QCD-Instantons in $e^{\pm}P$ Scattering

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– DIS '97, Chicago, April 1997 –

1. Introduction

- [redacted] [Belavin *et al.* '75]:
  - Tunnelling transitions **in YM theories, connecting degenerate vacua of different topology;**
  - induce processes forbidden in perturbation theory, **yet bound to exist due to** Adler-Bell-Jackiw **anomalies of certain fermionic quantum numbers** ['t Hooft '76]:

    $B + L$ in QFD; Chirality (Q_5) in (massless) QCD.

  - **suppressed at low energies,** $\sim \exp\{-4\pi/\alpha\}$,

  ♣ **may become unsuppressed, i. e. observable at** high energies **via production of additional gauge bosons** $A^I_\mu \sim 1/g$
  [Ringwald '90; Espinosa '90]

♣ An experimental discovery of such a novel, non-perturbative manifestation of non-abelian gauge theories would clearly be of basic significance.

[illegible]

♣ Experimentally accessible: ⇒ HERA

- QCD-instantons less suppressed than QFD-instantons ($\alpha_s \gg \alpha_W$).
- Unique window to detect QCD-instanton induced processes through their characteristic multi-particle final-state signature
  [Ringwald & F. Sch. '94 '96; Gibbs, Ringwald & F. Sch. '95; Gibbs, Greenshaw, Milstead, Ringwald & F. Sch. '96].
- c.f. instanton talks in WG III $\left\{ \begin{array}{ll} \text{A. Ringwald} & \text{(phenomenology)} \\ \text{T. Carli (H1)} & \text{(exp. searches at HERA)} \end{array} \right.$

♣ Theoretically:

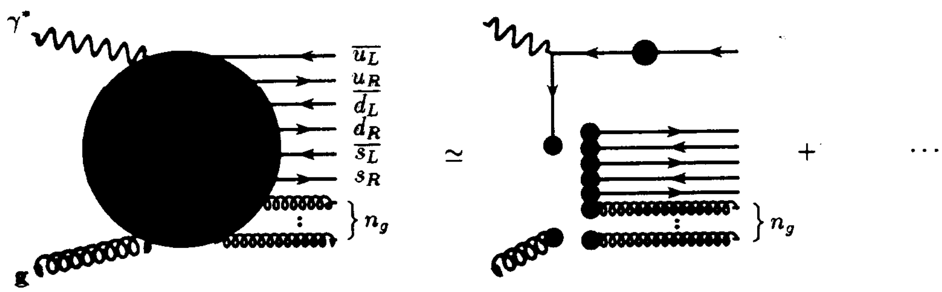
- Possible to make contact with experiment! [Balitsky & Braun '93; Ringwald & F. Sch '94 '96; Moch, Ringwald & F. Sch. '96 and in prog.]
- Absence of generic IR divergencies from integration over I-size ρ, since hard scale $\mathcal{Q}$ provides dynamical cutoff, $\rho \lesssim \mathcal{O}(1/\mathcal{Q})$, .
- Instanton-induced amplitudes well-defined and calculable for small $\alpha_s(\mathcal{Q})$ and fixed scattering angles.
- Inclusive framework to systematically calculate properties of I-induced multi-parton final state; accounts for exponentiation of produced gluons including final-state tree-graph corrections

2. Instanton-Induced Processes in Leading Semi-Classical Approximation

- Consider Instanton-induced amplitude for $\Delta Q_5 = 2\, n_f$ process,

$$\gamma^* + \mathrm{g} \Rightarrow \sum_{\text{flavours}}^{n_f} [\overline{\mathrm{q}_L} + \mathrm{q}_R] + n_{\mathrm{g}}\, \mathrm{g} \ :$$

vanishing to any order of conventional P.T in (massless) QCD:



- Expand (Euclidean) path integral for relevant Green's functions about classical instanton solution.
- Fourier-transform (F. T.) with respect to external lines.
- LSZ amputate external legs.
- Analytically continue to Minkowski space.
- **Basic building blocks:** $\left(\Pi_x \equiv 1 + \frac{\rho^2}{x^2}\right)$,
  - Classical instanton gauge field (singular gauge) [Belavin *et al.* '75]:

$$A_{\mu'}^{(I)}(x) \;=\; -\mathrm{i}\, \frac{2\,\pi^2}{g_s}\, \rho^2\, U \left(\frac{\sigma_{\mu'}\, \overline{x} - x_{\mu'}}{2\,\pi^2\, x^4}\right) U^\dagger\, \frac{1}{\Pi_x},$$

collective coordinates: $\rho = I$-size, $\quad (U)^k_\alpha = I$-color orientation

(color: $k = 1, 2, 3$ and spinor: $\alpha = 1, 2$ indices).

– Quark zero modes in the instanton background (Weyl basis)[1] ['t Hooft '76]:

$$
\begin{aligned}
\kappa^{(I)\,m}{}_{\dot\beta}(x) &= 2\,\pi\,\rho^{3/2}\,\epsilon^{\gamma\delta}\,(U)^m_{\ \delta}\,\frac{\overline{x}_{\dot\beta\gamma}}{2\,\pi^2\,x^4}\,\frac{1}{\Pi_x^{3/2}}\,,\\
\overline{\phi}^{(I)\,\dot\alpha}{}_l(x) &= 2\,\pi\,\rho^{3/2}\,\epsilon_{\gamma\delta}\,\left(U^\dagger\right)^\gamma_{\ l}\,\frac{x^{\delta\dot\alpha}}{2\,\pi^2\,x^4}\,\frac{1}{\Pi_x^{3/2}}\,,
\end{aligned}
$$

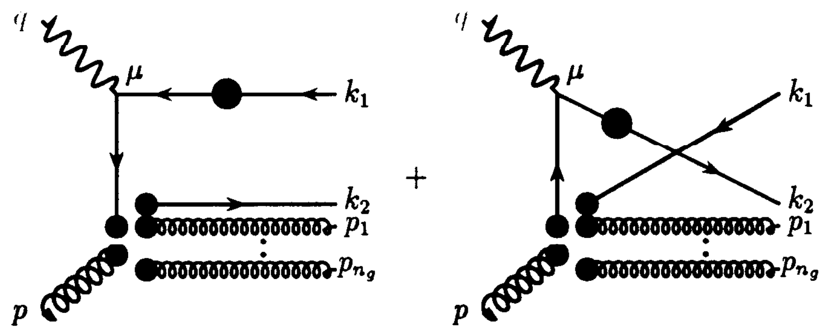
– (Non-zero mode) quark propagators in the instanton background [Brown *et al.* '78]:

$$
S^{(I)}(x,y) = \frac{1}{\sqrt{\Pi_x \Pi_y}}\left[\frac{x-y}{2\pi^2(x-y)^4}\left(1+\rho^2\frac{\left[U x\,\overline{y}\,U^\dagger\right]}{x^2 y^2}\right) + \frac{\rho^2\,\sigma_\mu}{4\pi^2}\,\frac{\left[U x\,(\overline{x}-\overline{y})\,\sigma_\mu\,\overline{y}\,U^\dagger\right]}{x^2(x-y)^2 y^4 \Pi_y}\right],
$$

$$
\overline{S}^{(I)}(x,y) = \cdots
$$

- Leading-order amplitude ($n_f = 1$) in Euclidean space after F.T. with respect to external lines [Moch, Ringwald & F. Sch. '96]:

[1] Standard notations: in Euclidean space, $\sigma_\mu = (-\mathrm{i}\,\vec\sigma, 1)$, $\overline{\sigma}_\mu = (\mathrm{i}\,\vec\sigma, 1)$, and in Minkowski space, $\sigma_\mu = (1, \vec\sigma)$, $\overline{\sigma}_\mu = (1, -\vec\sigma)$, where $\vec\sigma$ are the Pauli matrices. Furthermore, for any four-vector v_μ, we use the shorthand $v \equiv v\cdot\sigma$, $\overline{v} \equiv v\cdot\overline{\sigma}$.



$$\mathcal{T}_{\mu}^{(I)\,a\,a_1\ldots a_{n_g}} = -\mathrm{i}\,e_q\,d\,\left(\frac{2\,\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left[-\frac{2\,\pi}{\alpha_s(\mu_r)}\right]$$

$$\int dU \int_0^{\infty} \frac{d\rho}{\rho^5}\,(\rho\,\mu_r)^{\beta_0} \lim_{p^2\to 0} p^2\,\mathrm{tr}\left[\lambda^a\,\epsilon_g(p)\cdot A^{(I)}(p)\right]$$

$$\prod_{i=1}^{n_g} \lim_{p_i^2\to 0} p_i^2\,\mathrm{tr}\left[\lambda^{a_i}\,\epsilon_g^*(p_i)\cdot A^{(I)}(-p_i)\right]\;\times$$

$$\chi_R^{\dagger}(k_2)\left[\lim_{k_2^2\to 0}\,(\mathrm{i}\not{k}_2)\,\kappa^{(I)}(-k_2)\,\lim_{k_1^2\to 0}\,\mathcal{V}_\mu(q,-k_1)\right.$$

$$\left.+\lim_{k_2^2\to 0}\,\mathcal{V}_\mu^c(q,-k_2)\,\lim_{k_1^2\to 0}\,\overline{\phi}^{(I)}(-k_1)\,(-\mathrm{i}\,\not{k}_1)\right]\chi_L(k_1)\,,$$

- **Instanton-density with renormalization scale μ_r** ['t Hooft '76; Bernard '79],

$$d\,\left(\frac{2\,\pi}{\alpha_s(\mu_r)}\right)^6 \exp\left[-\frac{2\,\pi}{\alpha_s(\mu_r)}\right]\,(\rho\,\mu_r)^{\beta_0}\;;\;\textbf{d known constant}$$

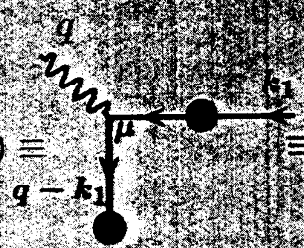
  - is 1-loop RG-invariant; in practice:⇒ 2-loop RG-invariance [Morris, Ross, Sachrajda '85] .

  - strongly IR-divergent if integrated over ρ

- F.T. and LSZ-amputation of instanton gauge field $A_{\mu}^{(I)}$, and quark zero modes κ and $\overline{\phi}$ is straightforward [SVZ '80; Ringwald '90; Espinosa '90],

- LSZ-amputation of current quark in F.T'ed photon-fermion vertices, $\mathcal{V}_\mu(q,-\boldsymbol{k}_1)$, $\mathcal{V}^c_\mu(q,-\boldsymbol{k}_2)$, e.g.

$$\mathcal{V}_\mu(q,-\boldsymbol{k}_1) \equiv$$



$$\equiv \int d^4x\, e^{-iq\cdot x}\left[\overline{\phi}^{(I)}(x)\overline{q}_\rho S^{(I)}(x,-\boldsymbol{k}_1)(-i\boldsymbol{k}_1)\right]$$

quite non-trivial and has important physical consequences.

- After a long and tedious calculation . . .

$$\lim_{\boldsymbol{k}_1^2\to 0} \mathcal{V}_\mu(q,-\boldsymbol{k}_1) = 2\pi i \rho^{3/2}\left[\epsilon\sigma_\mu \overline{V}(q,\boldsymbol{k}_1;\rho)\,U^\dagger\right],$$

$$\lim_{\boldsymbol{k}_2^2\to 0} \mathcal{V}^c_\mu(q,-\boldsymbol{k}_2) = 2\pi i \rho^{3/2}\left[U\,V(q,\boldsymbol{k}_2;\rho)\overline{\sigma}_\mu\epsilon\right],$$

where

$$V(q,\boldsymbol{k};\rho) = \frac{\boldsymbol{k}}{2\boldsymbol{q}\cdot\boldsymbol{k}}\rho\sqrt{q^2}\,K_1\left(\rho\sqrt{q^2}\right) + \left[\frac{(\boldsymbol{q}-\boldsymbol{k})}{(\boldsymbol{q}-\boldsymbol{k})^2} - \frac{\boldsymbol{k}}{2\boldsymbol{q}\cdot\boldsymbol{k}}\right]\rho\sqrt{(q-k)^2}\,K_1\left(\rho\sqrt{(q-k)^2}\right).$$

- The integration over the instanton size ρ in $\mathcal{T}^{(I)}_\mu$ is finite due to the exponential decrease of the form factors in $V(q,\boldsymbol{k};\rho)$,

$$\mathcal{Q}\rho K_1(\mathcal{Q}\rho) \overset{\mathcal{Q}\rho\to\infty}{\to} \sqrt{\frac{\pi}{2}}\sqrt{\mathcal{Q}\rho}\,\exp\left[-\mathcal{Q}\rho\right].$$

- After continuation to DIS-regime of Minkowski space: Hard scale,

$$\mathcal{Q} \equiv \min\left\{Q \equiv \sqrt{-q^2}, \sqrt{-(q-k_1)^2}, \sqrt{-(q-k_2)^2}\right\} \geq 0,$$

provides a dynamical IR cutoff for the instanton size,

$$\rho \lesssim \mathcal{O}(1/\mathcal{Q}).$$

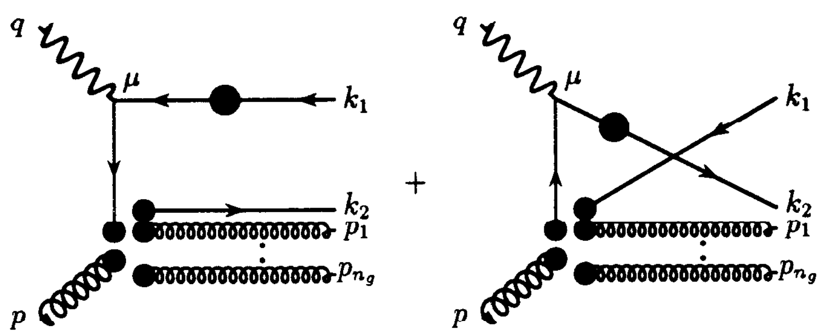
Therefore, deep-inelastic scattering very well suited for studying manifestations of QCD-instantons.

- Next steps:
  - Insert all LSZ-amputated F.T.s. into $\mathcal{T}_\mu^{(I)}$.
  - Continue to Minkowski space.
  - Perform integration over ρ analytically $\Rightarrow$ [redacted]
- Highly non-trivial check: Electromagnetic current conservation,

$$q^\mu \mathcal{T}_\mu^{(I)\, a\, a_1 \ldots a_{n_g}} = 0.$$

- Like in perturbative QCD:

  Leading-order I-induced amplitudes well-behaved as long as we avoid the collinear singularities, arising when the internal quark virtualities vanish, $t \equiv -(q-k_1)^2$ and/or $u \equiv -(q-k_2)^2 \to 0$.

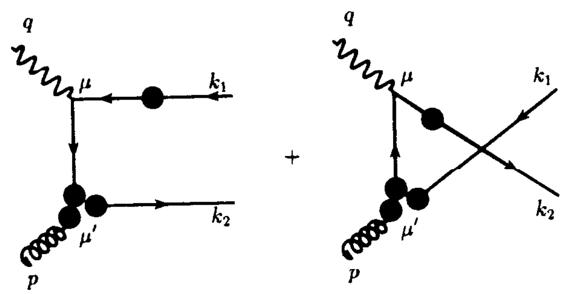


Fixed-angle scattering processes at high Q^2 and moderate multiplicity are reliably calculable in (instanton) perturbation theory.

[redacted]

[Moch, Ringwald & F. Sch. '96]

$$\gamma^* + \mathrm{g} \to \overline{\mathrm{q}_L} + \mathrm{q}_R\ ; \qquad (\triangle Q_5 = 2)$$



May be considered as lower bound on total instanton-induced contribution. Contains all essential features of dominant multi-gluon process.

$$\frac{d\sigma^{(I)}}{dt} = \pi^2\, \frac{x(1-x)}{Q^4}\, \frac{\alpha e_q^2}{2} \mathcal{N}^2 \left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{13} \exp\left[-\frac{4\pi}{\alpha_s(\mu_r)}\right] \left(\frac{\mu_r^2}{Q^2}\right)^b$$

$$\left[\left(\frac{Q^2}{-t}\right)^{b+1} + \left(\frac{Q^2}{-u}\right)^{b+1} + 2tu \frac{\left(\left(\frac{Q^2}{-t}\right)^{\frac{b+1}{2}} - 1\right)\left(\left(\frac{Q^2}{-u}\right)^{\frac{b+1}{2}} - 1\right)}{(t+Q^2)(u+Q^2)}\right],$$

$$\mathcal{N} \equiv \sqrt{\frac{2}{3}}\,\pi^2\, d\, 2^b\, \Gamma\left(\frac{b+1}{2}\right) \Gamma\left(\frac{b+3}{2}\right)\ ; \qquad x = \frac{Q^2}{2p\cdot q}$$

$$b \equiv \beta_0 + \frac{\alpha_s(\mu_r)}{4\pi}\,(\beta_1 - 12\,\beta_0)$$ $\Leftrightarrow$ 2-loop RG improvement of I-density.

- Dependence on renormalization scale μ_r very weak!

- Validity of approximation:

  Average instanton size $\langle\rho\rangle$ contributing for a given virtuality $\mathcal{Q}$,

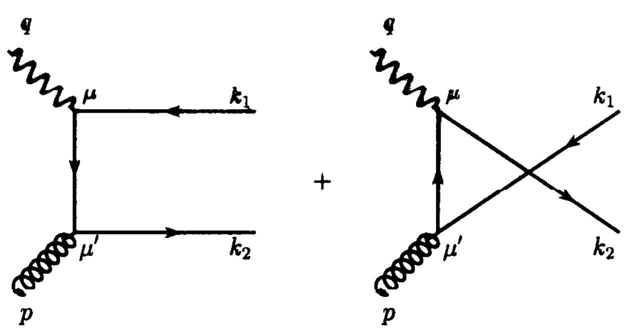
$$\langle\rho\rangle \equiv \frac{\int_0^\infty d\rho\,\rho\,\rho^{\beta_0+1}\,K_1(\rho\,\mathcal{Q})}{\int_0^\infty d\rho\,\rho^{\beta_0+1}\,K_1(\rho\,\mathcal{Q})} \simeq \frac{\beta_0+3/2}{\mathcal{Q}}\,.$$

  Should be small, in order that higher-order corrections of I-pert. theory can be neglected;
  [Callan et.al '78, Andrei & Gross '78, Appelquist & Shankar '78, SVZ '80]

$$\langle\rho\rangle \lesssim \frac{1}{500\ \mathrm{MeV}} < \frac{1}{\Lambda} \quad \Rightarrow \quad \mathcal{Q} \gtrsim 5\ \mathrm{GeV}.$$

- Compare with the appropriate leading-order perturbative QCD, chirality-conserving process,

$$\gamma^* + \mathrm{g} \to \overline{\mathrm{q}_L} + \mathrm{q}_L\ ; \qquad (\Delta Q_5 = 0)\ .$$

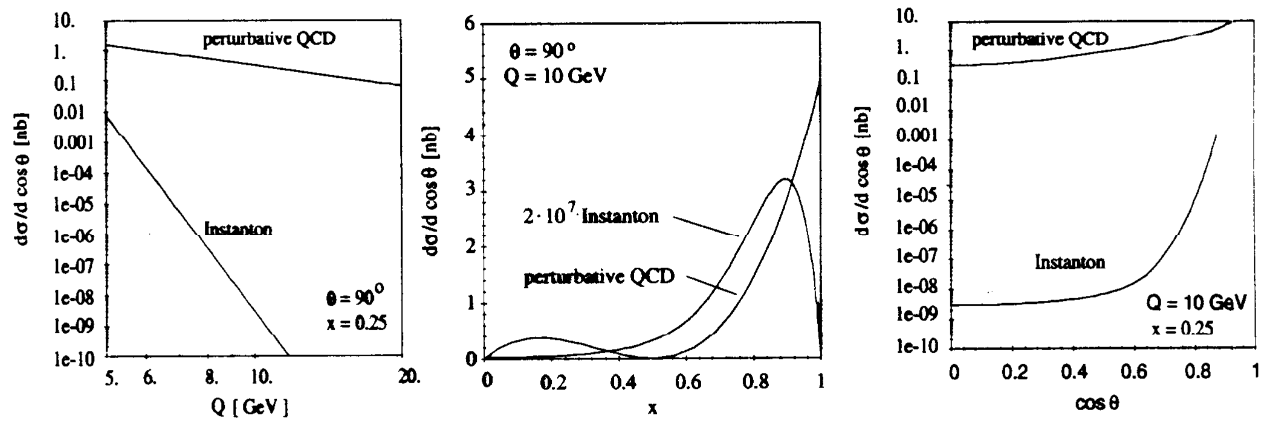


- Compare fixed-angle **differential cross-sections**,

$$\frac{d\sigma}{d\cos\theta}(Q, x, \cos\theta)\,, \quad \textbf{with scattering angle } \theta \textbf{ in } \gamma^* g \textbf{ c.m.s.}$$

Parameters: $e_q = 2/3$, $\Lambda = 234$ MeV, $\mu_r = Q$.

- Result



- Fixed-angle scattering at large enough Q^2 well under control **(left and middle Fig.)**.

- Collinear singularity ($\cos\theta \to 1$) for instanton case much stronger than perturbative one **(right Fig.)**.

- [illegible] [Moch, Ringwald & F. Sch., in prog.]
Study hard photoproduction limit, **with large γ^* virtuality Q^2 substituted by** large k_T of produced q-jet,

$$Q^2 \to 0, x \to 0, \frac{Q^2}{x} \to s \text{ (large)}, k_T = \frac{\sqrt{s}}{2}\sin\theta \text{ large}$$

3. Inclusive Approach to Multi-Particle Final State

- [redacted] Best theoretical framework for making contact with Experiment (HERA) without upsetting validity of I-P.T.?

- Best bet experimentally: hunting for I-"footprints" in multi-particle final state [Ringwald & F. Sch. '94 '96].

- **Momentum space picture** $\Rightarrow$ Control over I-approximations **also at** small $\{x_{Bj}, Q^2\}$ **through** kinematical cuts **on (reconstructed)** final-state momentum variables!

[redacted]

a) **Exclusive, "brute-force" method:** Multi-parton cross-sections by squaring $\gamma^* + \mathrm{g} \Rightarrow \sum_{\text{flavours}}^{n_f} [\overline{\mathrm{q}_L} + \mathrm{q}_R] + n_\mathrm{g}\, \mathrm{g}$ amplitudes from Sect.2 and summing over unobserved partons.

b) Prefer inclusive approach: Based on [Ringwald & F. Sch. '94 '96, Moch, Ringwald & F. Sch. in prog.]

Systematic characterization of final state in terms of $\sigma_{\text{tot}}^{(I)}$ and 1,2,... parton inclusive cross sections.

- Elegant, implicit summation over exponentiating leading order gluon emission and gluonic final-state tree-graph corrections.

optical theorem: $\sigma_{tot}^{(I)}$ ⇔ Disc{[illegible]}

Mueller- optical "theorem" [A.H. Mueller [illegible]]

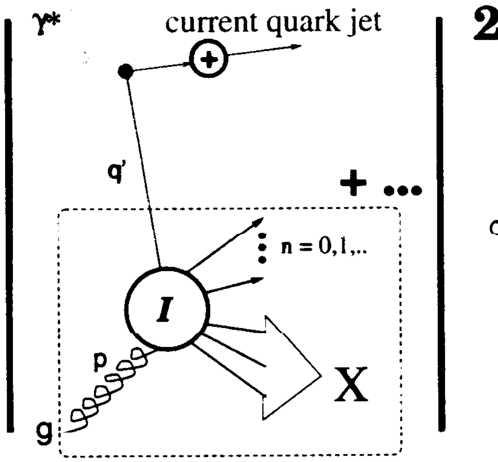
n=1,2,... particle inclusive cross sections ⇔ Disc{[illegible]$_{2+n}$}

⊕ $I\bar{I}$-valley method [Yung '88, Khoze & Ringwald '91, ...]

⊕ saddle-point integration over collective $I\bar{I}$ coordinates.

- Reduction to I-induced subprocess via

$$d\sigma^{\text{incl.}(I)}_{\gamma^* g \to \{n+1\}+X} \sim$$



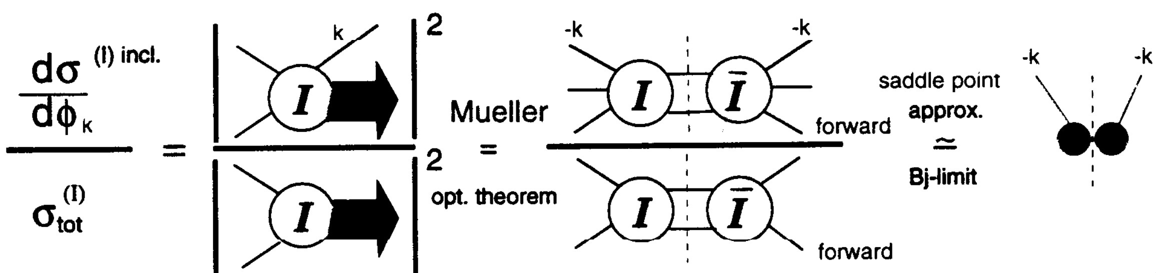
Instanton-induced subprocess

Bjorken variables: x', Q'^{2} = -q'^{2}

$$\propto \frac{\alpha}{Q^2}\int_x^1 \frac{dx'}{x'}\int^{Q^2\frac{x'}{x}} dQ'^2 P^{(I)}_{q^*/\gamma^*}\, d\sigma^{\text{incl.}(I)}(q^* g \to \{n\}+X)$$

- [redacted]

  – 1-parton inclusive ($n = 1$):

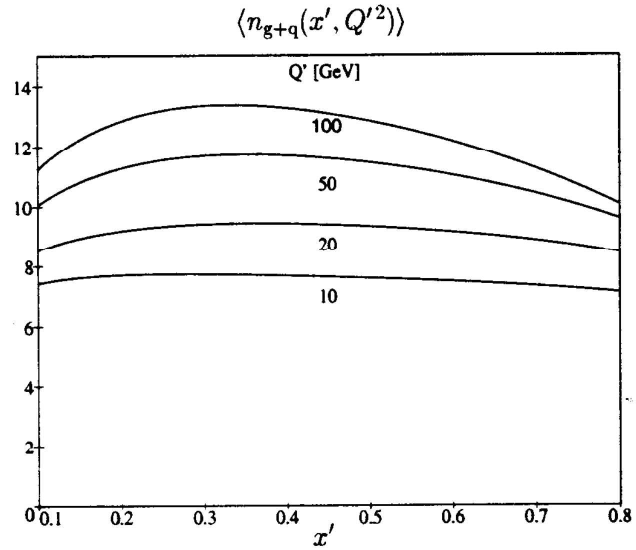


$I\bar{I}$ valley method

Product of amp'ed, FT'ed fields in bg. of $I\bar{I}$ valley configuration

Output:

[redacted]



– 2-parton inclusive $(n = 2)$:

  $*$ $\langle n^2 \rangle - \langle n \rangle^2 \Rightarrow \lim_{Q'^2 \to \infty} \sigma_n{}^{(I)\ \text{excl.}} = \text{Poisson}$

  $*$ momentum correlations, ...

- Highly [redacted]

[De Tar, Freedman & Veneziano '71; L. Brown '71]

[redacted]

e.g.

$$\sum_{\text{q,G}} \int d\phi_k \; k_\mu \; \frac{1}{\sigma_{\text{tot}}^{(I)}} \frac{d\sigma^{(I)}}{d\phi_k} \;=\; (p + q')_\mu$$

– $\sim$ o.k. in Bjorken limit $(Q'^2 \to \infty$, x' fixed$)$

[redacted]

4. Conclusions

- Distinguished rôle of $e^{\pm}P$ scattering involving hard momentum scale $\mathcal{Q}$.

- **Experimentally:**
  HERA offers unique window in DIS (and hard photoproduction?) to detect I-induced events through their characteristic multi-particle final-state signature.

- [illegible]

  - Absence of IR divergencies associated with integration over I-size ρ.

  - Chirality-violating Amplitudes well-defined and calculable for small $\alpha_s(\mathcal{Q})$ and fixed scattering angles.

  - Inclusive framework to systematically calculate properties of I-induced multi-parton final state;

    accounts for exponentiation of produced gluons including final-state tree-graph corrections:

    * Optical theorem and Mueller-optical theorems, to express various I-induced inclusive cross sections as discontinuities of elastic n $\rightarrow$ n forward amplitudes $T^{(I)}_{n\rightarrow n}$

    * Uniform evaluation by means of $I\bar{I}$ valley method and saddle-point integration over collective coordinates.

    * Stable results, consistent with stringent energy-momentum sum rules.