STATUS OF LATTICE STRUCTURE FUNCTION CALCULATIONS

G. Schierholz

With

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C. Best, A. Schäfer, S. Schramm

Aoki et al. JLQCD

Liu et al.

Negele et al.

Capitani & Rossi

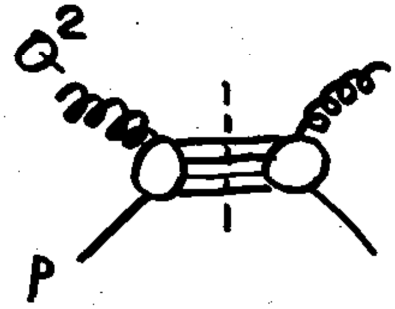
Theoretical framework

Lattice calculation

Operators & renormalization

Results, results, ...

Summary & outlook



$$Q^2 = -q^2$$

$$W_{\mu\nu} = i \int d^4y \, e^{iqy} \langle p, \vec{s} \,|\, [j_\mu(y), j_\nu(0)] \,|\, p, \vec{s} \rangle$$

$$= \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1 + \left(p_\mu - \frac{pq}{q^2} q_\mu\right)\left(p_\nu - \frac{pq}{q^2} q_\nu\right) F_2$$

$$+ i\epsilon_{\mu\nu\rho\sigma} q^\rho s^\sigma \frac{1}{pq} g_1 + i\epsilon_{\mu\nu\rho\sigma} q^\rho (pq\, s^\sigma - sq\, p^\sigma) \frac{1}{(pq)^2} g_2$$

$$- r_{\mu\nu}\, b_1 + \tfrac{1}{6}(s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu})\, b_2$$

$$+ \tfrac{1}{2}(s_{\mu\nu} - u_{\mu\nu})\, b_3 + \tfrac{1}{2}(s_{\mu\nu} - t_{\mu\nu})\, b_4$$

b_3, b_4 twist-4

$$S^\sigma = -\frac{i}{m^2}\,\epsilon^{\sigma\alpha\beta\gamma}\epsilon^*_\alpha \epsilon_\beta p_\gamma \quad ; \quad \epsilon p = 0\,, \quad \epsilon^2 = -m^2$$

$$r_{\mu\nu} = \frac{1}{(pq)^2}\left[\epsilon^* q\, \epsilon q - \frac{1}{3}\left((pq)^2 - m^2 q^2\right)\right] g_{\mu\nu}$$

$$s_{\mu\nu} = \frac{2}{(pq)^2}\left[\epsilon^* q\, \epsilon q - \frac{1}{3}\left((pq)^2 - m^2 q^2\right)\right] \frac{p_\mu p_\nu}{(pq)^2}$$

$$t_{\mu\nu} = \frac{1}{2(pq)^2}\Big[\epsilon^* q\,(p_\mu \epsilon_\nu + p_\nu \epsilon_\mu) + \epsilon q\,(p_\mu \epsilon^*_\nu + p_\nu \epsilon^*_\mu) - \frac{4}{3}(pq)^2 p_\mu p_\nu\Big]$$

$$u_{\mu\nu} = \frac{1}{(pq)^2}\left[\epsilon^*_\mu \epsilon_\nu + \epsilon_\mu \epsilon^*_\nu + \frac{2}{3} m^2 \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}\right)\right]$$

Hoodbhoy, Jaffe,
Manohar

Nucleon

$$q_{\uparrow}^{\frac{1}{2}}(x,Q^2) = q_{\downarrow}^{-\frac{1}{2}}(x,Q^2) \equiv q_{\uparrow}(x,Q^2)$$

$$q_{\downarrow}^{\frac{1}{2}}(x,Q^2) = q_{\uparrow}^{-\frac{1}{2}}(x,Q^2) \equiv q_{\downarrow}(x,Q^2)$$

Rho

$$q_{\uparrow}^{1}(x,Q^2) = q_{\downarrow}^{-1}(x,Q^2)$$

$$q_{\downarrow}^{1}(x,Q^2) = q_{\uparrow}^{-1}(x,Q^2)$$

$$q_{\uparrow}^{0}(x,Q^2) = q_{\downarrow}^{0}(x,Q^2)$$

OPE only quarks, leading twist

Spin independent:

$$2\int_0^1 dx\, x^{n-1} F_1(x,Q^2) = \sum_f C_{1,n}^{(f)}\overbrace{(\mu^2/Q^2, g(\mu))\; v_n^{(f)}(\mu)}^{\text{matching}} \qquad n \geq 2$$

$$\int_0^1 dx\, x^{n-2} F_2(x,Q^2) = \sum_f C_{2,n}^{(f)}(\mu^2/Q^2, g(\mu))\; v_n^{(f)}(\mu)$$

$$\frac{1}{2}\sum_{\vec{S}} \langle \vec{P},\vec{S}|\, \mathcal{O}^{(f)}_{\{\mu_1\cdots\mu_n\}}\, |\vec{P},\vec{S}\rangle = 2\, v_n^{(f)}\, [P_{\mu_1}\cdots P_{\mu_n} - \text{traces}]$$

$$\mathcal{O}^{(f)}_{\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{\psi}_f\, \gamma_{\mu_1} D_{\mu_2}\cdots D_{\mu_n}\psi_f \;-\text{traces}$$

$$v_n^{(f)} \equiv \langle x^{n-1}\rangle^{(f)} \doteq \int dx\, x^{n-1} q(x,Q^2)$$

$$= \int dx\, x^{n-1}\left(q_\uparrow(x,Q^2) + q_\downarrow(x,Q^2)\right)$$

Spin dependent:

$$2\int_0^1 dx\, x^n g_1(x,Q^2) = \frac{1}{2} \sum_f e_{1,n}^{(f)}(\mu^2/Q^2, g(\mu))\, a_n^{(f)}(\mu) \quad n \geq 0,\ \text{even}$$

$$2\int_0^1 dx\, x^n g_2(x,Q^2) = \frac{1}{2}\frac{n}{n+1} \sum_f e_{2,n}^{(f)}(\mu^2/Q^2, g(\mu))\left[d_n^{(f)}(\mu) - a_n^{(f)}(\mu)\right] \quad n \geq 2,\ \text{even}$$

$$\langle \vec{P},\vec{S} | O^{5(f)}_{\{\sigma\mu_1\cdots\mu_n\}} | \vec{P},\vec{S}\rangle = \frac{1}{n+1}\, a_n^{(f)} \left[S_\sigma P_{\mu_1}\cdots P_{\mu_n} + \cdots - \text{traces}\right]$$

$$\langle \vec{P},\vec{S} | O^{5(f)}_{[\sigma\{\mu_1]\cdots\mu_n\}} | \vec{P},\vec{S}\rangle = \frac{1}{n+1}\, d_n^{(f)} \left[(S_\sigma P_{\mu_1} - S_{\mu_1} P_\sigma) P_{\mu_2}\cdots P_{\mu_n} + \cdots - \text{traces}\right]$$

twist 3

$$O^{5(f)}_{\sigma\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^n \bar{\psi}_f \gamma_\sigma \gamma_5 D_{\mu_1}\cdots D_{\mu_n} \psi_f \ - \text{traces}$$

$$a_0^{(u)} \equiv 2\Delta u, \quad a_0^{(d)} \equiv 2\Delta d$$

$$a_n^{(q)} = 2\int dx\, x^n (q_\uparrow(x,Q^2) - q_\downarrow(x,Q^2))$$

Rho

$$\langle \vec{p},\vec{s} | \mathcal{O}^{(f)}_{\{\mu_1\cdots\mu_n\}} | \vec{p},\vec{s}\rangle = 2\Big[\underline{a_n}^{(f)} p_{\mu_1}\cdots p_{\mu_n}$$

$$+ \frac{1}{2n-3}\underline{d_n}^{(f)}(\epsilon^*_{\mu_1}\epsilon_{\mu_2} - \tfrac{1}{3}p_{\mu_1}p_{\mu_2})p_{\mu_3}\cdots p_{\mu_n} + \cdots$$

$$- \text{traces}\Big]$$

$$\langle \vec{p},\vec{s} | \mathcal{O}^{5(f)}_{\{\mu_1\cdots\mu_n\}} | \vec{p},\vec{s}\rangle = \frac{1}{n}\underline{r_n}^{(f)}\left[s_{\mu_1}p_{\mu_2}\cdots p_{\mu_n} + \cdots - \text{traces}\right]$$

$$\underline{a_n} \leftarrow v_n$$

$$\underline{d_n} \leftarrow \text{new}$$

$$\underline{r_n} \leftarrow a_{n-1}$$

$$2\int_0^1 dx\, x^{n-1} b_1(x,Q^2) = \sum_f h_{1,n}^{(f)}(\mu^2/Q^2, g(\mu))\ \underline{d_n^{(f)}(\mu)}$$

$$\int_0^1 dx\ x^{n-2} b_2(x,Q^2) = \sum_f h_{2,n}^{(f)}(\mu^2/Q^2, g(\mu))\ \underline{d_n^{(f)}(\mu)}$$

$$b_1(x,Q^2) = q^0(x) - q^1(x)$$

$$q^m(x) = \frac{1}{2}\left(q^m_\uparrow(x) + q^m_\downarrow(x)\right)$$

Gluons, higher twist

$$\mathcal{O}^{(g)}_{\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-2} \mathrm{Tr}\, F_{\alpha\mu_1} D_{\mu_2} \cdots D_{\mu_{n-1}} F_{\mu_n\alpha} \ - \text{traces}$$

$$\mathcal{O}^{5(g)}_{\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-2} \mathrm{Tr}\, F_{\alpha\mu_1} D_{\mu_2} \cdots D_{\mu_{n-1}} \tilde{F}_{\mu_n\alpha} \ - \text{traces}$$

$$\mathcal{O}^{(ff')}_{\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-2} \bar{\psi}_f \gamma_{\mu_1} D_{\mu_2} \cdots D_{\mu_{k-1}} \psi_f \, \bar{\psi}_{f'} \gamma_{\mu_k} D_{\mu_{k+1}} \cdots D_{\mu_n} \psi_{f'} \ - \text{traces}$$

$$\mathcal{O}^{(fg)}_{\mu_1\cdots\mu_n} = \left(\frac{i}{2}\right)^{n-1} \bar{\psi}_f \gamma_\alpha D_{\mu_1} \cdots D_{\mu_{n-1}} F_{\mu_n\alpha} \psi_f \ - \text{traces}$$

$\vdots$

mixing, renormalons

Lattice calculation

$$S_G = \beta \sum_{\substack{x \\ \mu<\nu}} \left(1 - \frac{1}{3}\,\mathrm{Re\,Tr}\, U_{\mu\nu}\right), \quad U_{\mu\nu} = e^{igF_{\mu\nu}}$$

$\beta = \frac{6}{g^2}$ $\qquad\qquad O(a^2)$ corrections

Wilson fermions

$$S_F = \sum_x \Big\{ \varkappa \sum_\mu \big[\bar\psi(x)(\gamma_\mu - 1) U_\mu(x) \psi(x+\hat\mu) + \bar\psi(x)(\gamma_\mu + 1) U_\mu^\dagger(x-\hat\mu)\psi(x-\hat\mu) \big] + \bar\psi(x)\psi(x) \Big\}$$

$$m_q a \approx 0.5 \left(\frac{1}{\varkappa} - \frac{1}{\varkappa_c}\right) \qquad O(a) \text{ corrections}$$

Quenched $\qquad \beta = 6.0, \quad \frac{\pi}{a} \approx 7\,\mathrm{GeV}$

	$16^3 32$			$24^3 32$			
$\varkappa$	0.1515	0.153	0.155	0.155	0.1558	0.1563	
m_q	190	130	70	70	40	25	MeV

chiral limit $\Rightarrow$

Improvement

Remove $O(a)$ cut-off effects by

in on-shell quantities

$$S_F \to S_F - \frac{i}{2} \varkappa g c_{SW} \sum_x \underbrace{\bar{\psi} \sigma_{\mu\nu} F_{\mu\nu} \psi}_{dim=5}$$

Sheikholeslami, Wohlert
Lüscher, Weisz, ...

$$O \to O + \Delta O$$

$\beta = 6.0$; $\beta = 6.2$, $\frac{\pi}{a} \approx 9\,\mathrm{GeV}$

$24^3 48$

Evolution

Correlation functions

$$B_\alpha(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\vec{x}} \epsilon_{abc} u^a_\alpha(x) [u^b(x) C\gamma_5 d^c(x)]$$

neutron: $u \leftrightarrow d$

$$C_\Gamma(t,\vec{p}) = \Gamma_{\beta\alpha} \langle B_\alpha(t,\vec{p}) \bar{B}_\beta(0,\vec{p}) \rangle \quad \propto e^{-E_{\vec{p}} t}$$

$$C_\Gamma(t,\tau,\vec{p}) = \Gamma_{\beta\alpha} \langle B_\alpha(t,\vec{p})\, \mathcal{O}(\tau)\, \bar{B}_\beta(0,\vec{p}) \rangle$$

$\Gamma = \frac{1}{2}(1+\gamma_4)$ unpolarized

$\Gamma = \frac{1}{2}(1+\gamma_4)\, i\gamma_5\gamma_2$ polarized in z-dir.

$$R(t,\tau,\vec{p}) = C_\Gamma(t,\tau,\vec{p}) / C_{\frac{1}{2}(1+\gamma_4)}(t,\vec{p})$$

$$= \frac{1}{2\kappa} \frac{E_{\vec{p}}}{E_{\vec{p}} + m_N} \frac{1}{4} \mathrm{Tr}[\Gamma N \hat{\mathcal{O}} N]$$

$$\langle \vec{p},\vec{s} | \mathcal{O} | \vec{p},\vec{s} \rangle = \bar{u}(\vec{p},s)\, \hat{\mathcal{O}}\, u(\vec{p},s), \quad N = \frac{\gamma_4 E_{\vec{p}} - i\vec{\gamma}\vec{p} + m_N}{E_{\vec{p}}}$$

picture: fig10074.ps

Wed Jan 4 15:54:57 MET 1995

fort.21

nucleon mom:(0 ,0 ,0) momq:(0 ,0 ,0) 1 1 nonrel:1

beta = 6.000 kappa = 0.1550

50 0.2100 50 0.2100

normalisation factor = -0.219655E+18

gam[0,0] = (1.000,0.000)
gam[1,1] = (1.000,0.000)

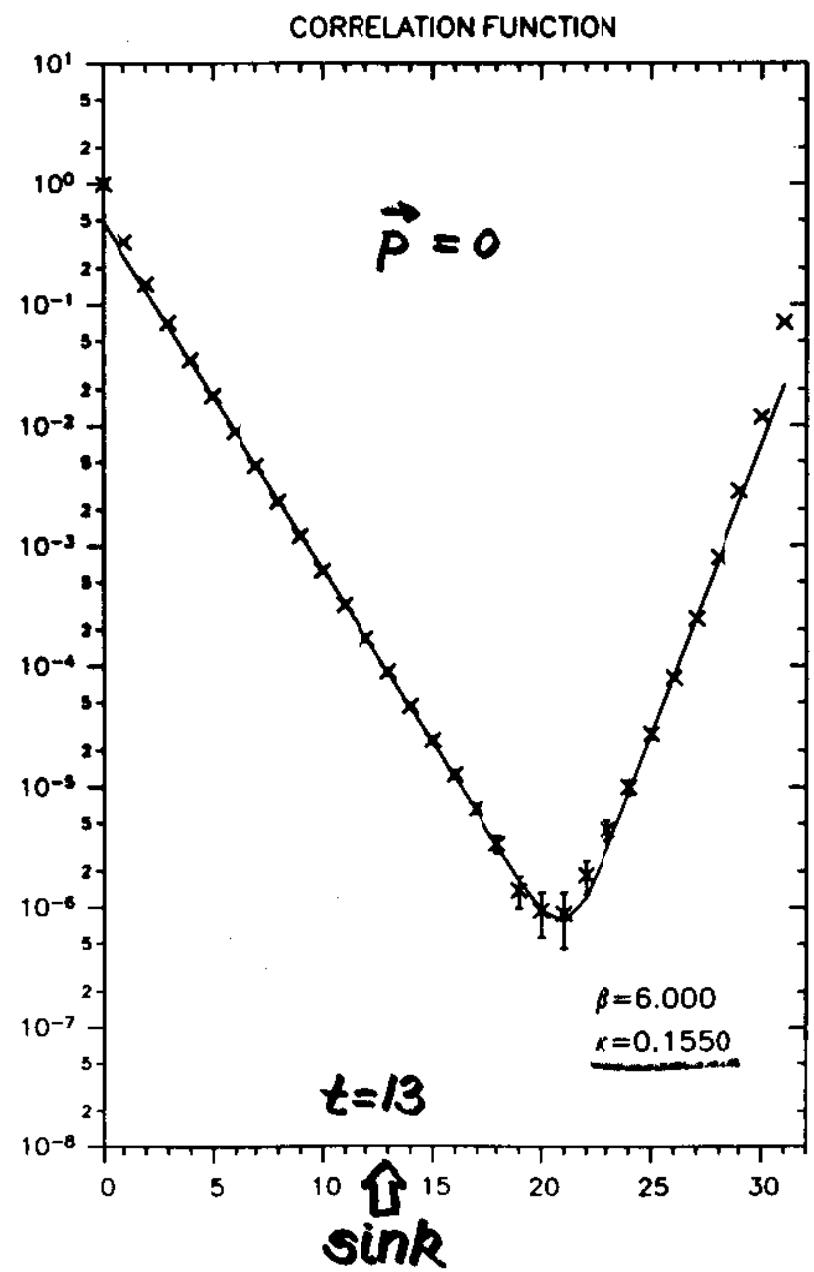
0A 32 810 0 RE

[4,28] 0.122767E+02
0.242217E+00+- 0.606881E-02 0.662863E+00+- 0.294227E-02
0.329852E-01+- 0.351665E-02 0.111261E+01+- 0.211112E-01

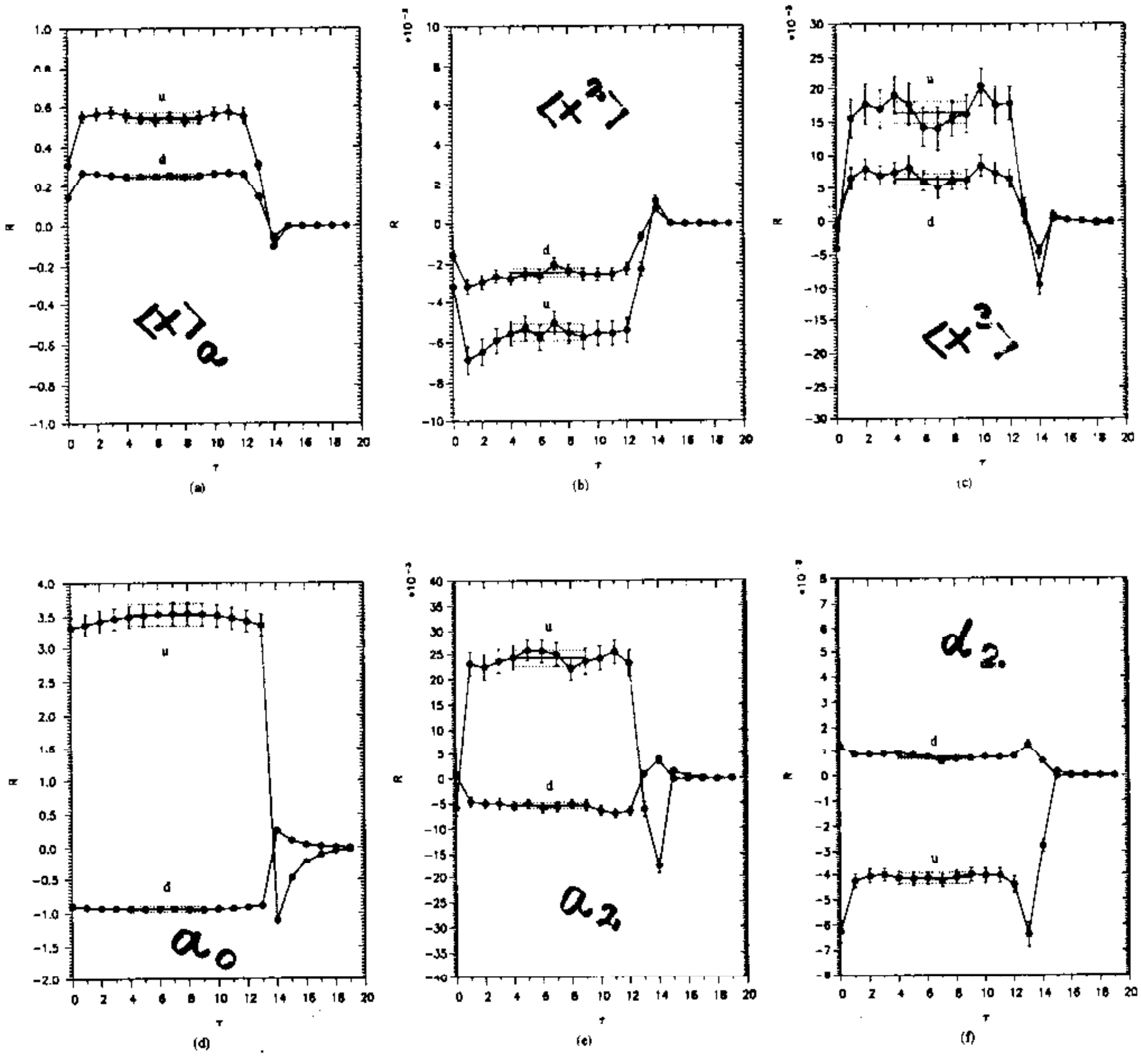
$$\psi \to \psi^s = \sum_{n=0}^{N_s} (\varkappa_s \vec{D})^n \psi$$

$N_s = 50, \varkappa_s = 0.21$

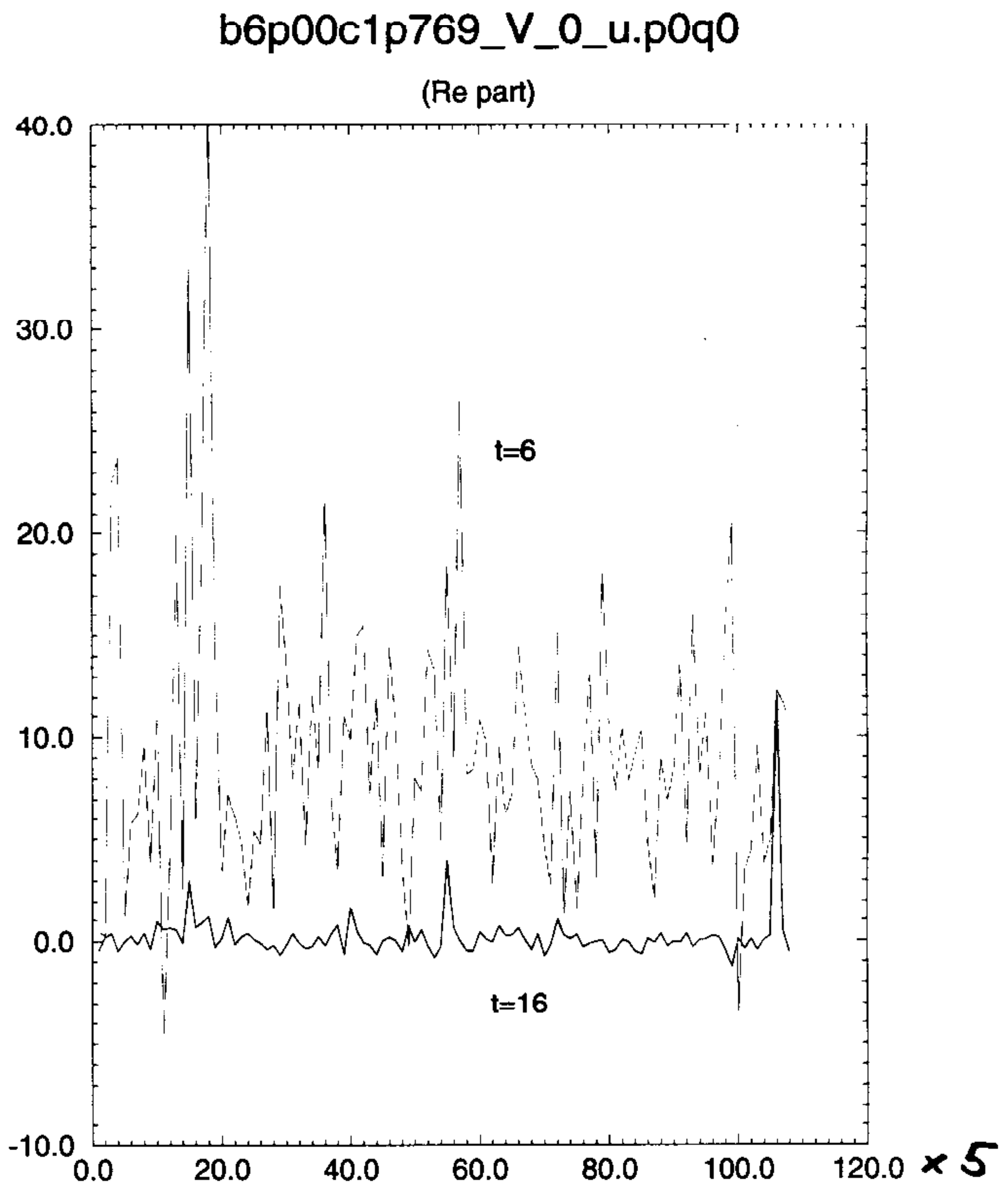
source and sink



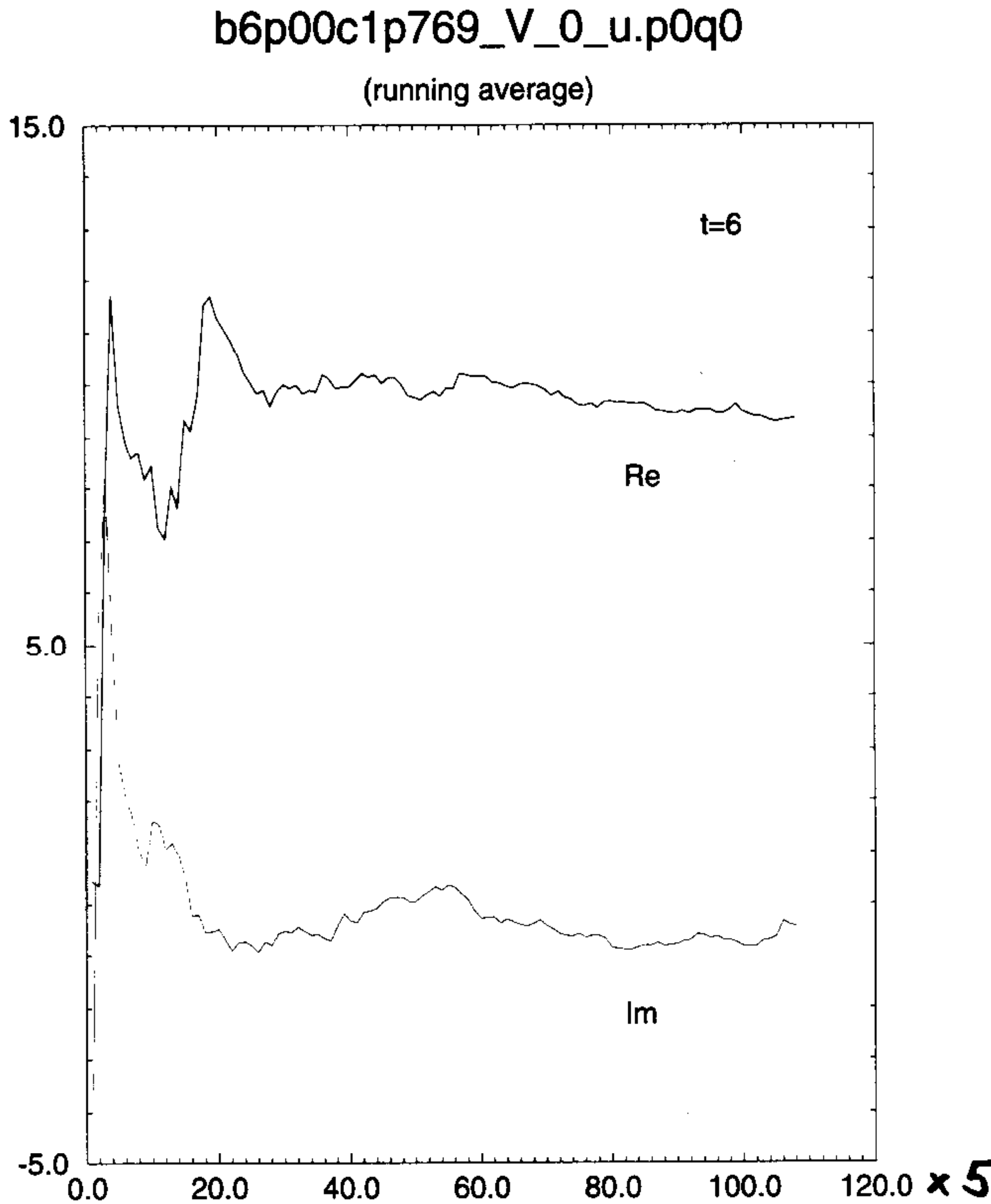
Raw data $\kappa = 0.153$



twist 3



Tue Apr 8 20:05:30 1997



Wed Apr 9 18:18:08 1997

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picture: fig1596C.ps
Tue Nov 14 16:08:48 MET 1995
fort.21
beta = 6.000 kappa = 0.1515
x1b->g   mom:(0 ,0 ,0 )    momq:(0 ,0 ,0 )  1 1  nonrel:1
 50 0.2100  50 0.2100  l+gam:1 mon-t
 C2pt(13) = -0.136713E+13
 <q> = (-0.5072E-01+- 0.7523E-01,-0.3115E-08+- 0.1116E-06)
gam[0,0] = ( 1.00, 0.00)
gam[1,1] = ( 1.00, 0.00)
DA 32   2834   0  RE  MCO
[ 8,15]  0.252305E+01
   -0.428837E+00+- 0.103049E+00  0.100000E+01+- 0.141421E+00
    0.000000E+00+- 0.000000E+00  0.000000E+00+- 0.000000E+00
[ 8,16]  0.260058E+01
   -0.417196E+00+- 0.941861E-01  0.100000E+01+- 0.141421E+00
    0.000000E+00+- 0.000000E+00  0.000000E+00+- 0.000000E+00
[ 8,17]  0.279793E+01
   -0.401779E+00+- 0.875596E-01  0.100000E+01+- 0.141421E+00
    0.000000E+00+- 0.000000E+00  0.000000E+00+- 0.000000E+00
[ 8,18]  0.347310E+01
   -0.377287E+00+- 0.823303E-01  0.100000E+01+- 0.141421E+00
    0.000000E+00+- 0.000000E+00  0.000000E+00+- 0.000000E+00
[ 9,15]  0.140791E+01
   -0.450263E+00+- 0.105028E+00  0.100000E+01+- 0.141421E+00
    0.000000E+00+- 0.000000E+00  0.000000E+00+- 0.000000E+00
[ 9,16]  0.153868E+01
   -0.434607E+00+- 0.956895E-01  0.100000E+01+- 0.141421E+00
    0.000000E+00+- 0.000000E+00  0.000000E+00+- 0.000000E+00
[ 9,17]  0.180006E+01
   -0.416333E+00+- 0.887636E-01  0.100000E+01+- 0.141421E+00
    0.000000E+00+- 0.000000E+00  0.000000E+00+- 0.000000E+00
[ 9,18]  0.256896E+01
   -0.389517E+00+- 0.833289E-01  0.100000E+01+- 0.141421E+00
    0.000000E+00+- 0.000000E+00  0.000000E+00+- 0.000000E+00
[10,15]  0.130336E+01
   -0.459435E+00+- 0.108792E+00  0.100000E+01+- 0.141421E+00
    0.000000E+00+- 0.000000E+00  0.000000E+00+- 0.000000E+00
[10,16]  0.145952E+01
   -0.441191E+00+- 0.985098E-01  0.100000E+01+- 0.141421E+00
    0.000000E+00+- 0.000000E+00  0.000000E+00+- 0.000000E+00
[10,17]  0.174546E+01
   -0.421019E+00+- 0.910008E-01  0.100000E+01+- 0.141421E+00
    0.000000E+00+- 0.000000E+00  0.000000E+00+- 0.000000E+00
[10,18]  0.254177E+01
   -0.392423E+00+- 0.851714E-01  0.100000E+01+- 0.141421E+00
    0.000000E+00+- 0.000000E+00  0.000000E+00+- 0.000000E+00
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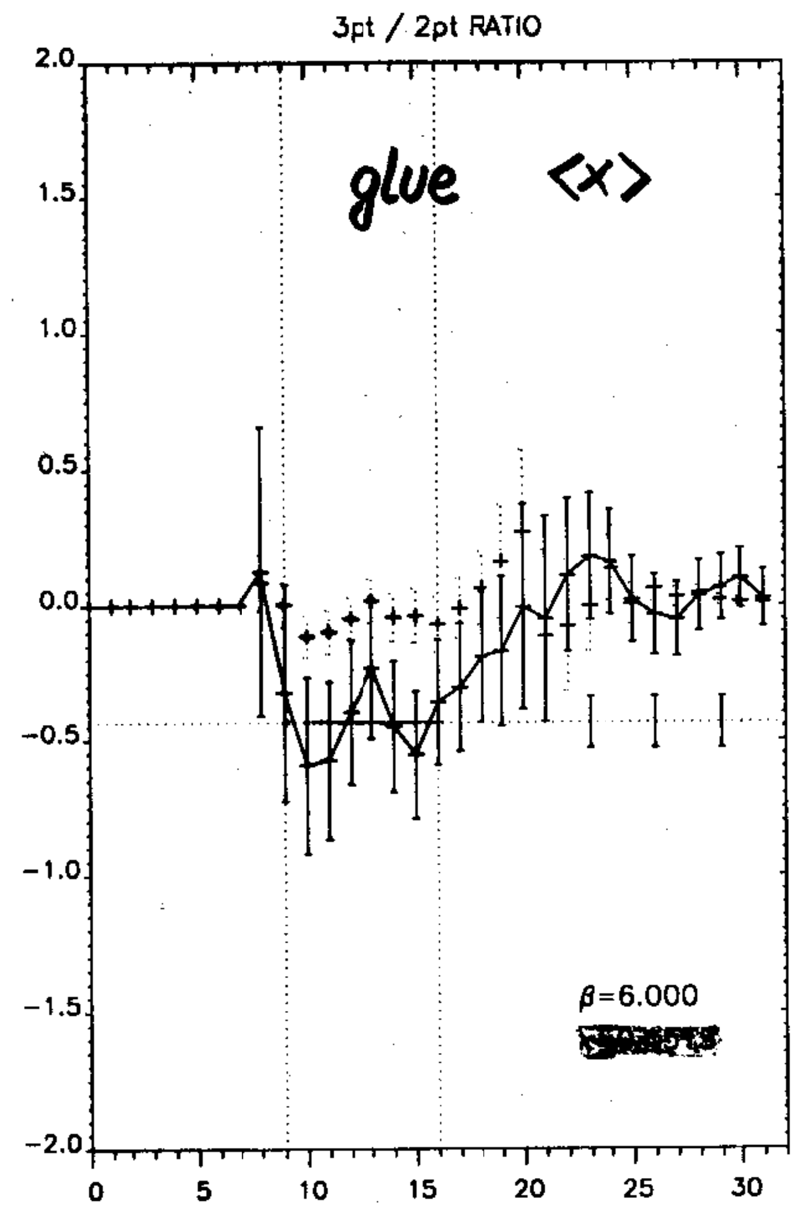


Rho

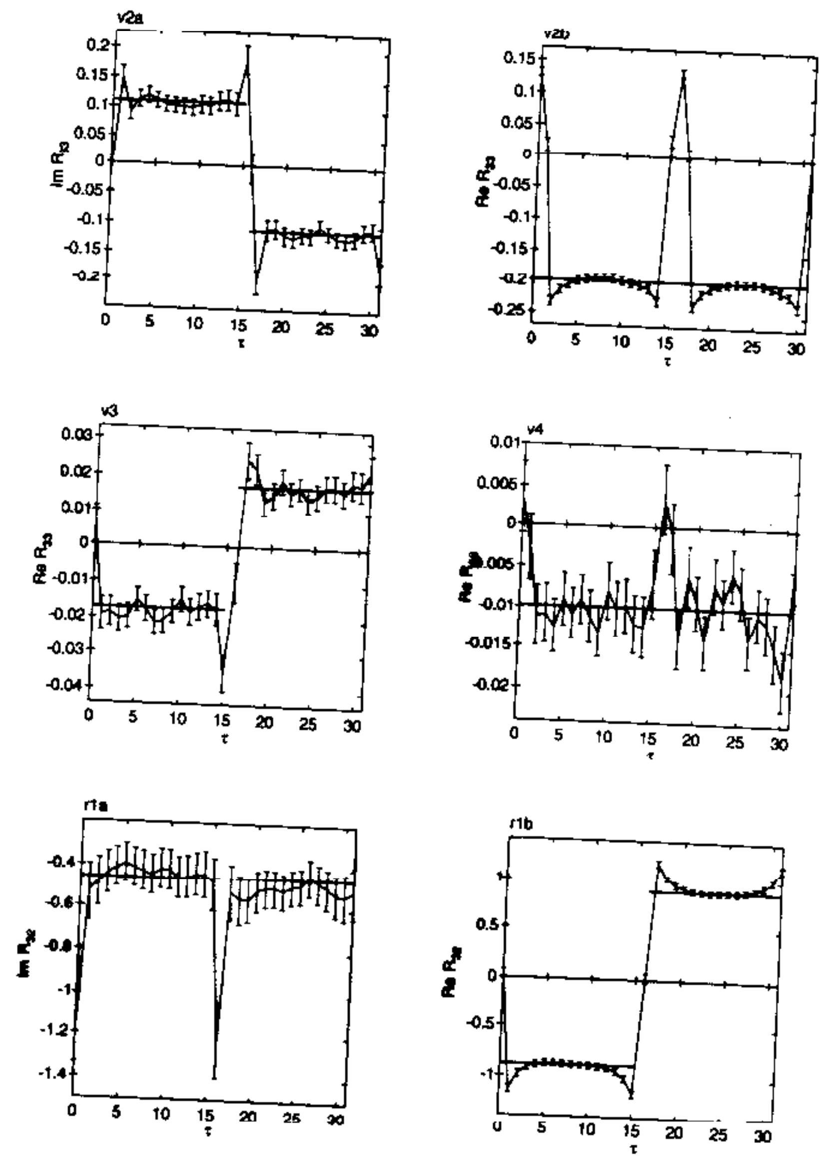


Figure 2: The ratios R_{33} and R_{32} for the rho operators $\hat{O}_{v2a}$, $\hat{O}_{v2b}$, $\hat{O}_{v3}$, $\hat{O}_{v4}$, $\hat{O}_{r1a}$, and $\hat{O}_{r1b}$ (left to right, top to bottom) at $\kappa = 0.153$.

⇨ O(4) ⇨ H(4)

$\langle \mathcal{O} \rangle$	Components	Representation		
		Ref. [13]	Ref. [14]	C
$v_{2,a}$	$\mathcal{O}_{\{14\}}$	$\tau_3^{(6)}$	$6^{(+)}$	+
$v_{2,b}$	$\mathcal{O}_{\{44\}} - \frac{1}{3}(\mathcal{O}_{\{11\}} + \mathcal{O}_{\{22\}} + \mathcal{O}_{\{33\}})$	$\tau_1^{(3)}$	$\ydiagram{3,1}^{(+)}$	+
v_3	$\mathcal{O}_{\{114\}} - \frac{1}{2}(\mathcal{O}_{\{224\}} + \mathcal{O}_{\{334\}})$	$\tau_1^{(8)}$	$8^{(+)}$	−
v_4	$\mathcal{O}_{\{1144\}} + \mathcal{O}_{\{2233\}} - \mathcal{O}_{\{1133\}} - \mathcal{O}_{\{2244\}}$	$\tau_1^{(2)}$	$\ydiagram{2,2}^{(+)}$	+
a_0	$\mathcal{O}_2^5$	$\tau_4^{(4)}$	$(\frac{1}{2},\frac{1}{2})^{(-)}$	+
a_2	$\mathcal{O}^5_{\{214\}}$	$\tau_3^{(4)}$	$(\overline{\frac{1}{2},\frac{1}{2}})^{(+)}$	+
d_2	$\mathcal{O}^5_{[2\{1]4\}}$	$\tau_1^{(8)}$	$8^{(+)}$	+

Table 2: The lattice operators and their representation. The momentum is taken to be $\vec{p} = (2\pi/16, 0, 0) = (p_1, 0, 0)$ in the case of $v_{2,a}, v_3, v_4, a_2, d_2$ and $\vec{p} = 0$ elsewhere.

3 Lattice Operators and their Renormalization

The bare lattice operators, $\mathcal{O}(a)$, are in general divergent. We define finite operators $\mathcal{O}(\mu)$, renormalized at the scale μ, by

$$\boxed{\mathcal{O}(\mu) = Z_{\mathcal{O}}((a\mu)^2, g(a))\mathcal{O}(a),} \tag{19}$$

where

$$\langle q(p)|\mathcal{O}(\mu)|q(p)\rangle = \langle q(p)|\mathcal{O}(a)|q(p)\rangle\,|^{tree}_{p^2=\mu^2} \tag{20}$$

with $|q(p)\rangle$ being a quark state of momentum p. In the limit $a \to 0$ this definition amounts to the continuum, momentum subtraction renormalization scheme.

The lattice operators transform under the discrete hypercubic group $H(4)$ [13, 14]. They must be constructed such that they belong to a definite irreducible representation of the latter. Moreover, they must not mix with lower-dimensional operators. This is prerequisite to the operators being multiplicatively renormalizable. Furthermore, from the more practical point of view, the operators should only require a non zero spatial momentum in at most one direction. We have considered the operators listed in Table 2. For the group theoretical classification of the lattice operators see Ref. [15]. The calculation of $v_{2,a}$, v_3, v_4, a_2 and d_2 requires non-vanishing nucleon momenta. Note that for the quenched theory there is no mixing with gluon operators. In the continuum limit the matrix elements $v_{2,a}$ and $v_{2,b}$ should be equal. At finite lattice spacing this provides us with a consistency check and gives information about possible lattice artifacts.

We have computed the renormalization constants for our operators in the quenched approximation for Wilson fermions in perturbation theory to one loop order. For this task we have developed packages of computer algebraic programs using *Mathematica* and *Maple*.

to $\overline{MS}$

$\langle\mathcal{O}\rangle$		$\gamma_{\mathcal{O}}$	$B_{\mathcal{O}}$	$Z_{\mathcal{O}}(1, g = 1.0)$	$B_{\mathcal{O}}^{\overline{MS}}$
$v_{2,a}$		$\frac{16}{3}$	-3.165(6)	1.0267(1)	$-\frac{40}{9}$
$v_{2,b}$		$\frac{16}{3}$	-1.892(6)	1.0160(1)	$-\frac{40}{9}$
v_3	{}{}:	$\frac{25}{3}$	-19.572(10)	1.1653(1)	$-\frac{67}{9}$
	{}():	0	0.370(10)	-0.0031(1)	
v_4		$\frac{157}{15}$	-37.16(30)	1.314(3)	$-\frac{2216}{225}$
a_0		0	15.795(3)	0.8666(0)	0
a_2		$\frac{25}{3}$	-19.560(10)	1.1652(1)	$-\frac{67}{9}$
d_2		$\frac{7}{3}$	-15.680(10)	1.1324(1)	$-\frac{13}{12}$

glue $v_{2,b}$ 1.296

Table 3: The renormalization constants in the quenched approximation. The errors quoted are a conservative estimate of the uncertainties in the numerical evaluation of the integrals involved. The numbers in the rightmost column represent the contribution of the continuum operators computed in the $\overline{MS}$ scheme.

we have developed packages of computer algebraic programs using *Mathematica* and *Maple* to such a level that all what is needed as input is to state the Feynman rules in symbolic form, both for the continuum and the lattice part of the calculation. We will summarize our results here. A detailed account of our calculation will be given elsewhere [16].

In the case of v_3 it turns out that the operator $\mathcal{O}_{\{114\}} - \frac{1}{2}(\mathcal{O}_{\{224\}} + \mathcal{O}_{\{334\}})$ mixes with the operator [15]

$$\mathcal{O}_{(114)} - \frac{1}{2}(\mathcal{O}_{(224)} + \mathcal{O}_{(334)}), \quad \mathcal{O}_{(\mu\mu\nu)} = \mathcal{O}_{\mu\mu\nu} + \mathcal{O}_{\mu\nu\mu} - 2\mathcal{O}_{\nu\mu\mu} \tag{21}$$

under renormalization. This operator is of mixed symmetry, is traceless and corresponds to the representation $8^{(+)}, C = -$ as well. Thus we have

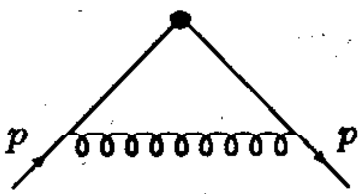
$$\mathcal{O}_{\{\}}(\mu) = Z_{\{\}\{\}}\mathcal{O}_{\{\}}(a) + Z_{\{\}()}\mathcal{O}_{()}(a), \tag{22}$$

where we have used a short-hand notation for the operators in Table 2 and eq. (21). We write ($C_F = 4/3$)

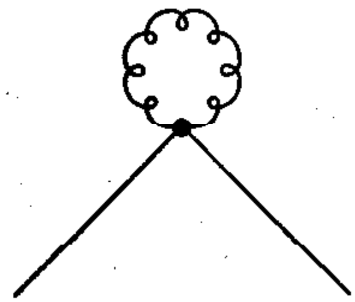
$$Z_{\mathcal{O}}((a\mu)^2, g) \;=\; 1\!\!1 - \frac{g^2}{16\pi^2}C_F[\gamma_{\mathcal{O}}\ln(a\mu) + B_{\mathcal{O}}]. \tag{23}$$

This is to be interpreted as a matrix equation in the case of v_3. Our results for the anomalous dimensions $\gamma_{\mathcal{O}}$ and the $B_{\mathcal{O}}$'s are given in Table 3 for $r = 1$. The renormalization constants $Z_{v_{2,b}}$ and Z_{a_0} have been given before in the literature [17, 18, 19]. We agree with the results of these authors. In the case of v_3 the off-diagonal component of Z is negligibly small.

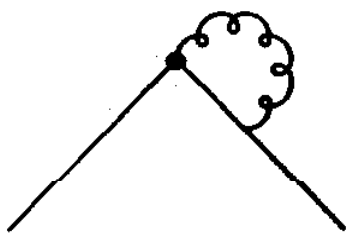
 $\mu^2 = Q^2 = a^{-2} \approx 4\ GeV^2$



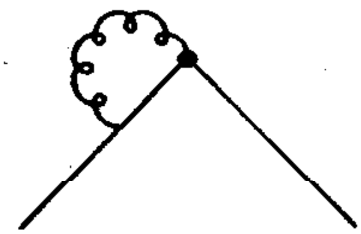
(a)



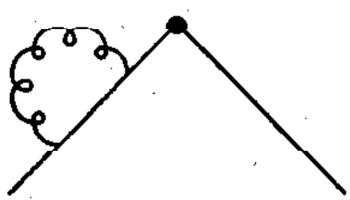
(b)



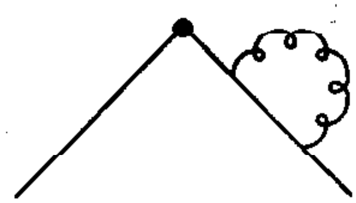
(c)



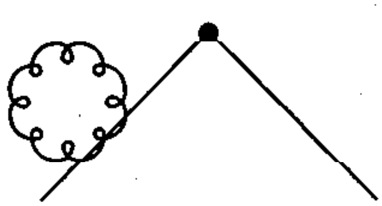
(d)



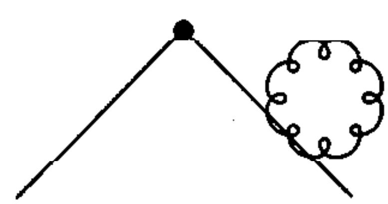
(e)



(f)



(g)



(h)

Impose

$$\langle q(p) | \mathcal{O}(\mu) | q(p) \rangle = \langle q(p) | \mathcal{O}(a) | q(p) \rangle^{\text{tree}}_{p^2=\mu^2}$$

nonperturbatively

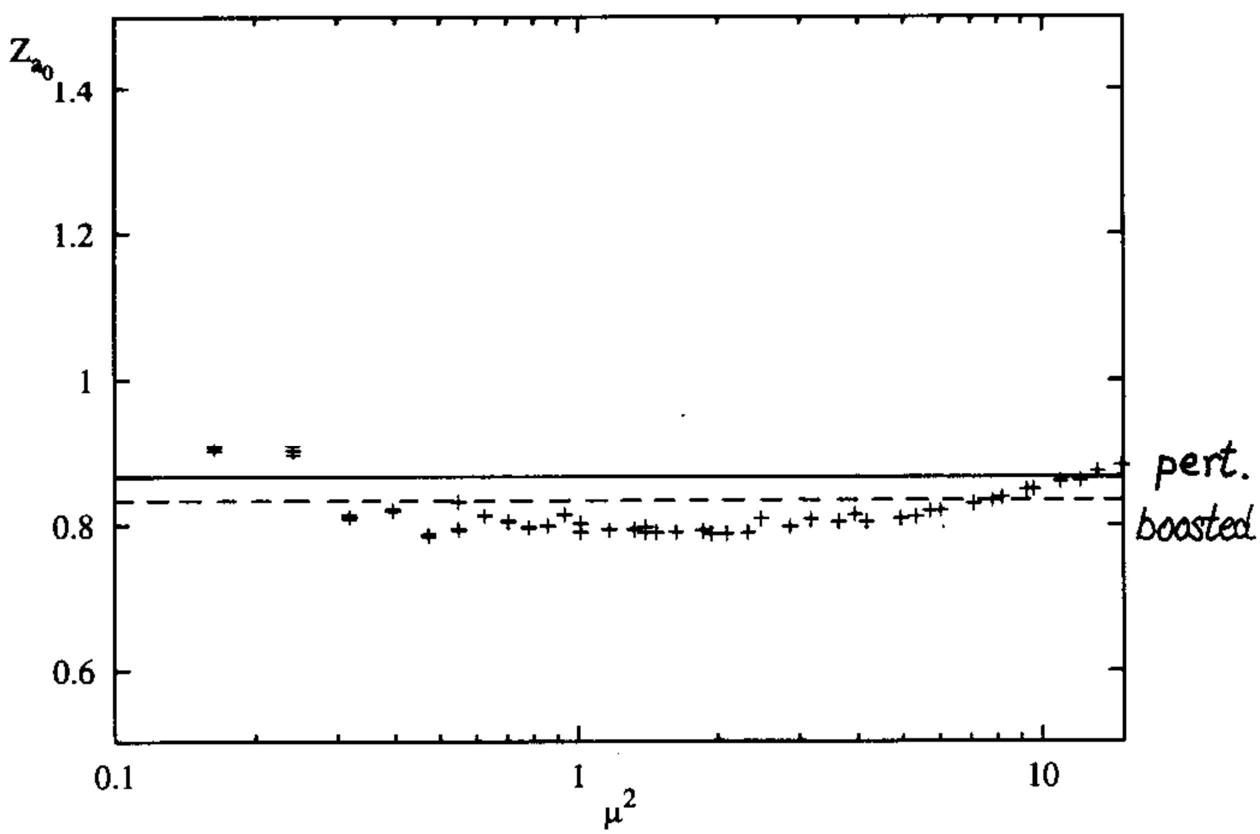
$$G_{\mathcal{O}}(p) = \sum_{x,y} e^{-ip(x-y)} \langle \psi(x)\, \mathcal{O}\, \bar{\psi}(y) \rangle$$

$$Z_{\mathcal{O}}(\mu^2) = \left. \frac{\mathrm{Tr}\, S(p)\, V^{\text{tree}}(p)\, S(p)}{\mathrm{Tr}\, \Gamma\, G_{\mathcal{O}}(p)} \right|_{p^2=\mu^2}$$

$$\mathcal{O} = \bar{\psi}\, V\, \psi$$

perturbative vs. non-perturbative

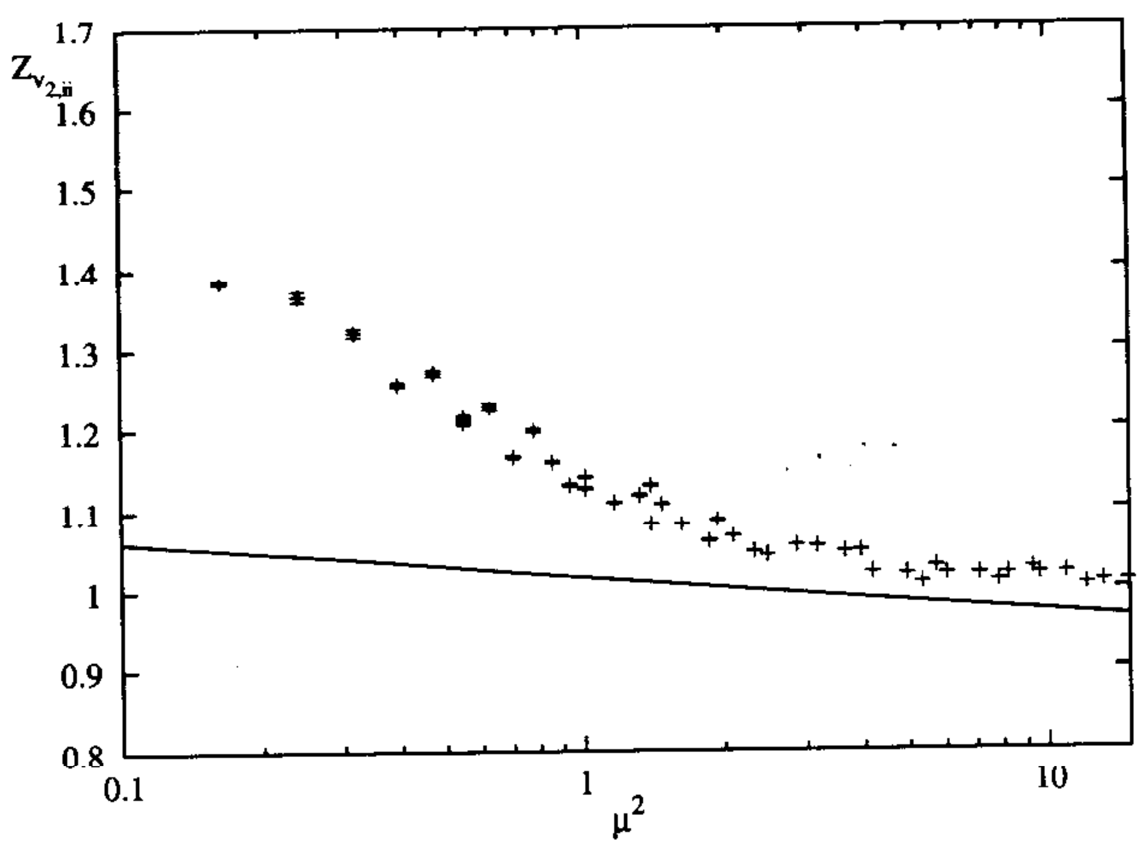
Axial vector current



Must match with
Wilson coefficient

$\Lambda_{\overline{MS}}$

$v_2 \Rightarrow \langle x \rangle$



All twist-2 & twist-3 operators
with $J \leq 4$
plus Z's

	Lattice (quenched, $\mu^2 \approx 5GeV^2$)	Experiment ($\mu^2 = 4GeV^2$)	
$\langle x \rangle_a^{(u)}$	0.369(26)	0.284 val.	*
$\langle x \rangle_b^{(u)}$	0.440(22)		
$\langle x \rangle_a^{(d)}$	0.169(13)	0.102 val.	*
$\langle x \rangle_b^{(d)}$	0.187(10)		
$\langle x \rangle_a^{(u-d)}$	0.200(39)	0.182	*
$\langle x \rangle_b^{(u-d)}$	0.253(32)		
$\langle x \rangle_a^{(u+d)}$	0.538(39)	0.561 $(u+d+S)$	*
$\langle x \rangle_b^{(u+d)}$	0.627(32)		
$\langle x^2 \rangle^{(u)}$	0.108(16)	0.083 val.	*
$\langle x^2 \rangle^{(d)}$	0.036(8)	0.025 val.	*
$\langle x^3 \rangle^{(u)}$	0.020(10)	0.032 val.	
$\langle x^3 \rangle^{(d)}$	0.000(6)	0.008 val.	
$\langle x \rangle^{(g)}$	0.52(22)	0.441	

val. CTEQ3M

	Lattice (quenched, $\mu^2 \approx 5GeV^2$)	Experiment ($\mu^2 = 4GeV^2$)	
Δu	0.841(52)	0.823 val.	*
Δd	-0.245(15)	-0.303 val.	*
g_A	1.086(67)	1.26	*
Δu	0.822(19)	0.823 val.	**
Δd	-0.262(9)	-0.303 val.	**
g_A	1.084(28)	1.26	**
$\int g_1^{p-n}$	0.176(14)	0.163(17)	**
$\Delta_1 u$	0.22(2)	0.14 val.	*
$\Delta_1 d$	0.05(1)	-0.05 val.	*
$\int x^2 g_1^p$	0.0150(32)	0.012(1)	
$\int x^2 g_1^n$	-0.0012(20)	-0.004(3)	
$\int x^2 g_2^p$	-0.0261(38)	-0.006(2)	
$\int x^2 g_2^n$	-0.0004(22)	0.005(8)	
$\Delta\bar{u} = \Delta\bar{d}$	-0.07(2)	-0.05	
$\Delta\bar{s}$	-0.06(2)	-0.05	
$\Delta\Sigma$	0.18(8)	0.223	

Improved

JLQCD $\beta=5.7$

val.. Gehrmann & Stirling

chiral limit

	$\kappa = 0.1515$	$\kappa = 0.153$	$\kappa = 0.155$	$\kappa = \kappa_c = 0.15717$
PION				
$v_{2,a} = \langle x \rangle_a$	0.301(20)	0.294(28)	0.290(71)	0.279(83)
$v_{2,b} = \langle x \rangle_b$	0.3239(70)	0.3150(71)	0.2910(75)	0.273(12)
$v_{2,bp} = \langle x \rangle_{bp}$	0.319(23)	0.316(33)	0.325(84)	0.318(98)
$v_3 = \langle x^2 \rangle$	0.1222(83)	0.116(12)	0.117(31)	0.107(35)
$v_4 = \langle x^3 \rangle$	0.0619(45)	0.0580(65)	0.054(18)	0.048(20)
RHO				
$a_2 = \langle x \rangle$	0.3555(80)	0.3531(93)	0.340(14)	0.334(21)
$a_3 = \langle x^2 \rangle$	0.1398(93)	0.144(14)	0.182(48)	0.174(47)
$a_4 = \langle x^3 \rangle$	0.0725(72)	0.069(12)	0.074(41)	0.066(39)
d_2	0.107(52)	0.128(75)	0.29(20)	0.29(23)
d_3	0.0145(32)	0.0135(49)	−0.002(14)	0.001(15)
d_4	0.0109(100)	0.004(17)	0.007(62)	−0.009(58)
$r_{1,a}$	0.709(56)	0.715(97)	0.42(34)	0.57(32)
$r_{1,b}$	0.721(17)	0.702(20)	0.627(32)	0.590(46)
$r_{1,bp}$	0.680(56)	0.62(13)	0.32(44)	0.33(42)
$r_{2,a}$	0.2743(62)	0.2631(70)	0.231(12)	0.212(17)
$r_{2,ap}$	0.257(17)	0.243(25)	0.216(69)	0.198(76)
$r_{2,b}$	0.242(20)	0.232(30)	0.210(89)	0.199(95)
r_3	0.1067(71)	0.099(11)	0.087(33)	0.077(34)

Exp.

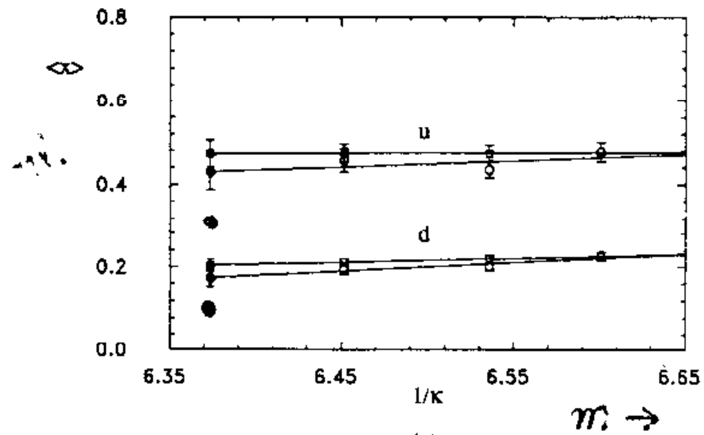
$\}$ 0.20(1)

$r_n = a_{n-1}$

Table 2: Result overview for a single flavor. The numbers refer to the $\overline{\text{MS}}$ scheme with a renormalization scale $\mu \approx 2.4\,\text{GeV}$. The last column gives the result of the extrapolation to the chiral limit.

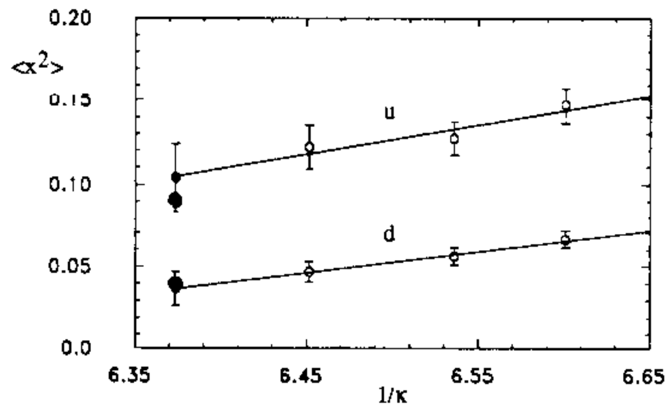
$$\int dx\, b_1(x) = \int dx\,(q^0(x) - q^1(x))$$

16^3

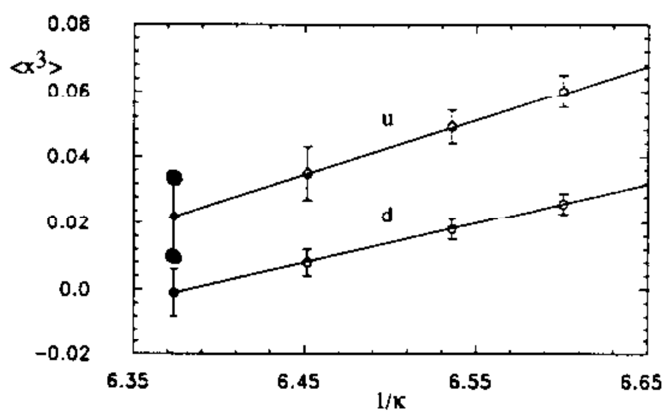


(a)

CTEQ3M valence

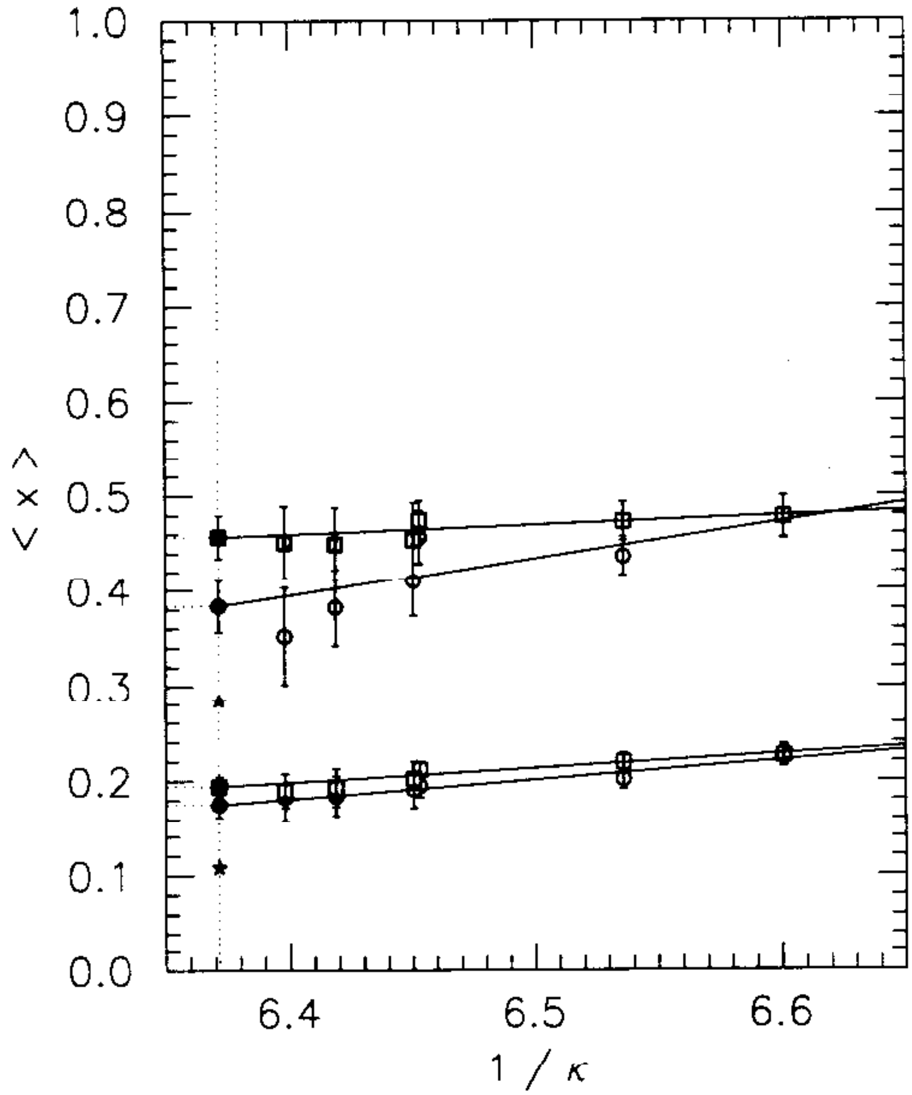


(b)

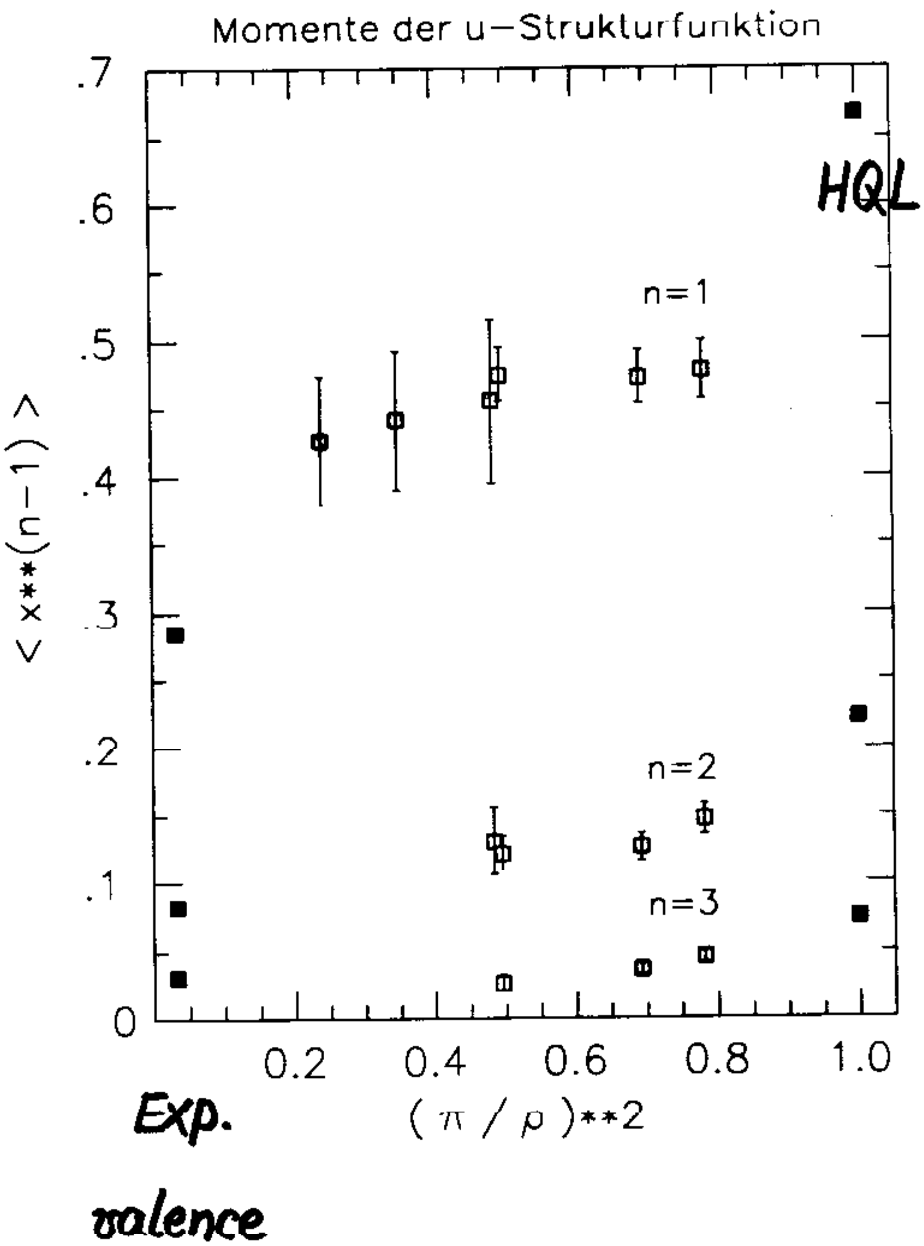


(c)

* O(2000) configs.



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
< x >
6.4
6.5
6.6
1 / κ



Momente der u−Strukturfunktion
HQL
n=1
n=2
n=3
Exp.
valence
< x**(n−1) >
(π / ρ)**2
.7
.6
.5
.4
.3
.2
.1
0
0.2
0.4
0.6
0.8
1.0

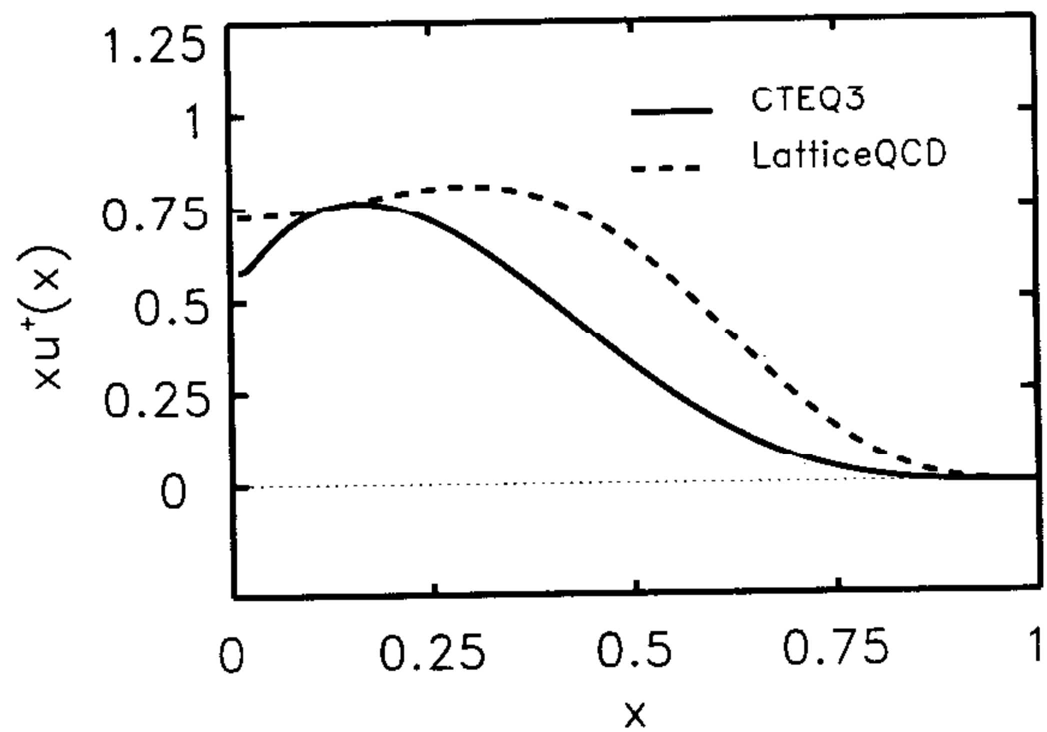


Fig. 3 Reconstruction of the u-quark distribution function using the lattice results for the first two moments [7] and the experimental normalization at small x. The solid line is the CTEQ parametrization [12] at $\mu^2 = 2.5$ GeV^2.

Weigl, Monkiewicz

Higher twist ?

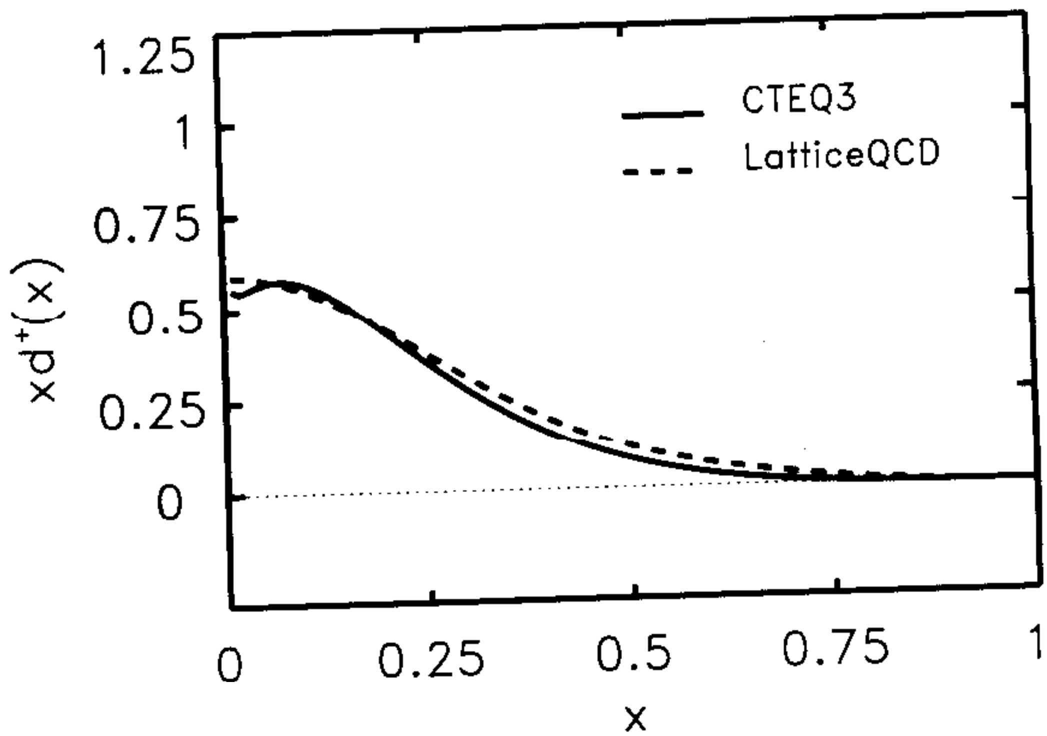


Fig. 4 Reconstruction of the d-quark distribution function, dashed line, using the lattice results for the first two moments [7] and the experimental normalization at small x. The solid line is the CTEQ parametrization [12] at $\mu^2 = 2.5$ GeV^2.

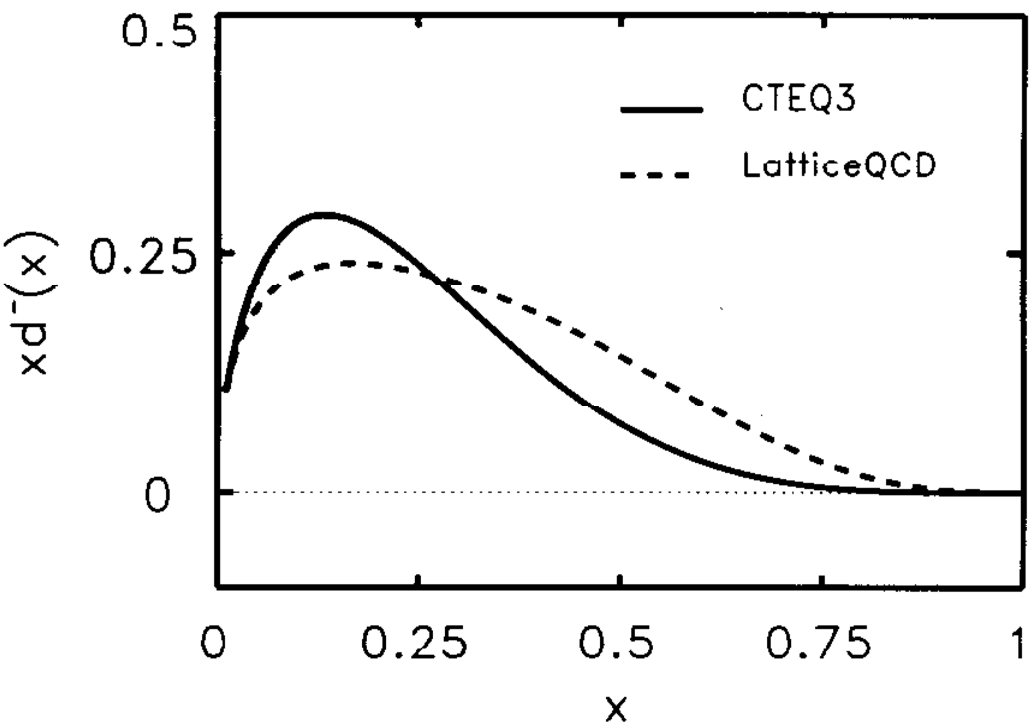


Fig. 6 Reconstruction of the valence d-quark distribution function using the lattice QCD [7] results for the first non-trivial moment and the experimental normalization at small x. The solid line is the CTEQ parametrization [12] at $\mu^2 = 2.5$ GeV^2.

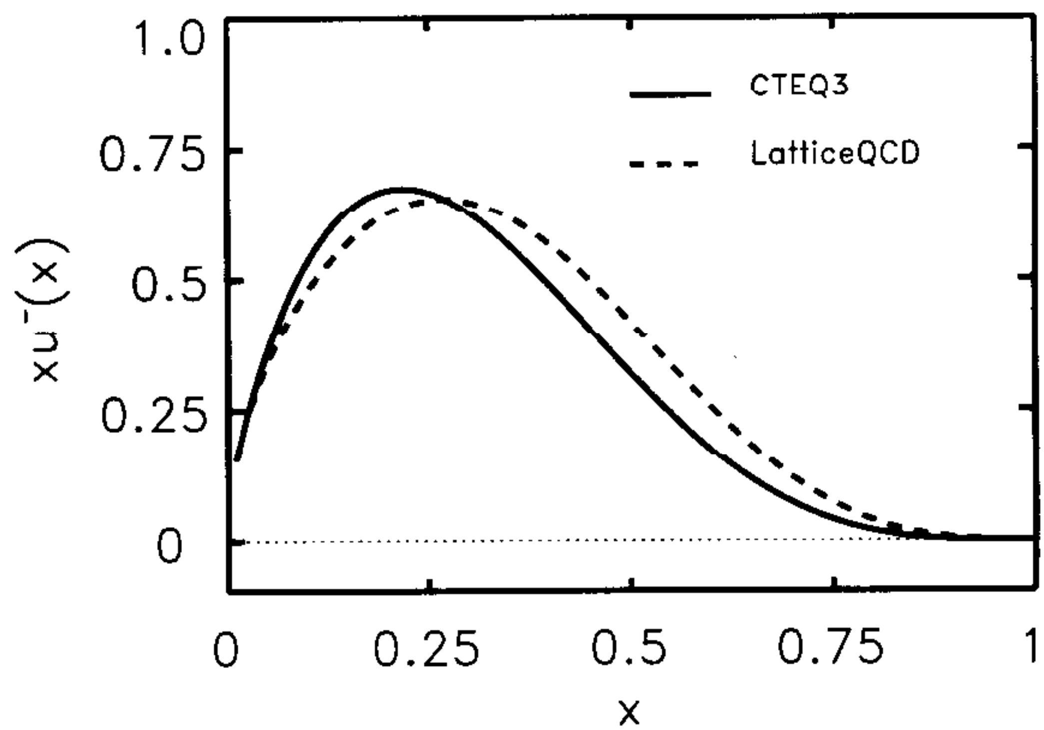
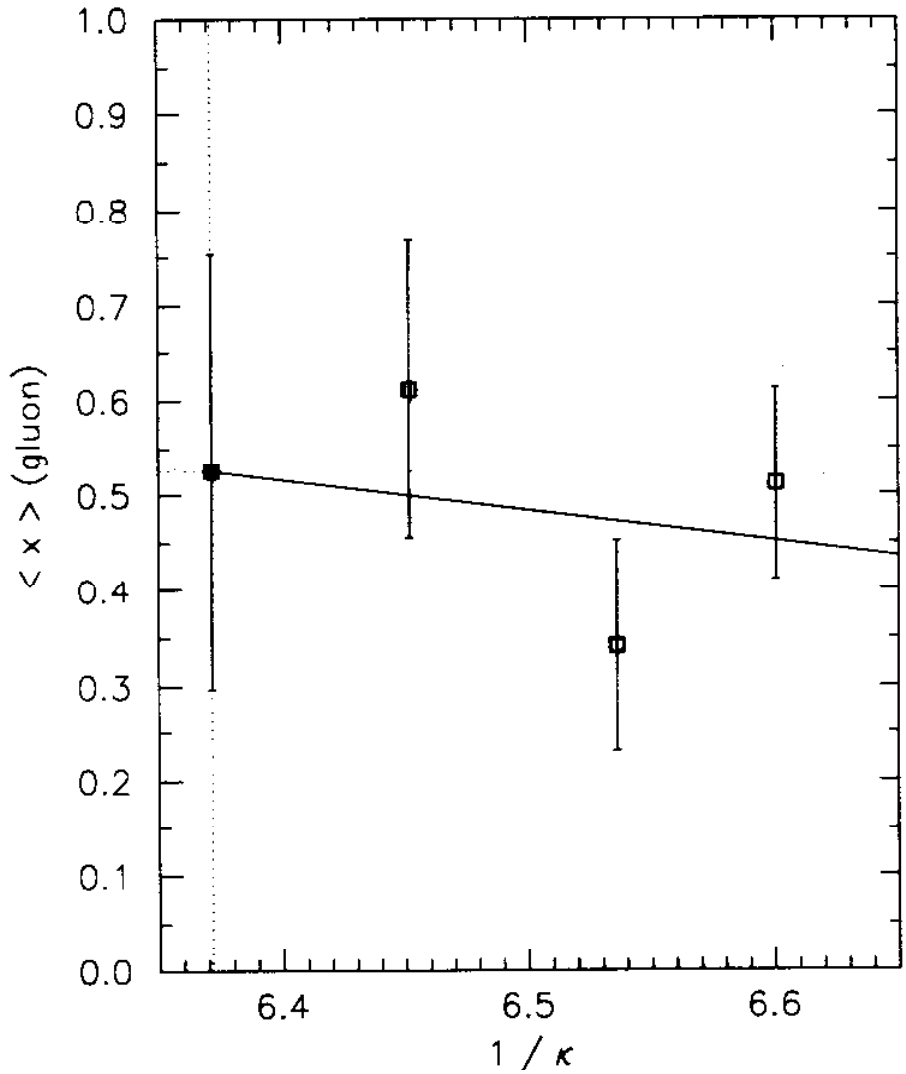


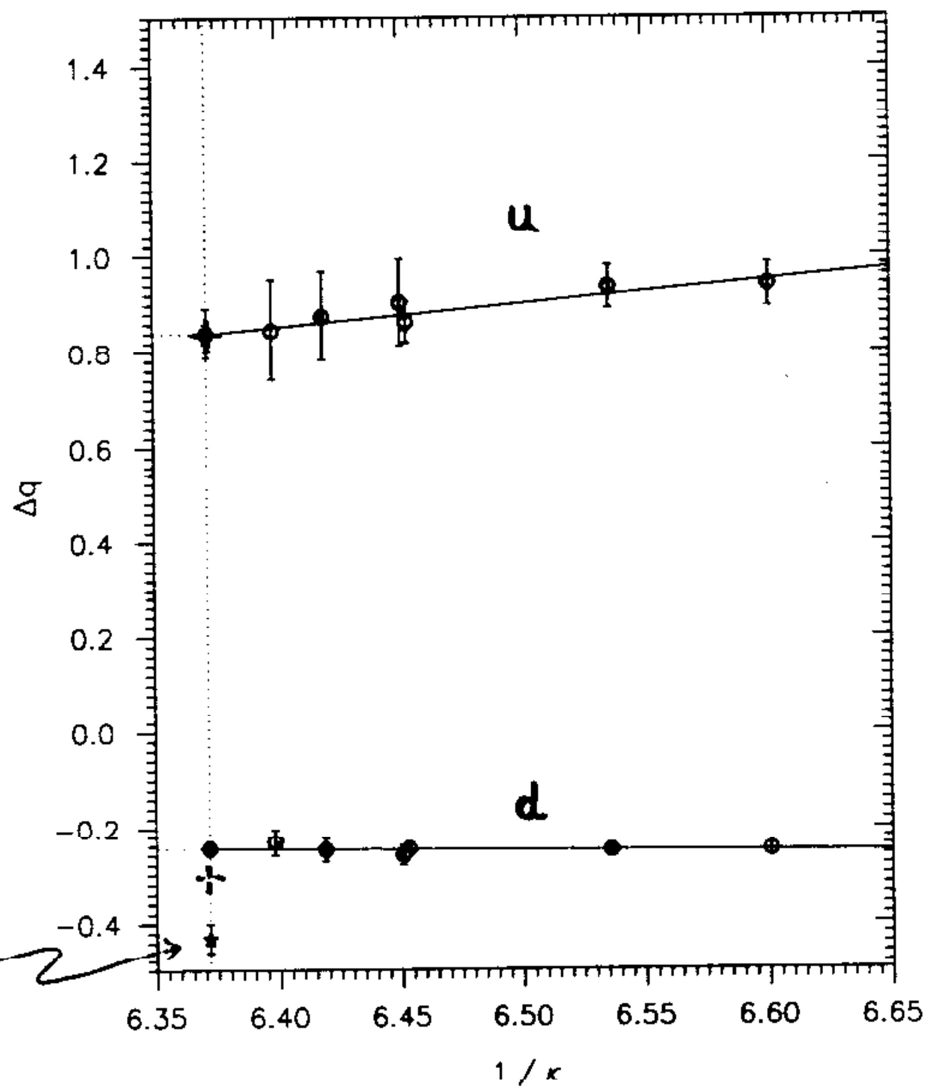
Fig. 5 Reconstruction of the valence u-quark distribution function using lattice QCD results for the first non-trivial moment and the experimental normalization at small x, dashed line. The solid line is the CTEQ parametrization [12] at $\mu^2 = 2.5$ GeV^2.

Glue



Σ-rule

$$\langle x \rangle_u = 0.44$$
$$\langle x \rangle_d = 0.19$$
$$\langle x \rangle_g = \underline{0.52}$$
$$1.16$$



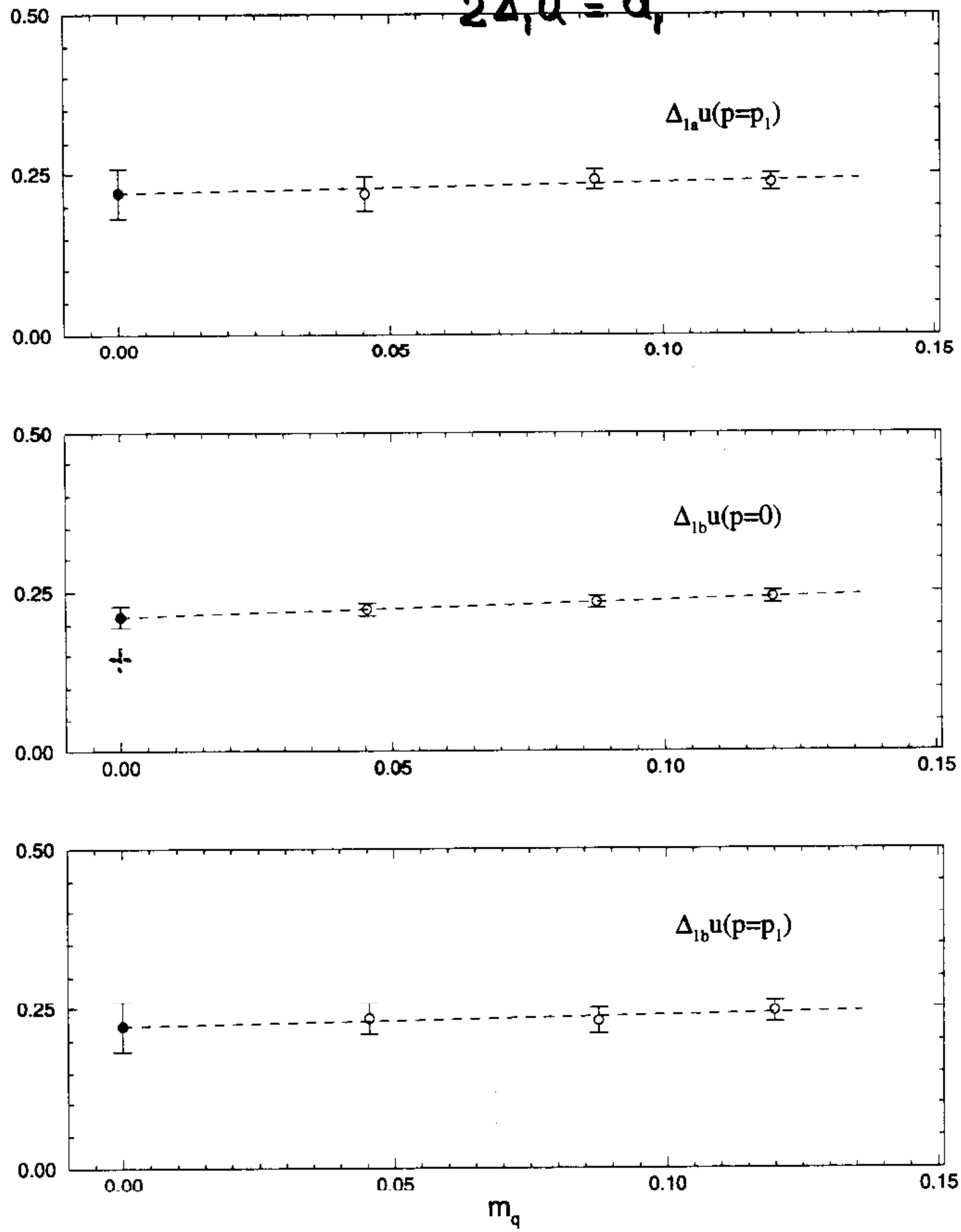
Ellis, Karliner

25 40 70 ⋯ m_q [MeV]

* O(2000) configs.

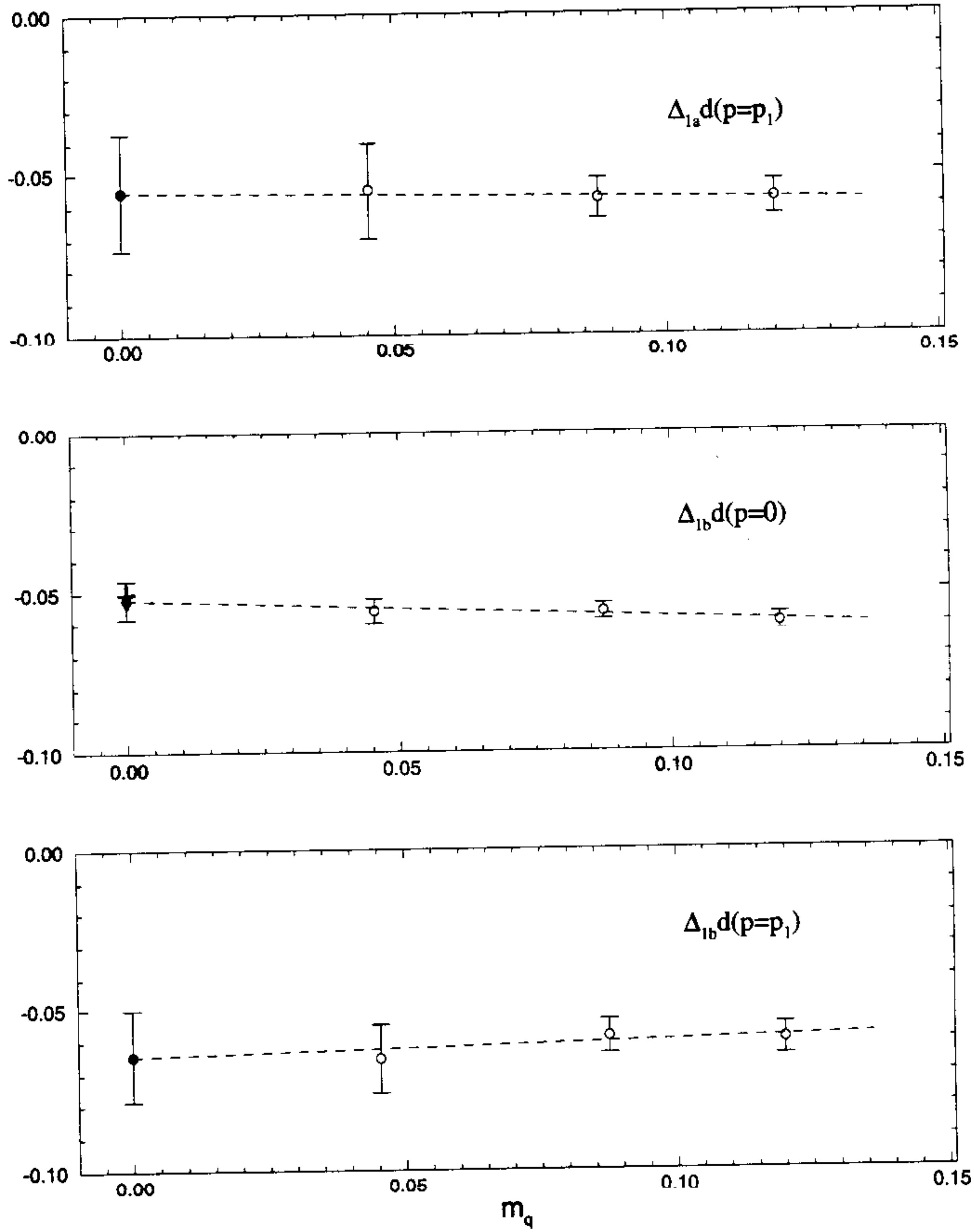
$$\Delta_1 u = \int_0^1 dx\, x\, \Delta u(x)$$

$$2\Delta_1 u = a_1^{(u)}$$

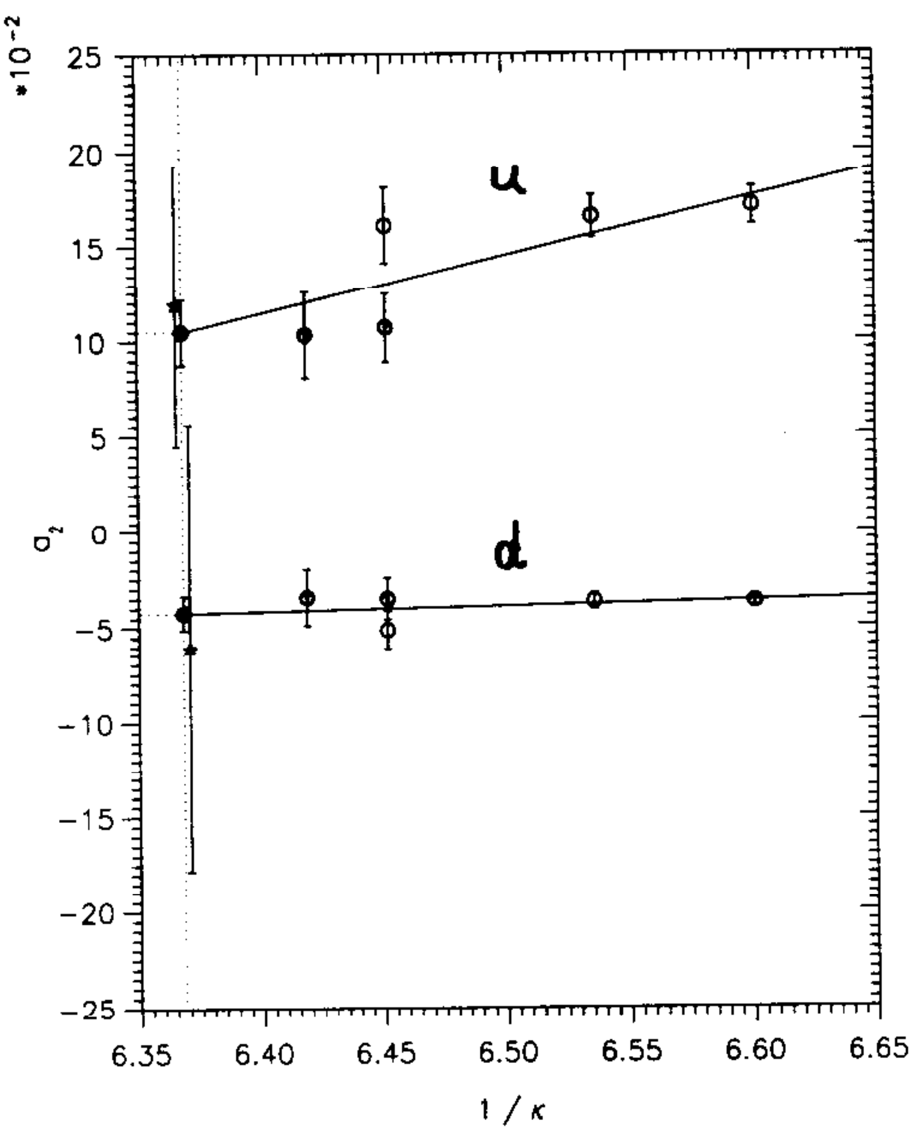


Thu Apr 3 10:51:42 1997

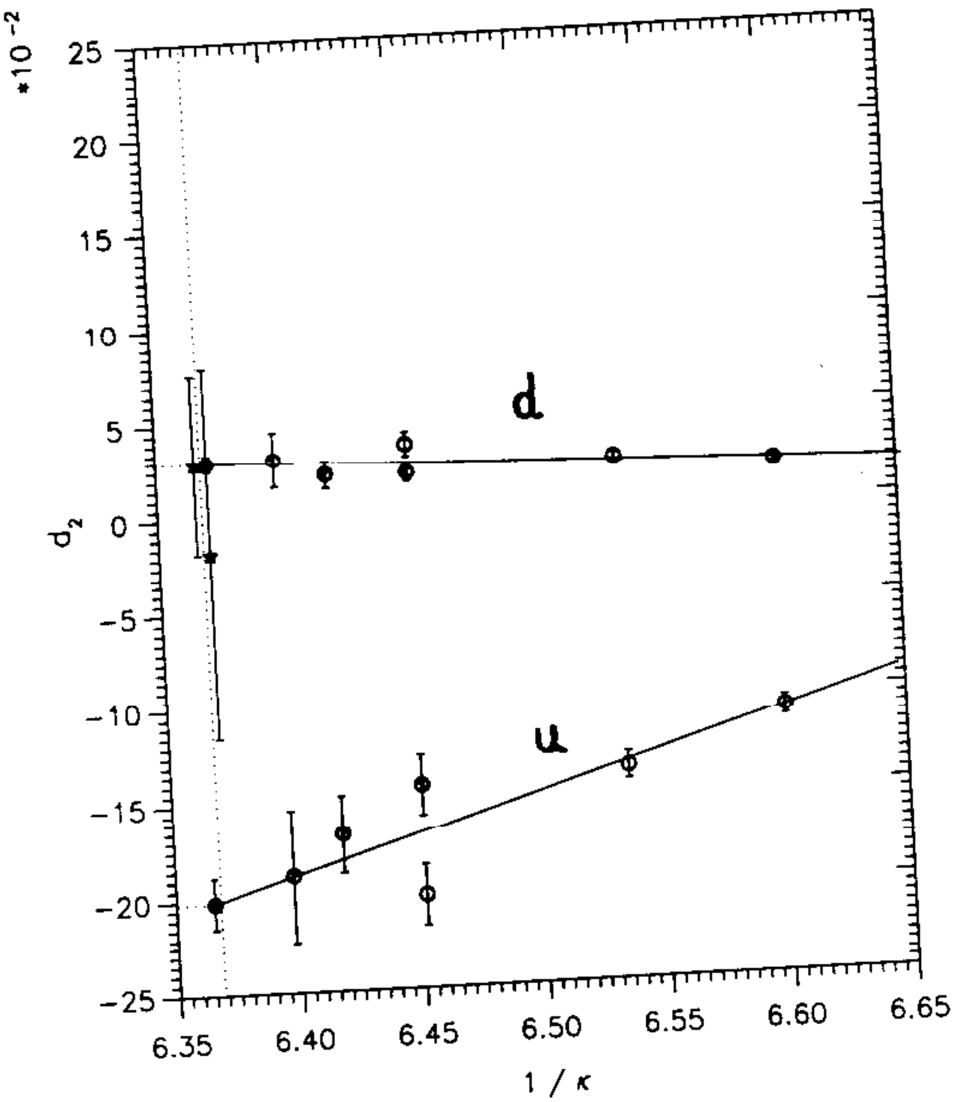
$\Delta_1 d$



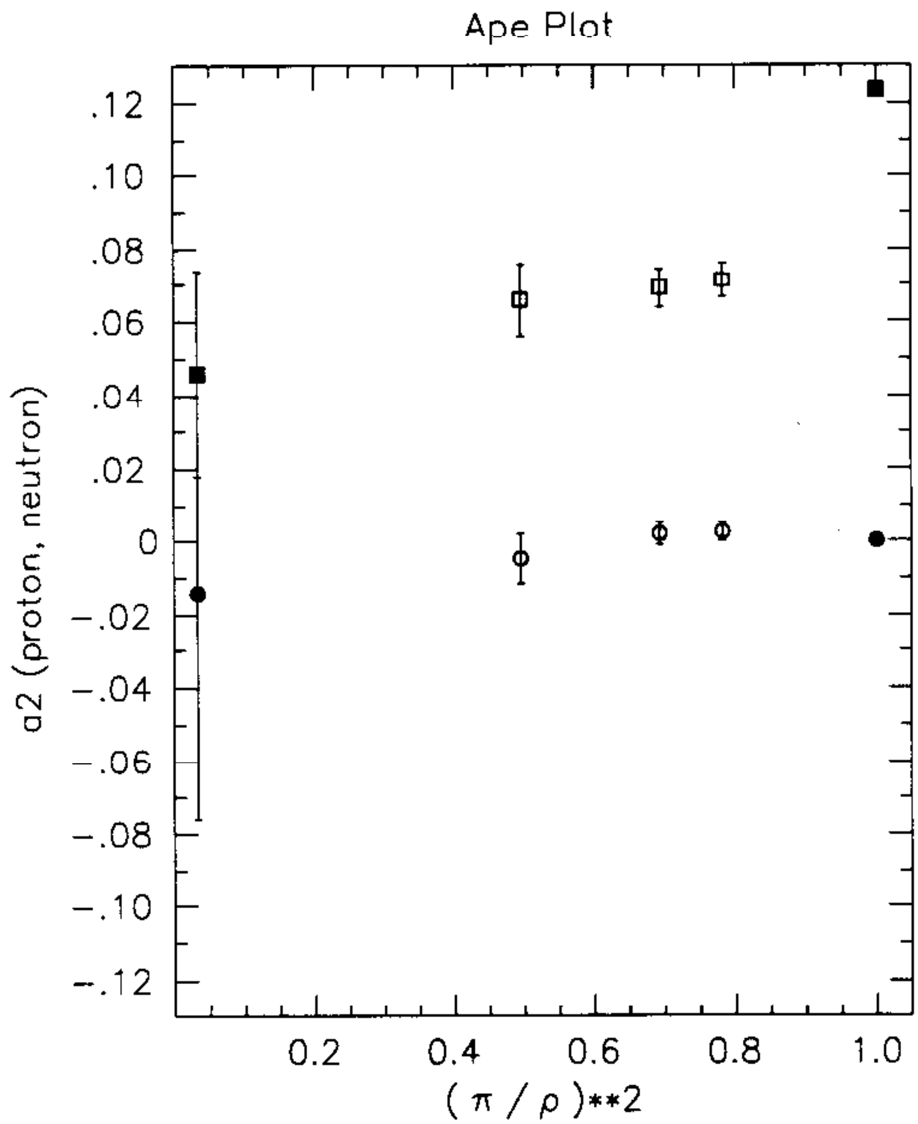
Thu Apr 3 10:50:20 1997



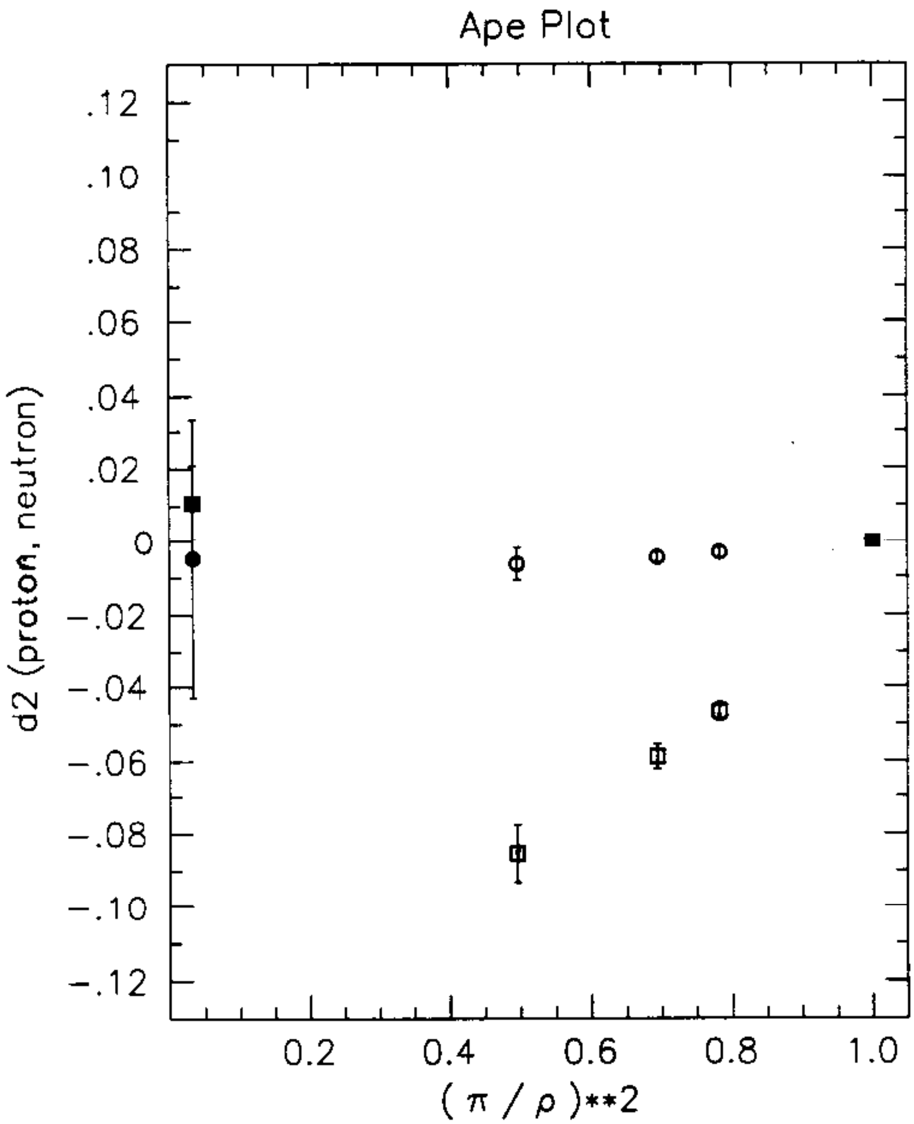
a_3 in progress ⇨ real structure functions

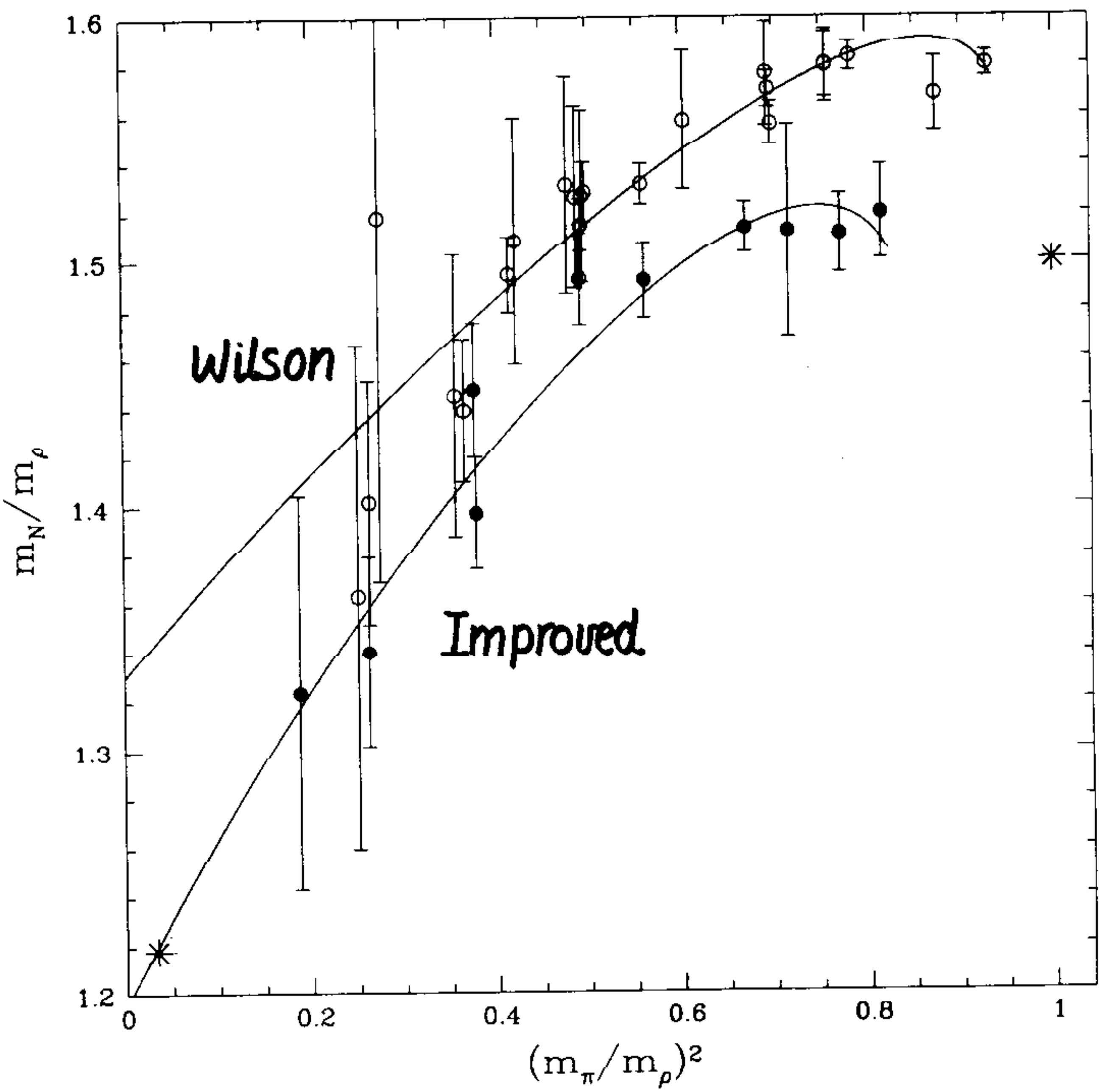


twist-3



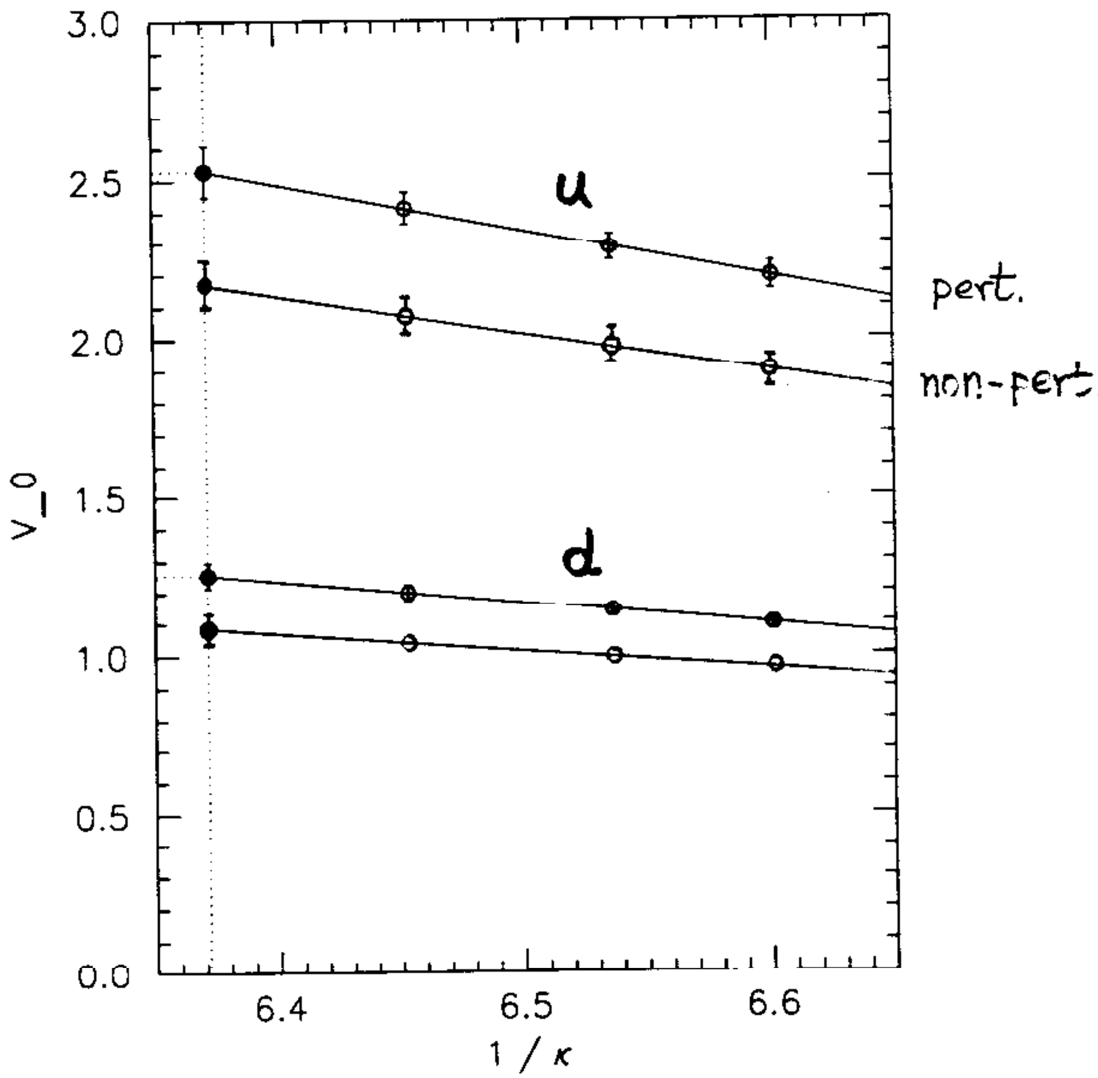
expect sea contribution to be negligible


Ape Plot
.12
.10
.08
.06
.04
.02
0
−.02
−.04
−.06
−.08
−.10
−.12
d2 (proton, neutron)
0.2
0.4
0.6
0.8
1.0
(π / ρ)**2



1.6
1.5
1.4
1.3
1.2
0
0.2
0.4
0.6
0.8
1
m_N/m_ρ
$(m_\pi/m_\rho)^2$
Wilson
Improved

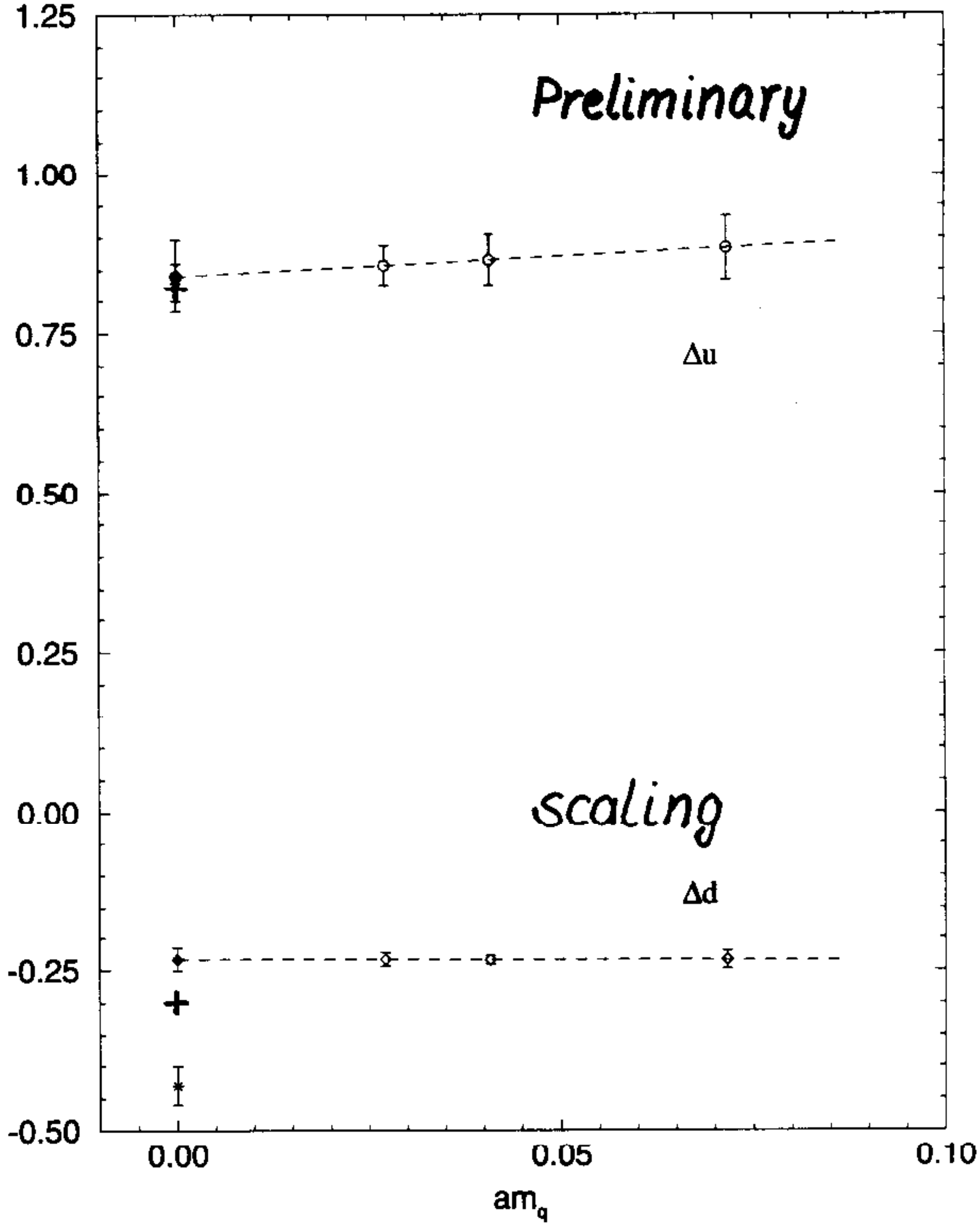
Wilson Local vector current



$$\int_0^1 dx\, q(x) = \begin{cases} 2 & u \\ 1 & d \end{cases}$$

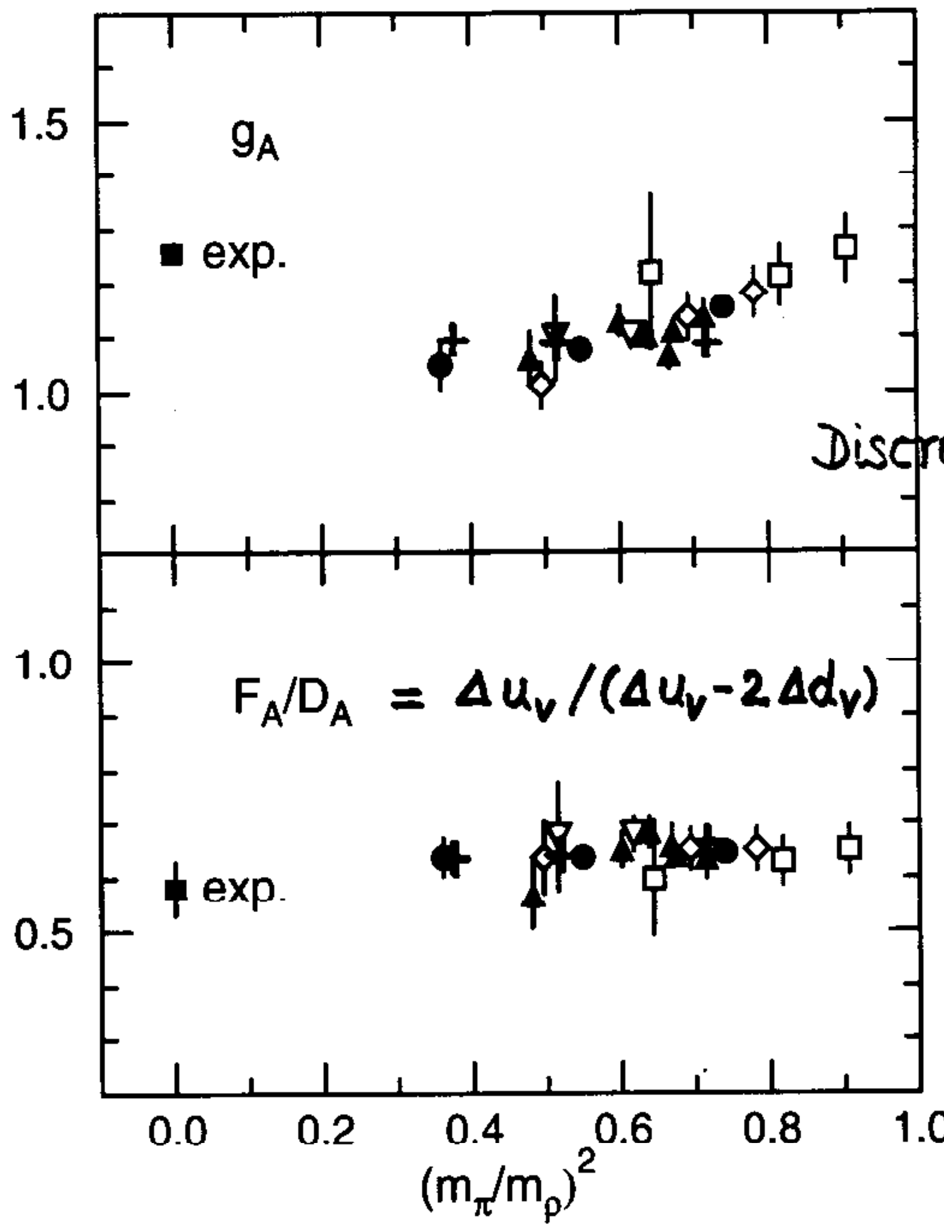
Δq (p=(0,0,0), q=(0,0,0))

($\beta = 6.2$, $c_{sw} = 1.614$)

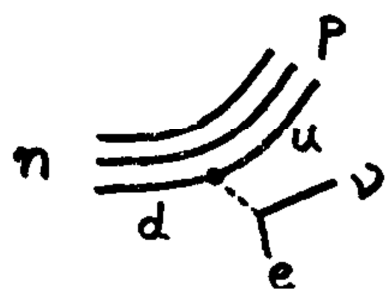


Fri Apr 11 18:08:09 1997

$g_A = \Delta u - \Delta d$ NS



Discrepancy ?



Rests on parton model

$L_q = 0$

$$\int_0^1 dx\, x^2 g_2(x,Q^2) = \begin{cases} -0.0161(16) - 0.0100(22) = -0.0261(38) & \text{proton} \\ -0.0013(9) + 0.0009(13) = -0.0004(22) & \text{neutron} \end{cases}$$

$\Uparrow$

d_2

twist-3

Wandzura-Wilczek.

$$\Delta\Sigma = \Delta u_v + \Delta d_v + 2\underbrace{(\Delta\bar{u} + \Delta\bar{d} + \Delta\bar{s})}_{\text{JLQCD}} = 0.18(8)$$

Where is the spin gone?

$$J_3 = \frac{1}{2}\epsilon_{3ij}\int d^3x\, M_{ij}$$

$$M_{ij} = x_i \underbrace{T_{0j}} - x_j T_{0i}$$

energy-momentum tensor

$$T_{\mu\nu} = \frac{1}{2}(\bar\psi\gamma_\mu\partial_\nu\psi + \bar\psi\gamma_\nu\partial_\mu\psi) - \delta_{\mu\nu}(\bar\psi\not{\partial}\psi + m\bar\psi\psi)$$
$$+ F_{\mu\rho}F_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma} + \cdots$$

$$\epsilon_{3ij}M_{ij} = \underbrace{\bar\psi\gamma_0(\vec{x}\times\vec{\nabla})_3\psi}_{L_q} + \underbrace{\bar\psi\gamma_3\gamma_5\psi}_{\Delta q \Rightarrow \Delta\Sigma}$$

$\langle\vec{p},s|\ldots|\vec{p},s\rangle$:

$$+ \underbrace{4\,\mathrm{Tr}\, E_k(\vec{x}\times\vec{\nabla})_3 A_k}_{L_g} + \underbrace{2\,\mathrm{Tr}(\vec{E}\times\vec{A})_3}_{\Delta g}$$

$(A_0 = 0)$

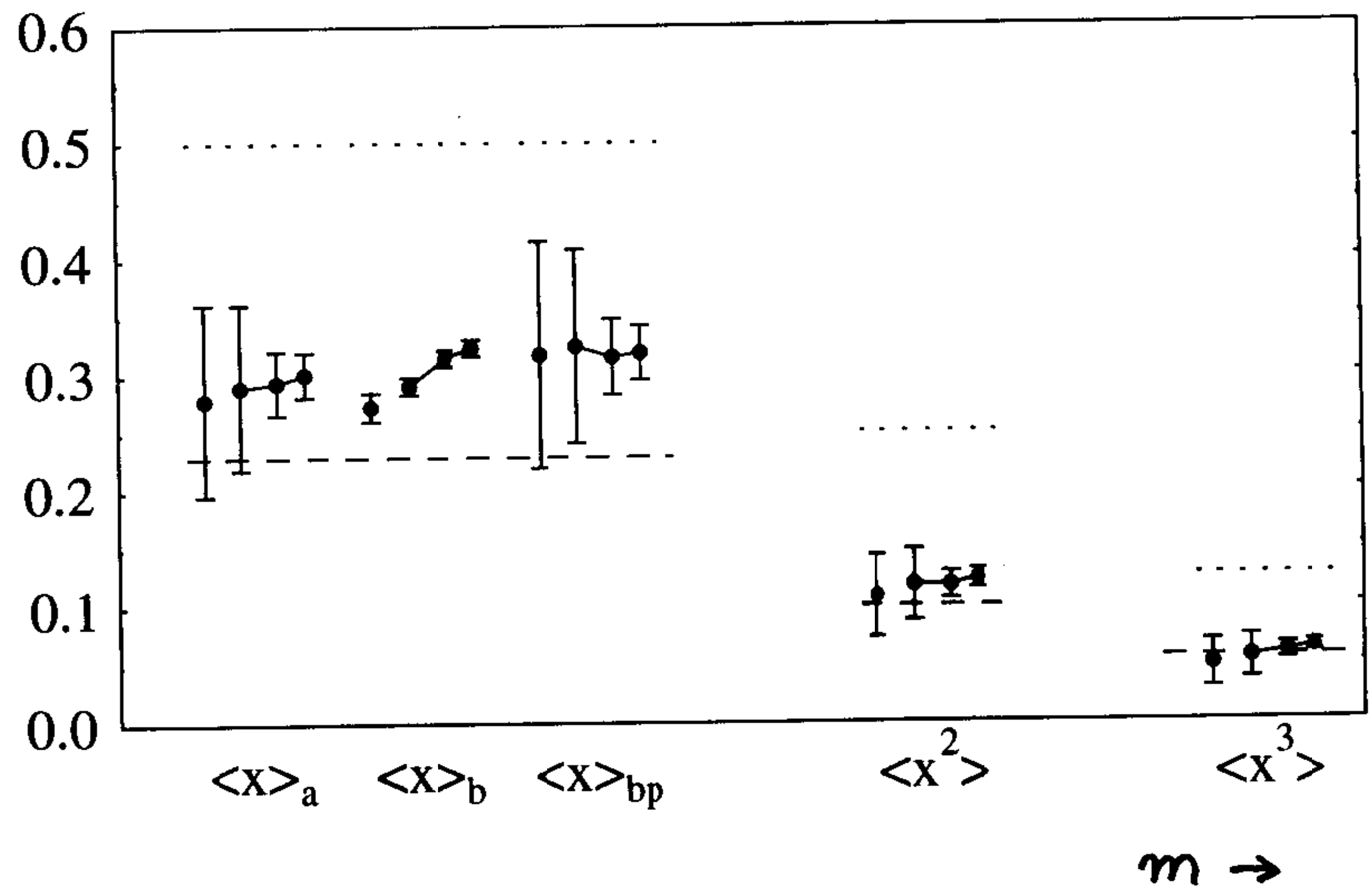
$+$ something $\langle\vec{p},s|\ldots|\vec{p},s\rangle = 0$

renormalization, mixing, gauge variance

$$\frac{1}{2} = \underbrace{L_q}_{} + \frac{1}{2}\Delta\Sigma + \underbrace{L_g}_{} + \Delta g$$

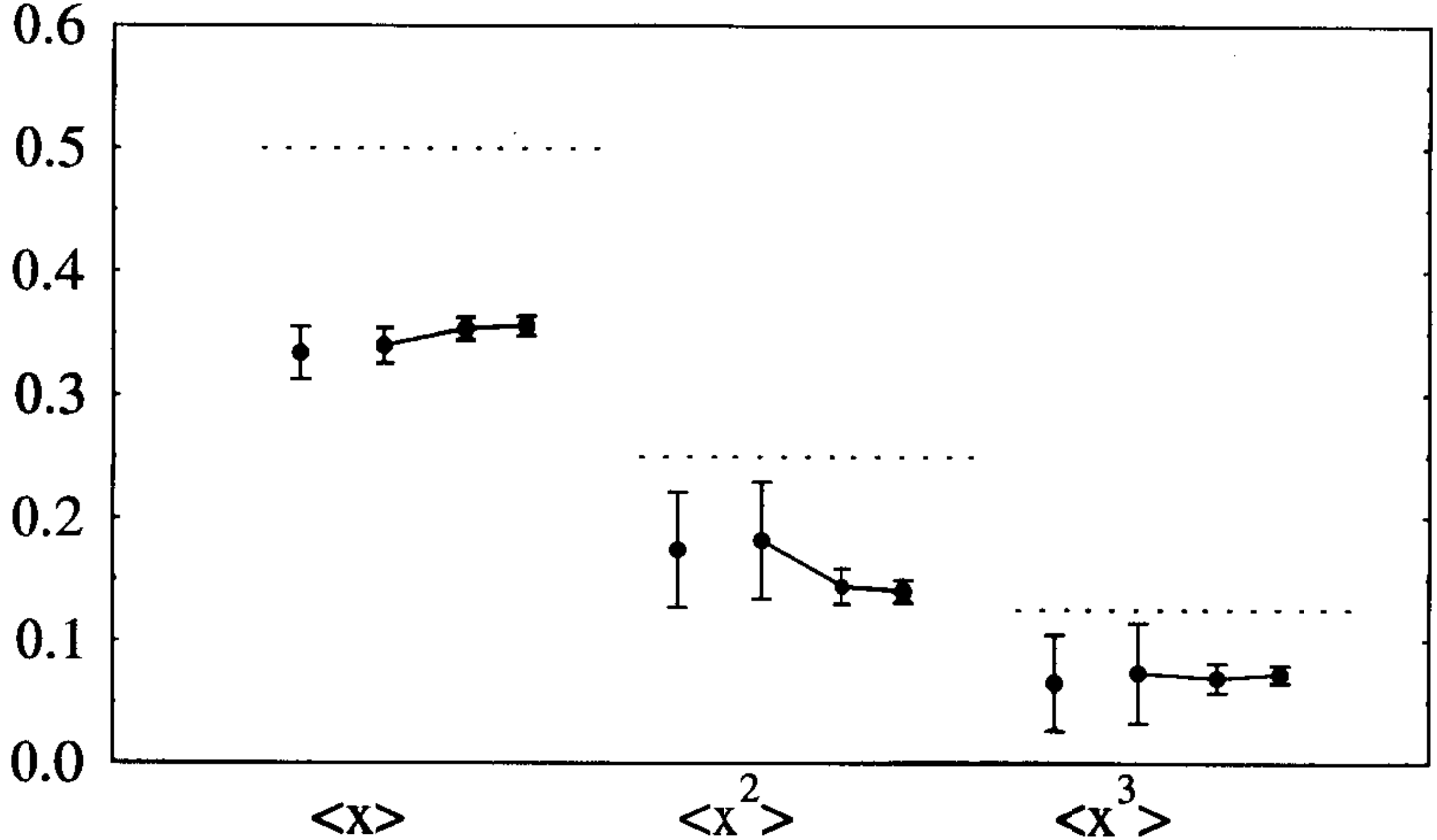
cannot be computed
in parton model

Pion

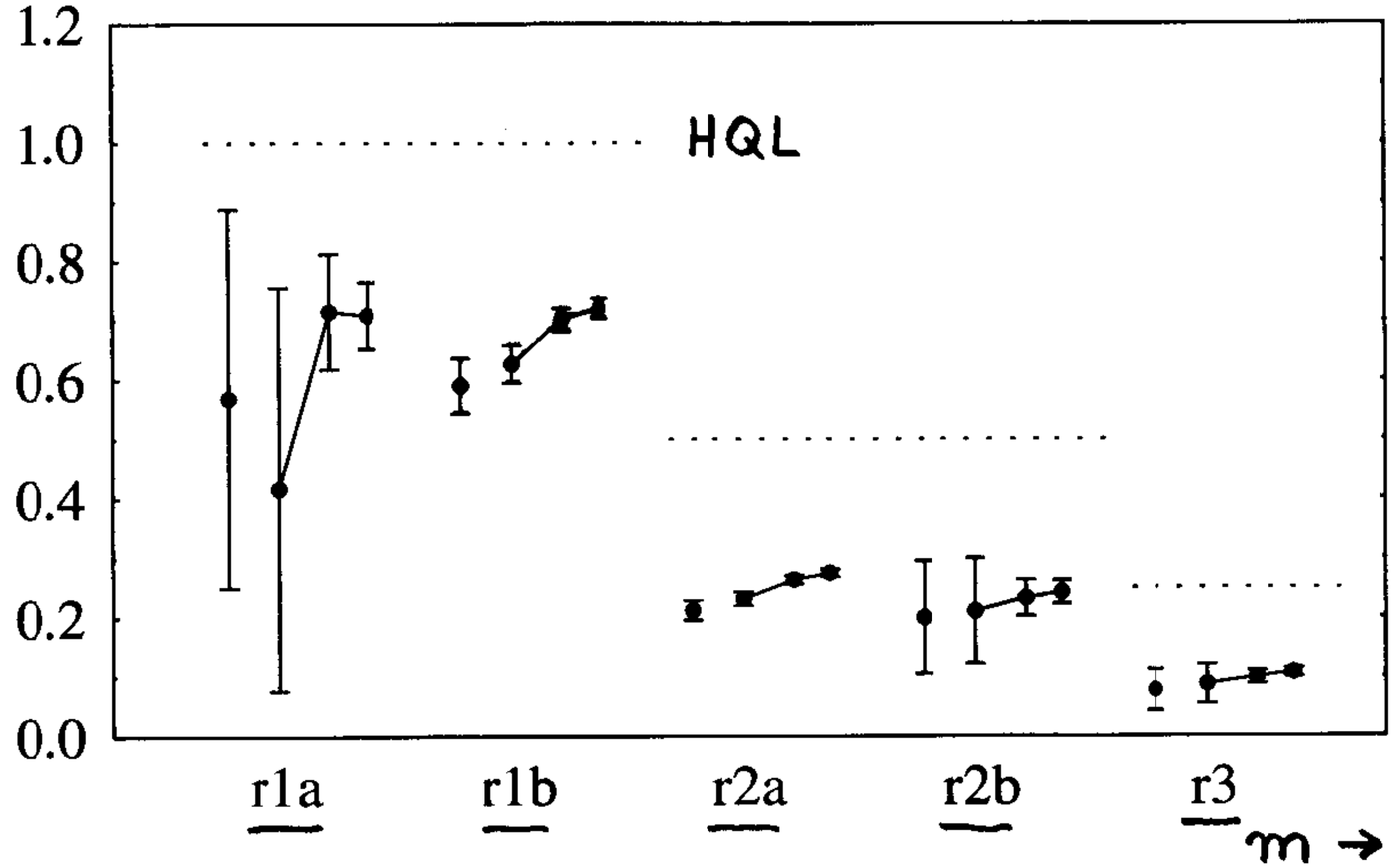


0.6
0.5
0.4
0.3
0.2
0.1
0.0
$\langle x \rangle_a$
$\langle x \rangle_b$
$\langle x \rangle_{bp}$
$\langle x^2 \rangle$
$\langle x^3 \rangle$
m →

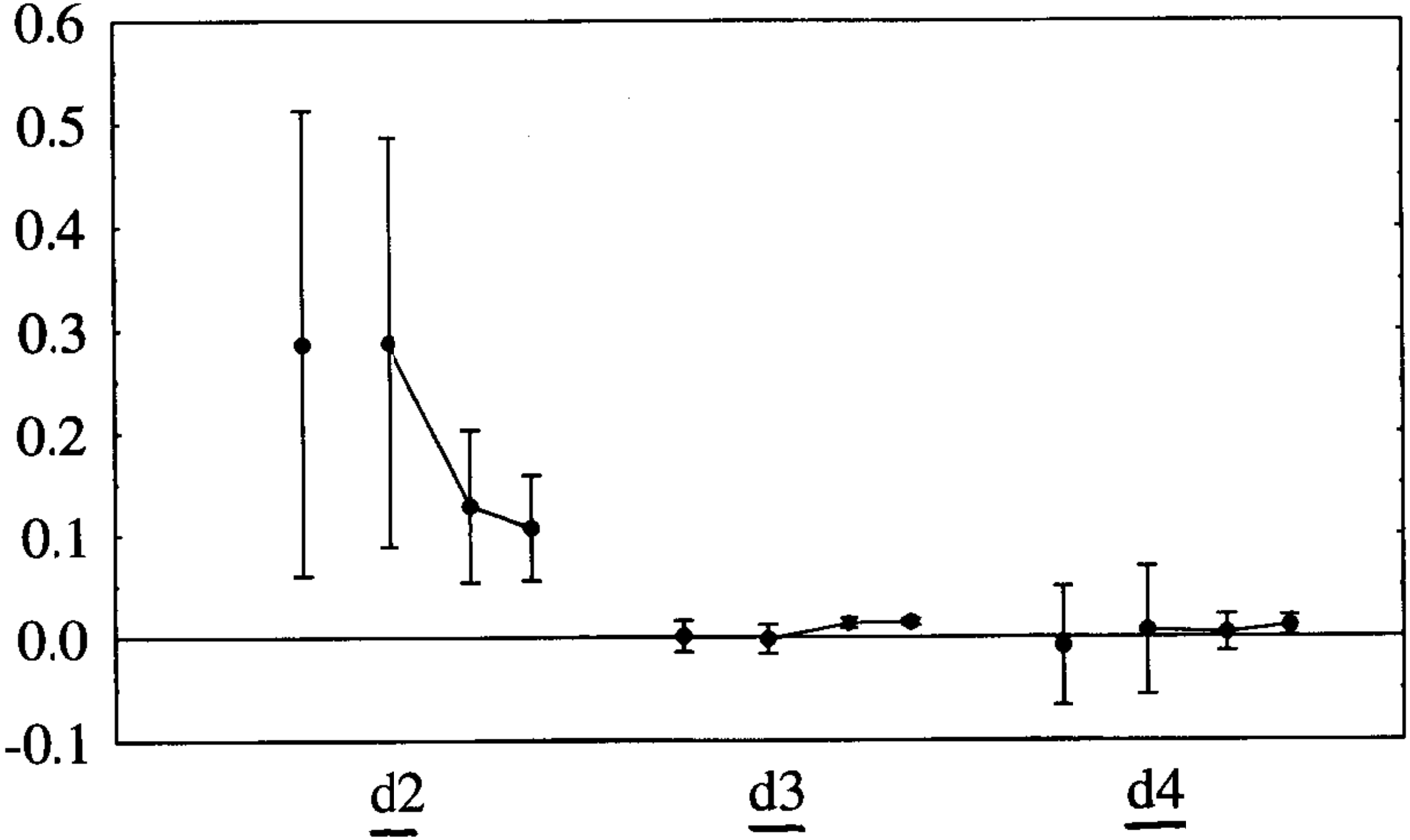
Rho



Rho



$r_n \leftarrow a_{n-1}$



$L_q \neq 0$

Summary & outlook

Precision calculation feasible

By and large agreement with experiment & theoretical considerations

Disagree on $\langle x \rangle_v$ for nucleon & pion

Room for higher twist

Finish ** calculation of valence distribution functions

Gluon distributions, J

Higher twist, renormalons

Dynamical fermions (***) require teraflop computer