Renormalon Variety in DIS

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(Work in Progress)

DIS '97

Akhoury

1. Infrared renormalons + Power corrections
2. Renormalon Models for x-dependance of $1/Q^2$ Corrections: Critique of approaches
   (i) Gluon mass, Dispersive approach
   (ii) IR Sensitivity in the OPE language.
   (iii) Role of higher order Perturbative Corrections: Arguments of running Coupling.

Example of F_L $(x \to 1)$.

Precision data on hard processes in QCD ask for theoretical description to power-like accuracy

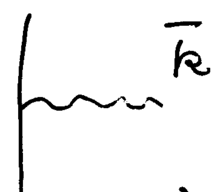
(GLS sum rule; event shapes, ..) $\ln Q + \frac{1}{Q^p}$

Can we improve perturbative predictions by including power like corrections?

Renormalons give indication of power corrections: overall coeff. undetermined.

Consider "improved" leading order prediction for some observable

$$\sigma \sim \int_0^Q d\bar{k}^2 \, \alpha_s(\bar{k}^2) \, F(\bar{k}^2, Q)$$

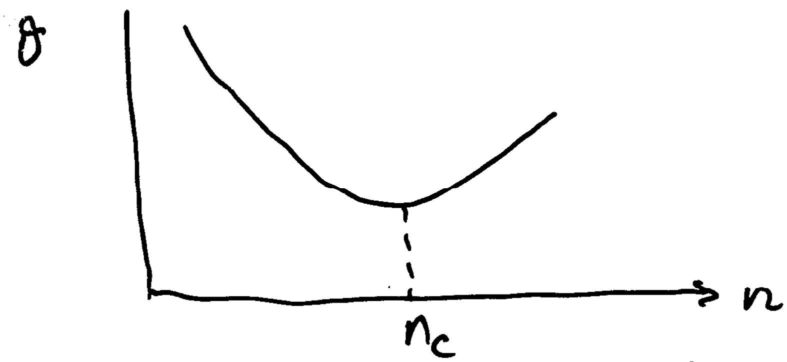


$$\alpha_s(\bar{k}^2) = \alpha_s(Q) + \frac{\beta_1}{p} \ln \frac{Q}{\bar{k}} \, \alpha_s(Q^2) + \cdots$$

$\bar{k}^2 \sim k^2, k_\perp^2, \ldots$

$$\sigma \sim \sigma_\ell \sum \left(\frac{\beta_1}{p}\right)^n \alpha_s^n \, n! \equiv \sum c_n \alpha_s^n$$

Perturbation theory at best asymptotic!



Perturbative calculations make sense only upto $n = n_c$. Effect of tail $\sim C_{n_c} \alpha_s^{n_c}$

$\sim \left(\frac{\Lambda^2}{Q^2}\right)^p$ for $n > n_c$

Terms with $n > n_c$ push us to mom. scales $k^2 \sim e^{-n} Q^2 < \Lambda^2$

remove such contributions from Coeff. functions and include $\left(\frac{\Lambda^2}{Q^2}\right)^p$ power corrections. Mueller.

For processes where OPE valid renormalons in agreement.
renormalons are models of matrix elements.
For situations where no OPE, renormalons only way to probe power corrections.
(Event shapes)
Another way to probe IR region is to give gluon mass λ & look for non-analytic in λ terms. ($\lambda^2 \ln \lambda, \ldots$)

Formalized in dispersive approach
(Marchesini's talk)

HIGHER TWIST ($1/Q^2$) in DIS

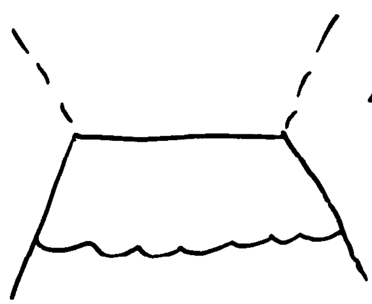
These can be addressed within the standard OPE. Renormalons in this case unify evaluation of Coeff. functions and of matrix elements.

Renormalons used to model the whole x-dependance of $1/Q^2$ correctains (Webber + Dasgupta; Stein, Meyer-Hermann, Mankiewicz, Schäfer)

How reasonable on theoretical ground.

$$F_i \sim \int_x^1 \frac{dz}{z}\, C(z, Q^2)\, q(x/z, Q^2)$$

$i = 2, L, 3$
(non-singlet)



Coeff. functions

$$F(x, \lambda^2) = -P(x) \ln \frac{\lambda^2}{Q^2} + C_0(x) - C_2(x) \frac{\lambda^2}{Q^2} \ln \frac{\lambda^2}{Q^2} + \cdots$$

$$C_{NP}(x) = C_2(x) \frac{A_2}{Q^2} \; ; \quad A_2 = \int_0^\infty \frac{d\lambda^2}{\lambda^2} \lambda^2 \ln \frac{\lambda^2}{\lambda_0^2}\, \delta\alpha_{eff}$$

F_2:

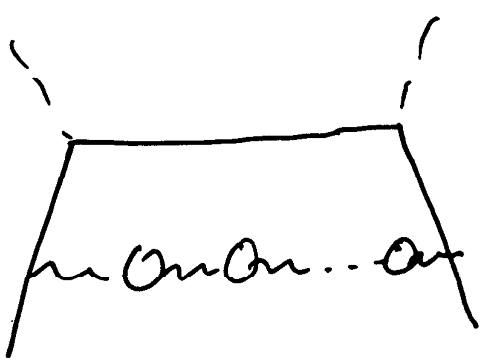
$$C_2(x) = -\frac{4}{(1-x)_+} + 2(2 + x + 6x^2) - 9\delta(1-x) - \delta'(1-x)$$

$$F(x,Q^2) = q(x,Q^2)\left(1 + \frac{D_2(x,Q^2)}{Q^2}\right)$$

$$D_2(x,Q^2) = \frac{A_2}{q(x,Q^2)}\int_x^1 \frac{dz}{z}\, C_2(z)\, q(x/z, Q^2)$$

$$C_2(x) = -\frac{4}{(1-x)_+} + \cdots$$

Shows characteristic $\frac{\Lambda^2}{Q^2(1-x)}$ behaviour of higher twist terms. Dominant in elastic limit ($x \to 1$). Above result equivalent to:



The Same $C_2(x)$ can obtained in a way closer to the OPE language: Actually we will obtain moments of $C_2(x)$:

Suppose we are interested in power corrections to F_3. First moment of F_3 relevant to GLS

For DIS, standard OPE for forward Compton amplitude:

$$\int d^4x \, \exp(iqx) \, T(J_\mu(x) J_\nu^\dagger(0)) = T_{\mu\nu}$$

$$\sum_{i,n} \left(\frac{2}{-q^2}\right)^n \Big(\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) q_{\mu_1} q_{\mu_2} C_L^i -$$

$$(g_{\mu\mu_1} g_{\nu\mu_2} q^2 - g_{\mu\mu_1} q_\nu q_{\mu_2} - g_{\nu\mu_2} q_\mu q_{\mu_1} + g_{\mu\nu} q_{\mu_1} q_{\mu_2}) C_2^i$$

$$- \varepsilon_{\mu\nu\mu_1\alpha} q_\alpha q_{\mu_2} C_3^i \Big) q_{\mu_3} \cdots q_{\mu_n} O^i_{\mu_1 \ldots \mu_n}$$

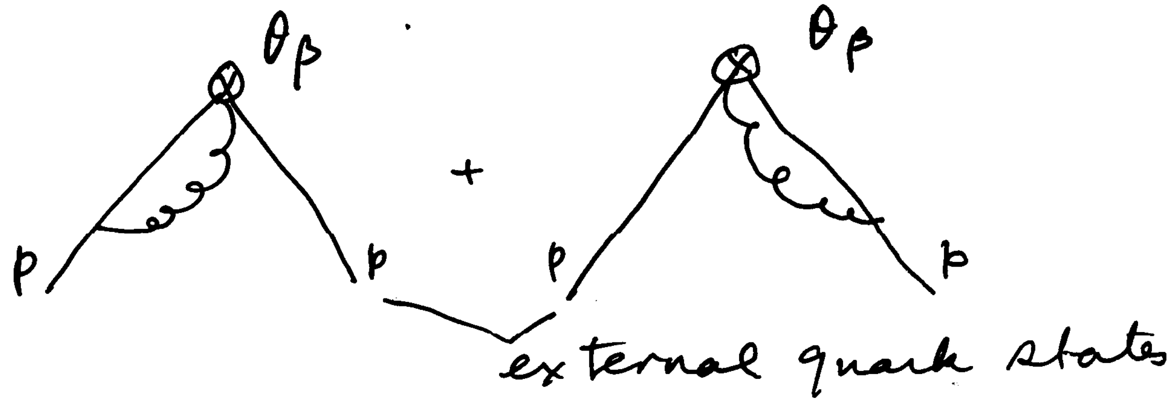
For the lowest moment of F_3, relevant operator (Shuryak + Vainshtein)

$$T^A_{\mu\nu} = \frac{2i}{q^2} \varepsilon_{\mu\nu\alpha\beta} q_\alpha \left(L_\beta + \frac{4}{q^2} \theta_\beta \right)$$

$$L_\beta = \bar{q} \gamma_\beta q$$

$$\theta_\alpha = g_s \bar{q} \tilde{G}^a_{\alpha\beta} t^a \gamma_\beta \gamma_5 q$$

Assume that matrix element of O_β containing gluon field can be reduced to that of L_β using perturbation theory: ie evaluate:



$$\langle p | T^A_{\mu\nu} | p \rangle \simeq \frac{2i}{q^2} \varepsilon_{\mu\nu\alpha\beta} q_\alpha \Big(\langle p | L_\beta | p \rangle + 2 \frac{\lambda^2}{q^2} \ln \frac{\lambda^2}{q^2} \; \frac{C_F}{2\pi} \frac{4\alpha_s}{3} \langle p | L_\beta | p \rangle \Big)$$

Same result as Coeff. function calculation with $\lambda^2 \neq 0$.
Similarly for higher moments.
Ambig in definition of higher twist op $\sim$ IR renormalon ambiguity.
However this also shows why the renormalons are at best a model for the x-dependance of higher twist structure functions:

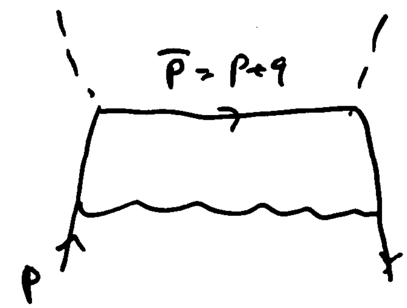
$$\gamma_L = 0 \qquad \text{and } \gamma_\theta = -\frac{\alpha_s}{4\pi}\left(\frac{32}{9}\right)$$

Thus above equality of two methods is at best true for particular Q^2. Indication of fact that relation is not IR safe. Different evolution ⇒ Power corrections governed by independant structure functions. (independant from leading twist)

Even viewed as a model, renormalon method for deducing x-dependance of $1/Q^2$ structure functions is difficult to justify using a quantitative approximation scheme.

Higher orders in perturbation theory could bring in unsuppressed contributions not present in leading order. Best seen in the example of F_L.

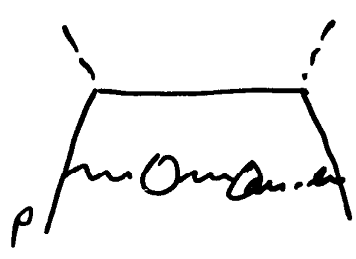
Coeff. fn for $F_L \sim \mathcal{O}(\alpha_s)$ from



In high orders, if either of quarks hit by virtual γ^* has no prior interaction $F_L \sim 0$. F_L is measure of struck constituent transverse momentum. F_L not log enhanced at leading order.

$$F_L(x) = C_F \frac{\alpha_s}{4\pi} \cdot 4x \qquad x = \frac{-q^2}{2p\cdot q}$$

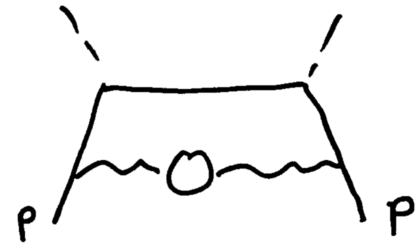
Renormalon Contribution to $1/Q^2$ can be obtained from



$$C_2^L(x) = 8x^2 - 4\delta(1-x)$$

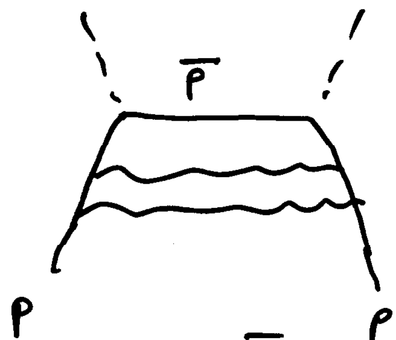
$$C_2(x) \frac{\lambda^2}{Q^2} \ln \frac{\lambda^2}{Q^2}$$

However at order α_s^2 in elastic limit $(x \to 1)$



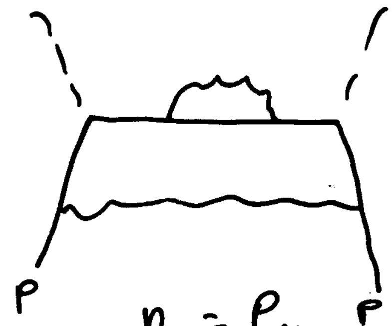
$\sim \alpha_s^2 \ln(1-x)$

Whereas, (depending on gauge)



$n_\mu = \bar{P}_\mu$

or



$n_\mu = P_\mu$

$\sim \alpha_s^2 \ln^2(1-x)$

(Devoto et al ; Sanchez - Guillen et. al)

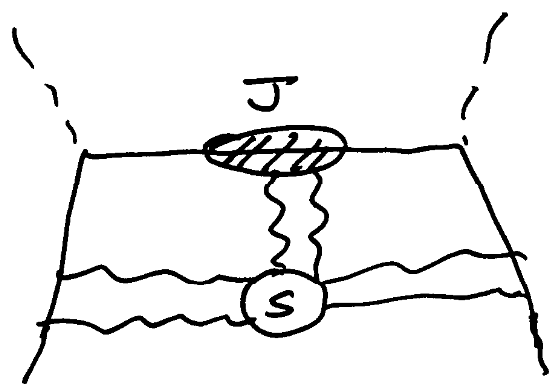
In fact the leading x dependance of F_L for large x can be determined from that of F_2 :

Using :

(1) Hadronic F_2, F_L determined in terms of single (non-singlet) quark distribution function

"universal"

(2) Factorization of "universal" Soft & collinear IR enhancements of Coeff. functions for $x \to 1$:



Sterman
Catani - Trentadue
⋮

we can write:

$$\hat{F}_L(x) = \frac{\alpha_s(Q^2)}{4\pi} C_F \int_x^1 \frac{dy}{y} \hat{F}_2(y, Q^2)\, 4\left(\frac{x}{y}\right)$$

$\uparrow$

$\alpha_s \left(\frac{\ln(1-y)}{1-y}\right)_+$

$\Downarrow$

$\alpha_s^2 \ln^2(1-x)$

$$+ \theta\left[\left(\frac{\alpha_s}{4\pi}\right)^2 \ln(1-x)\right] \int F_2$$

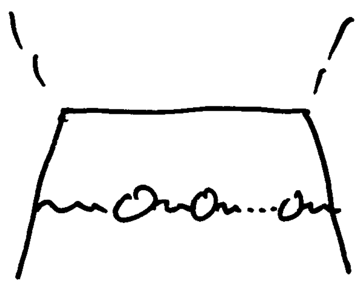
In addition, explicit $\theta(\alpha_s^2)$ calculation indicates that: (Devoto et. al. Sanchez-Guillen et. al)

$$\alpha_s = \alpha_s(Q^2(1-x))$$

L^{18}

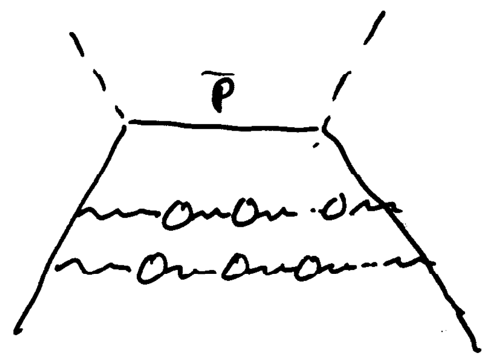
Since $F_2(x)\Big|_{1/Q^2} \sim \dfrac{\Lambda^2}{Q^2(1-x)}$

we see that at $x \to 1$



is not any more

dominant than



in $n_\mu = \bar{P}_\mu$ gauge

and so on ....

<u>Aside</u>: resummation of $\alpha_s^{n+1} \ln^{2n}(1-x)$ and $\alpha_s^{n+1} \ln^{2n-1}(1-x)$ at leading twist: in $\overline{MS}$ scheme.
(v Ritbergen, Sotiropoulos, Zakharov, RA)

Conclude with a Puzzle:

Can Compute x dependance of F_i using running of coupling and looking for renormalon poles in Borel plane.

As $x \to 1$, resummation of large IR corrections can be performed (for F_2)
(Sterman, Catani Trentadue, ...)
Explicit expressions at LL and NLL available

$$\ln F_N^{LL} = \frac{C_F}{\pi} \int_0^1 dz \, \frac{z^{N-1} - 1}{1-z} \int^{|Q^2|} \frac{dk_\perp^2}{k_\perp^2} \alpha_s(k_\perp^2) - \alpha_s(1-z) \ldots$$

one loop +

One can attempt to find $1/Q^2$ corrections from this, using:

$$\alpha_s(k_\perp^2) = \int_0^\infty d\sigma \left(\frac{k_\perp^2}{\Lambda^2}\right)^{-\sigma\beta_1}$$

and looking for poles in σ (Borel) plane.
However 2 differences with λ^2 method worth noting:

(1) For F_2 there are contributions like $\dfrac{\lambda^2/Q^2}{(1-x)^2}$ in evaluations of coeff. functions. We must be careful taking $\lambda^2/Q^2 \to 0$ limit and .

one defines "+ +" prescriptions:

$$\int_0^1 F(x)_{++} f(x)\, dx = \int_0^1 F(x)\left[f(x) - f(1) + (1-x) f'(1)\right] dx$$

Then we get non-analytic contributions from:

$$\frac{\lambda^2/Q^2}{(1-x)^2} \rightarrow \frac{\lambda^2/Q^2}{(1-x)^2_{++}} + \delta(1-x) + \underline{\frac{\lambda^2}{Q^2}\left(\ln\frac{\lambda^2}{Q^2} - \frac{\lambda^2}{Q^2}\right)} \times$$
$$\delta'(1-x) + \frac{1}{2}\frac{\lambda^2}{Q^2}\left(1 - \frac{\lambda^2}{Q^2}\right)\delta''(1-x)$$

In moments this gives contributions prop. to N. Finite Number of such terms. (Difference with fragmentation function: Beneke, Braun, Magnea.

No such contribution exists in approaches with $\lambda^2 = 0$.

Thus they will differ at least by such terms at λ^2/Q^2 level.

Perhaps they can be reinstated by enforcing Adler-Sum rule which may be violated in $\alpha_s(k_\perp^2)$ approach.

(Y. Dokshitzer)

2) After integration z_L of struck quark: $\lambda^2 \neq 0$ result reproduced only for $\alpha_s\left(\frac{k_\perp^2}{1-x}\right)$ for F_L.

Related to Problem(1)? $F_2: \sim N \ln N$ if $\alpha_s(k_\perp^2)$