DIS'97
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NLO evolution of polarized fragmentation functions

(Marco Stratmann, Dortmund)

(with W. Vogelsang)

- Derivation of NLO spin-dep. time-like spl. fcts. $\Delta P_{ij}^{(T),(1)}$ via analytic continuation
  - analytic cont. rule + breaking beyond LL
  - calculation of ACR violating pieces
- ACR breaking $\Leftrightarrow$ fact. scheme transformation
- 'SUSY - relation'

(with D. de Florian, W. Vogelsang)

- <u>Application</u>: LO + NLO analysis of Λ prod

General Framework

'TL' case completely anal. to 'SL' case:

Consider a 'TL' structure function $g_1^{(T)}$, e.g.:

$$\frac{d\Delta\sigma^{e^+\vec{e}^- \to \vec{\Lambda} X}}{dz\,dy} = \frac{6\pi\alpha^2}{Q^2}(2y-1)\, g_1^{(T)}(z,Q^2)$$

with $\equiv D_{q+}^{\Lambda+} - D_{q+}^{\Lambda-}$ usual convolution

$$g_1^{(T)}(z,Q^2) = \sum_q e_q^2 \Big\{ [\Delta D_q^\Lambda + \Delta D_{\bar{q}}^\Lambda] \otimes \Delta C_q^{(T)} + 2\,\Delta D_g^\Lambda \otimes \Delta C_g^{(T)} \Big\}(z,Q^2)$$

$\rightsquigarrow$ Need 'TL' hard subprocess cross sections $\Delta C_q^{(T)}$, $\Delta C_g^{(T)}$ and Q^2-evolution of ΔD_q, ΔD_g in the same scheme

$\Updownarrow$

phys. quant. $g_1^{(T)}$ scheme independent!

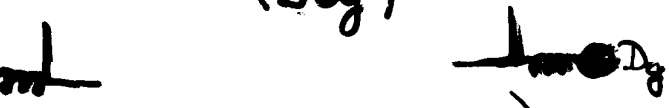
Q^2 - Evolution:

- 2 non-singlet eqs. for $\Delta D_{q\pm} \equiv \Delta D_q \pm \Delta D_{\bar{q}}$

  (evolve with $\Delta P^{(T)}_{qq+}$, $\Delta P^{(T)}_{qq-}$ $(= \Delta P_{qq}$ in LO$)$)

- a coupled singlet eq.:

$$\frac{d}{d\ln Q^2}\begin{pmatrix}\Delta D_\Sigma\\ \Delta D_g\end{pmatrix}(z,Q^2) = \left[\Delta\hat{P}^{(T)} \otimes \begin{pmatrix}\Delta D_\Sigma\\ \Delta D_g\end{pmatrix}\right](z,Q^2)$$



$$\nabla \quad \Delta\hat{P}^{(S)} = \begin{pmatrix}\Delta P^{(S)}_{qq} & \Delta P^{(S)}_{qg}\\ \Delta P^{(S)}_{gq} & \Delta P^{(S)}_{gg}\end{pmatrix} \longrightarrow \Delta\hat{P}^{(T)} = \begin{pmatrix}\Delta P^{(T)}_{qq} & 2n_f\,\Delta P^{(T)}_{gq}\\ \frac{1}{2n_f}\Delta P^{(T)}_{qg} & \Delta P^{(T)}_{gg}\end{pmatrix}$$

where $\Delta P_{ij} = \underbrace{\frac{\alpha_s}{2\pi}\Delta P_{ij}^{(0)}}_{LO} + \underbrace{\left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{ij}^{(1)}}_{NLO} + \dots$

LO results:

$\xi = (x,z) < 1$

- Gribov-Lipatov relation: $\Delta P^{(T),(0)}_{ij}(\xi) = \Delta P^{(S),(0)}_{ij}(\xi)$
- Anal. cont. rule (Drell, Levy, Yan): $\Delta P^{(T),(0)}_{ij}(z) = z \cdot AC\left[\Delta P^{(S),(0)}_{ji}(x=\right.$

  AC 'operator': $x \to \frac{1}{z} > 1$ + color factor and sign adjustme

E.g.: $\Delta P_{qq}^{(T),(0)}(z) = -z\,\Delta P_{qq}^{(S),(0)}\left(\frac{1}{z}\right)$,

$$\Delta P_{gq}^{(T),(0)}(z) = \frac{C_F}{2T_f}\, z\, \Delta P_{qg}^{(S),(0)}\left(\frac{1}{z}\right)$$

$\vdots$

<u>BUT</u>: ACR based on symmetries of tree diagrams under crossing

$\longrightarrow$ <u>broken in NLO</u> !

(depending on the regularization, etc.)

<u>NLO results:</u>

$$\Delta P_{ij}^{(S),(1)} \xrightarrow{?} \Delta P_{ij}^{(T),(1)}$$

Mertig, van Neerven
Vogelsang

ACR breaking essentially due to kinematics !

Curci, Furmanski
Petronzio

$\Updownarrow$

easy to detect within 'Light-cone gauge method' used for calc. of $(\Delta)P_{ij}^{(S)}$

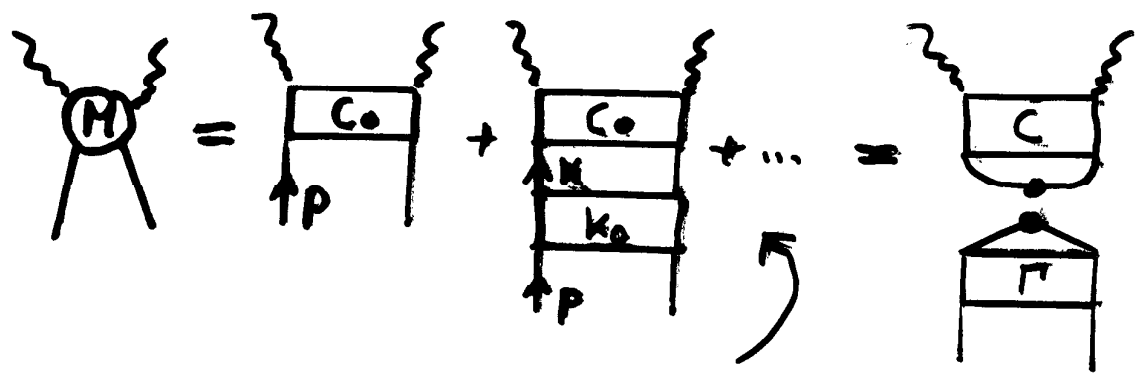
CFP
Ellis, Vogelsang
Vogelsang

P_{ij}^{S}
ΔP_{ij}^{S}

Reminder: 'Light-cone gauge method'

- close to 'parton picture' than formal OPE or 'cut vertex' methods
- based on fact. of mass sing. in axial gauge
- mass sing. regularized in $d = 4-2\varepsilon$ dim.

Strategy: rearrange pert. expansion — Ellis et al



axial gauge: 2PI kernels finite !

$\rightsquigarrow$ coll. sing. $\Leftrightarrow$ integr. over ext. lines

$$\Rightarrow \quad M_j = \sum_{i=q,g} C_i \otimes \Gamma_{ij}$$

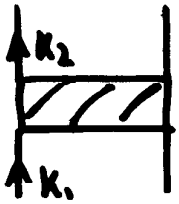
contains all (and only) mass sing.

e.g. $\Gamma_{ij}^{(1)}(x, \alpha_s, \frac{1}{\varepsilon}) = \delta(1-x)\delta_{ij} - \frac{1}{\varepsilon}\left[\frac{\alpha_s}{2\pi} P_{ij}^{(1),(0)}(x) + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(1),(1)}(x) + \ldots\right] + \mathcal{O}(\ldots$

ACR viol.: extra ε-pieces $\rightarrow$ contr. to '$\frac{1}{\varepsilon}$' from higher poles

<u>ACR violation:</u> $\exists$ 2 sources 'SL' $\to$ 'TL'

- extra phase space factor: Curci, Furmanski, Petronzio



$$: \left(\frac{k_2 \cdot n}{k_1 \cdot n}\right)^{-2\varepsilon} \equiv z^{-2\varepsilon}$$

gauge vector n spec. Long. direction! ↖ fraction of mom. transfer

- Spin averaging factors (<u>un</u>pol. case only!):

E.g. $\quad P_{qg}^{(S)} \longrightarrow P_{gq}^{(T)}$

in d dim.: $\quad \dfrac{1}{2(1-\varepsilon)} \xrightarrow{\nabla} \dfrac{1}{2}$

↳ <u>on aggregate</u>: (unpol. case)

$$\Gamma_{qq}^{(T)}(z, \alpha_s, \tfrac{1}{\varepsilon}) = -z^{1-2\varepsilon}\, \Gamma_{qq}^{(S)}(\tfrac{1}{z}, \alpha_s, \tfrac{1}{\varepsilon})$$

$$\Gamma_{qg}^{(T)}(z, \alpha_s, \tfrac{1}{\varepsilon}) = \frac{2T_f}{C_F}\, \frac{z^{1-2\varepsilon}}{1-\varepsilon}\, \Gamma_{gq}^{(S)}(\tfrac{1}{z}, \alpha_s, \tfrac{1}{\varepsilon})$$

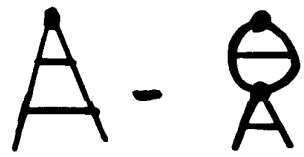
$\vdots$

extra ε-pieces $\longrightarrow$ add. contr. to $\frac{1}{\varepsilon}$-part of Γ!

e.g. $\quad \dfrac{1}{\varepsilon^2} z^{-2\varepsilon} = \dfrac{1}{\varepsilon^2} - \underline{\underline{\dfrac{1}{\varepsilon}\, 2 \cdot \ln z}} + \mathcal{O}(1)$

↳ go through NLO calc. of $\Gamma_{ij}^{(S)}$ graph by graph and pick up $1/\varepsilon^2$ pole terms $\to$ ACR viol. part!

Easy! Higher $\frac{1}{\varepsilon^2}$ terms ~ renorm. constants



'Ladder' 'Subtraction'

'Ladder': straightforward: $\frac{1}{\varepsilon^2}$ part ~ conv. of LO spl. fcts. for each rung $P^{(0)}_{jk} \otimes P^{(0)}_{ki}$

'Subtraction': similar but more subtle

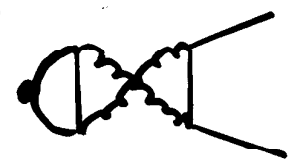
1) Kin. $z^{-2\varepsilon}$ corr. only in the upper part

Curci, Furmanski, Petronzio

2) Unpol.: E.g.

$P^{(S),(0)}_{qg,4-2\varepsilon} \otimes P^{(S),(0)}_{gq}$

'SL' → 'TL'



$P^{(T),(0)}_{gq,4-2\varepsilon} \otimes P^{(T),(0)}_{qg}$

→ spin-av. factors!

→ only source for ACR viol. from spin-av. for diagonal (qq, gg) splitting functions

↳ $(\Delta) P^{(T),(1)}_{ij}(z) = z \cdot AG'\left[(\Delta) P^{(S),(1)}_{ji}\left(x=\frac{1}{z}\right)\right] + \underline{(\Delta) P^{\varepsilon}_{ij}(z)}$

ACR viol. piece

Similarly

$$(\Delta)\, C^{(T),(1)}_{i}(z) = z \cdot AG'\left[(\Delta) C^{(S),(1)}_{i}\left(x=\frac{1}{z}\right)\right] + \underline{(\Delta) C^{\varepsilon}_{i}(z)}$$

Final step: carry out $AG'[\ldots] \to P^{(T),(1)}_{ij}$, $\Delta P^{(T),(1)}_{ij}$!

Two subtleties:

- 2PI Kernels finite $\sim \frac{1}{\varepsilon}$ part $\leftrightarrow$ integr. over mom. k

but $k^2<0$ ('SL') $\to k^2>0$ ('TL')
$\to$ not only AC $x \to x=\frac{1}{z}$ $\Big\}\sim$ extra π^2-term

(similar: $C^{(T)}$: $q^2<0 \to q^2>0$ $\sim$ extra $\pi^2\delta(1-z)$ term)

- AC of $\mathrm{Ln}^i(1-x)$ terms $\overset{\text{effectively}}{\longleftrightarrow}$ $\ln(1-x) \xrightarrow{x>1} \ln(x-1)$
$\ln^2(1-x) \xrightarrow{x>1} \ln^2(x-1)-\pi$

$\hookrightarrow$ **Final results:** ($\delta(1-z)$ contr. same as in 'SL' case)

Unpol.: $P_{ij}^{(T),(1)}$, $C_{i,k}^{(T)}$ agree with Curci, Furman, Petronzio; Furmanski, Pe

$i,j = q,\bar{q}$; $k = 1,2 \leftrightarrow F_1, F_2$

Pol.: $\Delta P_{ij}^{(T),(1)}$ NEW W. Vogelsang, H.

$\Delta C_i^{(T)}$ agree with de Florian, Sacco

Results can also be written as: (GLR-relation)

$$(\Delta) P_{ij}^{(T),(1)}(z) = (\Delta) P_{ij}^{(S),(1)}(z) + \underbrace{(\Delta)\,\delta_{ij}(z)}_{\text{GLR viol. piece}},$$

$\to$ $\boxed{\text{Complete NLO framework for pol. frag. functions now available}}$

A different way to understand ACR breaking:

W. Vogelsa
M. S

ACR fulfilled in LO

But: d-dim LO spl. fcts. do NOT fulfil ACR

$$(\Delta)\, P_{ij,4-2\varepsilon}^{(T),(0)} = z\, AG\left[(\Delta)\, P_{ji,4-2\varepsilon}^{(S),(0)}\left(x=\tfrac{1}{z}\right)\right] + \underline{(\Delta)\, P_{ij}^{\varepsilon,(0)}(z)}$$

E.g. $P_{qq}^{\varepsilon,(0)}(z) = (-2\ln z)\,\varepsilon\, P_{qq}^{(S),(0)}(z)$

$P_{gq}^{\varepsilon,(0)}(z) = (-2\ln z - 1)\,\varepsilon\, P_{gq}^{(S),(0)}(z)$

$\vdots$ ↑ Spin av.! (only unpol. case)

Idea:

→ ACR breaking driven by corr. breaking in the ε-part of $d=4-2\varepsilon$ dim. LO spl. fcts!

→ $\boxed{\text{ACR breaking} \leftrightarrow \text{Simple fact. scheme transf.!}}$

$$(\Delta)\, \hat{P}^{(T),(1)} \to (\Delta)\, \hat{P}^{(T)(1)} - \frac{\beta_0}{2}\, (\Delta)\hat{Z}^{(T)} + \left[(\Delta)\hat{Z}^{(T)}, (\Delta)\, \hat{P}^{(T),(0)}\right]_{\otimes}$$

$$(\Delta)\, C_i^{(T),(1)} \to (\Delta)\, C_i^{(T)(1)} - (\Delta) Z_{iq}^{(T)} \qquad (i = q, g)$$

where $(\Delta)\hat{Z}^{(T)} \equiv \begin{pmatrix} (\Delta)\, Z_{qq}^{(T)} & 2n_f\, (\Delta)\, Z_{gq}^{(T)} \\ \frac{1}{2n_f}\, (\Delta)\, Z_{qg}^{(T)} & (\Delta)\, Z_{gg}^{(T)} \end{pmatrix}$

$$Z_{ij}^{(T)} = (2\ln z + a_{ij})\, P_{ij}^{(S),(0)}, \quad a_{qq} = a_{gg} = 0,\ a_{gq} = 1,\ a_{qg} = -1$$

$$\Delta Z_{ij}^{(T)} = 2\ln z\ \Delta P_{ij}^{(S),0}$$

SUSY - relation

In $\mathcal{N}=1$ SUSY $(C_F = N_c = 2T_f \equiv N)$ see, e.g. Dokshits

$$(\Delta) S \equiv (\Delta) P_{qq} + (\Delta) P_{gq} - (\Delta) P_{qg} - (\Delta) P_{gg} \overset{!}{=} 0$$

→ Important check for any $(\Delta) P_{ij}$ calculation!

LO: ✓

NLO: OK only if reg. method respects SUSY

→ <u>violated</u> in dim. regularization

→ OK in dim. reduction (γ-Alg. in 4 dim.)

<u>But</u>: dim. reg. ⟷ dim. red. Körner, Tung, Kunszt, Signer, Trocsanyi, Kamal

related by simple fact. scheme trafo (ε-pieces of $(\Delta) P^{(0)}_{ij,4-2\varepsilon}$)

⇓

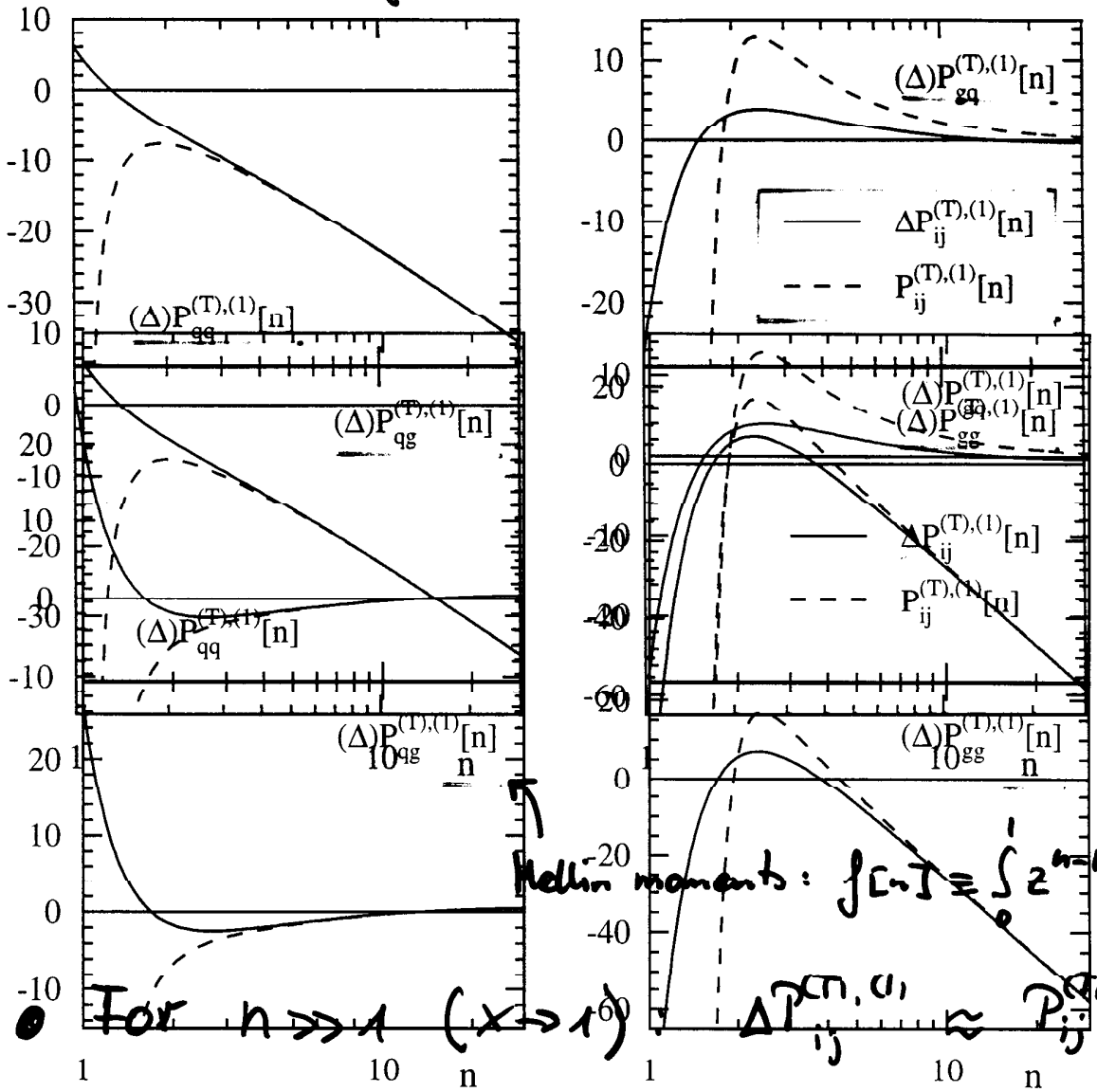
<u>'SL' case</u>: → unpol. ✓ Antoniadis, Floratos, Kunszt, Signer, Trocsanyi

→ pol. ✓ Mertig, van Neerven, Vogelsang

<u>'TL' case</u>: → unpol. ✓ W. Vogelsang, H. S.

→ pol. ✓

Comparison of $\Delta P_{ij}^{(T),(1)}$ and $P_{ij}^{(T),(1)}$:



Mellin moments: $f[n] \equiv \int_0^1 z^{n-1} f(z)\,dz$

- For $n \gg 1$ $(x \to 1)$ $\Delta P_{ij}^{(T),(1)} \approx P_{ij}^{(T),(1)}$

- $\Delta P_{ij}^{(T),(1)}$ have <u>finite</u> first moments

<u>Applications</u>: Λ-production ($\leftrightarrow \Delta D_q^\Lambda, \Delta D_g^\Lambda$)

- Λ-pol. can be 'easily' detected via weak decay $\Lambda \to \pi^- p$: $I(\theta) \simeq \frac{1}{4\pi}(1+0.64\cos\theta)$ see, e.g. Perkin

  decay dist. (Λ rest frame) $\measuredangle(\vec{p}_p, \vec{S}_\Lambda)$

- <u>Study</u> $e^+ \vec{e}^- \to \vec{\Lambda} X$ see: Burkardt, Jaffe, Kotzinian, Bravar, v. Harrach

  at high energies no pol. $\vec{e}$ required!

  ($q\bar{q}Z$-coup. parity viol. $\to$ net pol. of quarks!)

  Measured! (ALEPH, DELPHI) $\Rightarrow \Delta D_f^\Lambda$

  OR $\vec{e}p \to \vec{\Lambda} X$, $e\vec{p} \to \vec{\Lambda} X$

  Realistic! ($\to$ HERMES, COMPASS (μN))

- 'Naive' parton model: '$\Lambda(\uparrow) = (ud)\, s(\uparrow)$'

  $\longrightarrow \Delta D_u^\Lambda = \Delta D_d^\Lambda = 0$

<u>First step of analysis</u>: determine realistic set of unpol. frag. fcts (D_q^Λ, D_g^Λ)

<u>Needed</u> for $|\Delta D_f^\Lambda| \leq D_f^\Lambda$ ('<u>positivity</u>')

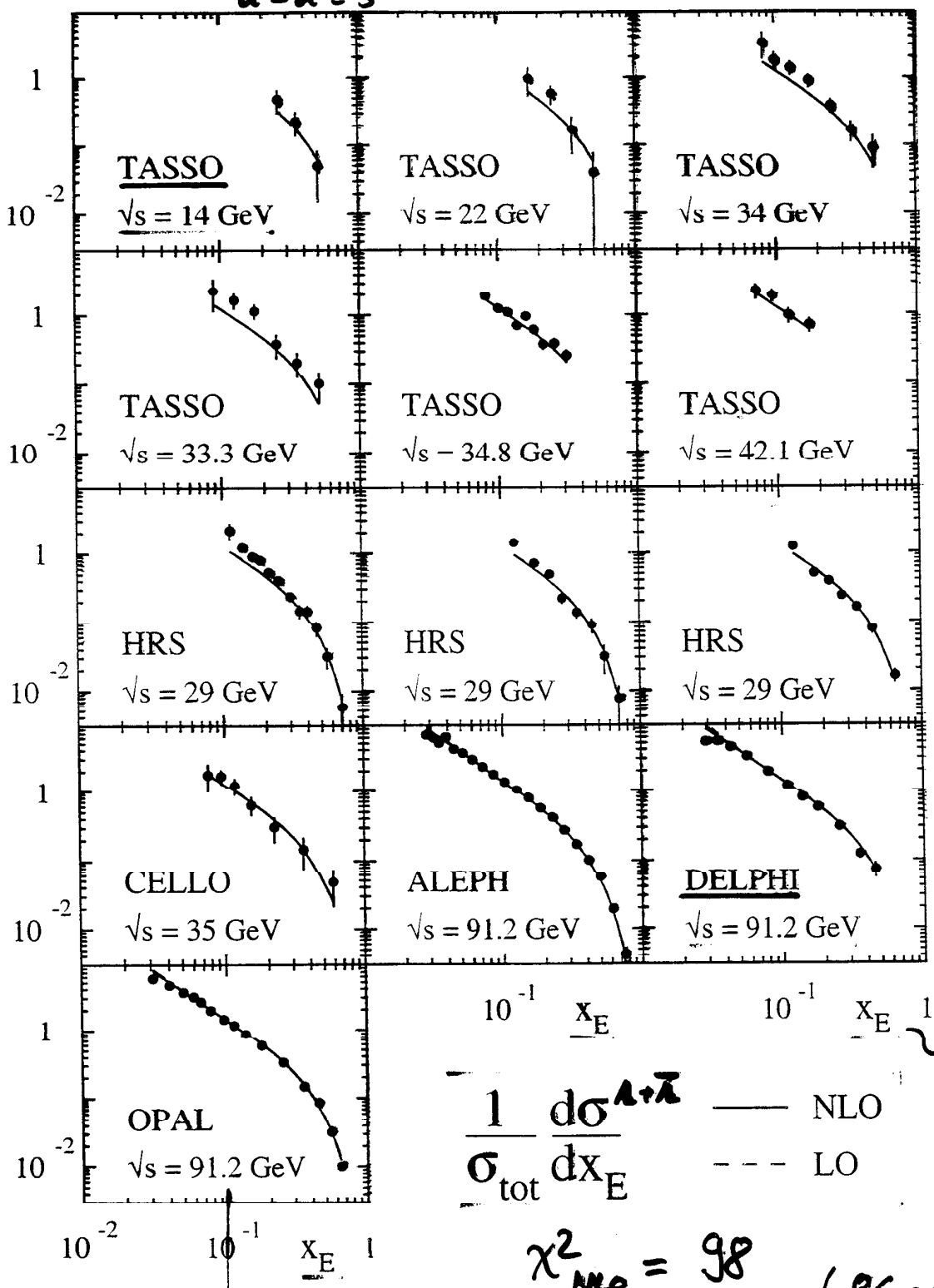
Result of unpol. fit:

D. de Florian, W. Vogelsang, M.S.

input: $D_{q\to\bar{q}}^{\Lambda+\bar{\Lambda}} = N x^{\alpha}(1-x)^{\beta}$, $D_g^{\Lambda+\bar{\Lambda}}$ similar at $\mu^2 = 0.3$ GeV

$\Lambda+\bar{\Lambda} \leftarrow$ measured

$u = d = s$



$= z = \frac{2}{\sqrt{\ }}$

$$\frac{1}{\sigma_{tot}}\frac{d\sigma^{\Lambda+\bar{\Lambda}}}{dx_E}$$

—— NLO

- - - LO

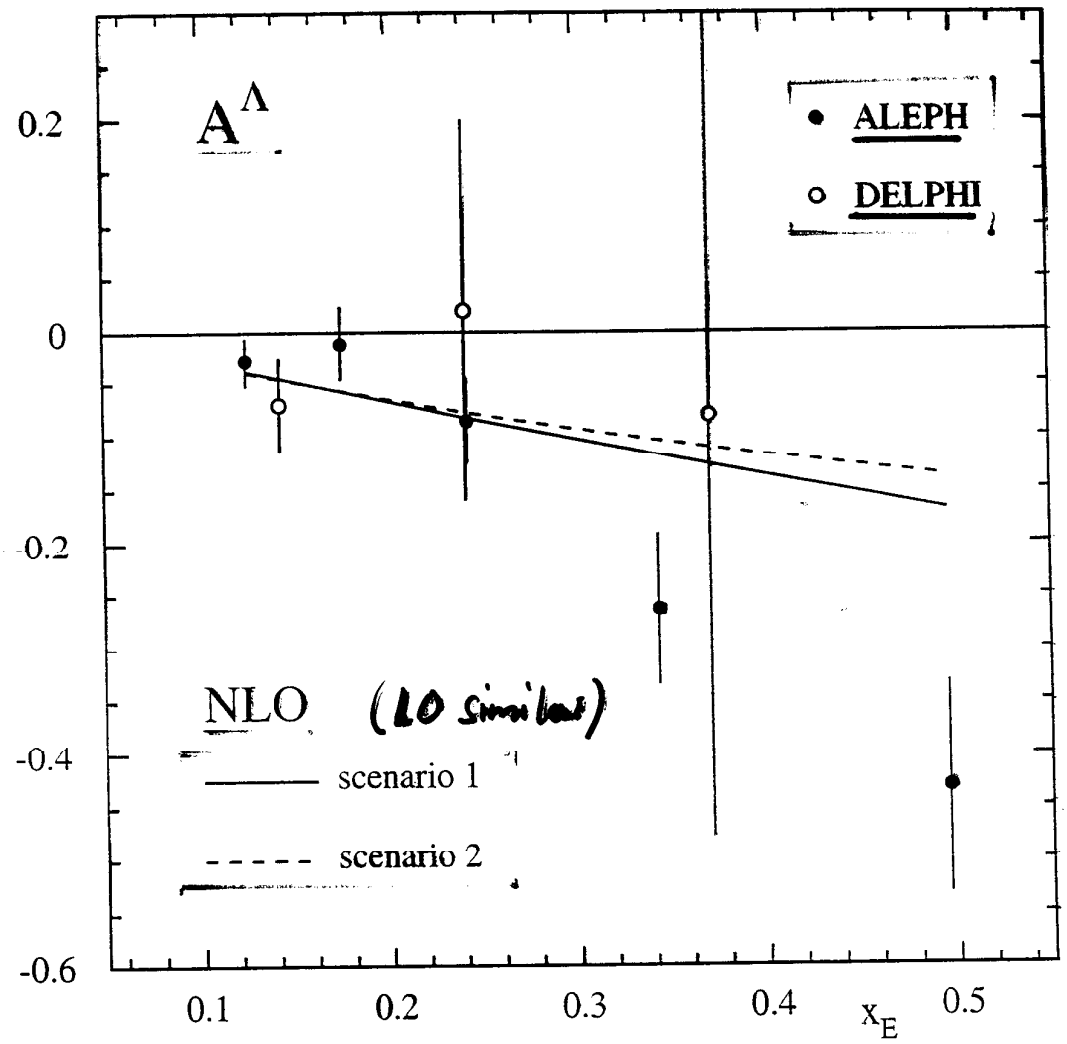
$\chi^2_{NLO} = 98$

$\chi^2_{LO} = 103$

(86 pts.)

only $x_E > 0.1$ included in fit

Result of pol. fit (preliminary): D. de Florian, W. Vogelsang, M.S.



At input scale μ^2:

Scen. 1 : ('naive') $\Delta D_s = x^\alpha D_s$, $\Delta D_u = \Delta D_d = 0$

Scen. 2 : ('Burkardt-Jaffe like') $\Delta D_s = x^\alpha D_s$, $\Delta D_{u,d} = -|N| \Delta I$ ($|N| \sim 0.2$)

Drawback: ALEPH/DELPHI fix only '$q-\bar{q}$' at M_Z^2! ($\Leftrightarrow g_3, g_4$)

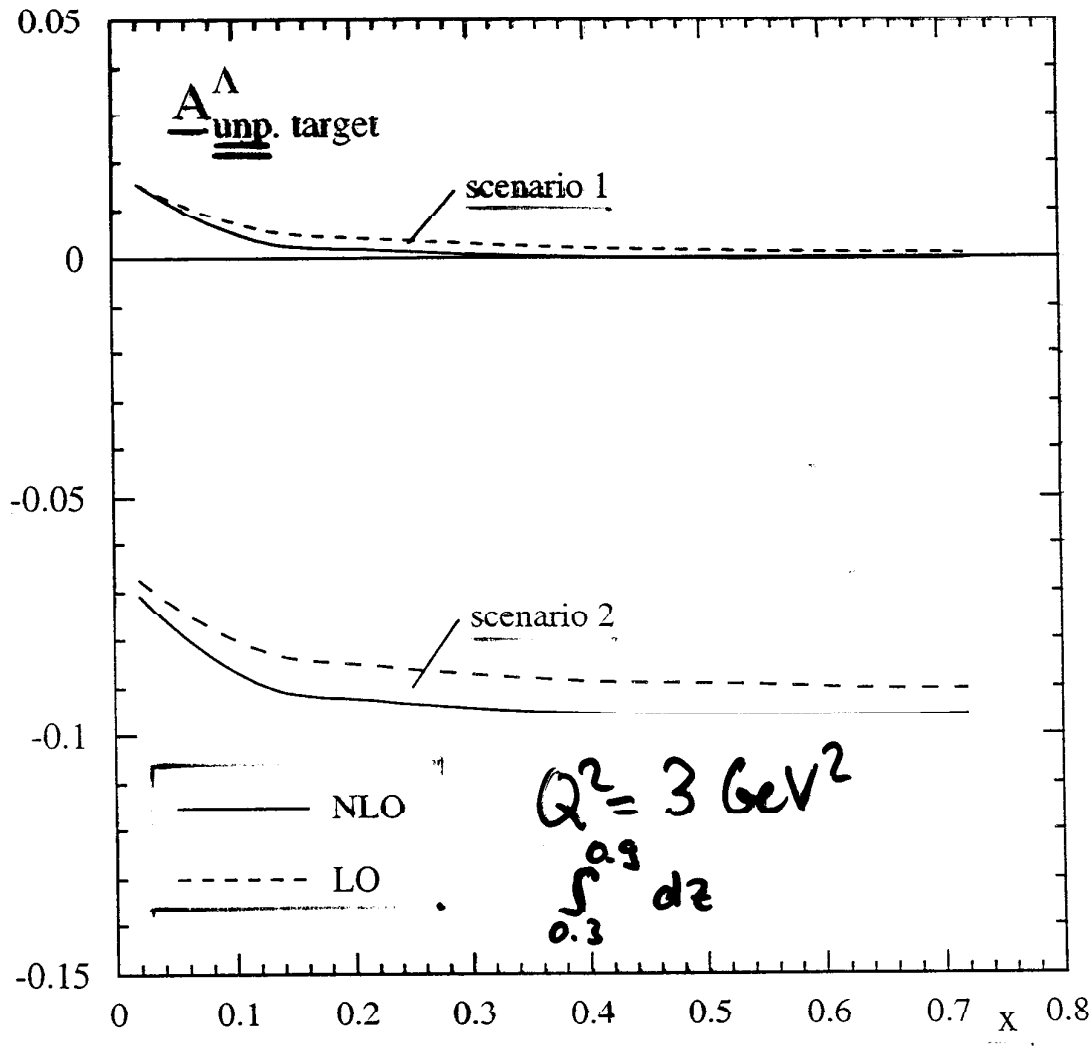
Need also '$q+\bar{q}$' + 'g' for $ep \to \Lambda X$

$\to$ Assume: $\Delta D_{\bar{u}}^\Lambda(\mu^2) = \Delta D_u^{\bar{\Lambda}} = \ldots = 0$, $\Delta D_g^\Lambda = 0$ (or $\Delta D_g^\Lambda = D_g$)

Prediction for HERMES (preliminary):

D. de Florian, W. Vogelsang, M

consider, e.g., $\vec{e}\, p \to \vec{\Lambda}\, X$ or $\vec{\bar{\Lambda}}\, X$



- 'Sc. 1' and 'Sc. 2' could be distinguished
- asym. perturbatively stable

Summary

- NLO 'TL' splitting functions $(\Delta) P_{ij}^{(T),(1)}$ calculated via analytic continuation of $(\Delta) P_{ij}^{(S),(1)}$
- ACR-violation can be understood

  'graph by graph' — simple fact. scheme transformation

- SUSY-relation fulfilled for $P_{ij}^{(T),(1)}$ and $\Delta P_{ij}^{(T),(1)}$

LO + NLO analyses of unpol. Λ-prod.

$\rightarrow$ unpol. frag. functions D_f^{Λ}

Pol. Λ-prod. analysed

ALEPH / DELPHI data (at Z-pole) $\rightarrow \Delta D_{q-\bar{q}}^{\Lambda}$ (two scen.)

$\rightarrow$ predictions for $\vec{e} p \rightarrow \vec{\Lambda} X$, ... (HERMES)