Kovner

BFKL equation from Wilson RG.

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Parton densities grow fast at low x.

$Q^2 \sim 20\ GeV^2$, $x \sim 10^{-5}$ ~ 30-gluons

Theory: both DGLAP and BFKL predict growth and violate unitarity at very small x.

Unitarization $\stackrel{?}{=}$ finite density effect.

Try to find a setup to handle these effects.

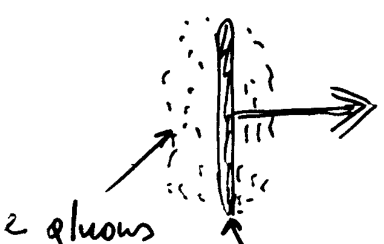
The Setup:

IMF: $p^+ \to \infty$; LCG: $A^+ = 0$

$$x = \frac{p^+}{P^+}$$

$$g(x, Q^2) = \frac{dN}{dx} = \int^{Q} dk_\perp \langle a^+(x,k)\, a(k) \rangle$$

Low x physics is dominated by the low p^+ modes of the glue field (gluons). They do not live in the vacuum: interact with high p^+ gluons & quarks



$$\left[\begin{array}{l} \text{high } p^+ \text{ stuff is} \sim \text{static} \\ p^- \propto \frac{p_\perp^2}{p^+} \ll k^- \propto \frac{k_\perp^2}{k^+} \end{array} \right]$$

The leading interaction is eikonal $a^- J^+(A)$

So: $S_{int} = g\, a^- J^+$;

$$J^+ = \rho(x_\perp)\, \delta(x^-)$$

$\rho(x_\perp)$ must have some distribution – generated on a long time scale: hadronic wave function, bremstrahlung of high p^+, etc.

$$Z = \int D\rho(x_\perp)\, DA\, e^{-F[\rho] + i S(A, \rho)}$$

Modeling $F[\rho]$: $F = \frac{1}{\mu^2}\int dx_\perp\, \rho^2(x_\perp)$ – if surface density is high and partons uncorrelated.

For us $F[\rho]$ is initial condition for the RG equation. Does not have to be specified to derive the equation itself.

Important! Gauge invariance:

$$\rho A^- \Rightarrow \mathrm{tr}\, \rho\, W[A]$$

$$W[A^-] = P \exp \int_{-\infty}^{+\infty} dx^+\, A_a^-(x^- = 0, x_\perp)\, T_a$$

So we are lead to the action:

$$Z = \int D\rho(x_\perp)\, DA\, e^{-F[\rho] - \frac{i}{4} G^2(A) - g\, \mathrm{tr}\, \rho(x_\perp)\, W[A^-(x_\perp, x^- = 0)]}$$

Semiclassical approximation:

1. Solve classical equations at fixed $\rho(x_\perp)$

$$D^\mu F^{\mu\nu} = gJ^\nu$$

$$\Downarrow$$

$$A^- = 0 \quad , \quad A^i = \theta(x^-)\, \alpha^i(x_\perp)$$

$$\alpha^i(x_\perp) = i\, U^+(x_\perp)\, \partial^i U(x_\perp) \quad ; \quad \partial_i \alpha^i = g\rho$$

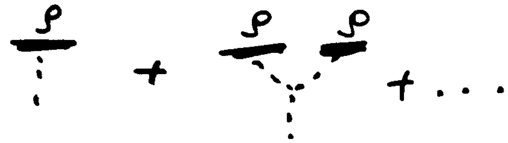
2. Calculate $g(x, k_\perp)$ by averaging over ρ:

$$g(x, k_\perp) = \langle a^{+i}_a(k)\, a^i_a(-k) \rangle_\rho \simeq$$

$$\frac{\alpha_s}{x\, k_\perp^2} \langle \rho^a(k_\perp)\, \rho^a(-k_\perp) \rangle + \ldots$$

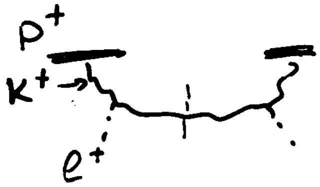
But! One loop corrections are big at low x – just like in standard PT.

Classical :

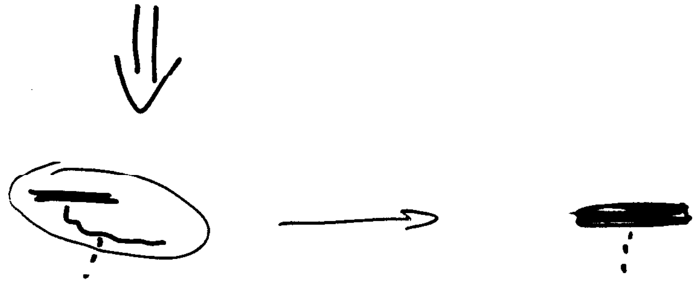
$\mathcal{A} =$ 

$g(x, k_\perp) = \langle$  $\rangle_\rho + \dots$

Quantum corrections :



$\ell^+ \ll k^+ \ll p^+ \quad . \quad \ln \frac{p^+}{\ell^+} \approx \int \frac{dk^+}{k^+}$



Effectively the charge density is modified!

Have to take this modification into account when going to very small x.

The change is small $\Longrightarrow$ RG.

Integrate out k^+: $\alpha \ln \frac{p^+}{k^+} < 1$; $\ln \frac{p^+}{k^+} \gtrsim 1$

The Renormalization Group

Decompose the field:

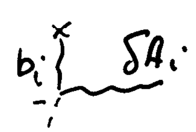
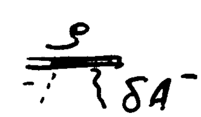
$$\mathcal{A}_\mu^a = b_\mu^a + \delta A_\mu^a + a_\mu^a$$

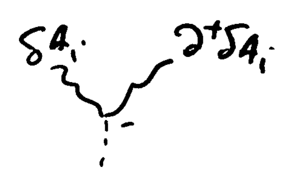
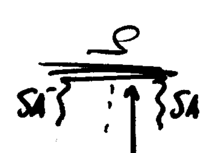
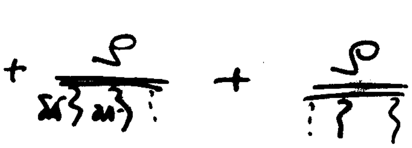
classical solution $b^- = 0,\ b^i = \theta(x^-)\alpha^i(x_\perp)$ — small $\Lambda < p^+ < P^+$ — not assumed small $p^+ < \Lambda$

Expand to second order in δA_μ^a

$$S = -\tfrac{1}{4} G^2(a) - \tfrac{1}{2}\delta A_\mu D^{-1}_{\mu\nu}\delta A_\nu + g a \rho' + O(a^{-3})$$

$$\rho' = \rho + \delta\rho_1 + \delta\rho_2$$

$\delta\rho_1 =$  $+$ 

$\delta\rho_2 =$  $+$  $+$ 

eikonal propagators — follow from $W[A]$

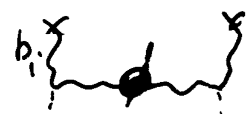
1. Integrate over δA^{μ} at fixed ρ & ρ'
If not explicitly, this can be represented diagrammatically:

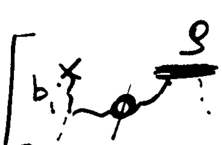
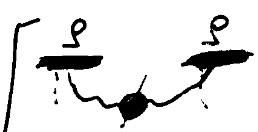
$$\int D\rho\, e^{iS[A,\rho]} \Longrightarrow$$

$$\Rightarrow \int d\rho' D\rho\; e^{-F[\rho] - \frac{1}{2}[\rho' - \rho - \alpha \ln\frac{1}{x} M[\rho]] \frac{1}{\alpha \ln\frac{1}{x} \chi[\rho]} [\rho' - \rho - \alpha \ln\frac{1}{x} M}$$

$$\times\, e^{iS(a,\rho')}$$

with $M[\rho] = \langle \delta\rho \rangle_{\delta A} =$  $+$ 

$\chi[\rho] = \langle \delta\rho\, \delta\rho \rangle_{\delta A} =$  $+$

$\Big[$  + permutations $\Big]$ + $\Big[$  + permutations $\Big]$

2. The integral over ρ at fixed ρ' should give $F'[\rho']$

$$\boxed{\frac{d}{d \ln 1/x} F[\rho] = \alpha \Delta[\rho]} \quad \left\{ \Delta(\rho) = \frac{1}{\alpha \ln 1/x} \left\{ F'[\rho] - F[\rho] \right\} \right\}$$

The BFKL (weak fields) limit.

$$x\, g(x, Q^2) = \int_0^{Q_2} \frac{dk_\perp}{k_\perp^2}\, \varphi(k_\perp)$$

identify this

In our semiclassical appr.:

$$g(x, Q^2) = \frac{1}{x}\int_0^{Q_2} \langle \alpha_a^i(k)\, \alpha_a^i(-k) \rangle_\rho \quad \overline{\overline{\text{small } \rho}}$$

$$\frac{1}{x}\int_0^{Q^2} \left\langle \frac{k^i}{k^2}\rho^a(k)\, \frac{k^i}{k^2}\rho^a(-k) \right\rangle$$

$$\Downarrow$$

$$\varphi(k) = \langle \rho^a(k)\, \rho^a(-k) \rangle$$

So, we need $\langle \rho' \rho' \rangle$ to leading order in ρ: $\quad \sigma(\rho) \sim O(\rho)$

$$\chi(\rho) \sim O(\rho^2)$$

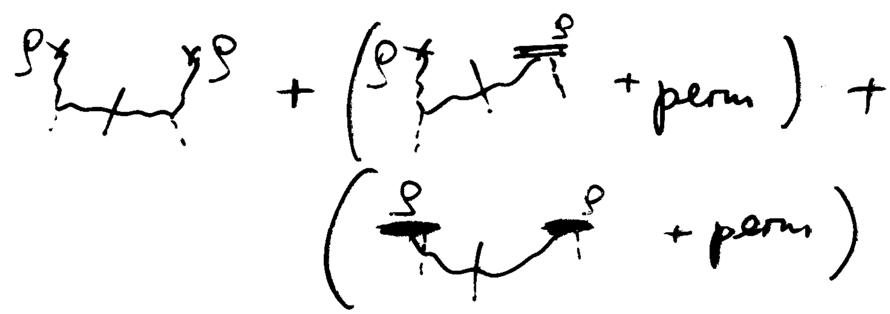
It is enough to perform only

δA^μ average:

$$\rho'(k)\rho'(-k) = \rho(k)\rho(-k) + \alpha \ln\tfrac{1}{x}\left[2\, M(\rho)\,\rho + \chi(\rho)\right] =$$

$$= \varphi(k) + K(k,q)\,\varphi(q)$$

$M(\rho) =$ 

$\delta\chi =$ 

Collecting all these diagrams we obtain BFKL

$$\frac{\partial \varphi(k)}{\partial \ln 1/x} = -\frac{\alpha N_c}{2\pi}\int d^2p_t\, \frac{k_t^2}{p_t^2 (p_t-k_t)^2}\left[\varphi(k_t) - 2\,\varphi(p_t)\right]$$

virtual (from M) real (from χ)

Conclusions

What next?

Full nonlinear RG:

fixed point in $\ln 1/x$? ([illegible])

low $k_\perp$ behaviour at fixed x, large ρ?

RG in time resolution rather than momentum?

$$k^- \sim \frac{k_\perp^2}{k^+} \Rightarrow \text{fixed } k^+, \quad k_\perp^2 \sim \frac{1}{\Delta t}$$

(DGLAP)

$$\text{fixed } k_\perp^2, \quad k^+ \sim \Delta t$$

(BFKL)

Can we unify?