Operator expansion for diffractive high-energy scattering

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Plan:

1. OPE for the high-energy amplitudes
2. Functional integral and operator language for cross sections
3. OPE for the cross sections of high-energy scattering
4. Diffractive high-energy scattering and 3-pomeron vertex.

Operator expansion and factorization for high-energy scattering

Small-x DIS

$p_A \equiv q = p_1 + \frac{q^2}{s} p_2$

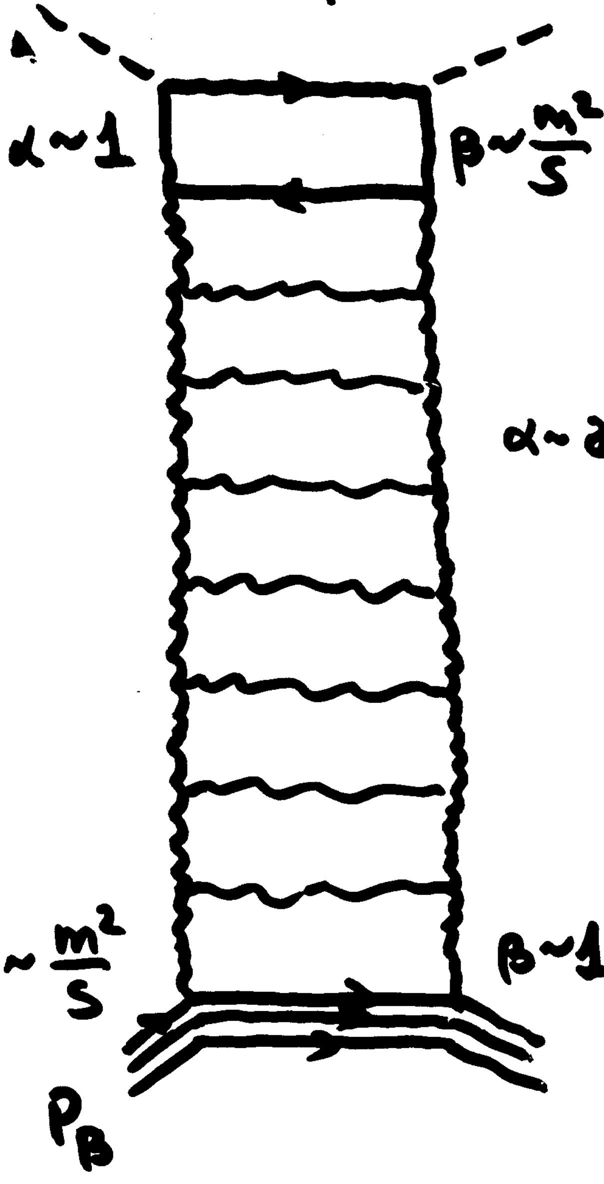
$p_B = p_2 + \frac{m_N^2}{s} p_1 \simeq p_2$

$p = \alpha p_1 + \beta p_2 + p_\perp$ „Sudakov variables"

($\eta = \ln\frac{\alpha}{\beta}$ - rapidity)

$p_1^2 = p_2^2 = 0$

$x = \frac{-q^2}{2pq} \ll 1 \Leftrightarrow s \gg |q^2|$ Regge limit



$1 \gg \alpha \gg \sigma$ - coefficient functions „fast" quarks & gluons

$\alpha \sim \sigma$

$m^2 \sim p_A^2, p_B^2$

$\sigma \gg \alpha \gg \frac{m^2}{s}$ - matrix elements „slow" quarks & gluons

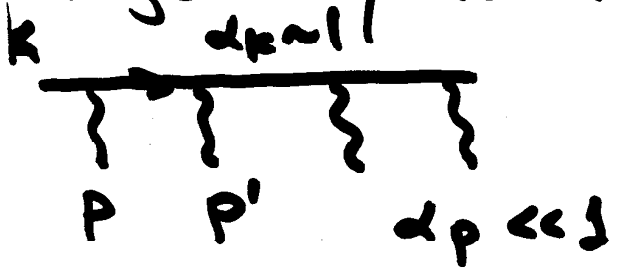
$$\ln\frac{1}{\sigma} + \ln\frac{\sigma}{m^2/s} = \ln\frac{s}{m^2}$$

At first, let us integrate over $\alpha\sim 1$

← fast-moving quarks ($\alpha\sim 1$)

↖ slow gluons ($\alpha\ll 1$) – „external field"

Quarks move fast ⇒ quark trajectory is a straight line



⇒

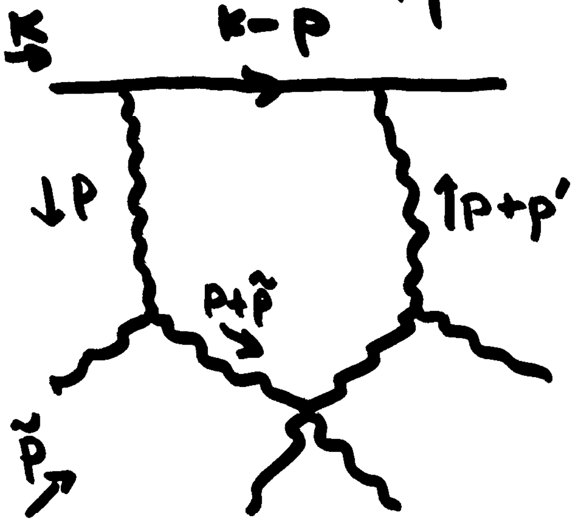
$$P\exp ig\int_{-\infty}^{\infty} du\, p_{1\mu} A_\mu(up_1 + x_\perp)$$

„Wilson line"

Proof of  $\rightarrow$ 

We assume that all $p_\perp^2 \sim m^2$ (to be verified a posteriori)

Consider typical integral over p



$$g_{\mu\nu} \rightarrow \frac{2}{s} p_{2\mu} p_{1\nu}$$

$$\hat{a} \equiv \not{a}$$

$$\frac{\hat{p}_2 (\hat{k}-\hat{p})\hat{p}_2}{(k-p)^2+i\epsilon} =$$

$$= \frac{\hat{p}_2 (\alpha_k - \cancel{\alpha_p})\hat{p}_1 \hat{p}_2}{(\alpha_k - \cancel{\alpha_p})(\beta_k-\beta_p)s-(k-p)_\perp^2+i\epsilon} =$$

$$= \frac{\hat{p}_2}{\beta_k - \beta_p - \frac{(k-p)_\perp^2}{\alpha_k s} + i\epsilon\alpha_k}$$

$\alpha_k \sim 1$ $\quad$ $\alpha_p, \tilde{\alpha}_p \ldots \ll 1$

$\beta_k \sim \frac{m^2}{\alpha_k s}$

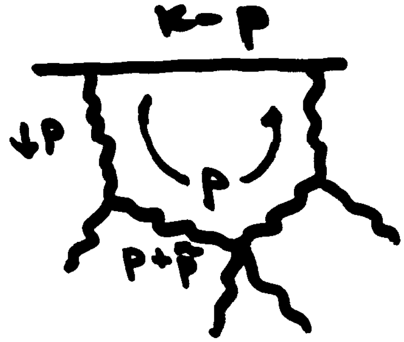
If I replace $\dfrac{1}{\beta_k - \beta_p - \frac{(k-p)_\perp^2}{\alpha_k s} + i\epsilon\alpha_k}$

by $\dfrac{1}{-\beta_p + i\epsilon\alpha_k}$

nobody in the loop integral over p will notice that

$$\frac{1}{\beta_k - \beta_p - \frac{(k-p)_\perp^2}{\alpha_k s} + i\epsilon\alpha_k} \rightarrow \frac{1}{-\beta_p + i\epsilon\alpha_k}$$

Proof:
Loop integral over p: residue in the quark propagator and/or residues in slow-gluon propagators



1. If I take the residue in the quar[k] propagator

$$\beta_p = \beta_k - \frac{(k-p)_\perp^2}{\alpha_k s} \qquad \beta_k \sim \frac{m^2}{\alpha_k s}$$

then typical slow-gluon denominator i[s]

$$(\alpha_p + \tilde\alpha_p)(\beta_p + \tilde\beta_p)s - (p_\perp + \tilde p_\perp)^2 =$$

$$= \underbrace{(\alpha_p + \tilde\alpha_p)\beta_p s - (p+\tilde p)_\perp^2}_{\sim m^2} + \underbrace{(\alpha_p + \tilde\alpha_p)\beta_k s}_{\sim m^2 \frac{\alpha_p}{\alpha_k} \ll m^2} - \underbrace{\frac{\alpha_p + \tilde\alpha_p}{\alpha_k}(k-p)_\perp^2}_{\sim \frac{\alpha_p}{\alpha_k} m^2 \ll}$$

2. If I take the residue in gluon propagator, then

$$\beta_p = -\tilde\beta_p + \frac{(p+\tilde p)_\perp^2}{(\alpha_p + \tilde\alpha_p)s} \Rightarrow$$

quark propagator reduces to

$$\frac{1}{\beta_k - \beta_p - \frac{(k-p)_\perp^2}{\alpha_k s}} \rightarrow \frac{1}{\beta_k + \tilde\beta_p + \frac{(p+\tilde p)_\perp^2}{(\alpha_p + \tilde\alpha_p)s} - \frac{(k-p)_\perp^2}{\alpha_k s}}$$

Functional integral and operator language for cross sections

$$W_{\mu\nu} = \frac{1}{\pi}\int dx\, e^{iqx} \langle N| j_\mu(x)\, j_\nu(0) |N\rangle$$

$$j(0) = \tilde{T} e^{i\int_{-\infty}^{0} dt\, H_I^{in}(t)}\, j_{in}(0)\, T e^{-i\int_{-\infty}^{0} dt\, H_I^{in}(t)}$$

$\tilde{T}$ ↑ inverse time ordering

$$\langle N| j_\mu(x)\, j_\nu(0) |N\rangle = \langle N| \tilde{T} e^{i\int_{-\infty}^{t} H_I\, dt}\, j_{in}(x)\, T e^{-i\int_{-\infty}^{t} H_I\, dt}\,\cdot$$

insert $1 = \tilde{T} e^{i\int_{-\infty}^{\infty} H_I dt}\, T e^{-i\int_{-\infty}^{\infty} H_I d t}$

$$\tilde{T} e^{i\int_{-\infty}^{0} H_I\, dt}\, j_{in}(0)\, e^{-i\int_{-\infty}^{0} H_I\, dt} |N\rangle =$$

$$= \langle N| \tilde{T}\left\{ j_{in}(x)\, e^{i\int_{-\infty}^{\infty} H_I\, dt}\right\} T\left\{ j_{in}(0)\, e^{-i\int_{-\infty}^{\infty} H_I\, dt}\right\} |N\rangle \equiv$$

$$\equiv \langle N| \tilde{T}\left\{ j_{in}(x)\, e^{-i\int \mathcal{L}_I(z) d^4z}\right\} T\left\{ j_{in}(0)\, e^{i\int \mathcal{L}_I(z) d^4 z}\right\} |N\rangle =$$

We can rewrite this formally as one T-product, but with two types of fields — + and − ones

$$= \langle N| T\left\{ j_-^{in}(x)\, j_+^{in}(0)\, e^{i\int dz\, (\mathcal{L}^+(z) - \mathcal{L}^-(z))}\right\} |N\rangle$$

By definition

$$T\{\varphi^+(x)\, \varphi^+(y)\} \equiv T\{\varphi(x)\, \varphi(y)\}$$

$$T\{\varphi^-(x)\, \varphi^-(y)\} \equiv \tilde{T}\{\varphi(x)\, \varphi(y)\}$$

and all the ⊖ fields stand to the left of all ⊕ ones.

Functional integral for a cross section

$$\sigma \sim \sum_{\substack{\text{intermediate}\\ \text{states}}} A^* \cdot A$$

$$A \sim \int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\,\mathcal{D}A\; e^{i\int \mathcal{L}_I(z)dz}$$

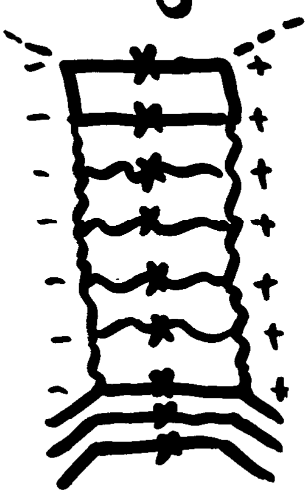
$$A^* \sim \int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\,\mathcal{D}A\; e^{-i\int \mathcal{L}_I(z)dz}$$

$$\Rightarrow \sigma \sim \sum_{\substack{\text{int.}\\ \text{states}}} \int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\,\mathcal{D}A\, e^{-i\int \mathcal{L}dz} \int \mathcal{D}\bar{\psi}\,\mathcal{D}\psi\,\mathcal{D}A\, e^{i\int \mathcal{L}dz}$$

$$= \int \mathcal{D}\bar{\psi}_-\,\mathcal{D}\psi_-\,\mathcal{D}A_-\;\mathcal{D}\bar{\psi}_+\,\mathcal{D}\psi_+\,\mathcal{D}A_+\; e^{i\int dz\,(\mathcal{L}^+(z)-\mathcal{L}^-(z))}$$

$$\Rightarrow \varphi_-(\vec{x},t) = \varphi^+(\vec{x},t)\Big|_{t\to+\infty}$$ – boundary condition

Feynman rules for this double functional integral are simply <u>Cutkosky rules</u>



⊖ fields are to the left from the cut

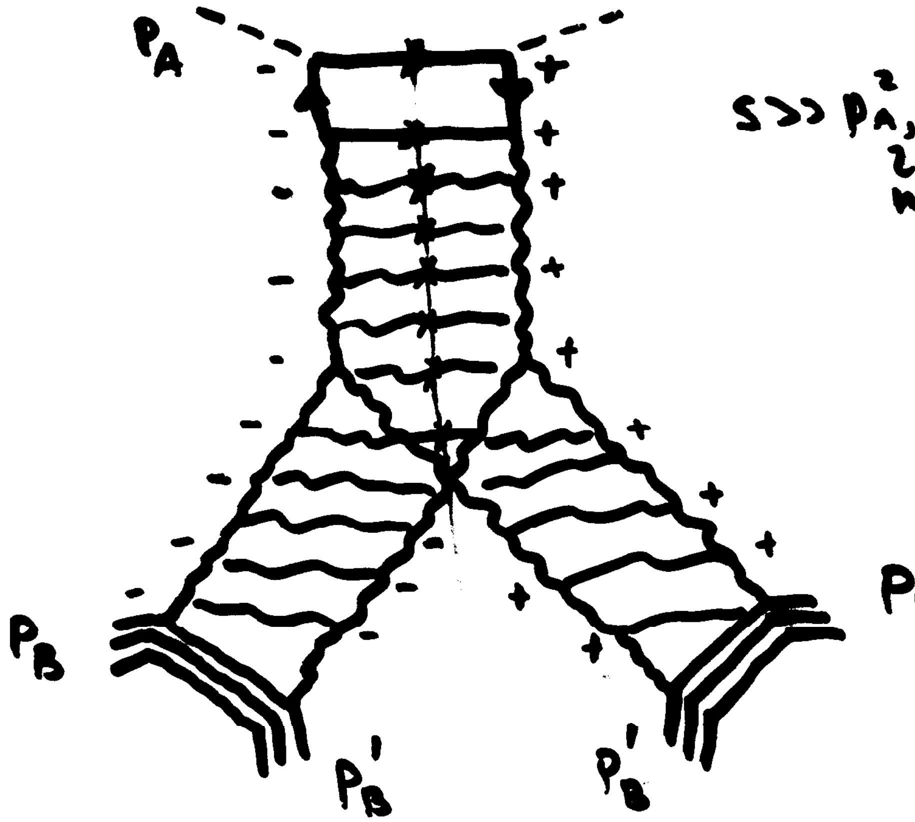
⊕ – to the right

$$\begin{matrix} \psi_-(x)\bar{\psi}_-(y) \\ \psi_+(x)\bar{\psi}_+(y) \\ \psi_-(x)\bar{\psi}_+(y) \end{matrix} = \int \frac{d^4p}{(2\pi)^4 i} \begin{cases} -\dfrac{m+\not{p}}{m^2-p^2+i\epsilon} \\ \dfrac{m+\not{p}}{m^2-p^2-i\epsilon} \\ (m+\not{p})\,2\pi i\,\delta(m^2-p^2)\cdot\theta(p_0) \end{cases}$$

To the right of the cut – Feynman propagators
To the left – c.c. ones
Between – „cutted" propagators —✕—

Operator expansion for cross sections at high energy

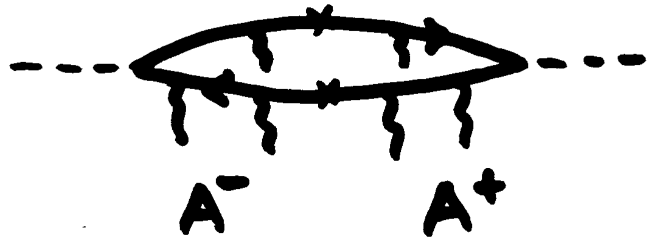
Consider smth like



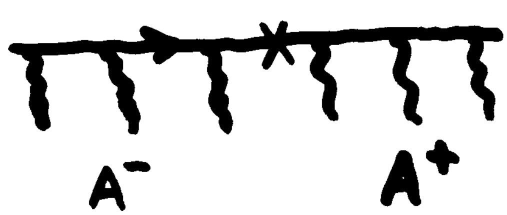
$s \gg p_A^2, p$
m^2

Same steps as above.

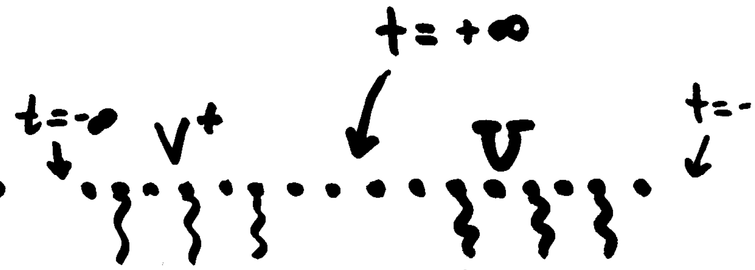
1. Integrate over $\alpha \sim 1$



We will prove that

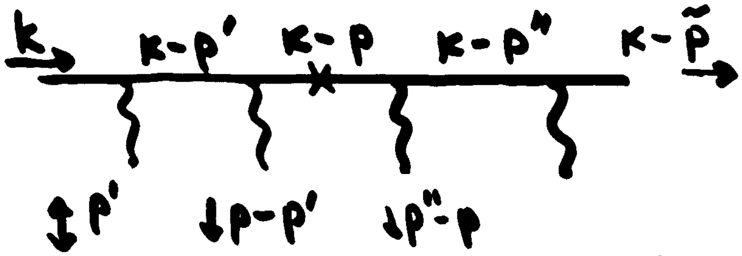


$\Rightarrow$



$U(x_\perp) = P\exp i\int_{-\infty}^{\infty} du\, A^+_\bullet(up_1+x_\perp$

$V(x_\perp) = P\exp i\int_{-\infty}^{\infty} du\, A^-_\bullet(up_1+x_\perp$



$\alpha_k \sim 1$

$\alpha_p, \alpha_p' \ldots \ll 1$

As we proved $\quad \dfrac{\hat{p}_2(k-p'')\hat{p}_2}{(k-p'')^2+i\epsilon} \to \dfrac{\hat{p}_2}{-\beta_p''+i\epsilon\alpha_k} \to U$

Similarly $\quad \dfrac{\hat{p}_2(k-p')\hat{p}_2}{(k-p')^2-i\epsilon} \to \dfrac{\hat{p}_2}{-\beta_p'-i\epsilon\alpha_k} \to V^+$

Also, we have proved that the replacement of

$$\delta(k-p)^2 = \delta((\alpha_k-\cancel{\alpha_p})(\beta_k-\beta_p)s-(k-p)_\perp^2)$$

by $\delta(-\alpha_k\beta_p s) \to \dfrac{1}{\alpha_k s}\delta(\beta_p)$

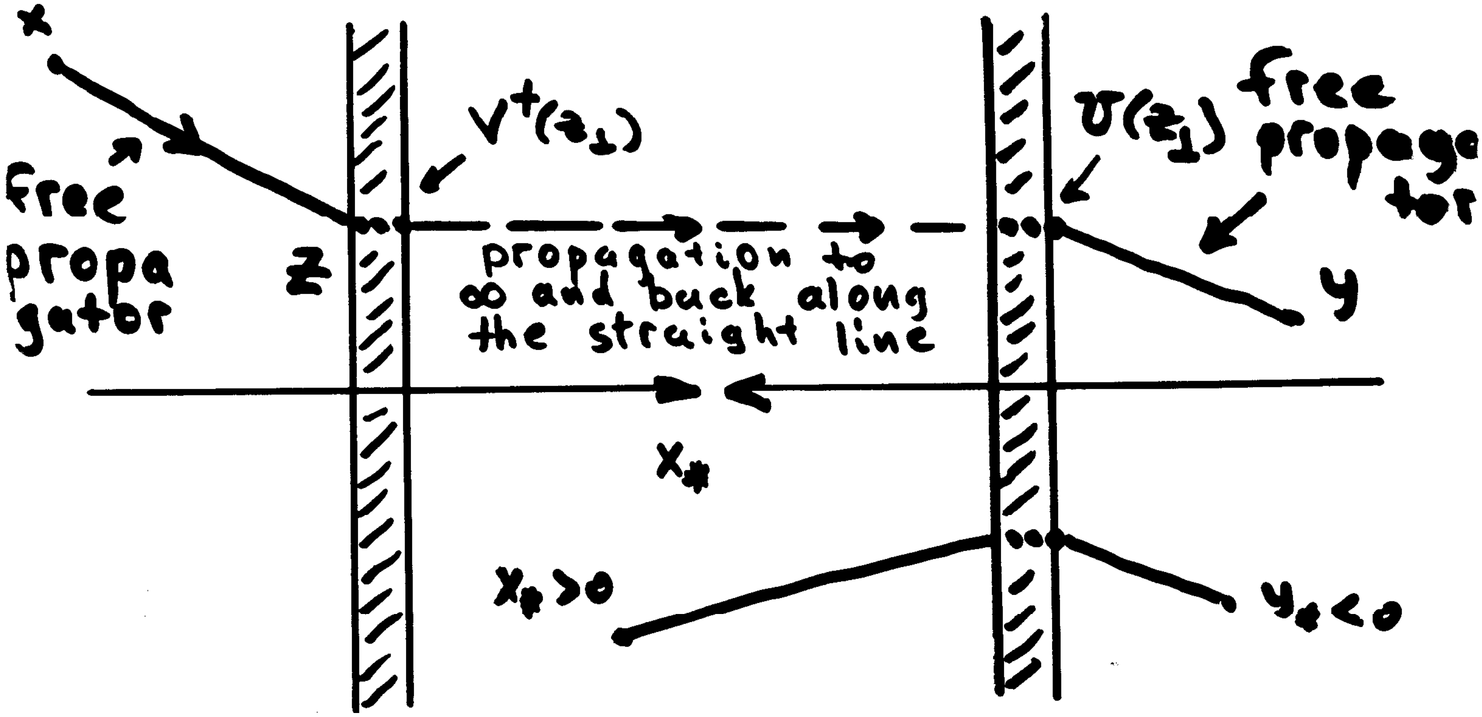
will cause no problems in loop integra over p.

In the coordinate space

$$\int d\beta_p\, d\beta_p'\, d\beta_p''\; A_\bullet^-(\beta_p')\frac{\hat{p}_2}{-\beta_p'-i\epsilon\alpha_k} A_\bullet^-(\beta_p-\beta_p')\,\hat{p}_1\, 2\pi\delta(\beta_p)\, A_\bullet^+(\beta''-\beta)\cdot$$

$$\cdot\frac{\hat{p}_2}{-\beta_p''+i\epsilon\alpha_k} A_\bullet^+(\tilde{\beta}_p-\beta_p'') \Longrightarrow \begin{array}{ll} V^+(x_\perp)U(x_\perp) & \alpha_k>0 \\ V(x_\perp)U^+(x_\perp) & \alpha_k<0 \end{array}$$

Space-time picture

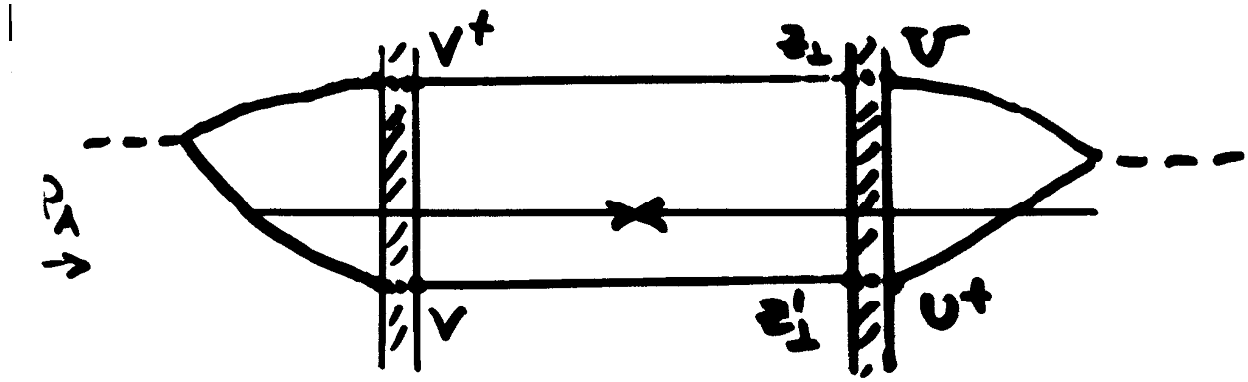


$$\overline{\psi_-(x)\bar{\psi}_+(y)}_{x_*,y_*<0} = \int dz\, \delta(z_*) \frac{\hat{x}-\hat{z}}{(x-z)^4} V^{\dagger}(z_\perp) U(z_\perp) \hat{p}_2 \frac{\hat{z}-\hat{y}}{(z-y)}$$

$$\overline{\psi_-(x)\bar{\psi}_+(y)}_{\substack{x_*>0\\ y_*<0}} = \overline{\psi_+(x)\bar{\psi}_+(y)} = \int dz\, \delta(z_*) \frac{\hat{x}-\hat{z}}{(x-z)^4} U \hat{p}_2 \frac{\hat{z}-\hat{y}}{(z-y)}$$

Similarly

$$\overline{\psi_-(x)\bar{\psi}_+(y)}_{\substack{x_*<0\\ y_*>0}} = \overline{\psi_-(x)\bar{\psi}_-(y)} = \int dz\, \delta(z_*) \frac{\hat{x}-\hat{z}}{(x-z)^4} V^{\dagger}(z_\perp) \hat{p}_2 \frac{\hat{z}-}{(z-}$$



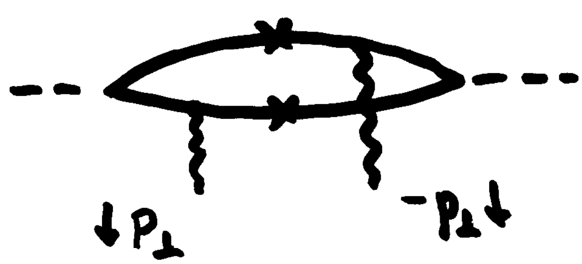
$$\to \int dp_\perp\, I^A(p_\perp) \int dz_\perp dz'_\perp\, e^{-i(p,z-z')_\perp} \,.$$

$$\cdot \operatorname{Tr}\{(V_{z'} V^+_z - 1)(U_z U^+_{z'} - 1)\}$$

$$I^A(p_\perp) \sim \int_0^1 d\alpha\, d\alpha' \, \frac{(e^\perp_A, e'_A)(1-2\bar{\alpha}\alpha)(1-2\alpha'\bar{\alpha}')\,p_\perp^2}{p_\perp^2 \alpha'\bar{\alpha}' - p_A^2 \alpha\bar{\alpha}}$$

$$\bar{\alpha} \equiv 1-\alpha$$

In lowest order in α_s this simply reduces to



$$\int dp_\perp\, I^A(p_\perp)\, A^-_0(p_\perp) A^+_0($$

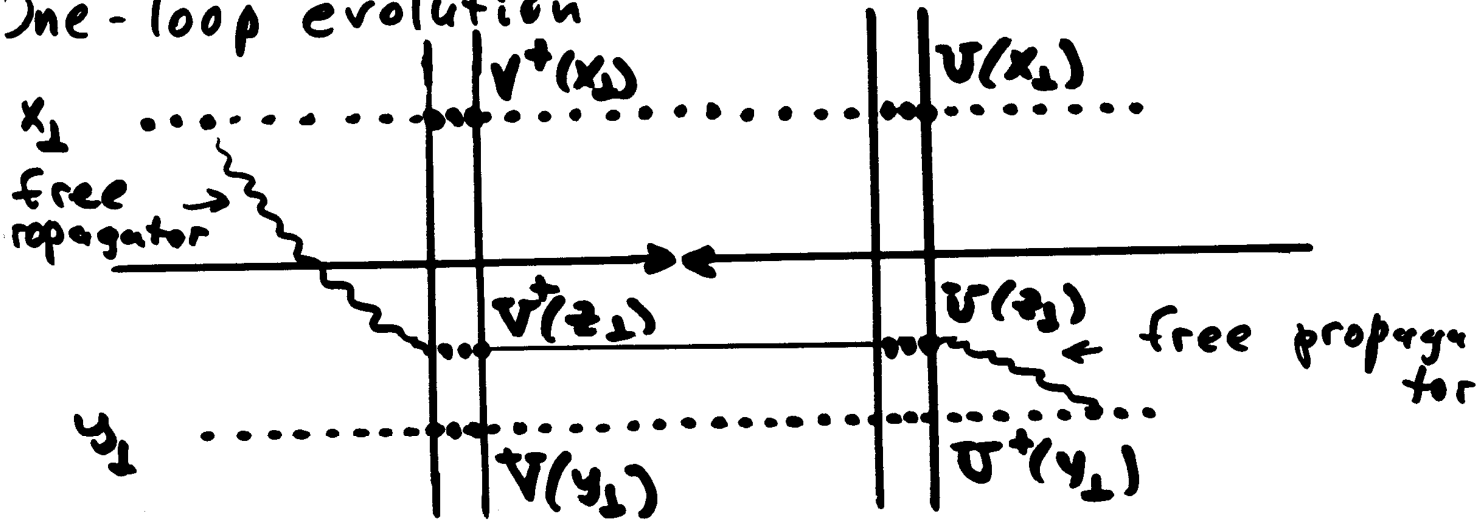
As was discussed above, in order to cut off the longitudinal divergence, one shoud replace

$$U(z_\perp),\, V(z_\perp) \longrightarrow U^\xi(z_\perp),\, V^\xi(z_\perp)$$

ordered along $p_1 + \xi p_2$ $\quad \xi = \frac{p_A^2}{s}$

Evolution equation

One-loop evolution



$$\xi \frac{d}{d\xi} \mathrm{Tr}\{(V_y V_x^+ - 1)(U_x U_y^+ - 1)\} =$$

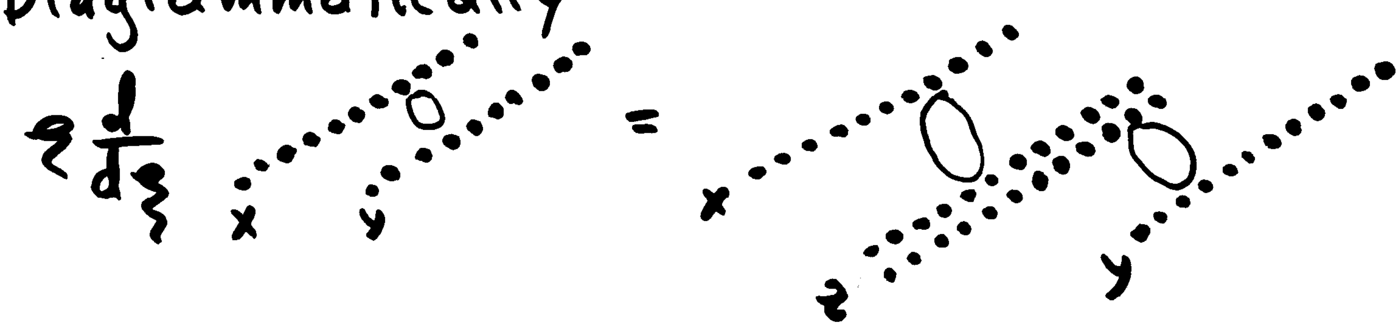
$$= -\frac{g^2}{16\pi^3} \int d^2 z \, \frac{(x-y)^2}{(x-z)^2 (y-z)^2} \cdot$$

$$\Big[\mathrm{Tr}\{(V_z V_x^+ - 1)(U_x U_z^+ - 1)\} \mathrm{Tr}\{(V_y V_z^+ - 1)(U_z U_y^+ - 1)\} -$$

$$- N_c \mathrm{Tr}\{(V_y V_x^+ - 1)(U_x U_y^+ - 1)\}\Big]$$

$O \equiv$ Trace

Diagrammatically



again, linearization gives BFKL eqn

$$\mathcal{O}(x,y) \equiv \mathrm{Tr}(V_y V_x^\dagger - 1)(U_x U_y^\dagger - 1)$$

$$\xi \frac{d}{d\xi} \mathcal{O}(x,y) = -\frac{g^2}{16\pi^3} \int dz \frac{(x-y)^2}{(x-z)^2(y-z)^2} \{ \mathcal{O}(x,z) + \mathcal{O}(z,y) - \mathcal{O}(x,y) \}$$

Solution

$$\mathcal{O}^{\xi}(x,y) = \int d\nu \int \frac{d^2 x_0}{\pi^4} \nu^2 \left(\frac{(x-y)^2}{\tilde{x}^2 \tilde{y}^2}\right)^{\frac{1}{2}-i\nu} \int dx' dy' \, \mathcal{O}(x',y') \underbrace{\left(\frac{(x'-y')^2}{\tilde{x}'^2 \tilde{y}'^2}\right)}_{\frac{1}{2}+i\nu} \cdot \left(\frac{\xi}{\xi_0}\right)^{\overbrace{2N_c \frac{\alpha_s}{\pi} \chi(\nu)}^{\omega(\nu)}}$$

$\tilde{x} \equiv x - x_0$
$\tilde{x}' \equiv x' - x_0$ etc

In terms of

$$\mathcal{O}(x_0,\nu) \equiv \int dx dy \frac{1}{(x-y)^4} \left(\frac{(x-y)^2}{(x-x_0)^2(y-x_0)^2}\right)^{\frac{1}{2}+i\nu} \mathcal{O}(x,y)$$

$$\mathcal{O}^{\xi}(x_0,\nu) = \left(\frac{\xi}{\xi_0}\right)^{\omega(\nu)} \mathcal{O}^{\xi_0}(x_0,\nu)$$

$$\omega(0) = 4N_c \frac{\alpha_s}{\pi} \ln 2$$

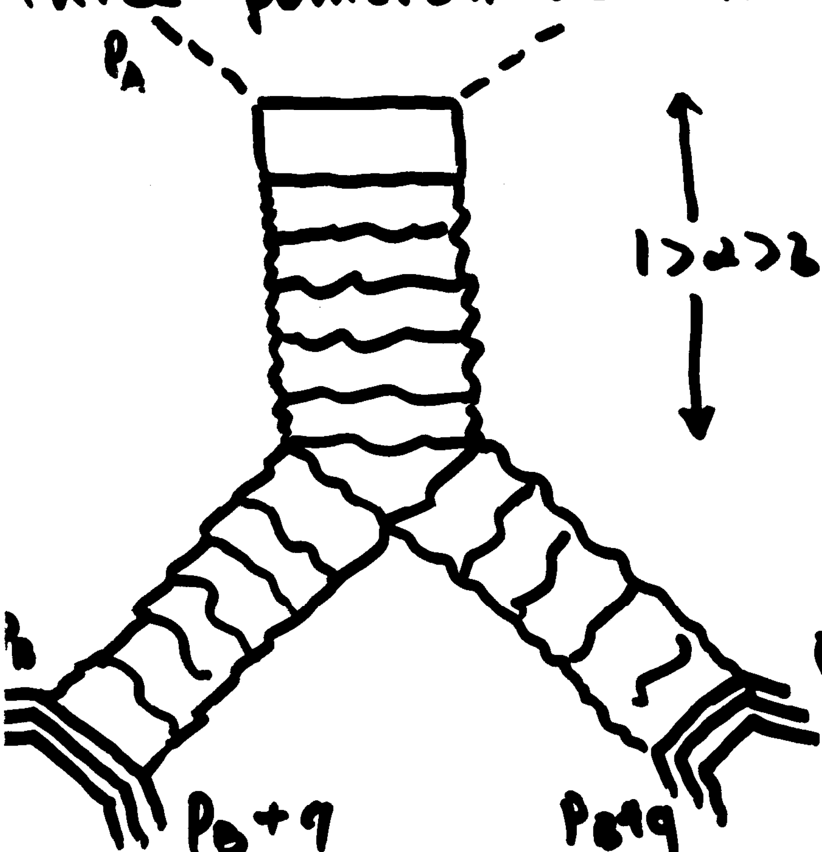
position of the BFKL pomeron

Non-linear eqn. at large N_c

$$\zeta\frac{d}{d\zeta}\mathcal{O}(x,y)=\frac{g^2N_c}{8\pi^3}\int d^2z\,\frac{(x-y)^2}{(x-z)^2(y-z)^2}\Big\{\mathcal{O}(x,z)+\mathcal{O}(z,y)-\mathcal{O}(x,y)+\frac{1}{N_c}\mathrm{Tr}(V_yV_z^\dagger-1)\mathrm{Tr}(U_xU_z^\dagger-1)+\frac{1}{N_c}\mathrm{Tr}(V_zV_x^\dagger-1)\,\mathrm{Tr}(U_zU_y^\dagger-1)\Big\}$$

$$\frac{m^2}{s\sigma^2}=\xi$$

Three-pomeron vertex



$1>\alpha>\zeta$

linear evolution (BFKL)

$\zeta>\alpha>\frac{m^2}{s}$

rapidity gap $\Rightarrow$ two linear evolutions

Approximation: leading logs + large N_c

Up to $\alpha=\zeta$ - linear evolution

At $\alpha=\zeta$ $\quad N_c\mathcal{O}(x,y)\to\langle p_B|\mathrm{Tr}\{V_y^\dagger U_x-1\}|p_B+q\rangle\cdot$

$\cdot\langle p_B+q|\mathrm{Tr}\{U_xU_y^\dagger-1\}|p_B\rangle$

$\mathrm{Tr}(V_yV_z^\dagger-1)\mathrm{Tr}(U_xU_z^\dagger-1)\to$ one-proton state + go

$\to\langle p_B|\mathrm{Tr}(U_yU_z^\dagger-1)|p_B+q\rangle\langle p_B+q|\,\mathrm{Tr}(U_xU_z^\dagger-1)|p_B\rangle$

Evolution:

$$\sigma^{3}(x_0,\nu) = \int \frac{d\nu_1}{\pi\gamma}\nu_1^2 \frac{d\nu_2}{\pi\gamma}\nu_2^2 \int dx_1 dx_2 \int \frac{dz}{\sigma}$$

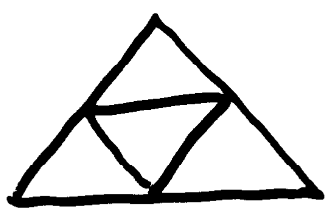
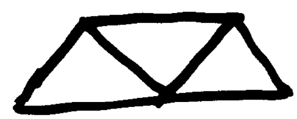
$$\left(\frac{1}{\sigma}\right)^{\omega(\nu)} \left(\frac{z}{\sigma_0}\right)^{\omega(\nu_1)+\omega(\nu_2)} (x_{10}^2)^{-\frac{1}{2}+i(\nu_2-\nu-\nu_1)} \cdot (x_{20}^2)^{-\frac{1}{2}+i(\nu_1-\nu-\nu_2)} \cdot (x_{12}^2)^{-\frac{1}{2}-i(\nu+\nu_1+\nu_2)}$$

$$\cdot V^-(x_1,\nu_1)\, U^+(x_2,\nu_2) \cdot f(\nu,\nu_1,\nu_2)$$

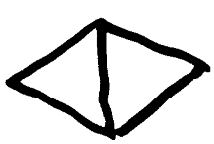
three-pomeron vertex

$$U(x_2,\nu_2) \equiv \int dx\,dy \frac{1}{(x-y)^4}\left(\frac{(x-y)^2}{(x-x_2)^2(y-x_2)^2}\right)^{\frac{1}{2}+i\nu_2} \cdot \mathrm{Tr}\, U_x U_y^\dagger,$$

similarly for $V(x_1,\nu_1)$

$f(\nu,\nu_1,\nu_2) \simeq$  $\rightarrow$ 

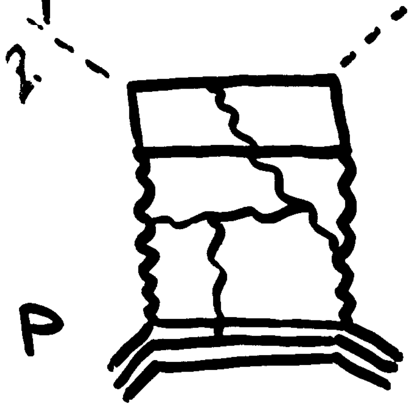
$+$



propagators are $((x-y)^2)^{-\frac{1}{2}+i\nu}$

$\Rightarrow$ answer for diffuctive cross section

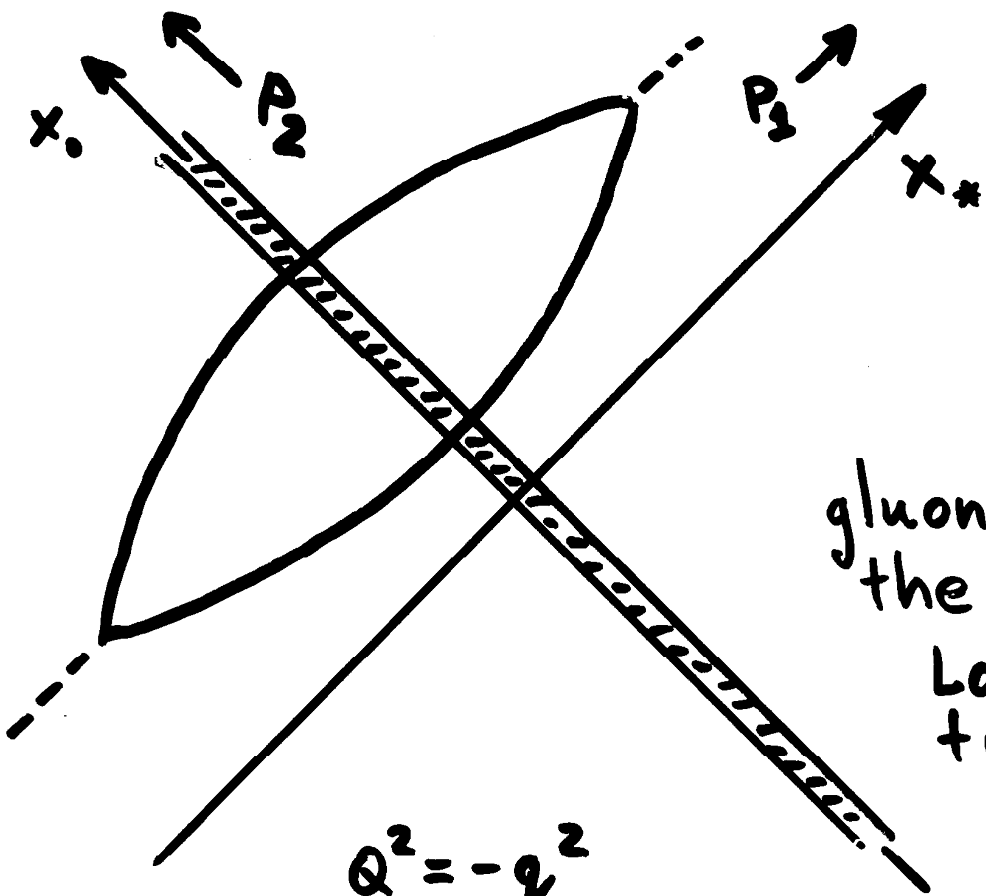
Space-time picture



Deep inelastic scatterin[g]
at small $x_B = \frac{-q^2}{2pq}$

large energy ⇒ gluon excha-
ges

In the frame of virtual photon:



$q = p_1 + \frac{q^2}{s} p$

$p \simeq p_2$

$x_\bullet \equiv x p_1$

$x_* \equiv x p_2$

gluonic „cloud" of
the nucleon

Lorentz contra-
tion:

$\Delta x_* \sim \sqrt{1-v^2} \sim x$

⇓

shock wave

$Q^2 = -q^2$

$x_* = \frac{s}{Q} x_+$

$x_\bullet = Q x_-$

x_+, x_- - usual l.c. coordinates

ropagator in a shock-wave background

$_A(x,y) = \langle x|\frac{1}{\hat{\mathcal{P}}}|y\rangle = -i\int_0^\infty d\tau\, \langle x|\hat{\mathcal{P}}\, e^{-i\tau\hat{\mathcal{P}}^2}|y\rangle =$

chwinger representation   proper time

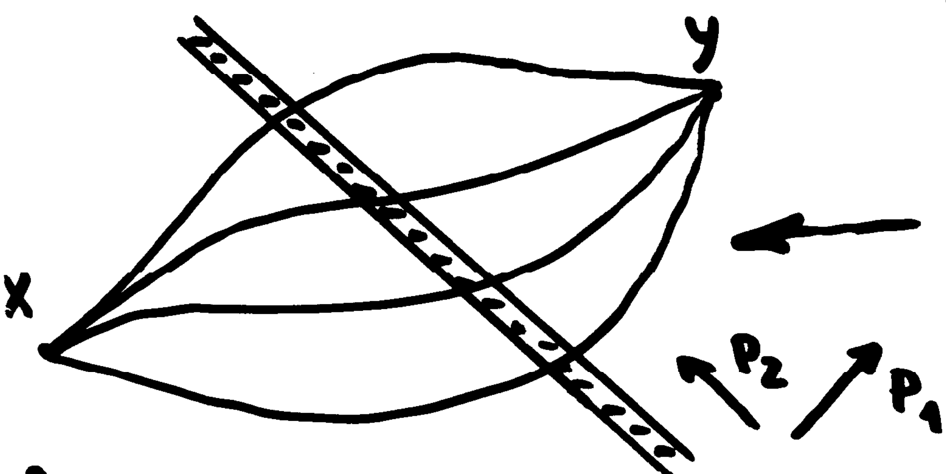
$-i\int_0^\infty d\tau \int \mathcal{D}x(t)\, \left(\frac{\hat{\dot{x}}}{2} + \hat{A}\right) e^{-i\int_0^\tau dt\, \frac{1}{4}\dot{x}^2(t)}$

$\hat{\mathcal{P}} \equiv \not{\mathcal{P}}$

$P\exp ig\{\int A_\mu dx_\mu + \frac{ig}{2}\sigma_{\mu\nu}G_{\mu\nu}\, dt\}$

eynman formula

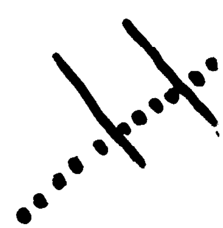
path integral over different quark trajectories



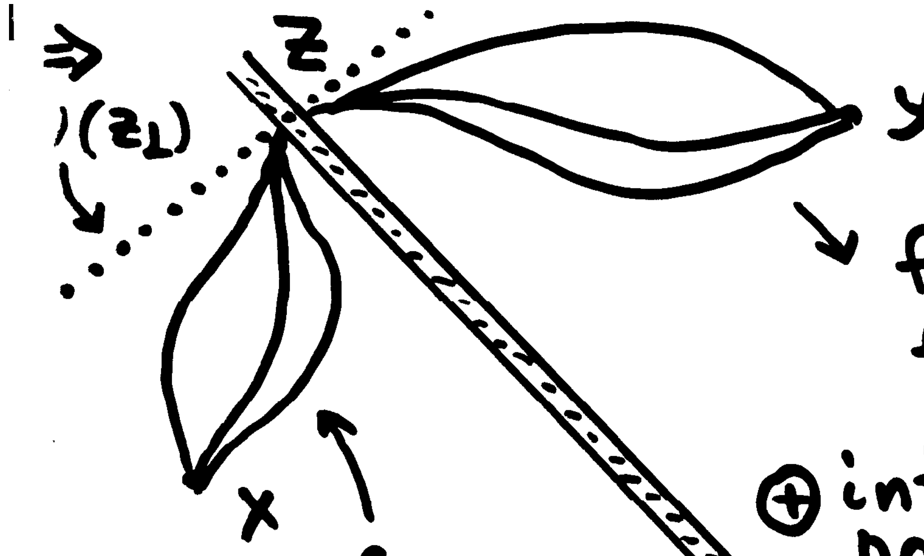
$\frac{\dot{x}_\perp(t)}{\dot{x}_*(t)} \sim O(\frac{1}{\lambda}) \Rightarrow$ quark trajectory insid the shock wave is a straight line $\uparrow\uparrow p_1$

$P\exp ig\int A_\mu dx_\mu \rightarrow P\exp ig\int du\, A_\bullet(x_\perp + up_1) \rightarrow$

$\rightarrow P\exp ig\int_{-\infty}^{\infty} du\, A_\bullet(x_\perp + up_1) \equiv U(x_\perp) =$



$\Rightarrow$



free propogatoi from z toy

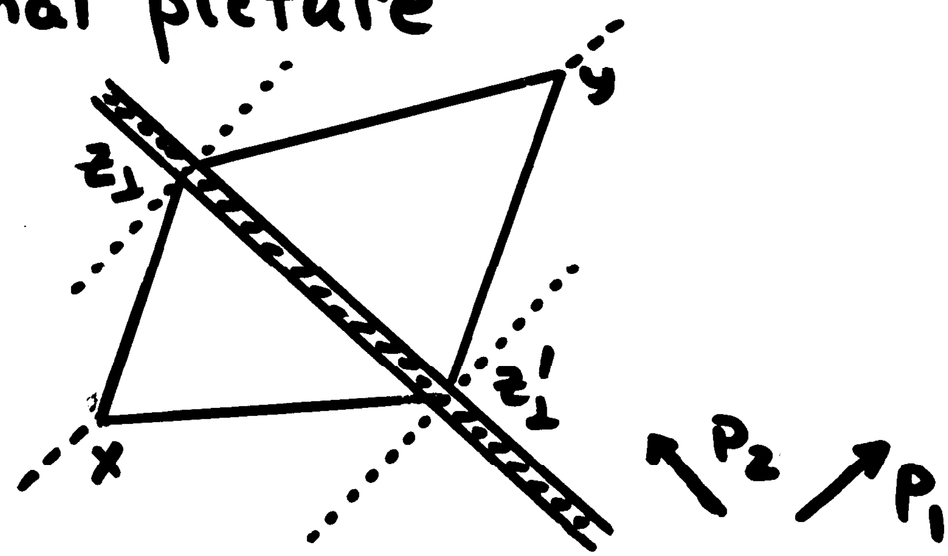
⊕ integration over position of the intersection

free propagator from x to z

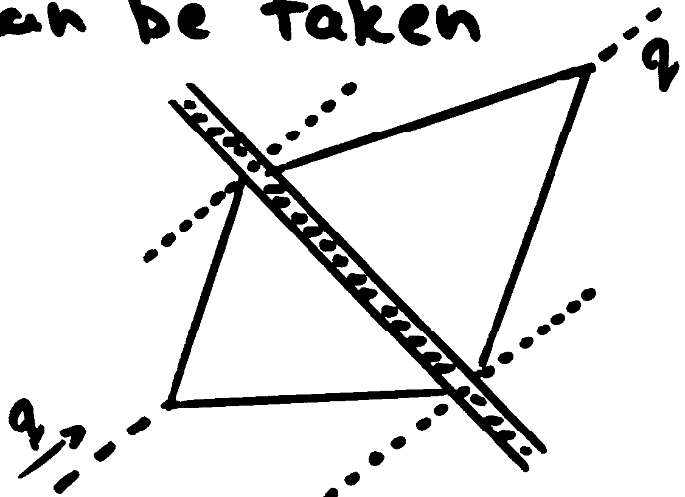
$$\langle x|\frac{1}{\hat{P}}|y\rangle = i\int \frac{dz}{4\pi^2}\,\delta(z_*)\,\frac{\hat{x}-\hat{z}}{(x-z)^4}\,\hat{p}_2\,U(z_\perp)\,\frac{\hat{z}-\hat{y}}{(z-y)}$$

For the antiquark $U \to U^\dagger$

Final picture



After Fourier transform all the integrals can be taken



$$= \int dk_\perp \, I(k_\perp) \, \mathrm{Tr}\, U_k U^\dagger_{-k}$$

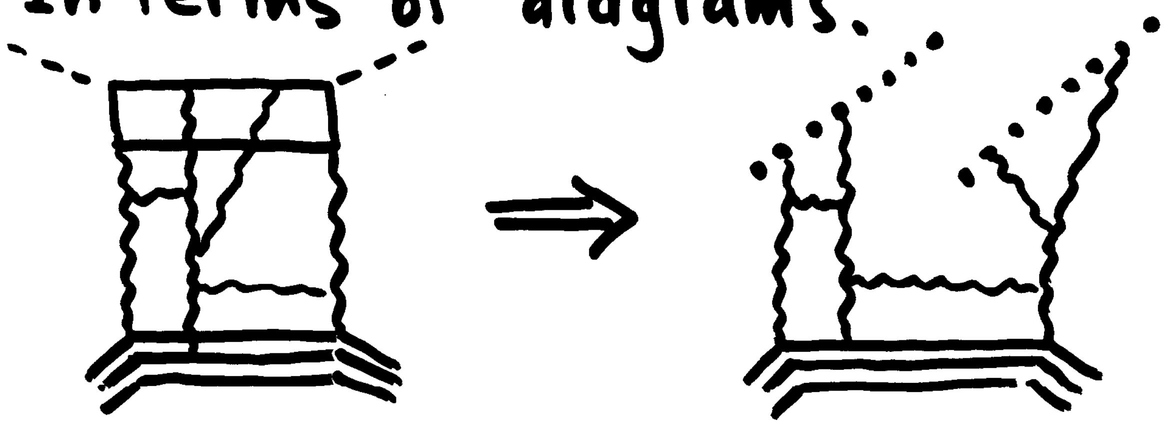
$$U_k \equiv \int dx_\perp \, e^{-i(kx)_\perp} \, U(x$$

$I(k_\perp)$ - „impact facto

$$I(k_\perp) \sim \int_0^1 du\, dv \, \frac{k_\perp^2}{k_\perp^2 u(1-u) + Q^2 v(1-v)}$$

$Q^2 \equiv -q$

In terms of diagrams:



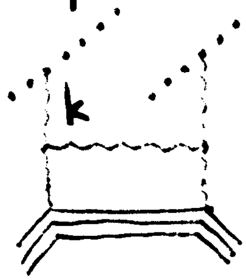
Structure function at small x_B ~

~ „matrix element" of a two-

-Wilson-line operator switched

between nucleon states

NB Matrix elements of light-like Wilson line operators diverge in the longitudinal directions



$$k = \alpha p_1 + \beta p_2 + k_\perp$$

$$p_1^2 = p_2^2 = 0$$

Sudakov variables ($\eta = \ln\frac{\alpha}{\beta}$)

$$q \equiv p_A \simeq p_1 + \frac{p_A^2}{s} p_2$$

$$p \equiv p_B \simeq p_2$$

$$\int_{m^2/s}^{\infty} \frac{d\alpha_k}{\alpha_k} \rightarrow \text{diverges} \qquad (m^2 \sim p_A^2, p_B^2)$$

Why: we have assumed that there are no fast ($\alpha_k \sim 1$) gluons in the nucleon „cloud" but they do exist and should be treated separately - like hard gluons in Wilson OPE

Factorization:

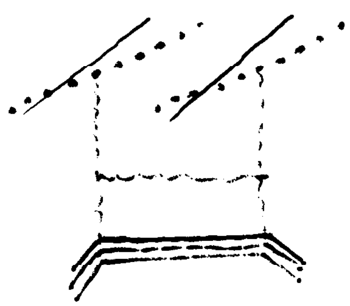
Fast gluons (and quarks) $\Rightarrow$ coefficient function $\alpha \sim 1$

Slow (nucleon „cloud") $\Rightarrow$ regularized matrix elements $\alpha < 1$

We regularize the light-like Wilson line operators by changing the slope of the line

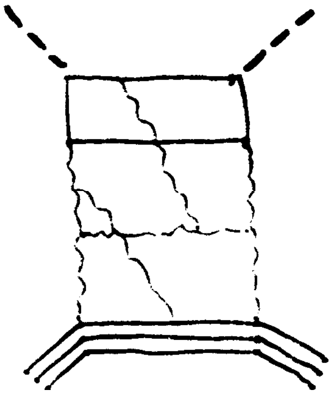
$$U^{\varsigma}(x_\perp) = P\exp ig\int_{-\infty}^{\infty} du\, p_\mu^{\varsigma} A_\mu(up^{\varsigma} + x_\perp)$$

$$p^{\varsigma} = p_1 + \varsigma p_2$$



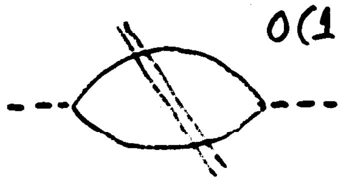
$$\int_{m^2/s}^{m^2/\varsigma s} \frac{d\alpha}{\alpha} \rightarrow \text{converges (and} \simeq \ln\frac{1}{\varsigma})$$

Factorization

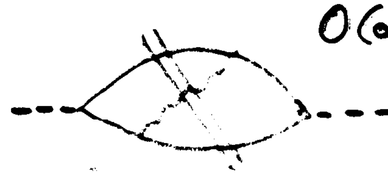


$(g^2 \ln\frac{s}{m^2})^n$ in the physical amplitude $\Rightarrow$

$\alpha \sim 1$
impact factor



$O(1)$

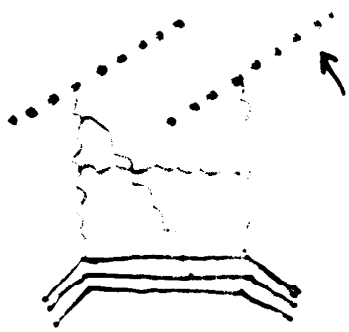


$O(\alpha_s)$

$\otimes$ $\otimes$

$\Rightarrow$

$\frac{m^2}{s} < \alpha < 1$
matrix element



$+$

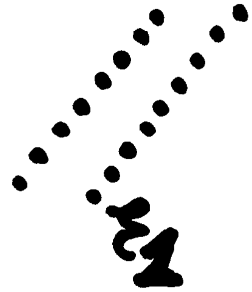
$\uparrow\uparrow$ $p_1 + \xi p_2$
$\xi \sim \frac{m^2}{s}$

$+ \dots$

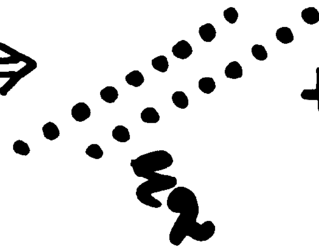
$(g^2 \ln\frac{1}{\xi})^n$ in the matrix elements of Wilson-line operators

Instead of the dependence of physical amplitude on s we can study a much simpler dependence of matrix elements of Wilson-line operators on the slope ξ

Evolution - one loop

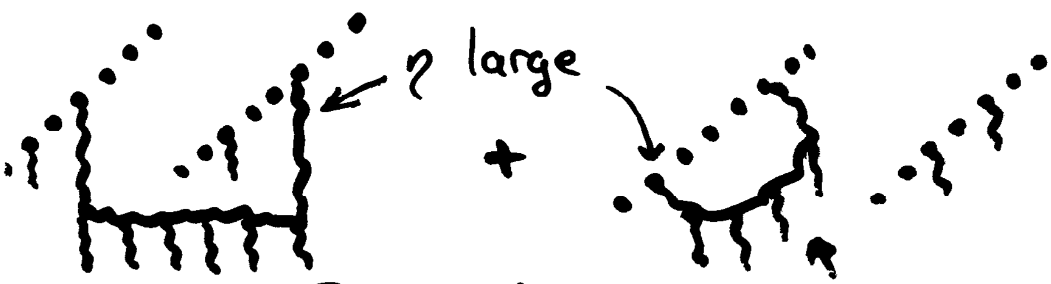


$\Longrightarrow$



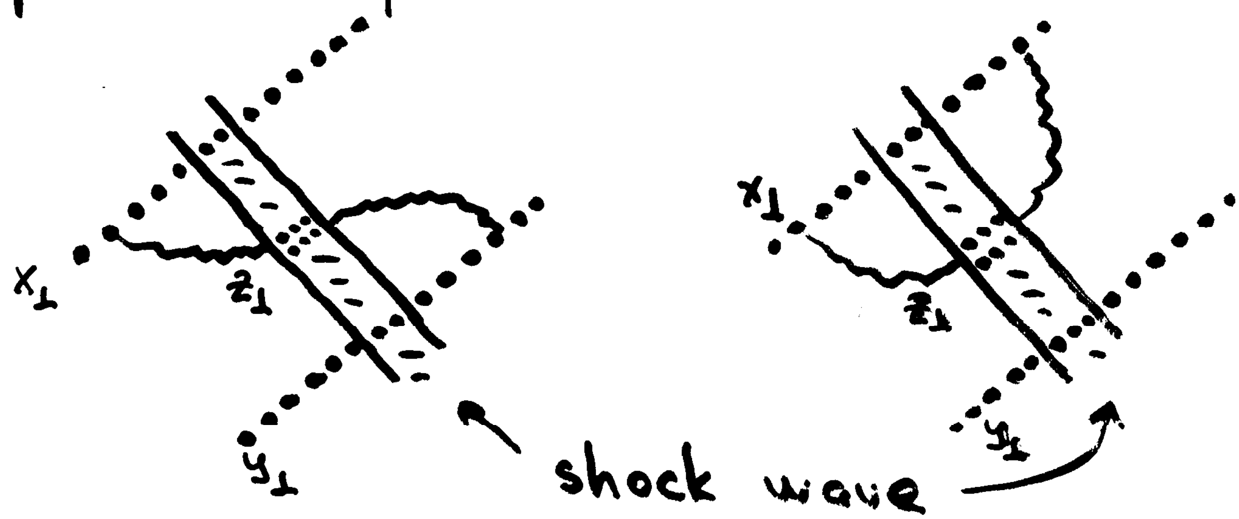
$+ \dfrac{\alpha_s}{\pi} \ln \dfrac{\xi_2}{\xi_1} \cdot \text{kernel} \cdot$

Kernel is determined by the loop integral over large rapidities $\left(\ln \frac{m^2}{s\xi_1} > \eta > \ln \frac{m^2}{s\xi_2}\right)$ in the background of gluons with small rapidities $\left(\ln \frac{m^2}{s\xi_2} > \eta\right)$

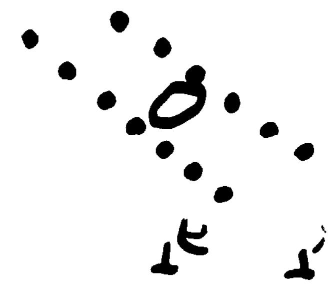
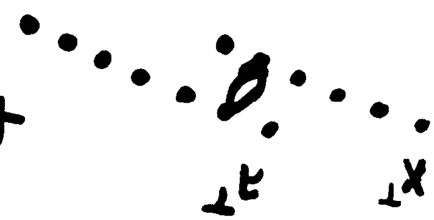


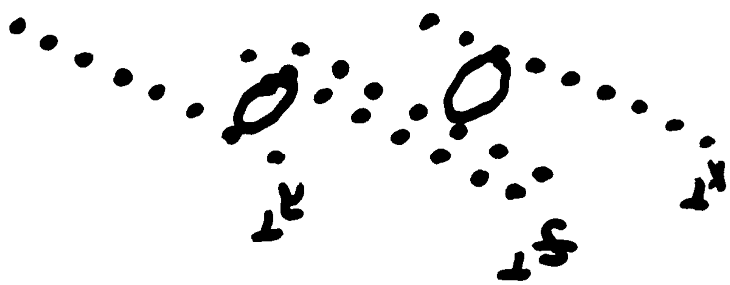
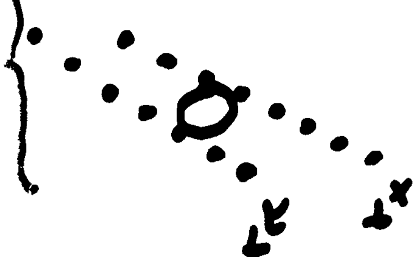
"external field" $\Leftrightarrow$ η small

Space - time picture



One-loop evolution:

 $\Rightarrow$  $+ \frac{\alpha_s}{\pi^2} \ln\frac{\sigma_2}{\sigma_1}$

$\Big\{$  $- N_c$  $\Big\}$

 $\equiv$ Trace

$\Rightarrow$ Evolution eqn:

$$\sigma\frac{d}{d\sigma}\mathrm{Tr}\,U_x U_y^\dagger = -\frac{\alpha_s}{4\pi^2}\int d^2z\, \frac{(x-y)^2}{(x-z)^2(y-z)^2}$$

$$\left\{\mathrm{Tr}\,U_x U_z^\dagger\, \mathrm{Tr}\,U_z U_y^\dagger - N_c\, \mathrm{Tr}\,U_x U_y^\dagger\right\}$$

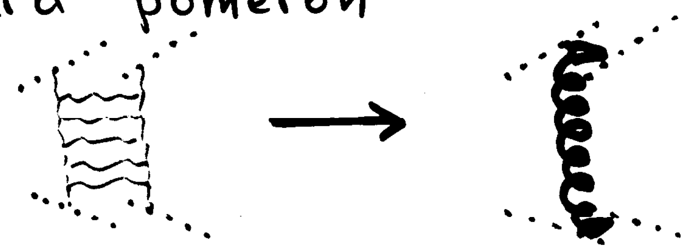
Linearization gives BFKL eqn:

Evolution equation $\quad \mathcal{V}(x,y) \equiv \frac{1}{(x-y)_\perp^2}\left(\frac{1}{N_c}\mathrm{Tr}\, U_x U_y^\dagger - 1\right)$

$$\xi \frac{d}{d\xi}\mathcal{V}(x,y) = -\frac{\alpha_s}{4\pi^2}\int d^2z \left[\left\{ \frac{\mathcal{V}(x,z)}{(y-z)^2} + \frac{\mathcal{V}(z,y)}{(x-z)^2} - \frac{(x-y)^2\mathcal{V}(x,y)}{(x-z)^2(z-y)^2}\right\} + \mathcal{V}(x,z)\mathcal{V}(z,y)\right]$$

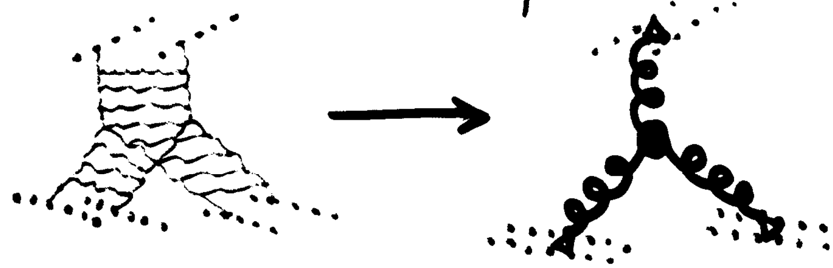
Without quadratic term — BFKL eqn

Linear terms describe propagator of the hard pomeron



$\sim s^{4N_c\alpha_s \ln 2}$

Non-linear term corresponds to the three-pomeron vertex



$\frac{(x-y)^2}{(x-z)^2(y-z)^2}$

(Bartels etc

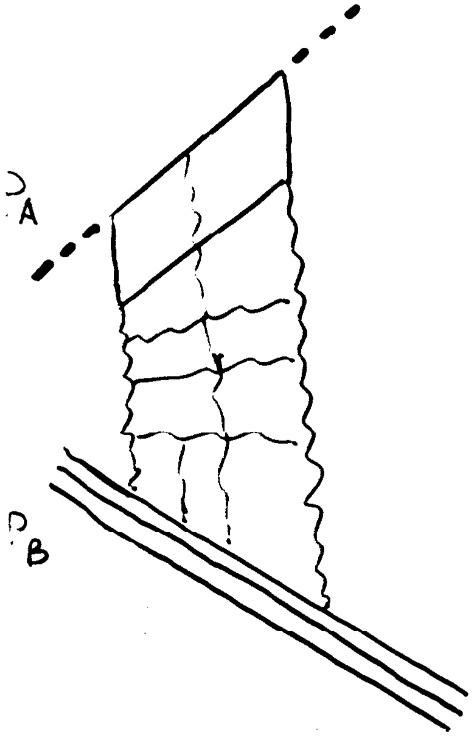
Linear evolution is enough (in leading logs) at:

1. Large N_c (Matrix element of $\mathcal{V}\otimes\mathcal{V}$ is $1/N_c$)
2. Special target (like heavy quarkonium)

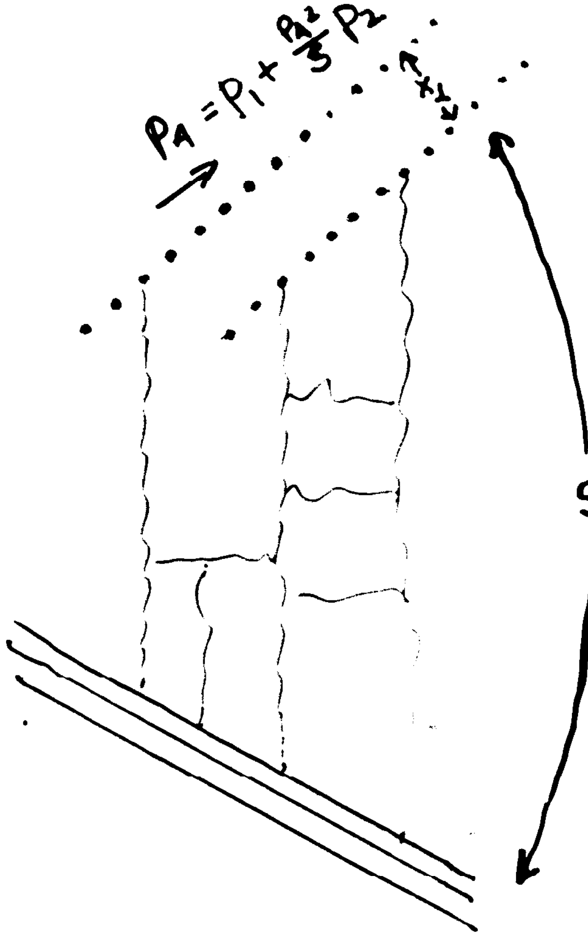
$\mathcal{V} \sim g^2$

$\mathcal{V}^2 \sim g^4$

3. ?

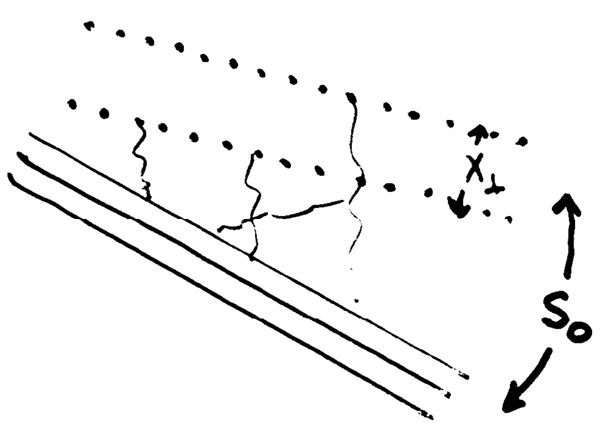


$\Longrightarrow$



$\Longrightarrow$

$$\left(\frac{s}{s_0}\right)^{\frac{\alpha_s}{\pi} N_c \ln 2} \cdot$$



$s_0 \gg m_N^2$,

$g^2 \ln \frac{s_0}{m_N^2} \ll 1$

$$\int d^2x\, e^{ikx} \langle N | \mathrm{Tr}\, U^{\xi_0}(x_\perp)\, U^{+\xi_0}(0) | N \rangle \sim \frac{1}{k_\perp^4} I^N(k_\perp)$$

$$F_2(x_B) \sim x_B^{-4\frac{\alpha_s}{\pi}N_c \ln 2} \underbrace{\int \frac{d^2p_\perp}{4\pi^2} \frac{I(p_\perp)}{|p_\perp|^3}}_{\text{known number}} .$$

$$\cdot \underbrace{\int \frac{d^2k_\perp}{4\pi^2} \frac{I^N(k_\perp)}{|k_\perp|^3}}_{\text{to be calculated (QCD sum rules?)}}$$

Relation to gluon parton density

$$\mathcal{D}_g^{M^2}(x_B) \simeq \int_0^{M^2} \frac{dk_\perp^2}{k_\perp^2} I^N(k_\perp) \qquad x_B = \sqrt{\frac{m^2}{\xi S}}$$

Conclusion:

Wilson-line approach gives an economic way to calculate perturbative diagrams at high energy. Besides that, it may serve as a bridge between perturbative and non-perturbative analysis of diffractive scattering

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