The Kernel of the BFKL Equation in the Next-to-Leading Approximation.

V. Fadin.

- Introduction
- Method of calculation
- Gluon Regge trajectory in the two-loop approximation
- Correction from real production in multi-Regge kinematics (RRG vertex)
- Correction from real production in quasi-multi-Regge kinematics (RRGG and $RRQ\bar{Q}$ vertices)
- Summary.

Introduction.

BFKL equation:

$$\frac{\partial}{\partial \ln 1/x} \mathcal{F}(x, \bar{q}^2) = \int d^2k\, \mathcal{K}(\bar{q}, \bar{k})\, \mathcal{F}(x, \bar{k}^2);$$

$$x\, g(x, Q^2) = \int_0^{Q^2} d\bar{k}^2\, \mathcal{F}(x, \bar{k}^2);$$

$\mathcal{F}(x, \bar{k}^2)$ - unintegrated gluon distribution.

x - momentum fraction,

Q^2 - virtuality;

$$\mathcal{K}(\bar{q}, \bar{q}') = 2\,\omega^{(1)}(t)\,\delta(\bar{q} - \bar{q}') +$$

$$+ \frac{N_c \alpha_s}{\pi^2} \frac{1}{(\bar{q} - \bar{q}')^2}; \qquad t = -\bar{q}^2;$$

$$\omega^{(1)}(t) = \frac{N_c \alpha_s}{4\pi^2}\, t \int \frac{d^2k}{\bar{k}^2 (\bar{q} - \bar{k})^2};$$

Regularization of infrared divergencies:

$$\frac{d^2 K}{(2\pi)^3} \to \frac{d^{D-2} K}{(2\pi)^{D-1}}; \quad D = 4 + 2\epsilon;$$

Approximation: leading $\log\frac{1}{x}$, only terms with the factor $\ln\frac{1}{x}$ for each extra power α_s are collected.

Maximal eigenvalue of $\mathcal{K}$:

$$\omega_0 = \frac{4\alpha_s}{\pi} N_c \ln 2.$$

$$g(x, Q^2) \sim x^{-1-\omega_0};$$

Region of applicability?

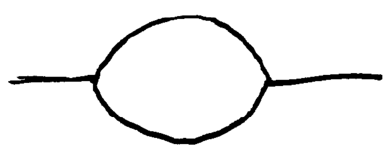
Corrections are necessary.

Method of calculation

is based on the gluon Reggeization

Gluon lies on the Regge trajectory with negative signature

$$j(t) = 1 + \omega(t); \qquad j(0) = 1;$$

$$\omega^{(1)}(t) = \frac{N_c t}{2}$$ 

$$= \frac{N_c t}{2} \frac{g^2}{(2\pi)^{D-1}} \int \frac{d^{D-2}k}{\vec{k}^2 (\vec{q}-\vec{k})^2}; \qquad t = -\vec{q}^{\,2}$$

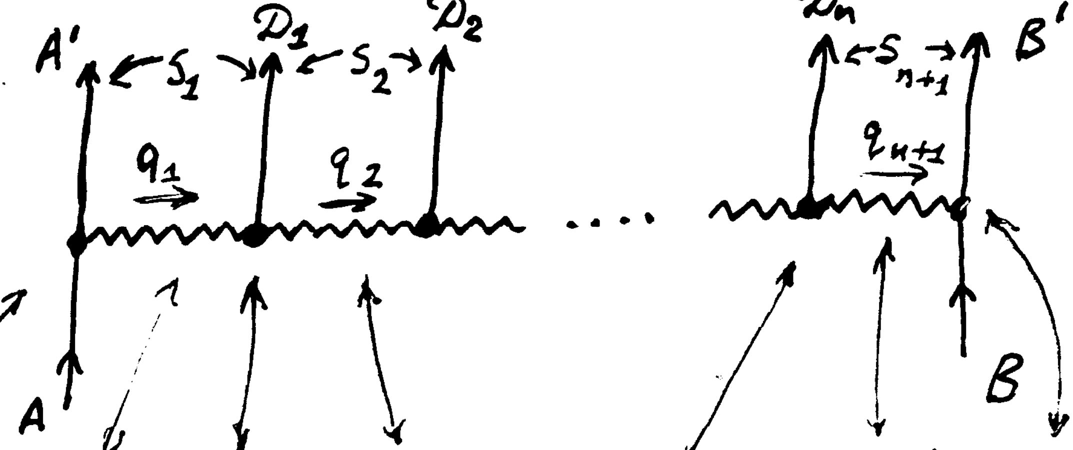
In any fixed order of perturbation theory negative signature contribution is dominant:

$$s \ln^n(-s) \mp u \ln^n(-u) \simeq$$

$$\simeq \begin{cases} 2 s \ln^n s & \text{— negative signature} \\ -i\pi n\, s \ln^{n-1} s & \text{— positive signature} \end{cases}$$

Productions amplitudes in Multi-Regge Kinematic (MRK)

$A_{2\to 2+n} =$



$$= 2s\, \Gamma^{c_1}_{A'A}\, \frac{s_1^{\omega(t_1)}}{t_1}\, \gamma^{D_1}_{c_1 c_2}\, \frac{s_2^{\omega(t_2)}}{t_2} \cdots \gamma^{D_n}_{c_n c_{n+1}}\, s_{n+1}^{\omega(t_{n+1})}\, \Gamma^{c_{n+1}}_{B'B}$$

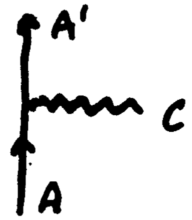
PPR vertex RRP vertices PPR vertex

$$t_i = q_i^2 \simeq -\vec{q}_{i\perp}^{\,2}\,; \quad \prod_{i=1}^{n+1} s_i = s \prod_{i=1}^{n} \vec{p}_{D_i\perp}^{\,2}\,;$$

In LLA – only gluon quantum numbers in all t_i-channels.

Only gluons are produced.

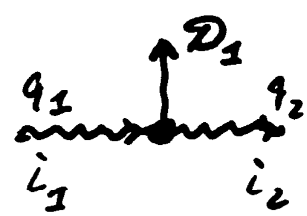
PPR vertices

$$\Gamma^{c}_{A'A} =$$  $$= g\langle A'|T^{c}|A\rangle\,\delta_{\lambda_{A'},\lambda_{A}} .$$

colour group generator.

Helicity conservation

RRP vertices

$$\gamma^{D_1}_{i_1 i_2} =$$  $$= g\,T^{d_1}_{i_2 i_1}\, e^{*\mu}_{D_1}\, C_{\mu}(q_2,q_1);$$

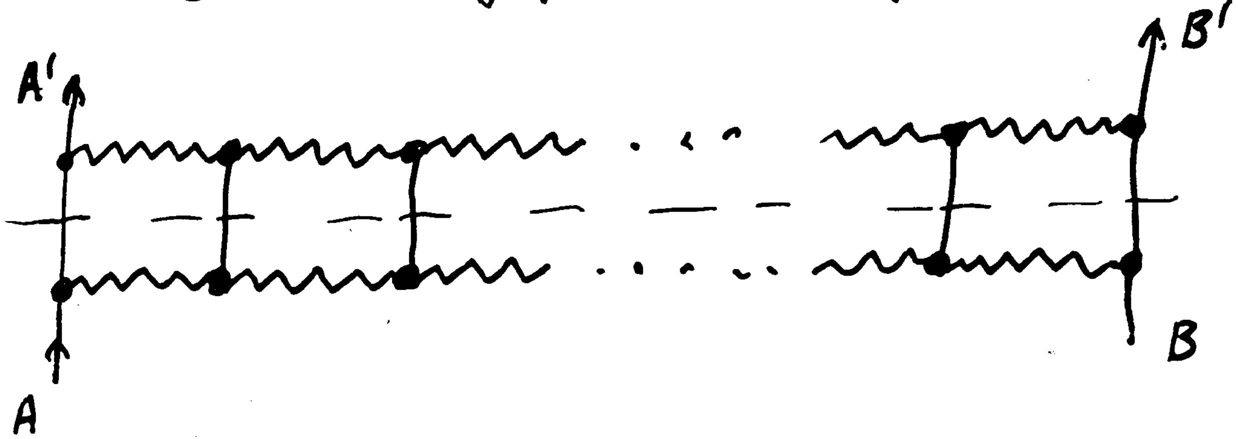
$$C^{\mu}(q_2,q_1) = -(q_1+q_2)^{\mu} + p_A^{\mu}\left(\frac{2kp_B}{p_A p_B} + \frac{q_1^2}{kp_A}\right) - p_B^{\mu}\left(\frac{2kp_A}{p_A p_B} + \frac{q_2^2}{kp_B}\right);$$

L. Lipatov, 1976

$$k = q_1 - q_2 . \qquad C^{\mu}(q_2,q_1)\,k_{\mu} = 0.$$

$$\mathrm{Im}_s A_{2\to 2} = \frac{1}{2}\sum_n A_{2\to n}\, \overset{*}{A}_{2\to n} \;;$$

Therefore, for the s-channel discontinuity of elastic scattering amplitude one has a Reggeized gluon ladder



in t-channel, which can be summed by the equation
V. F., E. Kuraev, L. Lipatov, Phys. Lett. B60 (1975) 5

$$\omega F_\omega(\vec{q}_1,\vec{q}) = F^{(0)}(\vec{q}_1,\vec{q}) + \int d^2 q_2\, K(\vec{q}_1,\vec{q}_2;\vec{q}) F_\omega(\vec{q}_2,\vec{q}),$$

$$K(\vec{q}_1,\vec{q}_2;\vec{q}) = \delta(\vec{q}_1-\vec{q}_2)\left(\omega^{(1)}(-\vec{q}_1^{\,2}) + \omega^{(1)}(-(\vec{q}_1-\vec{q})^2)\right)$$

$$- N_c \frac{\alpha_s}{4\pi^2} \frac{1}{\vec{q}_2^{\,2}(\vec{q}_2-\vec{q})^2} C_\mu(q_2,q_1)\, C^\mu(q_2-q, q_1-q),$$

$F_\omega(\vec{q}',\vec{q})$ – t-channel partial wave for the off-mass-shell elastic scattering amplitude.

r_c - colour factor; for the colourless state in t-channel $r_c = N_c$; for octet $r_c = \frac{N_c}{2}$.

For the singlet

$$\mathcal{K}(\bar{q}_1, \bar{q}_2; 0) = \mathcal{K}(\bar{q}_1, \bar{q}_2);$$

$$\mathcal{F}_\omega(\bar{K}, 0) = \bar{K}^2 \int_0^1 dx\, x^{\omega-1} \mathcal{F}(x, \bar{K}^2);$$

$$\mathcal{K}(\bar{q}, \bar{q}') = \delta(\bar{q}-\bar{q}')\, 2\,\omega^{(1)}(t) -$$

$$- \frac{N_c \alpha_s}{4\pi^2} \frac{\left(C_\mu(q_2, q_1)\right)^2}{\bar{q}_1^2\, \bar{q}_2^2};$$

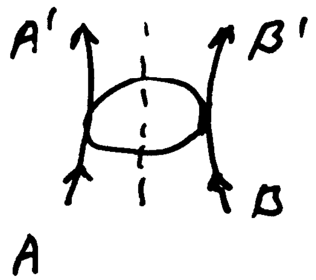
Corrections to the Kernel mean:

- to RRP vertex
- to gluon trajectory
- inclusion of new Kinematics (RRPP vertices).

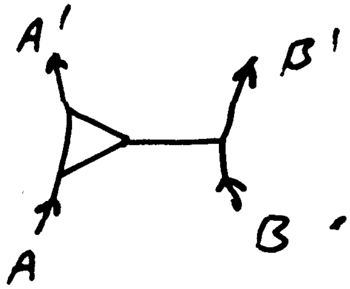
$$\boxed{\begin{aligned}&\mathcal{K}(\bar q,\bar q') = 2\omega(t)\,\delta(\bar q-\bar q') + \mathcal{K}_{real}(\bar q,\bar q').\\ &\omega(t) = \omega^{(1)}(t) + \omega^{(2)}(t);\\ &\mathcal{K}_{real}(\bar q,\bar q') = \mathcal{K}^{one\text{-}loop}_{RRG}(\bar q,\bar q') +\\ &+ \mathcal{K}^{Born}_{RRGG}(\bar q,\bar q') + \mathcal{K}^{Born}_{RRQ\bar Q}(\bar q,\bar q').\end{aligned}}$$

Since expressions for scattering amplitudes contain the PPR-vertices, the necessary intermediate step — their calculation.

It was performed using t-channel unitarity:



contains



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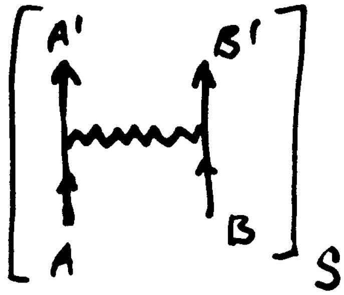
GGR-vertex: gluon contribution V.F., L. Lipatov, 199
quark contribution V.F., R. Fiore, 199

QQR-vertex: V. F., R. Fiore, A. Quartarolo, 1994.

Gluon Regge trajectory in the two-loop approximation

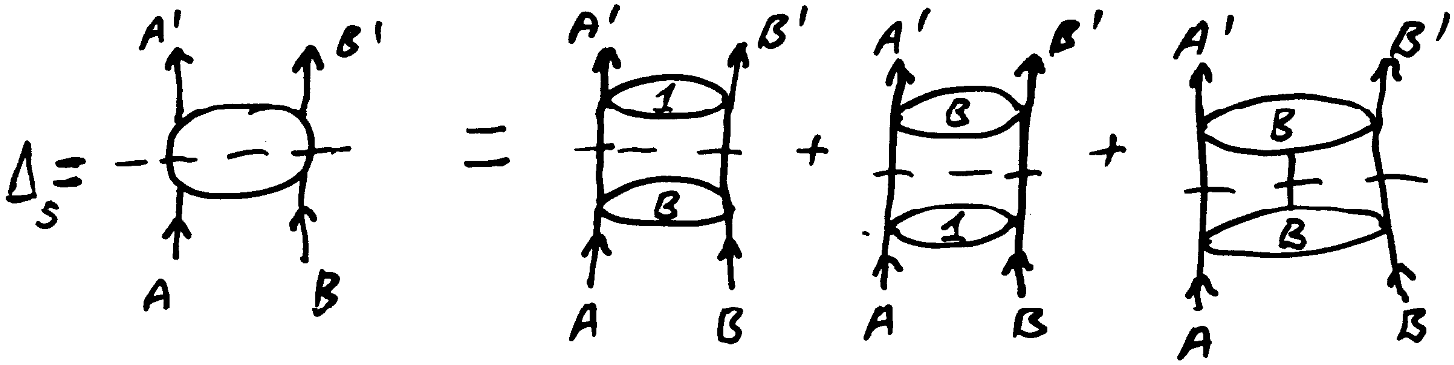
was calculated using s-channel unitarity.
Comparing



=

$$= \left[\Gamma^{i}_{A'A} \left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{-s}{-t}\right)^{j(t)} \right] \Gamma^{i}_{B'B} \right]_S$$

in the two-loop approximation with



we obtain expression for the two-loop contribution to the gluon Regge trajectory $\omega^{(2)}(t)$ in terms of Δ_s, $\omega^{(1)}(t)$ and one-loop corrections to the PPR vertices $\Gamma^{(1)}_{FI}(t)$.

Various cases: massless (V. F., 1994) and massive (V. F., R. Fiore, A. Quartarolo. 1995) quark-quark scattering gluon-gluon (V.F., M. Kotsky, 1995) and quark gluon (V.F., R. Fiore, M. Kotsky, 1995) scattering were considered.

All of them give the same $\omega^{(2)}(t)$, confirming the gluon Reggeization.

$\omega^{(2)}(t)$ can be presented by diagrams of $(D-2)$-dimensional field theory (like $\omega^{(1)}(t) = \frac{N_c}{2} t$), but we have to introduce "spoiled" propagators, for example

q q q_1

$$\frac{1}{\vec{q}_1^{\,2}} \ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}}\right).$$

Explicit form (V.F., R. Fiore, M. Kotsky, 1996)

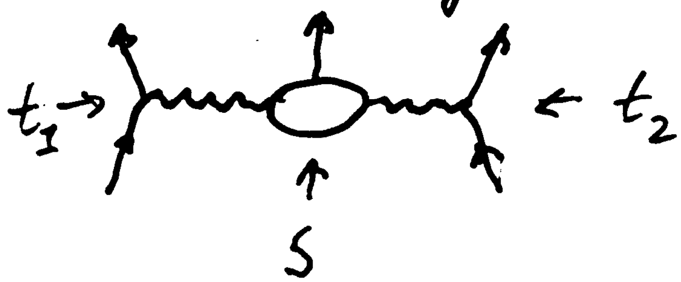
$$\boxed{\begin{aligned}\omega(t) = -\bar{g}_\mu^2\left(\frac{2}{\epsilon} + 2\ln\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)\right) - \bar{g}_\mu^4\Bigg[\underline{\left(\frac{11}{3} - \frac{2}{3}\frac{n_f}{N_c}\right)}\left(\frac{1}{\epsilon^2} - \ln^2\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)\right) \\ + \underline{\left(\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{n_f}{N_c}\right)}\left(\frac{1}{\epsilon} + 2\ln\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)\right) - \frac{404}{27} + 2\zeta(3) + \frac{56}{27}\frac{n_f}{N_c}\Bigg],\end{aligned}}$$

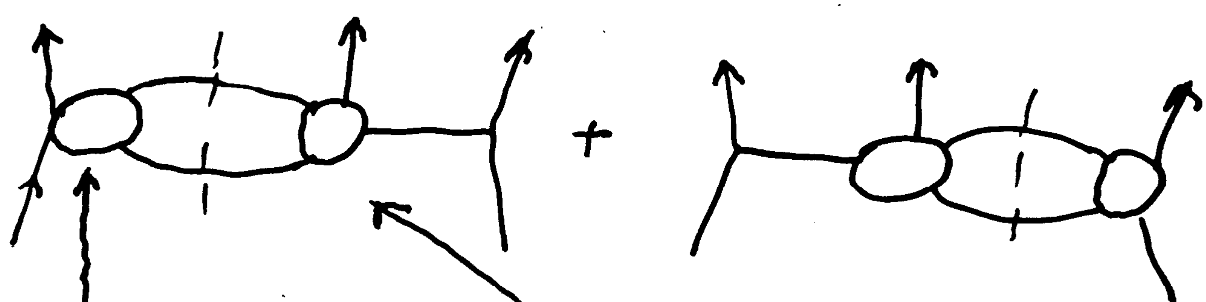
$$\bar{g}_\mu^2 = \frac{g_\mu^2 N_c \Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}.$$

I. Korchemskaya, G. Korchemsky, 1996: Evolution equation for $\omega(t)$.

Correction from real production in multi-Regge kinematics

comes from the RRG vertex calculated in the one-loop approximation. The calculation was performed using t_i-channel unitarity.



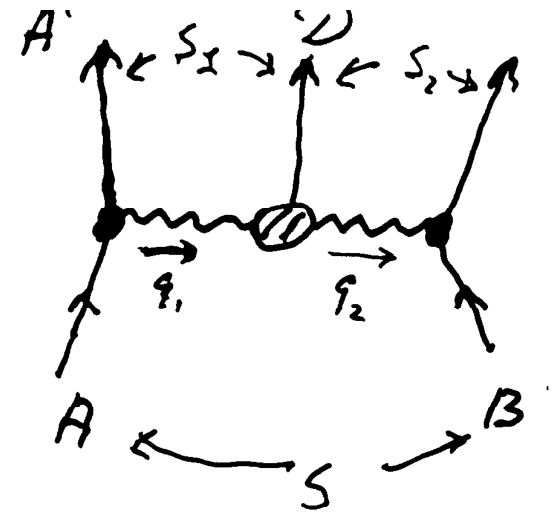


elastic scattering amplitude

amplitude of gluon production in the fragmentation region

L. Lipatov, V.F., 1989.

$$A_{2\to 3} =$$



$$=$$

$$= s\,\Gamma^{i}_{A'A}\,\frac{1}{t_1}\,T^{d}_{ij}\Bigg\{\frac{1}{4}\left[\left(\frac{s_1}{\mu^2}\right)^{\omega_1-\omega_2}+\left(\frac{-s_1}{\mu^2}\right)^{\omega_1-\omega_2}\right]\times$$

$$\left[\left(\frac{s}{\mu^2}\right)^{\omega_2}+\left(\frac{-s}{\mu^2}\right)^{\omega_2}\right]R_\lambda(t_1,t_2,\vec{\kappa}_\perp^2)+\frac{1}{4}\left[\left(\frac{s_2}{\mu^2}\right)^{\omega_2-\omega_1}+\right.$$

$$\left.+\left(\frac{-s_2}{\mu^2}\right)^{\omega_2-\omega_1}\right]\left[\left(\frac{s}{\mu^2}\right)^{\omega_1}+\left(\frac{-s}{\mu^2}\right)^{\omega_1}\right]L_\lambda(t_1,t_2\,\vec{\kappa}_\perp^2)\Bigg\}$$

$$\cdot\,\frac{1}{t_2}\,\Gamma^{j}_{B'B}\,; \qquad \omega_i \equiv \omega(t_i);\quad k \equiv p_D;$$

$$\vec{\kappa}_\perp^2 = \frac{s_1 s_2}{s}\,; \qquad t_i = q_i^2 = -\vec{q}_{i\perp}^{\,2}\,;$$

$$R_\lambda - L_\lambda = 4g\,C_\mu(q_2,q_1)\,e^{*\lambda}_\mu(k)\,N\,\frac{g^2}{(4\pi)^{D/2}}\,\Gamma\!\left(3-\frac{D}{2}\right)$$

$$\times\left[-\frac{2}{D-4}-\ln\vec{\kappa}_\perp^2\right]\frac{1}{[illegible]}\,;$$

$$R_\lambda + L_\lambda = 2 g_{\Delta\mu} C^\rho(q_2, q_1)\, e^{*\lambda}_\rho(k) \Big\{ 1 + \frac{g_\mu^2 N}{(4\pi)^{D/2}} \Gamma\left(3 - \frac{D}{2}\right)$$

$$\times \Big[\frac{1}{2} \ln^2 \frac{\bar{k}_1^2}{\mu^2} - \frac{4}{(D-4)^2} + \frac{1}{D-4}\left(\frac{11}{3} + 2 \ln \frac{t_1 t_2}{\mu^4}\right) +$$

$$+ \frac{1}{2} \ln^2 \frac{t_1 t_2}{\mu^4} - \frac{\bar{k}_1^2}{3} \frac{(t_1+t_2)}{(t_1-t_2)^6} + \frac{1}{6}\Big(\frac{11 (t_1+t_2)}{t_1 - t_2} +$$

$$+ 4 \bar{k}_1^2 \frac{t_1 t_2}{(t_1-t_2)^3} \Big) \ln \frac{t_1}{t_2} + \frac{\pi^2}{2} \Big) \Big] \Big\} +$$

V. F., L. Lipatov
1993

$$+ \left(\frac{p_B}{s_2} - \frac{p_A}{s_1}\right)_\mu e^{*\lambda}_\mu(k)\, 4 g^3_{\Delta\mu} \frac{N}{(4\pi)^{D/2}} \Gamma\left(3 - \frac{D}{2}\right) \times$$

$$\times \Big[\frac{t_1 t_2}{3 (t_1 - t_2)} \Big(11 + \bar{k}_1^2 \frac{(2\bar{k}_1^2 + t_1 + t_2)}{(t_1-t_2)^2} \Big) \ln \frac{t_1}{t_2} -$$

$$- \frac{\bar{k}_1^2}{6} \Big(1 + \frac{t_1+t_2}{(t_1-t_2)^2} (2\bar{k}_1^2 + t_1 + t_2) \Big] ;$$

new spin structure

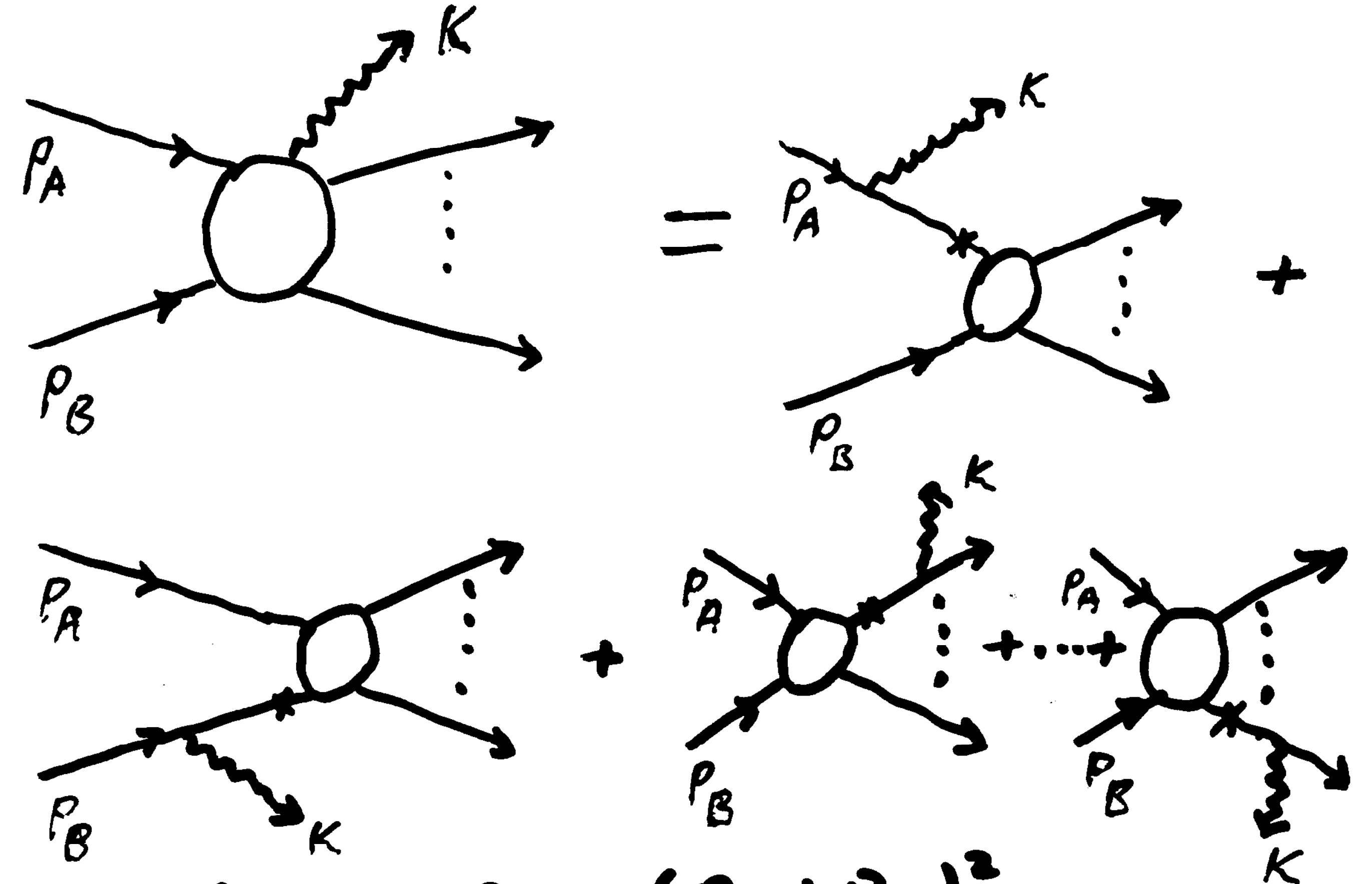
Quark contribution:

V. F., R. Fiore, A. Quartarolo,
1994.

$$\frac{11}{3} N - \frac{2}{3} n_f$$

But terms of order $\epsilon = \frac{(D-4)}{2}$ are omitted.

V. N. Gribov, 1967:



for large $s = (p_A + p_B)^2$
in the region:

$$\frac{2(p_A K)}{s} \ll 1, \quad \frac{2(p_B K)}{s} \ll 1,$$

$$\vec{k}_\perp^{\,2} \simeq \frac{2(p_A K)\cdot 2(p_B K)}{s} \ll \mu^2;$$

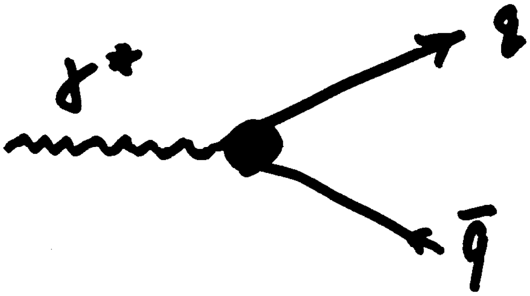
μ is a typical hadron mass.
This region is much greater tha

$$2(p_A K) \ll \mu^2, \; 2(p_B K) \ll \mu^2.$$

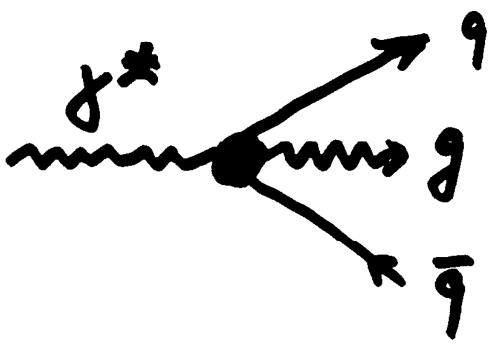
The theorem is not applicable to QCD.

Even in DL approximation

(E. Kuraev, V.F., !!!



$$\sim \exp\left[-\frac{1}{2}\left(w_q(\epsilon,\epsilon)+w_{\bar{q}}(\epsilon,\epsilon)\right)\right]$$



$$\sim g\langle t^a\rangle\left(\frac{(e^* p_q)}{(k p_q)}-\frac{(e^* p_{\bar{q}})}{(k p_{\bar{q}})}\right)\times$$

$$\times \exp\left[-\frac{1}{2}\left(w_q(\epsilon,\epsilon)+w_{\bar{q}}(\epsilon,\epsilon)+\underline{w_g(\omega,k_\perp)}\right)\right]$$

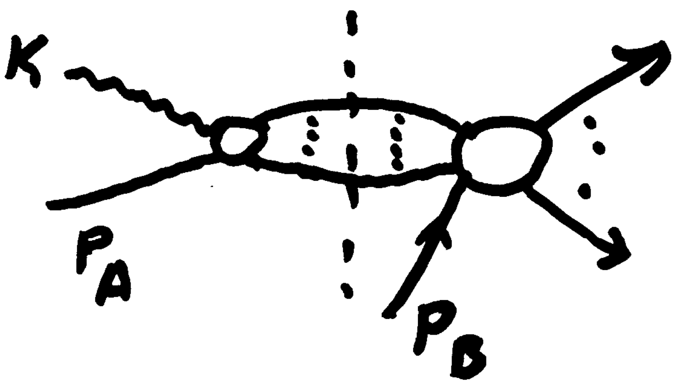
$$w_q(\epsilon,p_\perp)=\frac{2\alpha_s}{\pi}C_F\int^{\epsilon}\frac{d\omega_v}{\omega_v}\int^{\frac{\omega_v}{\epsilon}p_\perp}\frac{dk_\perp^v}{k_\perp^v};$$

$$w_g(\omega,k_\perp)=\frac{2\alpha_s}{\pi}C_V\int^{\omega}\frac{d\omega_v}{\omega_v}\int^{\frac{\omega_v}{\omega}k_\perp}\frac{dk_\perp^v}{k_\perp^v};$$

$- N^2 - 1 \qquad - N$

Reason for the violation of the theorem:
masslessness of particles having colour charge.

The proof of the Gribov's theorem:



$\sim \dfrac{k_\perp}{k_\perp^V}$;

In massive theories essential
$k_\perp^V \gtrsim \mu$.
For massless particles, essential $k_\perp^V$ can be arbitrary small.
Therefore, the theorem can not be literally applied.

Nevertheless, one can make use of it. For DL — strongly ordering transverse momenta L. Lipatov, B. Kinghy

In the region

$$|\vec{k}_\perp| \ll |\vec{q}_\perp|, \quad q \equiv \frac{q_1+q_2}{2};$$

$$|t_1 - t_2| \ll |t| \simeq \vec{q}_\perp^{\,2}$$

we have

the factor of accompanying radiation

$$C(q_2, q_1) = t\left(\frac{p_A}{p_A k} - \frac{p_B}{p_B k}\right);$$

Therefore, in the Born approximation the Gribov's theorem does work.

$$A_{AB}^{A'gB'}(\text{Born}) = \underbrace{\Gamma^{(0)i}_{A'A}\,\frac{2s}{t}\,\Gamma^{(0)j}_{B'B}}\;\underbrace{g\,T^{c}_{ji}\,e^{*}_{\mu}(k)\left(\frac{p_A}{p_A k} - \frac{p_B}{p_B k}\right)^{\mu}}$$

Evidently, it has to work.

But not for loop corrections.

Nevertheless, it is possible to use the theorem

The idea is to use the soft insertion formula for contribution of virtual particles with

$$K_\perp^v \gg K_\perp .$$

The contribution of the region

$$K_\perp^v \lesssim K_\perp$$

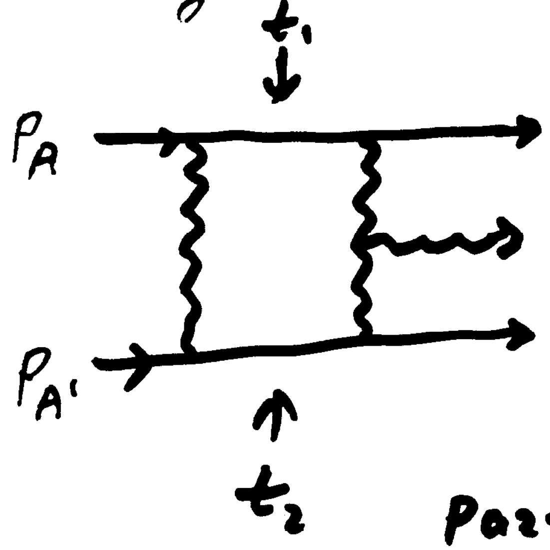
has to be calculated separately.

How can the contribution be separated?

It occurs to be a rather simple task at least in the one-loop approximation.

For $D > 4$ integrals over $K_\perp^v$ are convergent; there are two different scales where [illegible] a, and $K_\perp$.

A simple inspection of the Feynman diagrams shows that only



+ crossings in t_1 and t_2 channels (independently are particles A, A', B, B' gluons or quarks)

have contributions from the region $K_\perp^{\gamma} \lesssim K_\perp$.

Splitting

$$A_{AB}^{A'GB'} = A_{AB}^{A'GB'}(f) + A_{AB}^{A'GB'}(nf),$$

we have for octet in t_1, t_2 channels

$$A_{AB}^{(8)\,A'G\,B'} = \underbrace{\Gamma^{i}_{A'A}\left[\left(\frac{-s}{t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma^{j}_{B'B}}\ *$$

$$* \underbrace{g\, T^{c}_{i_2 i_1}\, e^{*}_{\mu}(K)\left(\frac{p_A}{p_A K} - \frac{p_B}{p_B K}\right)^{\mu}}$$

$$A^{(8)\,A'GB'}_{AB}(uf) = \Gamma^{(0)i}_{A'A}\,\frac{2s}{t}\,\Gamma^{(0)j}_{B'B}\, g T^{c}_{ji}\, e^{*}_{\mu}(k)\left(\frac{p_A}{p_A k} - \frac{p_B}{p_B k}\right)^{\mu}$$

$$* \left(\frac{-g^2 N_c}{8(4\pi)^{D/2}}\right)(\vec{k}_\perp^{\,2})^{\frac{D}{2}-2}\left[3\exp\left(-i\pi\left(\frac{D}{2}-2\right)\right) + \exp\left(i\pi\left(\frac{D}{2}-2\right)\right)\right]$$

$$* \frac{\Gamma^2\left(3-\frac{D}{2}\right)\Gamma^3\left(\frac{D}{2}-2\right)}{\Gamma(D-4)};$$

For RRG vertices it gives

$$R - L = g e^{*}_{\mu}(k)\left(\frac{p_A}{p_A k} - \frac{p_B}{p_B k}\right)^{\mu}\frac{t}{\omega_1 - \omega_2}\left(-\frac{2g^2 N_c}{(4\pi)^{D/2}}\right)(\vec{k}_\perp^{\,2})^{\frac{D}{2}-2} *$$

$$* \frac{\Gamma^2\left(3-\frac{D}{2}\right)\Gamma^3\left(\frac{D}{2}-2\right)}{\Gamma(D-4)}\,\frac{\sin\left(\pi\left(\frac{D}{2}-2\right)\right)}{\pi},$$

$$R + L = 2g e^{*}_{\mu}(k)\left(\frac{p_A}{p_A k} - \frac{p_B}{p_B k}\right)^{\mu} t\left\{1 - \omega(t)\ln\left(\frac{-t}{\mu^2}\right) - \right.$$

$$- \frac{g^2 N}{(4\pi)^{D/2}}\,\frac{\Gamma^2\left(3-\frac{D}{2}\right)\Gamma^3\left(\frac{D}{2}-2\right)}{2\Gamma(D-4)}(\vec{k}_\perp^{\,2})^{\frac{D}{2}-2}\left[\cos\left(\pi\left(\frac{D}{2}-2\right)\right) - \right.$$

$$\left.\left. - \frac{\sin\left(\pi\left(\frac{D}{2}-2\right)\right)}{\pi}\ln\left(\frac{k_1^2}{k_2^2}\right)\right]\right\}; \qquad \omega_i \equiv \omega^{(1)}(t_i);$$

Fortunately, the analytic properties of the vertex are not important in the next-to-leading approximation (only real part does contribute).

We obtain

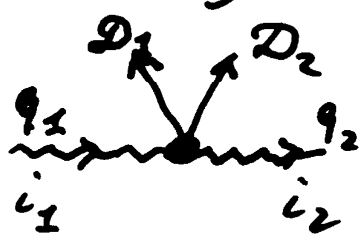
$$\boxed{\begin{aligned}
\mathcal{K}^{\text{one-loop}}_{RRG}(\bar q_1,\bar q_2) &= \frac{\bar g_\mu^2\,\mu^{-2\epsilon}}{\pi^{1+\epsilon}\,\Gamma(1-\epsilon)}\,\frac{4}{\bar k^2}\Bigg(1+ \\
&+\bar g_\mu^2\Bigg[-2\left(\frac{\bar k^2}{\mu^2}\right)^{\epsilon}\left(\frac{1}{\epsilon^2}-\frac{\pi^2}{2}+2\epsilon\,\zeta(3)\right)+\frac{11}{3\epsilon}\left(1-\epsilon^2\frac{\pi^2}{6}\right)- \\
&-\frac{2n_f}{3N_c\epsilon}+\frac{3\bar k^2}{(\bar q_1^2-\bar q_2^2)}\ln\left(\frac{\bar q_1^2}{\bar q_2^2}\right)-2\ln\left(\frac{\bar q_1^2}{\bar q_2^2}\right)\ln\frac{\bar k^2}{\mu^2}-\ln^2\left(\frac{\bar q_1^2}{\bar q_2^2}\right)+ \\
&+\left(1-\frac{n_f}{N_c}\right)\left(\frac{\bar k^2}{(\bar q_1^2-\bar q_2^2)}\left(1-\frac{\bar k^2(\bar q_1^2+\bar q_2^2+4\bar q_1\bar q_2)}{3(\bar q_1^2-\bar q_2^2)^2}\right)\ln\left(\frac{\bar q_1^2}{\bar q_2^2}\right)- \right.\\
&\left.-\frac{\bar k^2}{6\bar q_1^2\bar q_2^2}(\bar q_1^2+\bar q_2^2+2\bar q_1\bar q_2)+\frac{\bar k^4(\bar q_1^2+\bar q_2^2)}{6\bar q_1^2\bar q_2^2(\bar q_1^2-\bar q_2^2)^2}(\bar q_1^2+\bar q_2^2+4\bar q_1\bar q_2)\right)\Bigg]\Bigg);
\end{aligned}}$$

$$\bar k=\bar q_1-\bar q_2\,;\qquad \bar g_\mu^2=\frac{g_\mu^2 N_c\,\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\,;$$

Correction from real productions in quasi-multi-Regge kinematics.

In the next-to-leading approximation we have to consider the region where a pair of produced particles has fixed (not growing with s) invariant mass.

It means, we have to find the RRPP vertices

$$\gamma_{i_1 i_2}^{\mathcal{D}_1 \mathcal{D}_2} =$$ 

and to calculate

$$\mathcal{P}^{(0)}_{ii'} \mathcal{P}^{(0)}_{jj'} \sum_{pol} \int d\Phi \; \gamma_{ij}^{\mathcal{D}_1\mathcal{D}_2}(q_1, q_2) \, \gamma^{*\,\mathcal{D}_1\mathcal{D}_2}_{i'j'}(q_1, q_2);$$

$$\mathcal{P}^{(0)}_{ii'} \mathcal{P}^{(0)}_{jj'} = \frac{\delta_{ii'}\delta_{jj'}}{N_c^2 - 1}; \quad \mathcal{P}^{(8)}_{ii'} = \frac{f^{K}{}_{ii'}}{\sqrt{N}};$$

$$d\Phi = \frac{1}{2(2\pi)^{D-1}} \int_0^1 \frac{dx}{x(1-x)} \int d^{D-2} k_{1\perp};$$

$p_{\mathcal{D}_i} \equiv k_i$; $q_1 = \beta p_A + q_{1\perp}$; $q_2 = -\alpha p_B + q_{2\perp}$.

$$\gamma^{\mathcal{D}_1\mathcal{D}_2}_{ij}(q_1,q_2) = g^2 T^{d_1}_{ik} T^{d_2}_{kj} \overset{*}{e}^{\mu_1}_{\mathcal{D}_1} \overset{*}{e}^{\mu_2}_{\mathcal{D}_2} A_{\mu_1\mu_2} +$$

$$+ \begin{pmatrix} k_1 \leftrightarrow k_2 \\ i_1 \leftrightarrow i_2 \end{pmatrix};$$

L. Lipatov, V. F., 1989

– QED-like gauge invariance;

In the gauge $e_1 P_A = 0$; $e_2 P_D = 0$;

$$e_1 = \bar{e}_{1\perp} + \frac{(\bar{k}_{1\perp}\bar{e}_{1\perp})}{k_1 P_A} P_A,$$

$$e_2 = \bar{e}_{2\perp} + \frac{(\bar{k}_{2\perp}\bar{e}_{2\perp})}{(k_2 P_B)} P_B;$$

$$A_{\mu_1\mu_2} \overset{*}{e}^{\mu_1}_1 \overset{*}{e}^{\mu_2}_2 = \Big\{ \frac{4}{t}\Big[(q_1-k_1)_i (q_1-k_1)_j +$$

$$+(1-x)\left(\bar{k}_1\bar{q}_1 - \frac{\bar{q}_1^2}{2}\right)\delta_{ij}\Big] + \frac{4}{x}\Big[k_i q_{1j} - \frac{q_{1i}(k_1 - x\bar{\Delta})_j}{1-x} +$$

$$+ \bar{q}_1(\bar{k}_1 + x\bar{q}_2)\delta_{ij}\Big] + \frac{2\delta_{ij}\bar{q}_1^2 x(1-x)}{Z} - \frac{4x\, k_{1i}(q_1-k_1)_j}{\bar{k}_1^2} +$$

$$+ \frac{4x\bar{q}_1^2}{xz}\Big[k_{2i}k_{2j} + (1-x)\bar{k}_1\bar{\Delta}\,\delta_{ij}\Big] - \frac{4x k_{1i}}{x\bar{k}_1^2}\Big[\bar{q}_2^2 k_1 + \bar{\Delta}^2(q_1-k_1) -$$

$$- \frac{(\bar{q}_1^2 - 2\bar{k}_1\bar{q}_1)}{1-x} k_2\Big]_j\Big\} \overset{*}{e}^i_1 \overset{*}{e}^j_2$$ – all vectors are transverse

$$\Delta \equiv k_1 + k_2; \quad z = \frac{(\bar{k}_1 - x\bar{\Delta})^2}{x(1-x)}; \quad Z = -x(1-x)(x+\bar{\Delta}^2);$$

$$t = -\bar{k}_1^2(1-x) - (\bar{q}_1 - \bar{k}_1)^2.$$

Possible simplifications: V. F., L. Lipatov, 1995

- transverse dimension $D-2=2$;
- helicity basis (also suggested by V. Delduca, 1995)

$$\bar{e}_1 = \bar{e}^{(+)}; \quad \bar{e}_2 = \bar{e}^{(-)}; \quad \bar{e}^{(\pm)} = \frac{\mp i}{\sqrt{2}}(\bar{e}_x \pm i\bar{e}_y);$$

$$-\frac{1}{2} A_{\mu_1\mu_2} \overset{*}{e}_1^{\mu_1} \overset{*}{e}_2^{\mu_2} = -\frac{x k_1^* (k_1 - x\Delta)^+ q_1^- q_2^+}{\bar{k}_1^2 (\bar{k}_1 - x\bar{\Delta})^2} \equiv C^{+-}(k_1, k_2)$$

$$\bar{e}_1 = \bar{e}^{(+)}; \quad \bar{e}_2 = \bar{e}^{(+)};$$

$$-\frac{1}{2} A_{\mu_1\mu_2} \overset{*}{e}_1^{\mu_1} \overset{*}{e}_2^{\mu_2} = \frac{(q_1-k_1)^+(q_1-k_1)^+}{t} + \frac{x\, q_2^-(q_2(2-x)-k_1)^+}{\Delta^- k_1^-} -$$

$$- \frac{x\, q_1^- q_2^+ k_1^+}{\bar{\Delta}^2 (k_1 - x\Delta)_-} - \frac{x(1-x)^2 q_1^+ q_2^- \Delta^+}{k_1^- k_1^- (k_1 - x\Delta)^+} - \frac{x\bar{q}_1^{\,2} k_2^+ k_2^+}{\bar{\Delta}^2 z} \equiv C^{++}(k_1, k_2)$$

$$a^\pm \equiv a_x \pm i a_y;$$

$$
\begin{aligned}
\mathcal{K}^{(Born)}_{RRGG} = {} & \frac{\bar g_\mu^4\,\mu^{-2\epsilon}}{\pi^{1+\epsilon}\,\Gamma(1-\epsilon)}\,\frac{4}{\vec k^2}\Biggl\{2\left(\frac{\vec k^2}{\mu^2}\right)^{\epsilon}\Biggl(\frac{1}{\epsilon^2}-\frac{11}{6\epsilon}\Bigl(1-\epsilon^2\frac{\pi^2}{6}\Bigr)-\frac{2\pi^2}{3}+\frac{67}{18}-\epsilon\Bigl(\frac{404}{54}-9\zeta(3)\Bigr)\Biggr)-\frac{\vec k^2(\vec q_1^{\,2}+\vec q_2^{\,2})}{8\vec q_1^{\,4}\vec q_2^{\,4}}\cdot \\
& \cdot\Bigl(2\vec q_1^{\,2}\vec q_2^{\,2}-3(\vec q_1\vec q_2)^2\Bigr)-\Biggl(\frac{11}{3}\,\frac{\vec k^2}{\vec q_1^{\,2}-\vec q_2^{\,2}}+\frac{\vec k^2(\vec q_1^{\,2}-\vec q_2^{\,2})}{16\vec q_1^{\,4}\vec q_2^{\,4}}\Bigl(2\vec q_1^{\,2}\vec q_2^{\,2}-3(\vec q_1\vec q_2)^2\Bigr)\Biggr)\ln\left(\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}}\right) \\
& -\frac{2}{3}\,\frac{\vec k^2}{(\vec q_1^{\,2}-\vec q_2^{\,2})^3}\Biggl[\Biggl(1-\frac{2(\vec q_1\vec q_2)^2}{\vec q_1^{\,2}\vec q_2^{\,2}}\Biggr)\Biggl(\vec q_1^{\,4}-\vec q_2^{\,4}-2\vec q_1^{\,2}\vec q_2^{\,2}\ln\left(\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}}\right)\Biggr)+(\vec q_1\vec q_2)\Biggl(2(\vec q_1^{\,2}-\vec q_2^{\,2}) \\
& -(\vec q_1^{\,2}+\vec q_2^{\,2})\ln\left(\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}}\right)\Biggr)\Biggr]-\vec k^2\Biggl[4+\frac{(\vec q_1^{\,2}-\vec q_2^{\,2})^2}{4\vec q_1^{\,2}\vec q_2^{\,2}}+\frac{1}{16}\Biggl(2-\frac{3\vec q_1^{\,2}}{\vec q_2^{\,2}}-\frac{3\vec q_2^{\,2}}{\vec q_1^{\,2}}\Biggr)\Biggl(2- \\
& -\frac{(\vec q_1\vec q_2)^2}{\vec q_1^{\,2}\vec q_2^{\,2}}\Biggr)\Biggr]\int_0^\infty\frac{dx}{(\vec q_1^{\,2}+x^2\vec q_2^{\,2})}\ln\left|\frac{1+x}{1-x}\right|+\frac{2(\vec q_1^{\,2}-\vec q_2^{\,2})}{(\vec q_1+\vec q_2)^2}\Biggl[\ln\left(\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}}\right)\times \\
& \times\ln\left(\frac{\vec q_1^{\,2}\vec q_2^{\,2}}{(\vec q_1^{\,2}+\vec q_2^{\,2})^2}\right)+L\left(1-\frac{\vec k^2}{\vec q_2^{\,2}}\right)-L\left(1-\frac{\vec k^2}{\vec q_1^{\,2}}\right)+L\left(-\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}}\right)-L\left(-\frac{\vec q_2^{\,2}}{\vec q_1^{\,2}}\right)\Biggr]+ \\
& +2\vec k^2\Biggl[\int_0^1\frac{dt}{(\vec q_2^{\,2}t^2-2(\vec q_1\vec q_2)t+\vec q_1^{\,2})}\Biggl(\frac{(\vec q_2\vec k)}{\vec k^2}-\frac{\vec q_2^{\,2}(\vec q_1^{\,2}-\vec q_2^{\,2})}{\vec k^2(\vec q_1+\vec q_2)^2}(1+t)\Biggr)\times \\
& \times\ln\left(\frac{\vec q_2^{\,2}\,t(1-t)}{\vec q_1^{\,2}(1-t)+\vec k^2 t}\right)+\bigl(\vec q_1\leftrightarrow-\vec q_2\bigr)\Biggr]\Biggr\};
\end{aligned}
$$

$$
L(x)=\int_0^x\frac{dt}{t}\ln(1-t);\qquad \vec k=\vec q_1-\vec q_2;
$$

$$
\bar g_\mu^2=\frac{g_\mu^2 N_c\,\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}};
$$

$\mathcal{K}^{(Born)}_{RRQ\bar{Q}}$ — G. Camici, M. Ciafaloni, 1996

Summary

The kernel of the BFKL equation in the next-to-leading approximation is available now for those who want to deal with.