt'}{|2\eta|} \left[\gamma\left(\frac{t}{\eta}, \frac{t'}{\eta}\right)\right]_+ q^a(t', \eta, Q^2). \tag{4.16}$$

Here, the distribution function depends on variables with a clear physical interpretation. It coincides formally with the definition (4.8) so that no further proof is needed. Never

Here, we discuss the general solution of the problems stated above. Starting from the support properties of the anomalous dimension $\gamma(w_+, w_-)$ and from the definition of the evolution kernel (4.9), we study its domain of definition. Afterwards, we study the extension procedure. It turns out that the restricted BL-kernel contains already the essential information and that an explicit continuation procedure can be prescribed.

The first question is: What it the correct region in the (t, t')-plane where the kernel $\gamma(t, t')$ is defined in fact. To answer it, we start from the general representation (4.9) of this kernel

$$\gamma(t, t') = \iint dw_- \, dw_+ \, \delta(w_+ - t + t' w_-) \, \gamma(w_+, w_-)$$

and use all known symmetry properties and support restrictions:

$$\gamma(w_+, w_-) = \gamma(-w_+, w_-), \quad |w_\pm| \leqq 1, \quad |w_+ \pm w_-| \leqq 1.$$

After some algebra we obtain the following representation in the (t, t')-plane [7]

$$\begin{aligned}
\gamma(t, t') = &\left[\theta(t - t')\,\theta(1 - t) - \theta(t' - t)\,\theta(t - 1)\right] f(t, t') \\
&+ \left[\theta(t' - t)\,\theta(1 + t) - \theta(t - t')\,\theta(-t - 1)\right] f(-t, -t') \\
&+ \left[\theta(-t, -t')\,\theta(1 + t) - \theta(t' + t)\,\theta(-t - 1)\right] g(-t, t') \\
&+ \left[\theta(t + t')\,\theta(1 - t) - \theta(-t' - t)\,\theta(t - 1)\right] g(t, -t'),
\end{aligned} \qquad (5.$$

where the functions $f(t, t')$ and $g(t, t')$ are given by

$$f(t,t') = \int_0^{\frac{1-t}{1-t'}} dw_- \, \gamma(w_+ = t - w_- t', w_-),$$

$$g(-t,t') = \int_0^{\frac{1+t}{1-t'}} dw_- \, \gamma(w_+ = -t - w_- t', -w_-). \tag{5.2}$$

Note, that for $|t|$, $|t'|>1$ because of support restrictions the lower boundary in the integral representations (5.2) is not attained. However, in these regions only the differences

$$f(t,t') - f(-t,-t') \quad \text{and} \quad g(-t,t') - g(t,-t')$$

appear in Eq. (5.1), so that such undetermined contributions eliminate each other. It is clear, therefore, that the evolution kernel is defined in the complete (t, t')-plane as shown in Fig. 3.

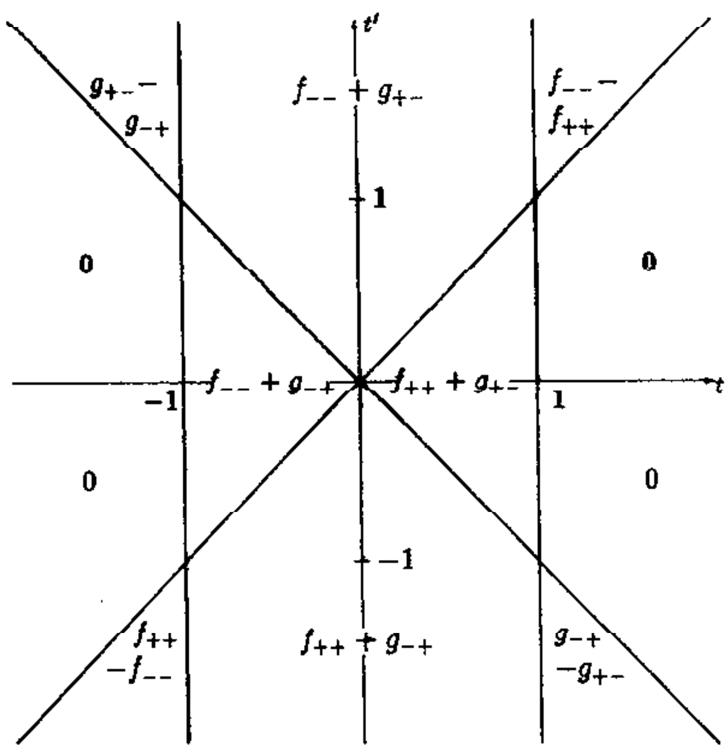


Fig. 3. Support of $\gamma(t, t')$, where $f_{\pm\pm} = f(\pm t, \pm t')$, and $g_{\pm\mp} = g(\pm t, \mp t')$ are defined by Eqs. (5.2).

Representations

Use • $F^q(z_+, z_-, \mu^2) = \int \frac{d\tilde{x}p_+ \kappa}{2\pi \tilde{x} p_+} \int \frac{d\tilde{x}p_- \kappa}{2\pi} e^{i\kappa\tilde{x}p_+ z_+ + i\kappa\tilde{x}p_- z_-}$

$$\langle p_2 | O^{qq}(-\kappa, +\kappa) | p_1 \rangle$$

• $\mu^2 \frac{d}{d\mu^2} \langle p_2 | O^q(\kappa_1, \kappa_2) | p \rangle = \int dw_1 \, dw_2 \, \hat{\gamma}(w_1, w_2 |$

$$\langle p_2 | O^{qq}(\kappa_1', \kappa_2') | p_1 \rangle$$

$$\mu^2 \frac{d}{d\mu^2} F^q(z_+, z_-, \mu^2) = \int \frac{dz_+'}{|z_+'|} dz_-' \, \hat{\gamma}\left(z_- - \frac{z_+}{z_+'} z_-', \frac{z_+}{z_+'}\right)$$

$$F^q(z_+', z_-')$$

Radyushkin 1996/97

M.A. Ahmed, G.G. Ross	1976	LCE
A.P. Bukhostov, E.A. Kuraev L.N. Lipatov	~1984	Matrix el.
E.V. Shuryak, A.I. Vainshtein	1982	LCE
J. Kodaira, Y. Yasui, T. Uematsu K. Tanaka, ..	1979- 1996	LCE EOM
A.V. Efremov, O.V. Teryaev P.G. Ratcliffe	1986	Matrix el.
I.I. Balitzky, V.M. Braun	1989	NLCE
X. Ji, C. Chou	1990 ⋮	LCE

and others!

Problems:

choice of operators

equations of motion EOM

different methods LCE, NLCE, Matrix el.

to understand it better for applications!

$T\, j^\mu(x)\, j^\nu(y) =$  $+$  $+ \cdots$

lowest order, antisymmetric part:

$$\left[T\, j^\mu(x)\, j^\nu(y)\right]^{[\mu\nu]} = \varepsilon^{\mu\nu\rho\sigma} \Big\{$$

$$+ i \partial_\rho^x D^c(x,y)\, \bar\psi(x)\, \gamma_\sigma \gamma_5\, U(x,y)\, \psi(y)$$

$$- g D^c(x,y)\, \bar\psi(x)\, \gamma_\sigma \int_0^1 dt\, U(x,z) \Big\{ (t - \tfrac{1}{2}) F_{\rho\lambda} \gamma_5 - \tfrac{i}{2} \tilde F_{\rho\lambda} \Big\} (x-y)^\lambda U(z,y)\, \psi$$

$$+ g^2 \ldots \quad + (x,\mu) \leftrightarrow (y,\nu) \Big\} \qquad z = (x-y)t + y$$

Calculation: Propagator in external field A_μ, exact to order g, gauge covariant Gross / Treiman; B. Geyer, D.R., O. Teryaev

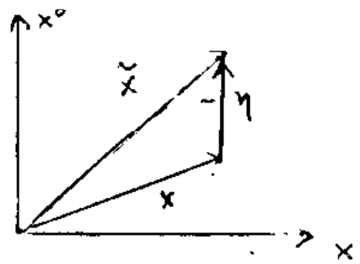
classical external field $\rightarrow$ quantum field (correct!)

similar: Balitzky / Braun.

first step for nonlocal Light-Cone Expansion.

Simplification $y=0$

Approach the Light-cone: $\tilde x = x + a\eta$, η fixed



$\tilde x^2 = 0$

second vector $\underset{\sim}{x}$: $\tilde x \underset{\sim}{x} = 1$

$\underset{\sim}{x}^2 = 0$

R1

$\psi(\kappa_1 x)\, \gamma_\sigma \gamma_5\, \psi(\kappa_2 x)$

$$\bar\psi(\kappa_1\tilde x)\, \gamma_\sigma \int_0^1 d\tau \left\{ \gamma_5 \left(\tau - \tfrac{1}{2}\right) F_{\varrho\lambda}(t\tilde x) - \tfrac{i}{2} \tilde F_{\varrho\lambda}(t\tilde x) \right\} \tilde x^\lambda\, \psi(\kappa_2 \tilde x)$$

$$t = (\kappa_1 - \kappa_2)\tau + \kappa_2$$

<u>problems:</u>

- evolution equations follow from the renormalization group equations
- under renormalization further operators appear of the same dimension and twist

  example: $m\, \bar\psi(\kappa_2\tilde x)\gamma_5[\gamma_\sigma, \gamma_\lambda]\psi(\kappa_2\tilde x)\, \tilde x^\lambda, \ldots$

  $\bar\psi(\kappa_1\tilde x)\gamma_\sigma\gamma_5\psi(\kappa_2\tilde x)$ has no definite twist!
- the operators are related by the eq. of motion!

choice of operators:

$$S^\pm_\sigma(\kappa_1, \tau, \kappa_2) = g\, \bar\psi(\kappa_1\tilde x)\left[F_{\sigma\lambda}(\tau\tilde x)\gamma_5 \pm i \tilde F_{\sigma\lambda}(\tau\tilde x)\right]\tilde x \gamma\, \psi(\kappa_2\tilde x)\, \tilde x^\lambda$$

$$\Omega^m_\sigma = m\, \psi(\kappa_1\tilde x)\gamma_5[\gamma_\sigma, \tilde x_\lambda\gamma^\lambda]\psi(\kappa_2\tilde x)$$

calculation:

- here: eq. of motion taken into account
- light-cone gauge
- result: support Balitzki-Braun

$$\bar\psi(\kappa_2\tilde x)\,\gamma\tilde x\, \underset{(\tau\tilde x)}{F_{\sigma\lambda}}\, \tilde x^\lambda\, \psi(\kappa_1\tilde x), \quad \bar\psi(\kappa_2\tilde x)\gamma_5\gamma\tilde x\, \tilde F_{\sigma\lambda}\, \tilde x^\lambda\psi(\kappa_1\tilde x)$$

R 2b

$$O_{1\sigma} = \bar{\psi}(0)\, \gamma_\sigma \gamma_5\, \psi(\kappa_2 \tilde{x}) + \text{sym}$$

no definite twist
traces

$$= \left(\check{\Omega}^{(2)}_\sigma + \tilde{\Omega}^{(3)}_\sigma \right)$$

$\tilde{\Omega}^{(3)}_\sigma \to \Omega^{(3)}_\sigma$
change of normalization

$$\Omega^{(3)}_\sigma = \bar{\psi}(0)\, \gamma_5 \gamma_\sigma (\tilde{x}\partial)\, \psi(\kappa_2 \tilde{x}) -$$

$$- \bar{\psi}(0)\, \gamma_5 (\tilde{x}\gamma) \left(\partial^\sigma - ig \frac{1}{\kappa_2} \int_0^{\kappa_2} d\tau\, A^\sigma(\tau \tilde{x}) \right) \psi(\kappa_2 \tilde{x}) +$$

- Operators of twist 2 and 3, howeve trace exist!
- $\left(\frac{d}{d\kappa_2}\right)^n \Omega^3{}_\sigma(0,\kappa_2)\Big|_{\kappa_2 = 0}$ = local operator of twist 3
- axial gauge!
- local operators – reordered, eq. of motion
  → Shuryak / Vainshtein operator

$$\boxed{\Omega^{(3)}_\sigma = \Omega^{SV}} + \Omega^{EOM} + \Omega^{Rem}$$

$$\Omega^{SV}_\sigma = -\frac{i}{2} g\, \bar{\psi}(0) \int_0^{\kappa} d\tau \left\{ i \tilde{F}_{\sigma\rho} + \gamma_5\, F_{\sigma\rho}(\tau\tilde{x}) \left(1 - 2\frac{\tau}{\kappa_2}\right) \right.$$

$$\cdot\, \tilde{x}^\rho\, \gamma_\rho (\tilde{x}^\nu)\, \psi(\kappa_2 \tilde{x}) + \text{sym}$$

A simila operator appeared however already in
NLCE (second terms!)
(not contracted with $\tilde{x}^\rho$, antisymmetrized!)

$$\frac{2C_F - C_A}{2}\Big[y\delta(z)S^-(-\kappa_1 y, \kappa_2 - \kappa_1 y) - 2zS^-(\kappa_1 - \kappa_2(1-z), -\kappa_2 y) +$$

$$K(y,z)S^-(\kappa_1(1-y) + \kappa_2 y, \kappa_2(1-z) + \kappa_A z)\Big] +$$

$$\frac{C_A}{2}\Big[(2(1-z) + L(y,z))\,S^-(\kappa_1 - \kappa_2 z, \kappa_2 y) + L(y,z)S^-(\kappa_1 y, \kappa_2 - \kappa_1 z)\Big] -$$

$$\mu^2\frac{d}{d\mu^2}S^+(\kappa_1,\kappa_2) = \frac{\alpha_s}{2\pi}\int_0^1 dy\int_0^{1-y} dz$$

$$\frac{2C_F - C_A}{2}\Big[y\,\delta(z)S^+(\kappa_1 - \kappa_2 y, -\kappa_2 y) - 2zS^+(-\kappa_1 y, \kappa_2 - \kappa_1(1-z)) +$$

$$K(y,z)S^+(\kappa_1(1-y) + \kappa_2 y, \kappa_2(1-z) + \kappa_A z)\Big] +$$

$$\frac{C_A}{2}\Big[(2(1-z) + L(y,z))\,S^+(\kappa_1 y, \kappa_2 - \kappa_1 z) + L(y,z)S^+(\kappa_1 - \kappa_2 z, \kappa_2 y)\Big] -$$

$$K(y,z) = \left[1 + \delta(z)\,\frac{1-y}{y} + \delta(y)\,\frac{1-z}{z}\right]_+$$

$$L(y,z) = \left[\delta(1-y-z)\,\frac{y^2}{(1-y)} + \delta(z)\,\frac{y}{(1-y)}\right]_+ - \frac{7}{4}\delta(1-y)\delta(z)$$

$$S^\pm(\kappa_1,\kappa_2) = \bar\psi(\kappa_1\tilde x)\left[\gamma^5 F^{\rho\sigma}(0) \pm i\tilde F^{\rho\sigma}(0)\right]\hat{\tilde x}\tilde x_\sigma\psi(\kappa_2\tilde x)$$

$$O_m(\kappa_1,\kappa_2) = m\frac{d}{d\kappa_1}\bar\psi(\kappa_1\tilde x)\gamma^5\sigma^{\rho\sigma}\tilde x_\sigma\psi(\kappa_2\tilde x)$$

Calculation by B. Geyer, D. Müller, D.R.
Proceedings Rheinsberg 1996

Confirms the result of Balitsky, Braun 1989