White. 513

THE POMERON BEYOND BFKL

- **Basic Ingredients of Multi-Regge Theory**

  - angular variables, multi-Regge limits, partial-wave expansions, dispersion relations, Sommerfeld-Watson representations, t-channel unitarity in the J-plane.

- **t-channel Unitarity $\rightarrow$ Reggeon Interactions**

  $\leftrightarrow$ *scale-invariant $k_\perp$ integrals*
  $\rightarrow$ *BFKL in leading-order*
  $\rightarrow$ *Conformally Symmetric Contributions at NLO and beyond*

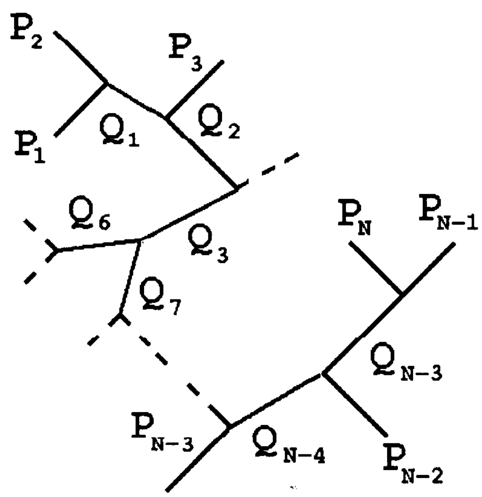
  $$\leftrightarrow \; ln^4 \left| \frac{\rho_{11'}\rho_{22'}}{\rho_{12'}\rho_{1'2}} \right| \; \textbf{etc.}$$

- **Massless Quarks $\rightarrow$ towards Physical Pomeron**

  - reggeon diagrams for multi-regge kinematics
  - quark triple-Regge vertices $\rightarrow$ infra-red anomaly
  => infra-red divergences at zero quark mass
  (when gauge symmetry is SU(2))
  - divergent amplitudes $\leftrightarrow$
  "single gluon" SuperCritical Pomeron,
  confinement and chiral symmetry breaking ... ?

MULTI-REGGE THEORY - the key ingredients are

i) *Angular Variables*



$M_N(P_1, \ldots, P_N)$

$\equiv M_N\left(t_1, \ldots, t_{N-3}, g_1, \ldots, g_{N-3}\right)$

$g_j \in$ **SO(3) - little group of** Q_j

$$\begin{array}{llll} \to & (N-3) & t_i & (\equiv Q_i^2) \\ & (N-3) & z_j & (\equiv \cos\theta_j) \\ & (N-4) & u_{jk} & (\equiv e^{i\omega_{jk}}) \end{array}$$

ii) *Multi-Regge Limits* $z_1, \ldots, z_{N-3} \longrightarrow \infty$
and Helicity-Pole Limits $u_{jk} \longrightarrow \infty$

iii) *Partial-wave Expansions*

$$f(g) = \sum_{J=0}^{\infty} \sum_{|n|,|n'|<J} D^J_{nn'}(g) a_{Jnn'}$$

$$\to \quad M_N(\underset{\sim}{t}, \underset{\sim}{g}) = \sum_{\underset{\sim}{J}, \underset{\sim}{n}, \underset{\sim}{n}'} \prod_i D^{J_i}_{n_i n_i'}(g_i)\; a_{\underset{\sim}{J}, \underset{\sim}{n}, \underset{\sim}{n}'}(\underset{\sim}{t})$$

iv) *Asymptotic Dispersion Relations (disperse in* $z_1, \ldots, z_{N-3}$*)*

$$M_N(p_1, \ldots p_N) = \sum_C M_N^C(p_1, \ldots, p_N) + M^0$$

$$M_N^C(p_1, \ldots, p_N) = \frac{1}{(2\pi i)^{N-3}} \int \frac{ds_1' \ldots ds_{N-3}' \Delta^C(\underset{\sim}{t}, \underset{\sim}{w}, s_1', s_2', \ldots, s_{n-3}')}{(s_1' - s_1)(s_2' - s_2) \ldots (s_{N-3}' - s_{N-3})}$$

Σ_C **is over all sets of (N-3) asymptotic cuts.**

v) *Sommerfeld-Watson Representations of Spectral Components* **e.g.**

$$M_4^C(z_1, z_2, z_3, u_1, u_2, t_1, t_2, t_3)$$
$$= \frac{1}{8} \int_{C_{n_2}} \frac{dn_2\, u_2^{n_2}}{\sin \pi n_2} \int_{C_{n_1}} \frac{dn_1\, u_1^{n_1}}{\sin \pi(n_1 - n_2)} \int_{C_{J_1}} \frac{dJ_1\;\; d^{J_1}_{0,n_1}(z_1)}{\sin \pi(J_1 - n_1)}$$
$$\times \sum_{\substack{J_2 - n_1 = N_1 = 0 \\ J_3 - n_2 = N_2 = 0}}^{\infty} d^{J_2}_{n_1,n_2}(z_2) d^{J_3}_{n_2,0}(z_3) a^C_{N_2 N_3}(J_1, n_1, n_2, \underset{\sim}{t})$$

$\longrightarrow$ **multi-Regge asymptotic behaviour.**

vi) *t-channel Unitarity in the J-plane*

Multiparticle phase-space

$$\leftrightarrow \quad i \int d\rho(t, t_1, \ldots) \int dg_L \prod_j dg_j$$

Unitarity

$$M^+(g) - M^-(g) = i \int d\rho \int dg_L \prod_j dg_j\, M^+(g_L, g_1..) M^-(g_L^{-1} g, g_1..)$$

Partial-wave projection $\rightarrow$ diagonalization

$$a_J^+ - a_J^- = i \int d\rho \sum_{\underset{\sim}{N}, \underset{\sim}{n}} a^+_{J\underset{\sim}{N}\underset{\sim}{n}} a^-_{J\underset{\sim}{N}\underset{\sim}{n}}$$

Continuation to complex J

$$a_J^+ - a_J^- = i \int d\rho \int \frac{dn_1 dn_2}{sin\pi(J - n_1 - n_2)} \int \frac{dn_3 dn_4}{sin\pi(n_1 - n_3 - n_4)} \cdots$$
$$\sum_{\underset{\sim}{N}} a^+_{J\underset{\sim}{N}\underset{\sim}{n}} a^-_{J\underset{\sim}{N}\underset{\sim}{n}}$$

"Nonsense poles" at $J = n_1 + n_2 - 1, n_1 = n_3 + n_4 - 1, \ldots$ *combined with Regge poles and phase-space* $\int d\rho$

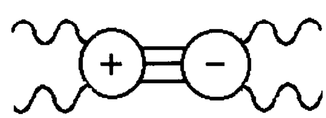
$\longrightarrow$ **Reggeon Unitarity** - *the* J*-plane regge cut discontinuity due to* M *Regge poles* $\underset{\sim}{\alpha} = (\alpha_1, \alpha_2, \cdots \alpha_M)$

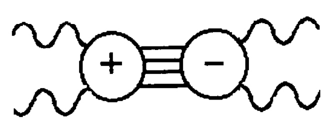
$$\underset{J=\alpha_M(t)}{disc} \quad a_{\underset{\sim}{N}\underset{\sim}{n}}(J) = \xi_M \int d\hat{\rho}\; a_{\underset{\sim}{\alpha}}(J^+) a_{\underset{\sim}{\alpha}}(J^-) \frac{\delta\left(J - 1 - \Sigma_{k=1}^{M}(\alpha_k - 1)\right)}{\sin\frac{\pi}{2}(\alpha_1 - \tau_1') \ldots \sin\frac{\pi}{2}(\alpha_M - \tau_M')}$$

Because the gluon "reggeizes" ($\leftrightarrow$ *Regge pole*) $\rightarrow$ *strong constraint on higher-order multigluon exchange.*

$\longrightarrow$ **Particle Thresholds in Reggeon Interactions**
Inserting gauge group via Regge pole vertices
$\rightarrow$ *leading and nonleading log results* -

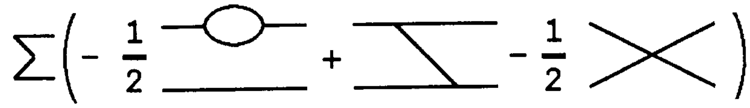
gluon reggeization $\leftrightarrow$ 

$O(g^2)$ **BFKL kernel** $\leftrightarrow$ **two-reggeon interaction** *via three-particle "nonsense" state.* $\leftrightarrow$ 

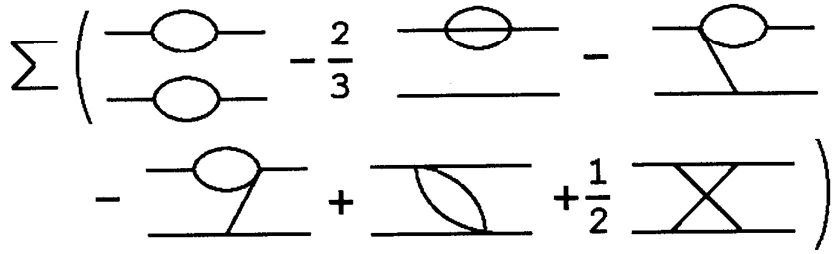
Four-particle state $\rightarrow$ $O(g^4)$ **contribution to BFKL kernel.** $\leftrightarrow$ 

$\int d\rho \;\rightarrow\; \int dt_1 dt_2 \lambda^{-1/2}(t, t_1, t_2) = 2 \int d^2k \;\rightarrow\; k_\perp$ **integrals. Unitarity has no scale and so if results are infra-red finite they must be scale-invariant** $k_\perp$ **integrals.**

BFKL kernel $\leftrightarrow$



$O(g^4)$ **kernel** $K^{(4)}$ $\leftrightarrow$



$K^{(4)}$ **has** (C.Corianò, ARW) **the eigenvalue spectrum** $(\phi_{\nu,n}(k) = |k|^{\nu} e^{i\frac{n}{2}\theta})$

$$\mathcal{E}(\nu, n) \;=\; \frac{1}{\pi}[\chi(\nu, n)]^2 \;-\; \Lambda(\nu, n)$$

$\chi(\nu, n)$ **are the eigenvalues of the BFKL kernel and**

$$\Lambda(\nu, n) = -\frac{1}{4\pi}\Big(\beta'(\frac{|n|+1}{2} + i\nu) + \beta'(\frac{|n|+1}{2} - i\nu)\Big)$$

$$\beta(x) = \textstyle\int_0^1 dy \; y^{x-1}[1+y]^{-1}$$

$\Lambda(\nu, n)$ **is holomorphically separable**

$=>$ *a conformally invariant representation for* $K^{(4)}$ *??*

$K^{(4)}(k_1, k_2, k_{1'}, k_{2'}) \rightarrow$ **impact parameter** $\tilde{K}^{(4)}(\rho_1, \rho_2, \rho_{1'}, \rho_{2'})$

Wave-function gauge invariance *(c.f. BFKL)*

$$F(k_1, k_2) \underset{k_i \rightarrow 0}{\longrightarrow} 0 \quad \leftrightarrow \quad \int d^2\rho_i \tilde{F}(\rho_1, \rho_2) \;=\; 0 \qquad i \;= 1, 2$$

$=>$ *can add to* $\tilde{K}^{(4)}$ *terms independent of any of* $\rho_1, \rho_2, \rho_{1'}, \rho_{2'}$

Using

$$\int d^2k \frac{e^{ik.\rho}}{(k^2+m^2)} = K_0(m|\rho|) \underset{m \to 0}{\longrightarrow} -ln[m|\rho|/2] + \psi(1) + O(m)$$

gives nice ρ-space results (M.Wüsthoff, C.Coriano,ARW)

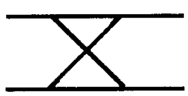
Two gluon propagator $(\omega = J-1,\ \rho_1 - \rho_{1'} = \rho_{11'})$.

$$\frac{1}{\omega}\,\frac{\delta^2(k_1-k_{1'})}{k_1^2}\frac{\delta^2(k_2-k_{2'})}{k_2^2} \quad \leftrightarrow \quad \frac{4}{(2\pi)^4\omega}\ln|\rho_{11'}|\ln|\rho_{22'}|$$

symmetrizing

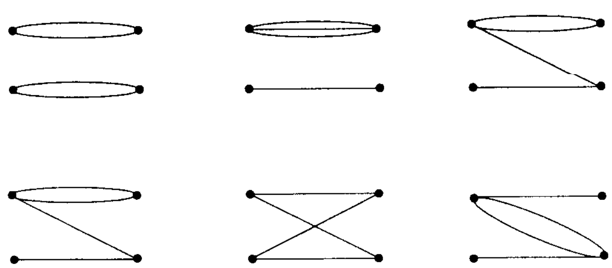
$$\rightarrow \quad \frac{4}{(2\pi)^4\omega}\ln^2 R \qquad\qquad R \;=\; \ln\left|\frac{\rho_{11'}\rho_{22'}}{\rho_{12'}\rho_{1'2}}\right|$$

Evaluating the $k_\perp$ diagrams of $K^{(4)}$ e.g.



$$\rightarrow \quad ln|\rho_{12'}|ln|\rho_{21'}|ln|\rho_{11'}|ln|\rho_{22'}| \;+\; \ldots.$$

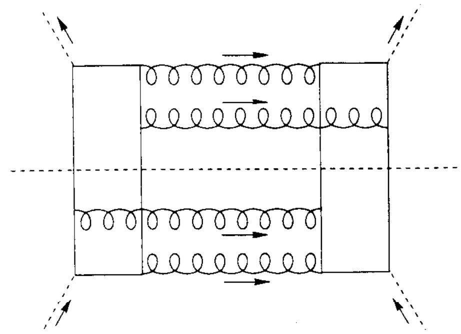
$\rightarrow$ **each $k_\perp$ diagram has simple ρ space analog.** $\tilde{K}^{(4)} \rightarrow$



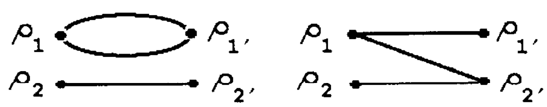
Each line $\leftrightarrow$ "propagator" $log[\rho_i - \rho_{j'}]$. The sum of all diagrams is very simple and manifestly conformally invariant.

$$\tilde{K}^{(4)} \leftrightarrow \frac{1}{24}\ln^4 R$$

The simple $ln^4 R$ **representation of** $\tilde{K}^{(4)}$ **was actually found** (Wüsthoff) **via the** *Feynman diagram NNLO calculation of the large rapidity scattering of two virtual photons*



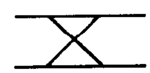
$ln^3 R$ **contains the diagrams**



$\leftrightarrow$ **BFKL?**

but is antisymmetric under $1 \leftrightarrow 2$ **or** $1' \leftrightarrow 2'$

$\Lambda(\nu, n)$ $\leftrightarrow$ **IR finite component** $\mathcal{K}_2$ **of**



The spectrum of $K^{(4)}$ => *(formally)*

$$K_{BFKL} \;=\; c_1 \, ln^3 \, R \;+\; c_2 \left[ln^4 \, R \;-\; \mathcal{K}_2 \right]^{\frac{1}{2}} ,$$

$ln^3 \, R$ *is antisymmetric,* $\left[ln^4 \, R \;-\; \mathcal{K}_2 \right]^{\frac{1}{2}}$ *is symmetric.*

(BFKL is holomorphic extension of $ln^3 R$ *?)*

$ln^m R$ $\leftrightarrow$ **2 pairs of points joined by** m **propagators.**
$\leftrightarrow$ **high-order** K_{BFKL} **in conformal approximation ??**
$\leftrightarrow$ t**-channel unitarity ??** $\rightarrow$ **power(s) of** R **???**

How $K^{(4)}$ $\leftrightarrow$ **to exact NLO should help.**

Spectra of $ln^m \, R$ *can be obtained* (Wüsthoff) *via the generating function* $\mathcal{G}(\mathcal{R}, \delta) \;=\; \mathcal{R}^{\delta}$.

Multi-Regge theory → perturbative Regge results.

More ambitious → *dynamical Pomeron which includes (derives?) confinement and chiral symmetry breaking.*

Outline Below -

QCD with massless quarks → infra-red effects ↔ triangle anomaly *(≡ instanton interactions) => "non-perturbative" physics ↔ (very special) reggeon interactions.*

Multi-Regge *reggeon unitarity* **is very powerful.**

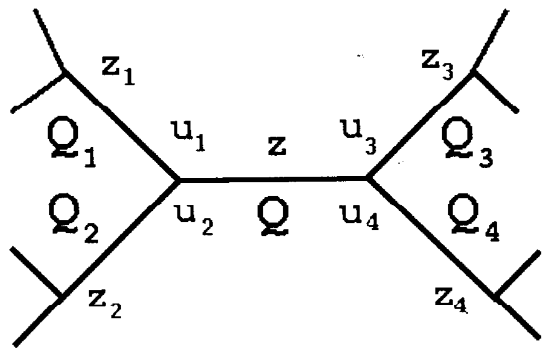
e.g.

8-pt angular variables → "helicity-pole limit"

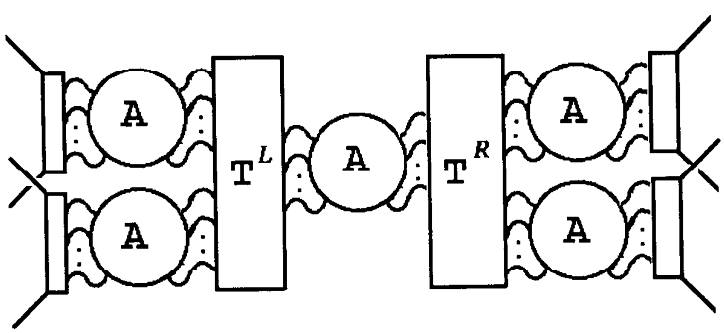
$$u_1, u_2, u_3, u_4 \quad \to \infty$$

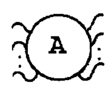
S-W representation

→ one partial-wave.



Reggeon unitarity in all channels => multi-Regge $k_\perp$ integrals



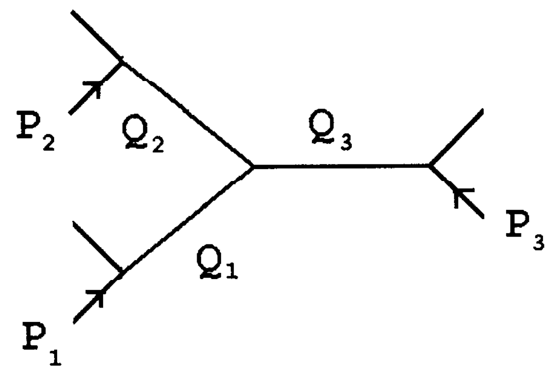


contains all elastic scattering reggeon diagrams. T^L, T^R **contain connected and disconnected interactions that involve both elastic scattering (helicity non-flip) reggeon vertices and also** *new (helicity-flip) vertices.*

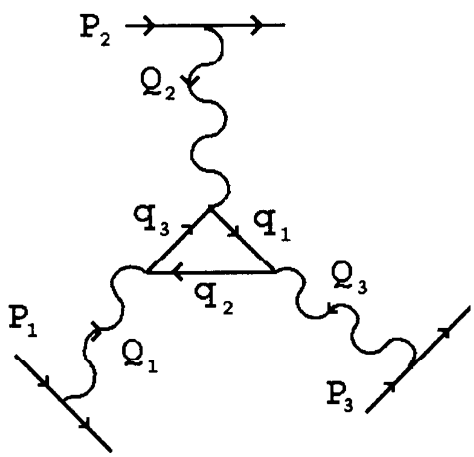
New vertices can be studied in "non-planar" triple-regge limit involving three light-cone momenta -

$$P_1 \to (p_1, p_1, 0, 0) \quad p_1 \to \infty$$
$$P_2 \to (p_2, 0, p_2, 0) \quad p_2 \to \infty$$
$$P_3 \to (p_3, 0, 0, p_3) \quad p_3 \to \infty$$

$$Q_1 \to (0, 0, q_2, -q_3)$$
$$Q_2 \to (0, -q_1, 0, q_3)$$
$$Q_3 \to (0, q_1, -q_2, 0)$$



Consider three quarks scattering via gluon exchange



- triple-gluon coupling $\leftrightarrow$ quark loop.

$$\to \ g^6 \, \frac{p_1 p_2 p_3}{t_1 t_2 t_3} \, \Gamma_{1^+ 2^+ 3^+}(q_1, q_2, q_3)$$

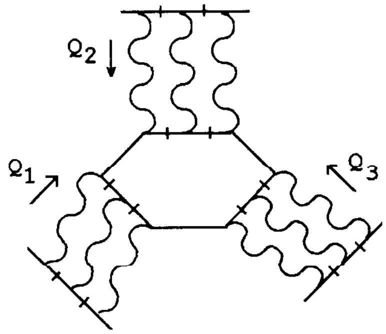
where $t_i = Q_i^2 \quad \gamma_{i^+} = \gamma_0 + \gamma_i$
$\Gamma_{\mu_1 \mu_2 \mu_3} \leftrightarrow$ **quark triangle diagram** $\to$

$$\Gamma_{\mu_1\mu_2\mu_3} = i \int \frac{d^4k \ Tr\{\gamma_{\mu_1}(\not{q}_3 + \not{k} + m)\gamma_{\mu_2}(\not{q}_1 + \not{k} + m)\gamma_{\mu_3}(\not{q}_2 + \not{k} + m)\}}{[(q_1+k)^2 - m^2][(q_2+k)^2 - m^2][(q_3+k)^2 - m^2]}$$

For "$O(m^2)$" part of $\Gamma_{1^+2^+3^+}$ limits $q_1, q_2, q_3 \sim Q \to 0$, $m \to 0$ **do not commute because of "IR anomaly".**

$$\underset{Q \to 0}{\sim} \quad Q \ i \ m^2 \int \frac{d^4k}{[k^2 - m^2]^3} \quad \underset{m \to 0}{\longrightarrow} \quad R \ Q$$

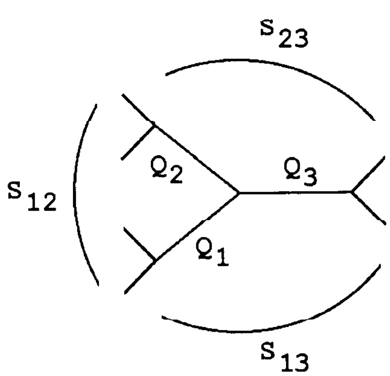
Adding color factors and summing diagrams - *"anomalous" part of $\Gamma_{1^+2^+3^+}$ in triple-regge vertex only if all reggeon states have "anomalous color parity"* **e.g.**



Each three reggeon state has odd signature but *even color parity,* **e.g.** $f_{ijk}d_{jrs}A^kA^rA^s$ **(c.f. winding-number current** $K^i_\mu = \epsilon_{\mu\nu\gamma\delta}f_{ijk}d_{jrs}A^k_\nu A^r_\gamma A^s_\delta$**)**

Survival of $O(m^2)$ helicity-flip processes as $m \to 0$ reproduces physics of instanton interactions.

Usual reggeon interactions have $[k_\perp]$-dimension - 2 with $\delta^2(k_\perp) \to$ scale/conformal invariance.

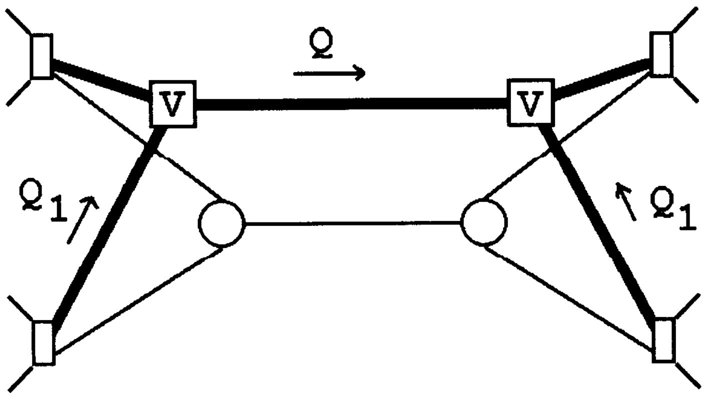


Anomalous vertex part has $[k_\perp]$-dimension - 1 $\leftrightarrow$ "special kinematics", i.e. large invariants $\to$ extra momentum factor e.g. non-planar triple-regge

$$p_1p_2p_3Q = (p_1p_3)^{1/2}(p_2p_3)^{1/2}(p_1p_2)^{1/2}Q$$
$$\equiv (s_{31})^{1/2}(s_{23})^{1/2}(s_{12})^{1/2}\ Q$$

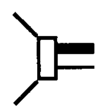
=> anomalous vertex part will not satisfy usual Ward identity property.
=> an infra-red divergence may appear as $m \to 0$ in non-planar multi-regge diagrams where $Q_1 \sim Q_2 \sim Q_3 \sim 0$ is part of the integration region.

A candidate diagram is -

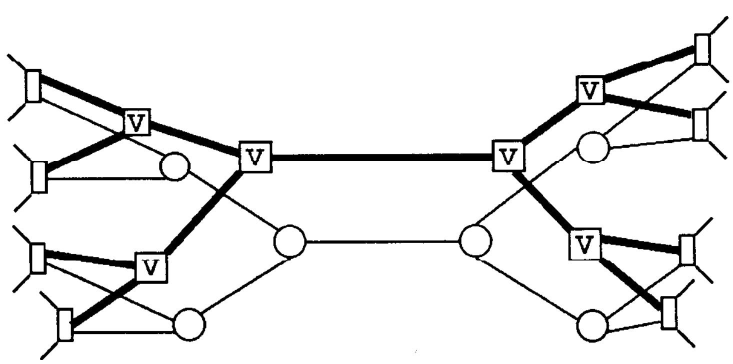


A divergence may occur for $Q, Q_1, Q_{1'} \sim 0$ as $m \to 0$ if V is anomalous $\leftrightarrow$ ▬ is anomalous color parity reggeon state.

To show divergence occurs requires a systematic analysis.
Must initially break the SU(3) gauge symmetry to SU(2)*(c.f. instantons are associated with SU(2) subgroup).*

Divergence occurs when — is an SU(2) singlet state containing massive gluon(s) (or quarks) and ▬ carries anomalous color parity. *Couplings*  *are also important.*

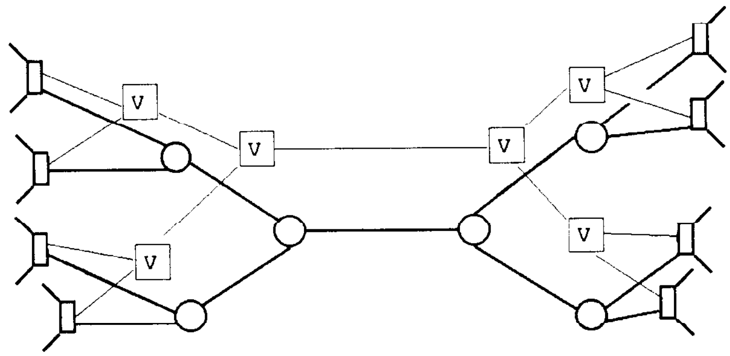
Higher-order diagrams containing a divergence include



Divergence is as all Q_i entering each V vertex vanish

$\rightarrow$ **an overall logarithmic divergence.**
Coefficient $\rightarrow$ *physical amplitudes (in which) all anomalous reggeon states carry zero* $k_\perp \leftrightarrow$ **"reggeon condensate".**

"Parton model process" $\rightarrow$ *dynamical interaction (in "condensate" background)*



Reggeon condensate $\leftrightarrow$ **quark triangle anomaly (**$\equiv$ **instanton interactions) coupling reggeon channels.**

Much remains to be done but I am very optimistic that this is how the "Super-Critical Pomeron" of Reggeon Field Theory is realized in QCD and that many other properties follow -

- **in first approximation, the Pomeron is a reggeized gluon** *(in condensate)*
- **hadrons are "constituent quark" reggeon state** *(in condensate)* **with confinement + chiral symmetry breaking spectrum.**
- **Critical Pomeron is related to the restoration of SU(3) gauge symmetry**
- $\cdots$