Renormalon Phenomenology:
Questions and Directions

with some

Observations on Power Corrections to Fragmentation Processes in e^+e^- Annihilation *

M. Beneke
DIS 97, Chicago

(a) Approaches

(b) Topics

(c) Fragmentation

* Work with V. Braun and L. Magnea
(hep-ph/9701309 + in preparation)

The basic idea

Parametrize infrared sensitive contributions to HARD processes

Generalization of ...

$$\int_{\Lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \sim \ln \frac{Q}{\Lambda}$$

perturbative IR logs $\downarrow$ nonperturbative parton densities

... to power-like IR sensitivity

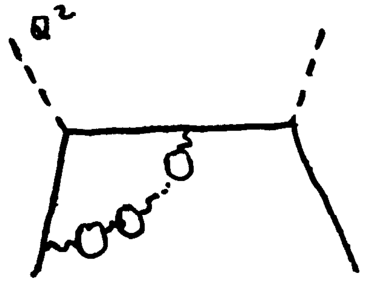
$$\frac{1}{Q^2}\int_{\Lambda^2}^{Q^2} dk_\perp^2 \ni \frac{\Lambda^2}{Q^2} \Longrightarrow$$

Higher twist in DIS

'Power Corrections' in general

Method 1 : The large-order / Borel transform approach

t'Hooft, Parisi, David, Mueller, Zakharov



$$C(Q) \sim \sum_n \alpha_s \int d^4k\, F(k,Q)\, \left[\alpha_s(Q)\,\beta_0 \ln \frac{k^2}{Q^2}\right]^n$$

$$\sim \sum c_n \alpha_s^{n+1}(Q)$$

$$B[C](u) \equiv \sum_n \frac{c_n}{n!}\left(\frac{u}{-\beta_0}\right)^n \sim \int d^4k\, F(k,Q) \left(-\frac{Q^2}{k^2}\right)^u \longleftarrow$$ analytic regularization

IR renormalon poles from loop momenta $k^2 \ll Q^2$ obstruct unambiguous reconstruction of

$$C(Q) = \left(-\frac{1}{\beta_0}\right)\int_0^\infty dt\, e^{-t/(-\beta_0 \alpha_s)}\, B[C](u)$$

$$\frac{1}{u} \longleftrightarrow \frac{d^4k}{k^4} \longleftrightarrow \ln\frac{Q^2}{\Lambda^2}$$

$$\frac{1}{1-u} \longleftrightarrow \frac{d^4k}{k^4}\cdot\frac{k^2}{Q^2} \longleftrightarrow \frac{\Lambda^2}{Q^2}$$

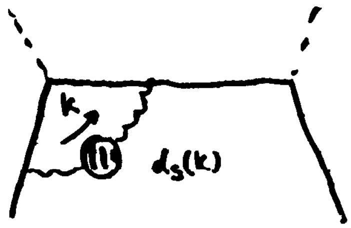
$$\vdots$$

"Ambiguity" of perturbative series

Ambiguities ("prescription dependence") must be compensated non-perturbatively $\Longrightarrow$ power behaviour

Method 2: The "dispersive / gluon mass" approach

MB, Braun, Zakharov
Ball, MB, Braun
Neubert
Dokshitzer, Marchesini, Webber



$\longrightarrow$ Effective gluon propagator $\frac{1}{(k^2)^{1-u}}$ for Borel transform

$$\frac{1}{k^2-\lambda^2} = \frac{1}{2\pi i k^2}\int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} du \cdot \Gamma(u)\Gamma(1+u) \cdot \left(-\frac{\lambda^2}{k^2}\right)^u$$

$$B[C](u) = -\frac{\sin \pi u}{\pi u}\int_0^\infty d\lambda^2 \left(\frac{\lambda^2}{Q^2}\right)^{-u} r_0'(\lambda^2)$$

dispersion relation for running coupling

$$C(\alpha) = \int_0^\infty d\lambda^2 \phi(\lambda^2)\, r_0'(\lambda^2)$$

+ Landau pole contribution

$r_0(\lambda^2)$: One-loop diagram with finite gluon mass

$\phi(\lambda^2)$: IR finite effective coupling

$$r_0(\lambda^2) = a_0 \ln\frac{\lambda^2}{Q^2} + a_1\sqrt{\frac{\lambda^2}{Q^2}} + a_2 \frac{\lambda^2}{Q^2}\ln\frac{\lambda^2}{Q^2} + \ldots + \text{analytic}$$

Non-analytic terms $\longleftrightarrow$ Poles in u

$$\sqrt{\lambda^2} \longleftrightarrow \frac{1}{1-2u} \longleftrightarrow \frac{\Lambda}{Q}$$

$$\lambda^2 \ln \lambda^2/Q^2 \longleftrightarrow \frac{1}{1-u} \longleftrightarrow \frac{\Lambda^2}{Q^2}$$

Note: Dispersive approach always works, but the distribution fn. $r_0(\lambda^2)$ coincides with finite gluon mass calculation only for quantities inclusive over ... [illegible]

Interpretation: The IR/UV connection & factorization

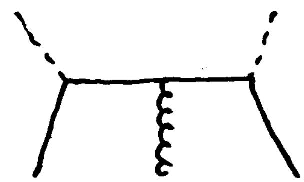
The dispersive approach over-emphasizes the rôle of the effective coupling. In a general context, renormalons are related to "higher-twist" matrix elements:

DIS:

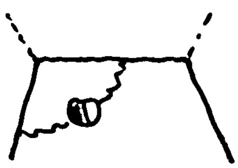
$$F_L(x,Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi}\, C_i(x,\xi,Q^2)\, f_i(\xi,Q^2) \quad \text{twist-2}$$

$$+ \sum_j \frac{1}{Q^2} \int d\xi_1 d\xi_2 \; C_j(x,\xi_1,\xi_2;Q^2)\, T_j(x,\xi_1,\xi_2,Q^2) \quad \text{twist-4}$$

$$+ \cdots$$



IR contribution to twist-2

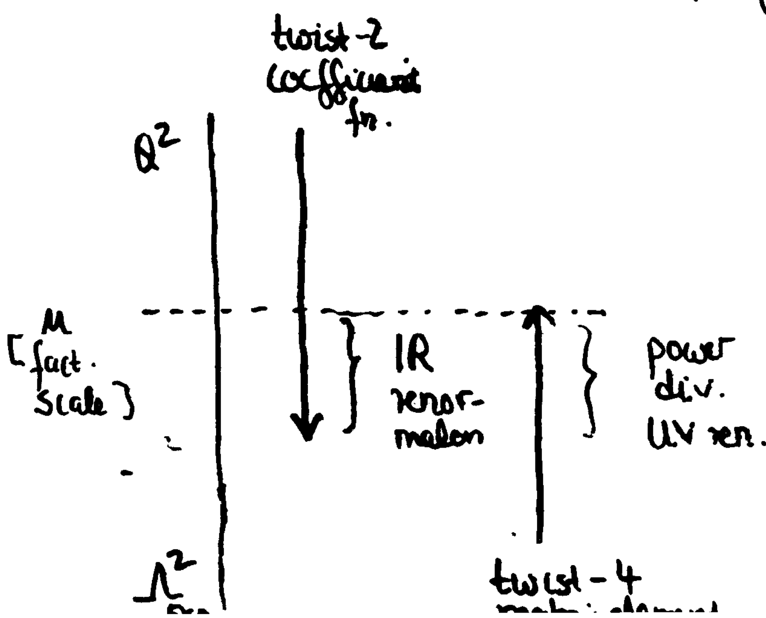


$$O_7(v,x) = \bar\Psi(x) \not x\, x_\beta D_\alpha G_{\alpha\beta}(vx) \Psi(-x)$$

$$\frac{F_L}{x}\Big|_{1/Q^2} = \frac{\text{const}}{1-u} \cdot \left\{-8x^2 + 4\delta(1-x)\right\} \xleftarrow{(-1)\cdot} \text{power UV divergent contribution to twist-4}$$

unique?

MB, Braun, Nagrea



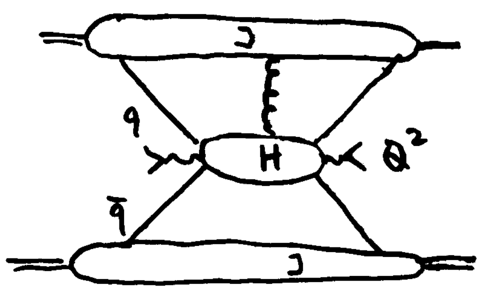
In DIS

$$Q^2 \gg \lambda^2, \mu^2 \gg \Lambda^2, m_q^2, p^2$$

$$Q^2 \gg m_q^2 \gg \Lambda^2, p^2 \quad ?$$

(i) Power Corrections to Hard Processes without OPE

Drell-Yan production



$1/Q^2$ [factorizable in terms of DIS multi-parton correlations] Sterman, Qiu (1991)

collinear

Role of soft gluons?

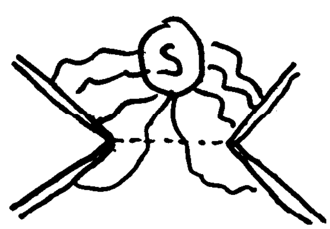
Renormalons : $1/Q$ or $1/Q^2$ corrections?

- No $1/Q$ from one-loop graphs
- True to all orders in abelian theory

Contopanagos, Sterma
Korchemsky, Sterma
MB, Braun
Akhoury, Sotiropoul
Zakharov

In general?

$Q^2 \to s$



Moment space

$$\sigma(N) \underset{Q^2 \to s}{\sim} F_{\gamma p}(N)^2 \cdot H(Q) \cdot S(Q/N)$$

Wilson line expectation value
Expand in N/Q.

$1/Q$ at 2 loops?
Korchemsky, Sterman

Explicit 2-loop calculation of leading IR - sensitive terms needed

Interpretation of $1/Q^{(1,2)}$ terms in terms of operators?

Event Shapes & Universality of $1/Q$ corrections

[Manohar, Wise]
Dokshitser, Webber
Akhoury, Zakharov
Nason, Seymour

$$\langle S \rangle_{power} = K_S \cdot \frac{\langle \mu_{had} \rangle}{Q} + \mathcal{O}\left(\frac{1}{Q^2}\right)$$

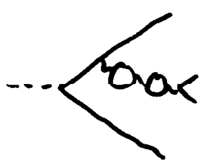
$\langle 1-T \rangle$, $\langle \frac{m_H^2}{Q^2} \rangle$, $\langle C \rangle$, $\frac{\sigma_L}{\sigma_{tot}}$, ...

$\frac{1}{Q}$ fits data well only from soft region

UNIVERSALITY of $1/Q$ power corrections?
UNIQUENESS of coefficient function?

Universality assumption:

Dokshitser, Web.
Akhoury, Zak.



$\longrightarrow \quad K_S \cdot \frac{1}{1-2u}$

NOT calculable
all graphs contribute

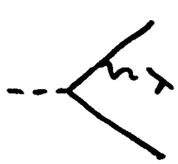
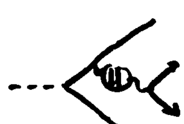
$\frac{1}{1-2u} \longleftrightarrow \langle \frac{\mu_{had}}{Q} \rangle$

If $\langle \mu_{had} \rangle$ is universal: $\quad K_{S_1}/K_{S_2} =$ calculable $+ \mathcal{O}(\alpha_s)$ at 2 loops

* Seems to work well phenomenologically
  e^+e^- and DIS [Dasgupta, Webber ; H1 coll. see WG III]
* Theoretical justification remains to be found

Uniqueness problem:

Ball, MB, Braun
Nason, Seymour

 vs  give different K_{S_1}/K_{S_2}

because $1/Q$ correction comes from soft & large angle radiation (even for $T \to 1$ e.g.)

MB, Braun

(ii) A MODEL for the x-dependence of twist-4 in DIS & fragmentation

DIS: Dokshitzer, Marchesini, Webber; Stein et al.; Dasgupta, Webber; Maul et al.

Fragmentation: Dasgupta, Webber; MB, Braun, Magnea

DIS

$$F(x,Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi} f_{i/p}(\tfrac{x}{\xi},Q^2)\cdot\left[C^i(\xi,Q^2) + A^i(\xi)\frac{\Lambda^2}{Q^2}\right] + \dots$$

Assumption: Substitute x-dependence of power-sensitive contribution in PT for x-dependence of all twist-4 multiparton correlations.

Why should / does this work?

* "Ultraviolet dominance" of higher-twist corrections, i.e. HT is dominated by its cut-off piece. That is, $A^i(\xi)$ effectively parametrizes higher orders in PT

* We are mainly seeing "parametric" x-dependences e.g. $A^i(x)/C^i(x) \underset{x\to 1}{\sim} \frac{1}{1-x}$

Certainly, no insight in hadron structure

$$\left.\frac{M_n^{twist-4}}{M_n^{twist-2}}\right|_{hadron\,1} - \left.\frac{M_n^{twist-4}}{M_n^{twist-2}}\right|_{hadron\,2} \equiv 0$$

In calculations so far $A^i(\xi)$ is Q^2-independent while twist-4 and twist-2 have different log-scaling behaviour

Dokshitser, Marchesini, Webber;
Dasgupta, Webber

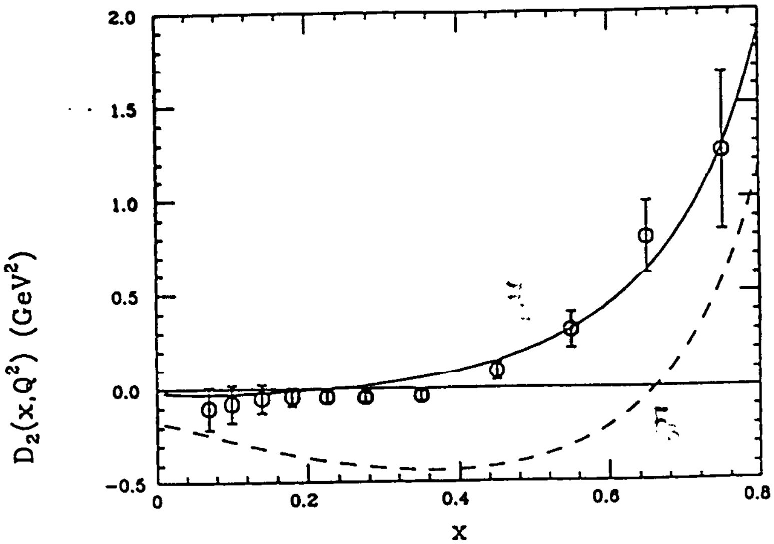


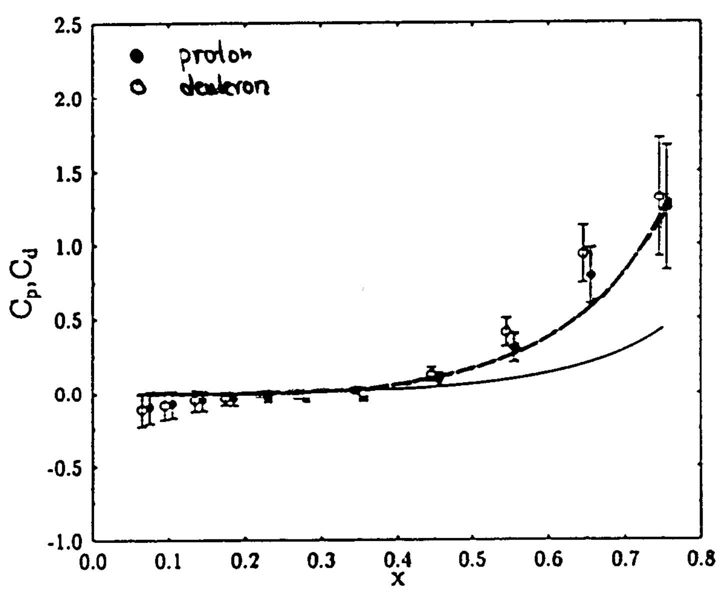
Figure 1: Coefficients of $1/Q^2$ contributions to F_2 (solid), and to F_1 and F_3 (dashed).

x - dependence of twist-4 corrections normalized to twist-2

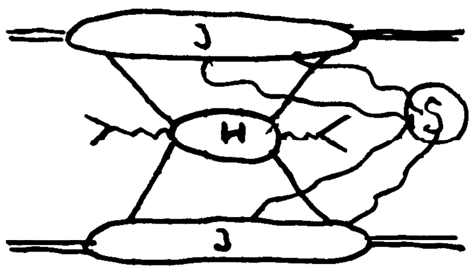
$$F_i(x,Q^2) = F_i^{\text{twist-2}}(x,Q^2)\left[1+\frac{C_i(x)}{Q^2}\right]$$

DIS

Maul et al.



(iii) Theoretical challenge: soft-gluon cancellations beyond logs



Tough part in factorization proofs:
cancellation of soft-gluon divergences $\ln\lambda^2$

$1/Q$ corrections, if present, arise from soft gluons.
Under which circumstances can one prove cancellation of subleading $\sqrt{\lambda^2}$ (IR finite but IR sensitive)

Drell-Yan:

Akhoury, Zakharov
Akhoury, Stodolsky, Zakharov
Akhoury, Sotiropoulos, Zakharov

(1) Extended KLN cancellations

$$\sum_{\substack{i \in D(I) \\ f \in D(F)}} |S_{i \to f}|^2$$ free of $\ln\lambda^2$ and $\sqrt{\lambda^2}$
for sum over degenerate initial and final states

(2) Sum over initial states not required
[up to collinear div.]
"Low theorem"
Relies on soft photon emission being independent.

Can this be generalized to the non-abelian theory?

Power Corrections to Fragmentation Processes in e^+e^- Annihilation

MB, Braun, Magnea
[see also Dasgupta, Webber

Event Shapes, Jet Observables .. : $1/Q$
DIS, Drell-Yan, $R_{e^+e^-}$ .. : $1/Q^2$ or less
Fragmentation ?

$$e^+e^- \to \gamma^*, Z \to H(p) + X$$

$$\frac{d\sigma^H}{dx\,d\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\frac{d\sigma_T}{dx} + \frac{3}{4}\sin^2\theta\,\frac{d\sigma_L}{dx} + \text{asym.}$$

$x = \frac{2p\cdot q}{q^2}$
Scaling variable

$$\frac{d\sigma_L^H}{dx} = \sum_{i=q,g}\int_x^1 \frac{dz}{z}\, C_L^i(z)\, D_i^H\!\left(\frac{x}{z}\right) + \text{higher twist}$$

$$\sigma_L = \sum_H \frac{1}{2}\int_0^1 dx\, x\,\frac{d\sigma_L}{dx} = \sigma_{born}\left[\frac{\alpha_s}{\pi} + \ldots\right]$$

gluon contributes in LO

$$\sigma_L + \sigma_T = \sigma_{tot}^{e^+e^-}$$

Nason, Webber
(detailed perturbat. + MC analysis)

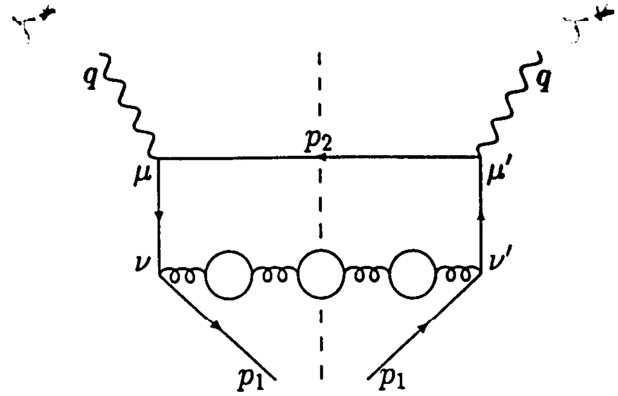
Leading twist formalism analogous to DIS.
Higher twist: Formal light-cone expansion

Balitsky, Braun

ϕ_+ right of cut
ϕ_- left of cut

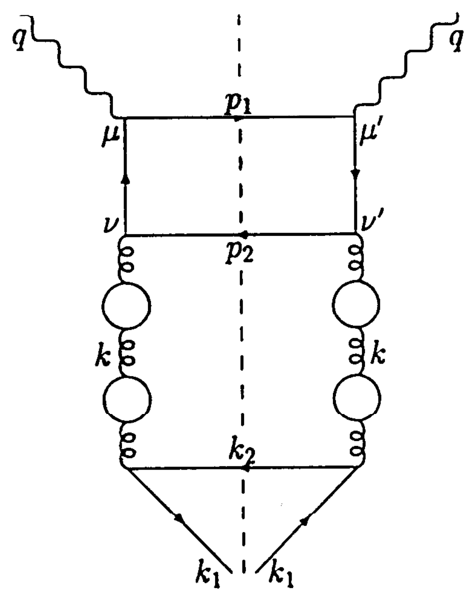
DIS : $\mathrm{Im}\, T(j^+_\mu(x) j^+_\nu(0))$ Fragmentation : $\mathrm{Im}\, T(j^+_\mu(x) j^-_\nu(0))$

$\Rightarrow$ Cutkosky rules in operator form ; suitable for external field techniques.
Twist-4 looks similar to DIS, except for doubling of fields + analyticity $\Rightarrow$ $1/Q^2$ only



$\gamma^* \longrightarrow q\bar{q} + g$

$\hookrightarrow q\bar{q}$



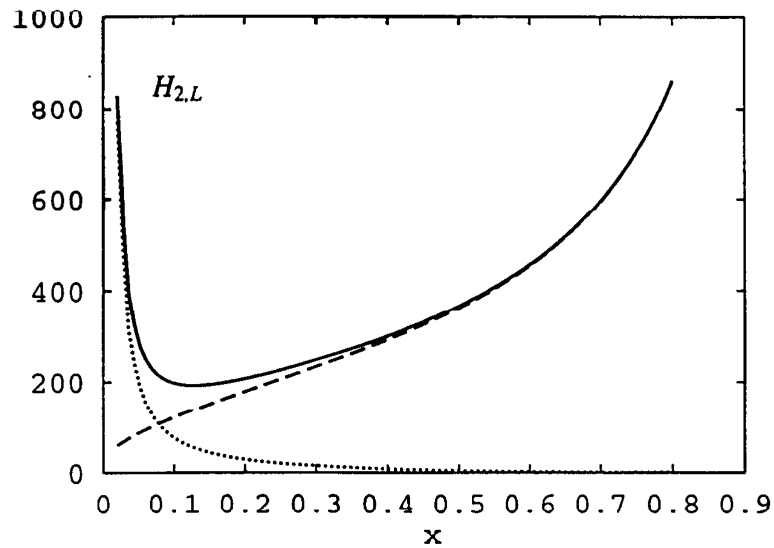


Figure 1: Shape of $1/Q^2$ power correction $H_{2,L}(x)$ to the longitudinal fragmentation cross section. Dashed line: primary quark contribution. Dotted Line: secondary quark contribution. Solid line: sum of both.

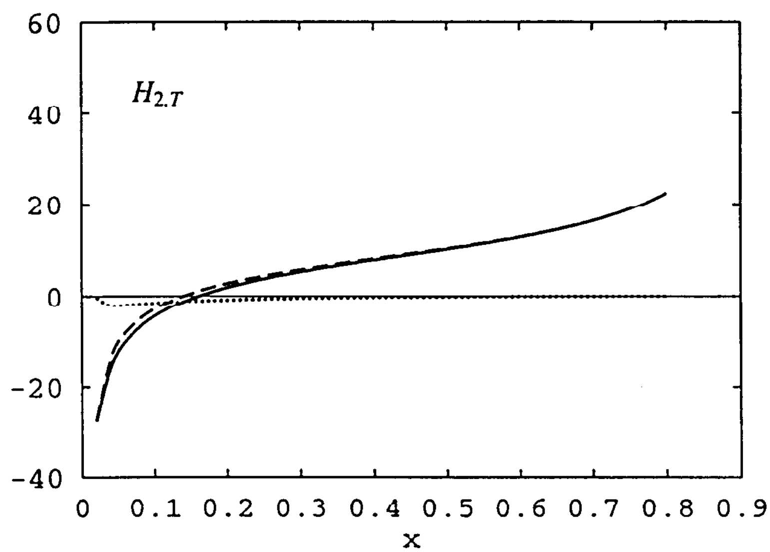


Figure 2: Shape of $1/Q^2$ power correction $H_{2,T}(x)$ to the transverse fragmentation cross section. Dashed line: primary quark contribution. Dotted Line: secondary quark contribution. Solid line: sum of both.

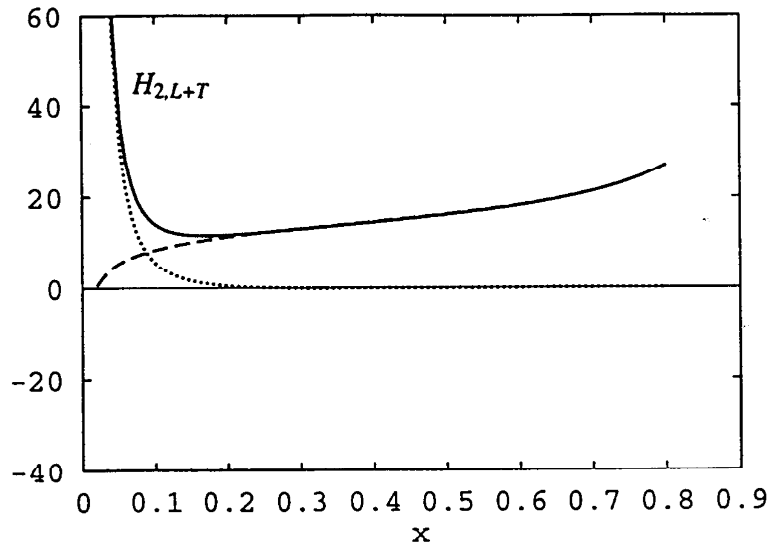


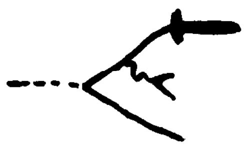
Figure 3: Shape of $1/Q^2$ power correction $H_{2,L+T}(x)$ to the sum of longitudinal and transverse fragmentation cross sections. Dashed line: primary quark contribution. Dotted Line: secondary quark contribution. Solid line: sum of both.

Renormalon Calculation: Power behaviour
x-dependence

First observable where different methods can be compared analytically!

For fixed x find only $1/Q^2$ $[+ 1/Q^4$ etc. ...$]$

Quark fragmentation

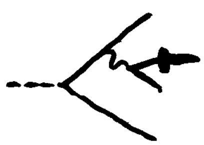


L $\quad \frac{4}{x} + 2\,\delta(1-x)$

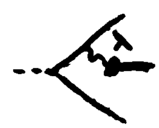
L+T $\quad \delta'(1-x) - \frac{4}{[1-x]_+} + 2$

coefficient of $1/Q^2$ correction

"Gluon" fragmentation



$* \; D_q$



$* \; D_g$

L $\quad \frac{6}{5x^3} + \frac{4}{x} - 6 + \frac{4}{5}x^2 + \frac{6\ln x}{x} \qquad\qquad \frac{8}{x^3} - \frac{8}{x}$

L+T $\quad \frac{6}{5x^3} - \frac{11}{x} + 9 + \frac{4}{5}x^2 - 6\ln x \qquad\qquad -\frac{8}{x} + 4 - 2\delta(1-x)$

* Longitudinal: x-dependence similar, but relative normalization to quark fragmentation different by a factor of 3
  $\rightarrow$ bad news for universality

* Both methods have artificial zeros:
  No $1/x^3$ in L+T with massive gluon, no $1/x^3$ in T with loops $\Rightarrow$ Generically see $\left(\frac{\Lambda}{Qx^2}\right)^n$ in L, T, L+T

σ_i and moments

$$\frac{d\sigma}{dx} = a_0(x) + a_1(x)\left(\frac{\Lambda}{Qx}\right)^2 + a_2(x)\left(\frac{\Lambda}{Qx}\right)^4 + \dots$$

consistent with Balitsky, Braun

Formal light-cone expansion is NOT an asymptotic expansion in the distribution sense

DIS: (integer) moments $\Rightarrow$ local operators always $1/Q^2$

Fragmentation:

Strong small-x (soft gluon) singularities which are <u>non-integrable</u>

$$\sigma_L \ni \int_{\Lambda/Q^2} dx \left(\frac{\Lambda^2}{Q^2x^2}\right)^n \sim \frac{\Lambda}{Q} \quad \text{for ALL } n$$

Large-x

$$\frac{1}{1-x}\left(\frac{\Lambda^2}{Q^2(1-x)}\right)^n \longrightarrow \frac{1}{[1-x]_{(n+1)+}} + \text{analytic} + \left(\frac{\Lambda^2}{Q^2}\right)^n \ln\frac{\Lambda^2}{Q^2}\,\delta^{(n)}(1-x)$$

Twist-expansion breaks down at large-x (DIS/Fragmentation) are the same in this respect, but the singularities are <u>integrable</u>

Total cross section

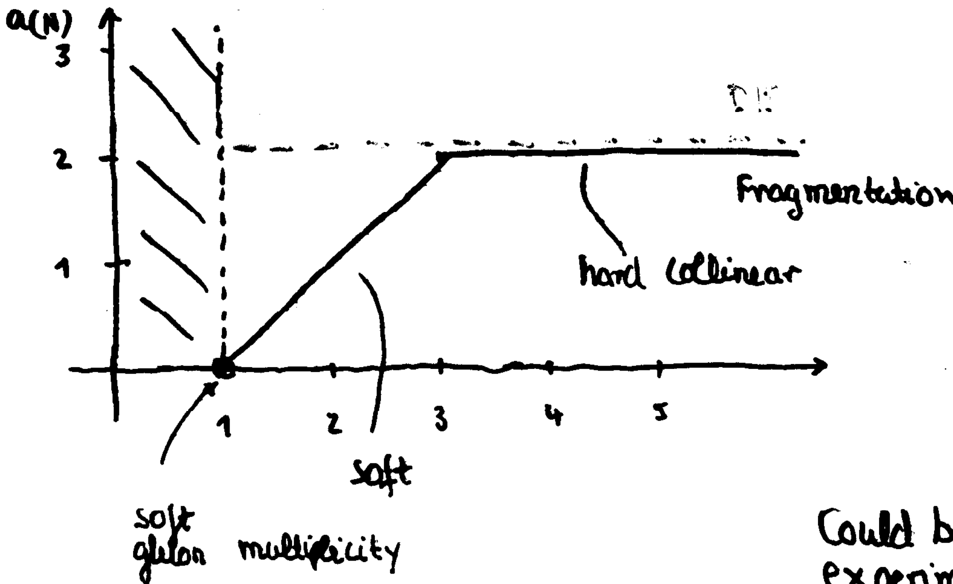
$$\sigma_{tot}\Big|_{1/Q} = 4\frac{\Lambda}{Q}\cdot\left\{2(1-\ln 2) - \sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2 n}\frac{2^{-2n}}{2n-1}\right\} = 0$$

from $a_n(x)\left(\frac{\Lambda^2}{Q^2}\right)^n$

Leading power corrections to moments

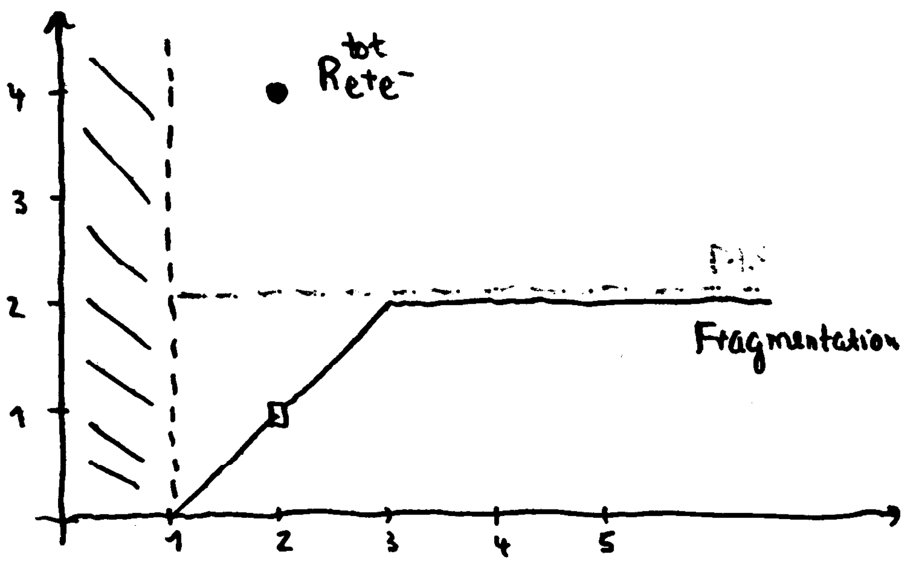
$$\sigma(N) \equiv \int_0^1 dx\, x^{N-1} \frac{d\sigma}{dx} = \text{leading power} + \left(\frac{\Lambda}{Q}\right)^{p(N)}$$

L



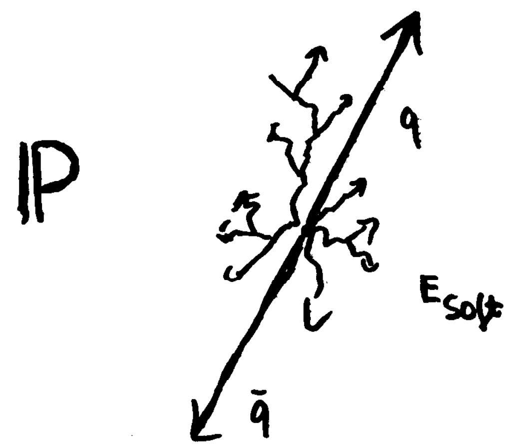
Could be checked experimentally for
$\langle (1-T)^{N-1} \rangle$
↑ thrust

L+T



Interpretation of $1/Q$ terms

MB, Braun, Ragnea
Sterman



$\Lambda_{QCD} \ll E_{soft} \ll Q$

$$F_{power} \sim \int_{|x_0| > \frac{Q}{\Lambda}} d^4x \; e^{iqx} \langle 0 | j(x) \, \mathbb{P} \, j(0) | 0 \rangle$$

Very non-local

Moments of $\frac{d\sigma}{dx}$

$$\mathbb{P} = \sum_X \left(\frac{2pq}{q^2} \right)^{N-1} |H(p)+X\rangle\langle H(p)+X|$$

Similar measures of MOMENTUM FLOW for event shapes in general

Universal in principle, but every observable takes only one particular weight on the distribution of soft momenta. Not a single number $\langle \mu_{had} \rangle$ [even in the two jet limit].

Phenomenology [Event Shapes , x-dep of twist-4 in DIS] is encouraging

BUT

there is some magic at work : Things work that needn't have to.

Theory : Open problems beyond one loop

- techniques for two loops
- all order arguments based on soft gluon cancellations
- identifying " operator structures " for power corrections