Intrinsic Glue @ very small x_{Bj}

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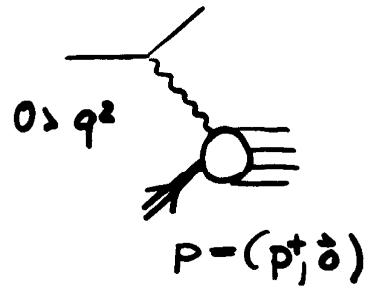
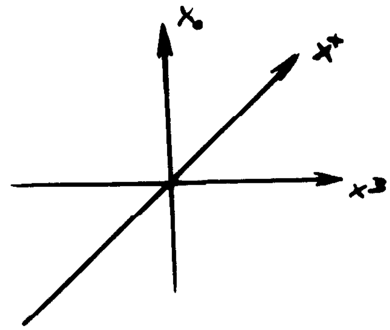
- small x_{Bj} & high ~~parton~~ densities
- classical color charge & classical gluon fields
  resummation tools
- R.G. program: status report
- summary

- **The problem**





$x_{Bj} = \frac{-q^2}{2p\cdot q}$

$\alpha_s \ln \frac{1}{x_{Bj}}$

BFKL resummation $\longrightarrow$ $x_{Bj}\, g(x_{Bj}, Q^2) \sim \frac{1}{x_{Bj}^{\#\alpha_s}}$

based on perturbatively leading $\alpha_s \ln \frac{1}{x_{Bj}}$

unitarity bounds ?

experimentally seen ?

recombination effects @ high densities

- **A solution ?**

perturbative

'nonperturbative'

- small α_s (?)
- simple (naive?) model for 'valence' part:

- resum high density effects

classical color charge $\longrightarrow$ full classical field

stat. weight

- Motivation:

► longitudinal properties
large x_{Bj} as seen by small x_{Bj}

- classical colored particles
- couple to smaller x_{Bj} eikonally
- growing numbers $\searrow x_{Bj}$
- independent on time scales resolved

► transverse properties

- large classical charge densities

$$\Downarrow$$

classical gluon fields

source of
color charge

Wilson lines

Wilson R.G.

statistical
ensemble

high density
resummation

Effective action

$$S_{p^+} = i\int d^4x\, F[\rho] - \int d^4x\, \frac{G^2[A]}{4} + \frac{i}{N_c}\int d^2x_\perp dx^-\, \delta(x^-)\, \mathrm{tr}\, \rho(x_\perp)\, W_{-\infty\infty}[A](x^-, x_\perp)$$

- $F[\rho]$ statistical weight — to be determined via R.G.

  inv. $\rho \to U(x_\perp, x^-)\, \rho\, U^{-1}(x_\perp, x^-)$

  Gaussian approx. $\dfrac{\mathrm{tr}\, \rho^2(x^-, x_\perp)}{2\mu(x^-)}$

- $\mathrm{tr}\, \rho(x^-, x_\perp)\, W_{-\infty\infty}[A]$

  gauge inv. generalization of $J^+ A^-$ — see below

$A^+ = 0$ gauge

(keep res. gauge inv.; consistency checks)

meaningful p^+ cutoffs $\updownarrow$ R.G.

- Y.M. Equations

$$0 = \int_{\rho^+} \mathcal{D}[\rho, A] \ \frac{\delta}{\delta A_\nu} \ e^{iS[\rho, A]}$$

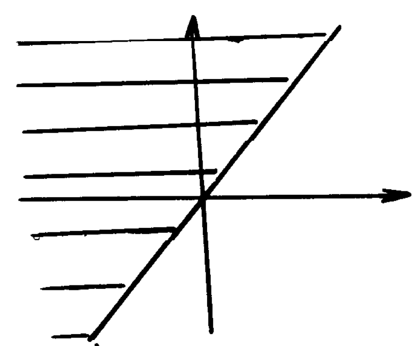
$$= \int_{\rho^+} \mathcal{D}[\rho, A] \Big([D_\mu[A], G^{\mu\nu}[A]] - J^\nu[\rho, A] \Big) \, e^{iS[\rho, A]}$$

$$J^+[\rho, A] = \frac{g}{N_c} \, tr\Big(\rho(x^-, x_\perp) \, W_{-\infty x^+}[A] \, t_a \, W_{x^+ -\infty}[A] \Big) t_a$$

$$[D_\mu[A], J^\mu[\rho, A]] = 0$$

- Classical solution reparametrized

$A^+ = 0$, static gauge



$$b_i = -\frac{1}{ig} U_{-\infty x^-}[\Lambda] \, \partial_i \, U_{x^- -\infty}[\Lambda]$$

$$U_{x^- -\infty}[\Lambda] := \mathcal{P}\exp -ig \int_{-\infty}^{x^-} dy^- \, \Lambda(y^-, x_\perp)$$

► new Y.M.:

$$\nabla_\perp^2 \Lambda(x^-, x_\perp) = U_{x^- -\infty}[\Lambda] \ \rho(x^-, x_\perp) \ U_{-\infty x^-}[\Lambda]$$

Gluon content of the classical field

- $$F[\rho] \xrightarrow[\text{Gaussian approx.}]{} \frac{\mathrm{tr}\, \rho^2(x^-, x_\perp)}{2\mu^2(x^-)}$$

- $$\int D[\rho] \; e^{-\int_0^\infty dy \int d^2x_\perp \frac{\mathrm{tr}\, \rho^2(y, x_\perp)}{2\mu^2(y)}}$$

$$= \int D[\Lambda] \; \exp - \int_0^\infty dy \int d^2x_\perp \frac{\mathrm{tr}\left(\nabla_\perp^2 \Lambda(y, x_\perp)\right)^2}{2\mu^2(y)}$$

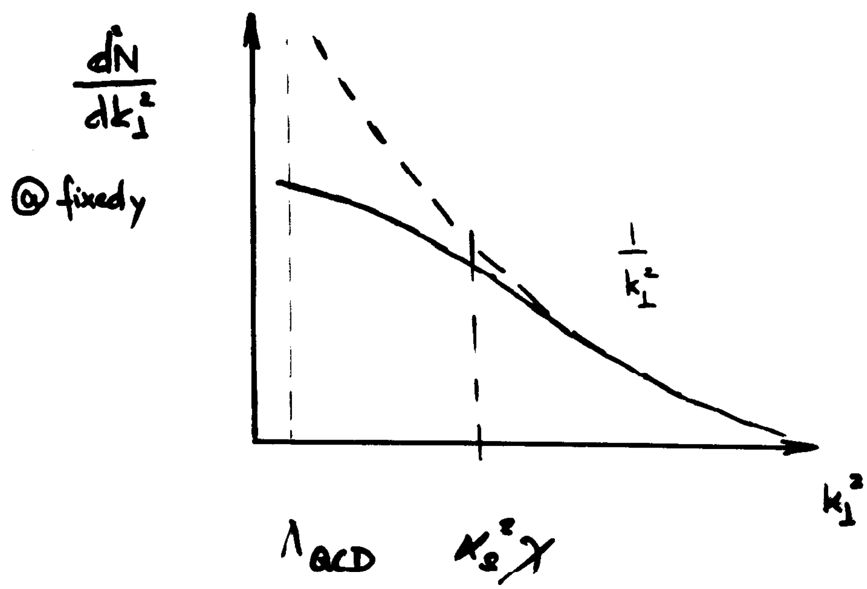
- $$G_{ij}^{\mathrm{class}}(y, x_\perp; y', x'_\perp) = \langle b_i(y, x_\perp) \, b_j(y', x'_\perp) \rangle$$

$$b_j = \int_y^\infty dy' \, U_{\infty y'}[\Lambda](x_\perp) \left(\nabla^i \Lambda(y', x_\perp)\right) U_{y'\infty}[\Lambda](x_\perp)$$

- $$\langle \Lambda(y, x_\perp) \, \Lambda(y', x'_\perp) \rangle = \mu^2(y, Q^2) \, \delta(y - y') \frac{1}{\nabla_\perp^4}(x_\perp, x'_\perp)$$

$$\frac{1}{4\pi} x_\perp^2 \ln x_\perp^2 \Lambda_{QCD} + \gamma_\perp(0) =: \gamma(x_\perp)$$

$$G^{ab}_{ij}(y,x_\perp;y',x'_\perp) = -\delta^{ab}\left(\nabla_i \nabla'_j \gamma_\perp(x_\perp - x'_\perp)\right) \cdot \frac{1-\exp g^4 N_c \chi(y)\left[\gamma_\perp(x_\perp-x'_\perp)-\gamma_\perp(0)\right]}{N_c\left[\gamma_\perp(x_\perp-x'_\perp)-\gamma_\perp(0)\right]}$$



$$\chi(y) := \int_y^\infty dy' \mu^2(y')$$

typical color charge at $y' > y$

Lessons to be learned

- classical fields $\longleftrightarrow$ new scale $\alpha_s^2 \chi(y)$
- pert. limit @ short distances
- softening at small $k_\perp$: $G^{aa}_{ii} \sim \ln \frac{k_\perp^2}{g^4 \chi(y)}$
- Λ_{QCD} dep. weak unless $k_\perp^2 \ll \alpha_s^2 \chi(y)$

■ R. G. improvement

$$0 = \int_{p^+} D[\rho, A] \Big([D_\mu[A], G^{\mu\nu}[A]] - J^\nu[\rho, A] \Big)\, e^{iS[\rho A]}$$

split

$$A = \underbrace{b}_{\text{class. sol.}} + \overbrace{a}^{k^+ < p'^+} + \underbrace{\delta A}_{p'^+ < k^+ < p^+}$$

$$= \int D[\rho, A] \int_{p'^+ < k^+ < p^+} D[\delta A] \Big\{ [D_\mu[b+a], G^{\mu\nu}[b+a]] - J^\nu[b+a]$$
$$- ig\, [\delta A_\mu, G^{\mu\nu}[b+a]] - ig\, [\delta A_\mu, G^{\mu\nu}[\delta A]]$$
$$+ \dots \Big\}\, e^{iS[\rho,\, b+a+\delta A]}$$

$$\delta J^\nu[b+a] \longrightarrow \delta\rho$$

induced current

perturbatively

$$\delta\rho = f^{abc} \left(\partial^+ b^i\, \delta A^i + \rho^b \text{"}\!\int dy^+\, \theta\text{"}\, \delta A^{-d} \right) +$$

$$+ f^{abc}\, \delta A_i^b\, \partial^+ \delta A_i^c$$

$$+ \frac{g^2}{N_c} \rho^b\, \text{"}\!\int dy^+ dz^+\, \theta\theta\, \mathrm{tr}\; tttt\text{"}\, \delta A^{-c}\, \delta A^{-d}$$

- procedure

$$\int D[\delta A] \;\; @ \text{ fixed } \rho \,\&\, \delta\rho$$

$$\int D[\rho] \;\; @ \text{ fixed } \rho' := \rho + \delta\rho$$

- new effective action

$$iS[\rho', a] = -F'[\rho'] - \tfrac{1}{4} G^2[a] + i/N_c \,\mathrm{tr}\, \rho' W[a]$$

with

$$\exp\left(-F'[\rho']\right) = \int D[\rho, \delta A]\, \delta\left(\rho' - \rho - \delta\rho[\delta A]\right) \exp\left(-F[\rho] - \tfrac{1}{2}\delta A D^{-1}[\rho]\, \delta A\right)$$

- to leading $\alpha_s \ln \frac{1}{x_{Bj}}$

$$\frac{d}{d \ln \frac{1}{x_{Bj}}} F[\rho] = \alpha_s \Delta\left(F[\rho]\right)$$

- tools needed

$$\Rightarrow \; = \; \Sigma$$ 

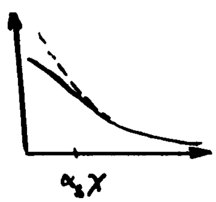
gauge inv. def of $\delta\rho$

- note: b_μ retains structure

- double log approx:

$$\frac{d}{d\ln\frac{1}{x_{Bj}}} F[\rho] \approx \Big\langle$$  $$\Big\rangle_\rho$$

$$= -\sum$$ 

$\approx$ 

$+$ qualitatively similar contributions

- summary

  - classical fields resum density effects
  - new scale ($\alpha_s^2 \chi$ in Gaussian approx) at large distances
  - perturbative at small distances
  - expansion in ρ (small densities) $\longrightarrow$ BFKL

- perspectives: TO DO list has become shorter

  - BG field propagator
  - BFKL derivation written up
  - gauge 'invariant' treatment of $\delta\rho$
  - ...