Threshold Resummation for Heavy Quark Production at Hadron Colliders

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E. L. Berger and H. Contopanagos
Phys. Lett. B361, 115 (1995)
Phys. Rev. D54, 3085 (1996)
& hep-ph/9610473

1. Inclusive Cross Section for $p\bar{p} \to t\bar{t}X$

$$\sigma^{t\bar{t}}_{p\bar{p}}(s) = \frac{4m^2}{s} \sum_{ij} \int_0^{\eta_{\max}} d\eta \; \Phi_{ij}\,(\eta)\hat{\sigma}^{t\bar{t}}_{ij}(\eta).$$

$m =$ top quark mass

distance from parton threshold $\boxed{\eta = \frac{\hat{s}}{4m^2} - 1.}$

i, j label parton types: $q\bar{q}, gg$

Parton flux

$$\Phi_{ij}(\eta) = \int_{x_{\min}}^{1} \frac{dx}{x} f_{i/p}\,(x)\; f_{j/\bar{p}}\,(x_{\min}/x)$$

$$x_{\min} = \frac{4m^2}{s}(1+\eta)$$

$f_{i/p}$: parton densities

Partonic hard cross sections $\hat{\sigma}^{t\bar{t}}_{ij}$ are computed from QCD perturbation theory; power series in α_s.

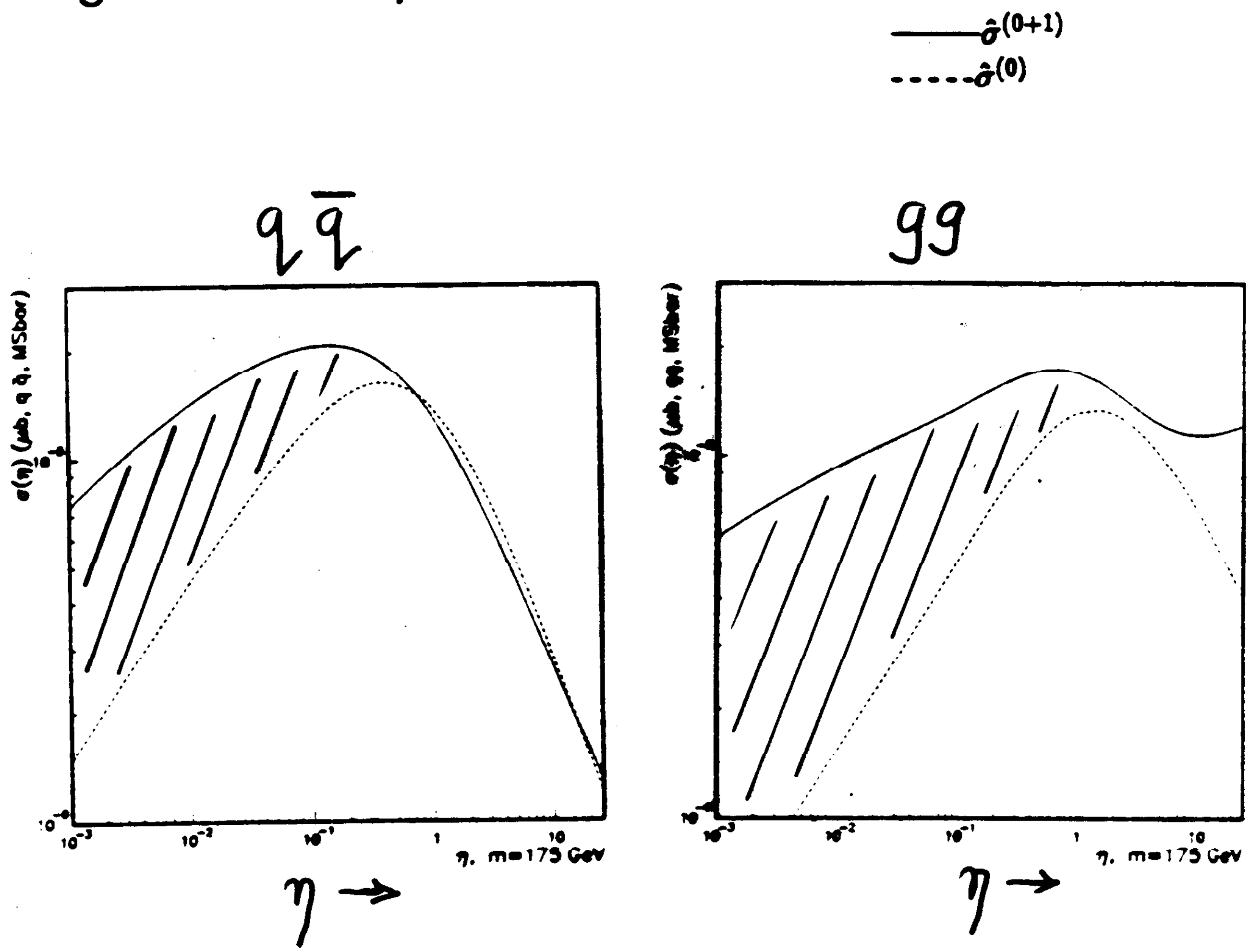
$$\hat{\sigma} = \hat{\sigma}^{(0)}_{ij} + \hat{\sigma}^{(1)}_{ij} + \cdots$$

$\hat{\sigma}^{(0)}$ is $O\left(\alpha_s^2\right)$ and $\hat{\sigma}^{(1)}$ is $O\left(\alpha_s^3\right)$

[$\hat{\sigma}^{(n\geq 2)}_{ij}$ have not been computed for $Q\bar{Q}$ production.]

Diagrams

Ratio of the partonic cross sections $\hat{\sigma}^{(0+1)}/\hat{\sigma}^{(0)}$ is large in some kinematic regions, notably the near-threshold region of small η



- Shading shows enhancement of next-to-leading order $\hat{\sigma}$ over lowest order $\hat{\sigma}$.
- K factor at parton level: $\hat{K}^{(n)}(\eta) \rightarrow \alpha^n \ell n^{2n}(\eta)$

- Near-threshold region is significant for t quark production at $\sqrt{s} = 1.8$ TeV; $2m_t/\sqrt{s} \simeq 0.2$

- After integral over η, ratio of the physical cross sections
$$\frac{\sigma_{\mathrm{NTLO}}}{\sigma_{\mathrm{BORN}}} \cong 1.25$$
at $m = 175$ GeV and $\sqrt{s} = 1.8$ TeV

- To obtain more reliable estimates in perturbative QCD, it is important
  - to identify/isolate the terms that provide the large next-to-leading order contributions and
  - to try to resum their effects to all orders in perturbation theory.

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2. Gluon Radiation and Resummation

- Origin of large threshold contributions in $O\left(\alpha_s^3\right)$.

After cancellation of soft singularities and factorization of collinear singularities, there are left-over logarithms associated with initial-state bremsstrahlung: $\ell n(1-z)$

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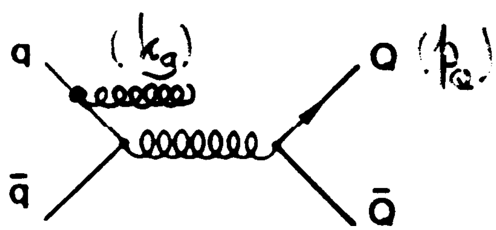
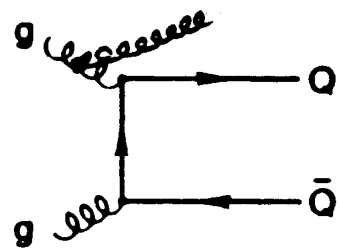




$(1-z) \equiv \frac{2k_g \cdot p_Q}{m^2}$

$z \rightarrow 1$: zero gluon momentum

[initial-state brem much larger than final-state brem – massless vs. massive propagator.]

- Work in the $\overline{\text{MS}}$ factorization scheme

Near threshold,

$$\hat{\sigma}_{ij}(\eta) \simeq \int_{z_{\min}}^{1} dz \{1 + 2\alpha C_{ij}\, \ell n^2 (1-z) + \alpha^2 [2 C_{ij}^2\, \ell n^4 (1-z) - \frac{4}{3} C_{ij}\, b_2\, \ell n^3 (1-z)]\} \times \frac{d}{dz} \sigma_{ij}^{(0)} (\eta, z) .$$

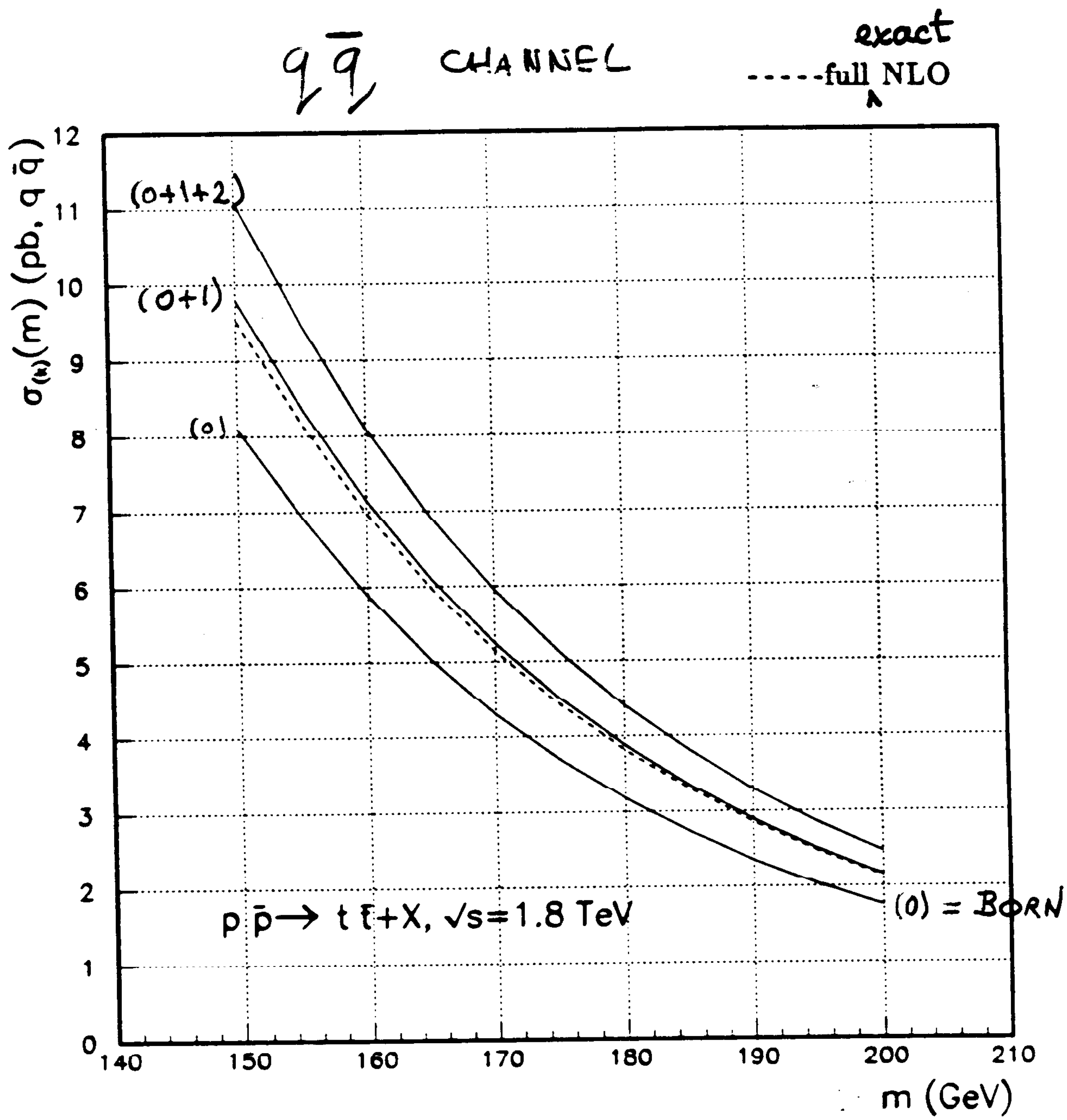
$$\alpha = \frac{\alpha_s}{\pi}; z_{\min} = 1 - 4(1+\eta) + 4\sqrt{1+\eta}$$

The $\alpha C_{ij}\, \ell n^2 (1-z)$ term is known explicitly from $O(\alpha_s^3)$ results for $ij \to Q\bar{Q}X$. The $O(\alpha^2)$ terms are taken over from $O(\alpha^2)$ calculations of massive lepton pair production (Drell-Yan process). Universality assumption for leading log terms.

Goal of gluon resummation for $\sigma(t\bar{t})$ is to sum the series in $\alpha^n\, \ell n^{2n} (1-z)$ to all orders. Procedure studied extensively for Drell-Yan.

Sterman(1987)
Catani and Trentadue(1989)
Appell, Mackenzie, and Sterman (1988)
Contopanagos and Sterman (1993)
Alvero and Contopanagos (1995)

Effect of the successive addition of $O(\alpha^n)$ leading logarithms on the physical cross section



at $m = 175\mathrm{GeV}$

$$\frac{\sigma^{(0+1)}}{\sigma^{(0)}} = 1.22; \quad \frac{\sigma^{(0+1+2)}}{\sigma^{(0+1)}} = 1.14$$

3. Perturbative Resummation

- Factorization and evolution lead to exponentiation in moment space (n). Sterman (1987), Catani & Trentadue (1989)
- In moment space, the exponent that resums the $\ell n^{2j}(1-z)$ terms is (principal value prescription)

Contopanagos & Sterman (1993)

$$E_{PV}(n, m^2) \equiv - \int_P d\zeta \frac{\zeta^{n-1} - 1}{1 - \zeta} \int_{(1-\zeta)^2}^{1} \frac{d\lambda}{\lambda} g\left[\alpha\left(\lambda m^2\right)\right]$$

$g(\alpha)$ is calculable perturbatively

  - E_{PV} is finite: Landau pole singularities in $\alpha\left(\lambda m^2\right)$ are by-passed by Cauchy principal-value prescription
  - All large threshold corrections are included by 2-loop running of α

- To retain only the perturbative content, we use a perturbative truncation of E_{PV}

Our perturbative truncation of the full exponent, restricted to leading logarithms only, is ($x = \ln n$)

$$E(x,\alpha,N) = 2C_{ij} \sum_{\rho=1}^{N(t)+1} \alpha^\rho s_\rho x^{\rho+1}$$

$$s_\rho = b_2^{\rho-1} 2^\rho / \rho(\rho+1)$$

$$C_{q\bar{q}} = C_F, \quad C_{gg} = C_A$$

$$\alpha \equiv \frac{\alpha_s(m)}{\pi} = \frac{1}{b_2 \ln(m^2/\Lambda^2)} - \frac{b_3}{b_2^3} \frac{\ln(\ln(m^2/\Lambda^2))}{\ln^2(m^2/\Lambda^2)}$$

$$b_2 = (11C_A - 2n_f)/12$$

$$t \equiv \frac{1}{2\alpha b_2} \sim \ln(m/\Lambda)$$

* It is valid in the moment-space interval

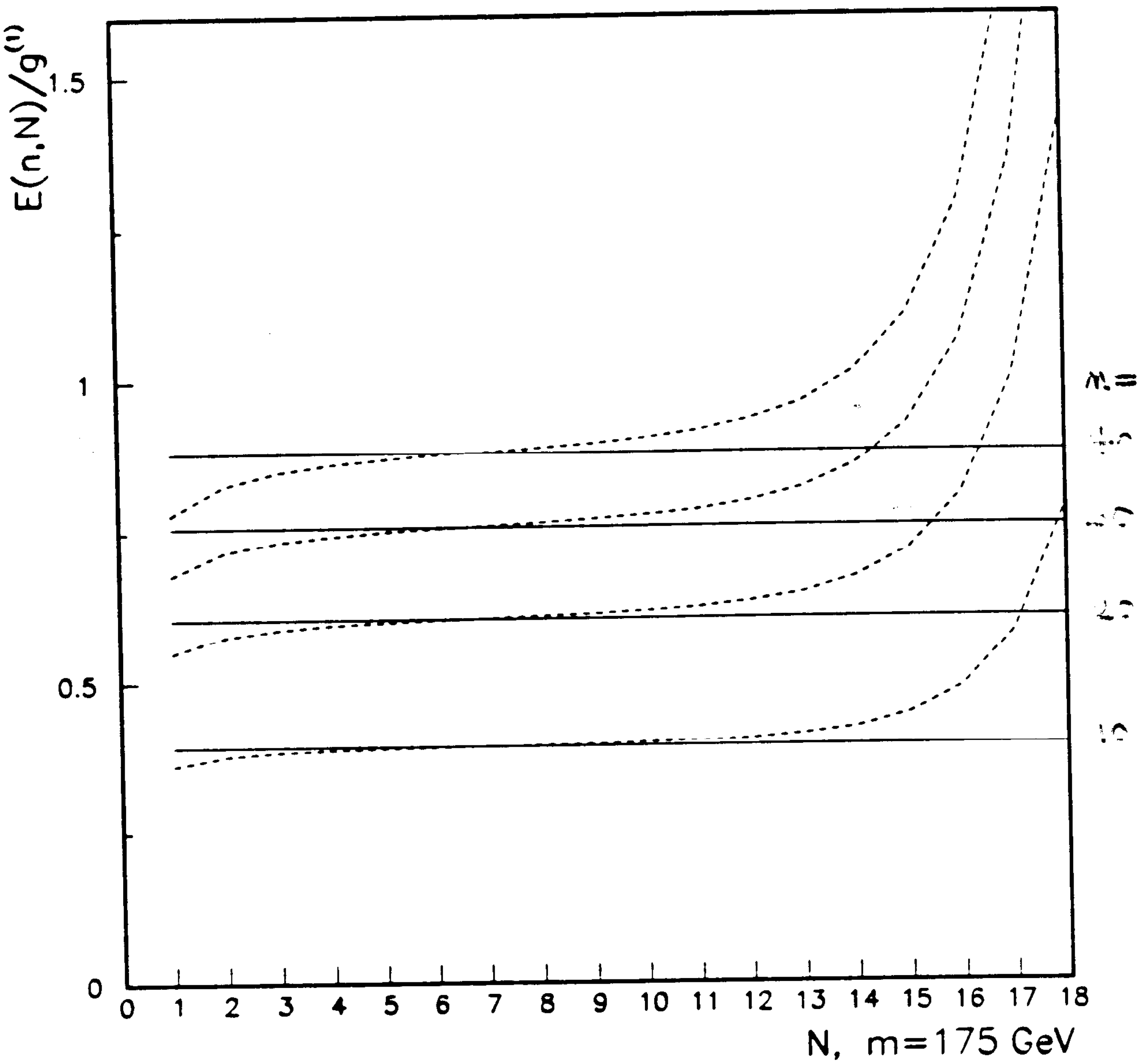
$$1 < x \equiv \ln n < t.$$

* $N(t)$ is a function of m, only, in the perturbative domain.

$N = 6$ at $m \simeq 175$ GeV

This expression contains no factorially growing (renormalon) terms. The perturbative region is far removed from the part of phase space in which renormalons could be influential.

Principal value exponent (—) and perturbative truncation (- - -)



Inversion of the Mellin transform to go from moment (n) to momentum space:

$$\hat{\sigma}_{ij}^{PV}(\eta, m^2) = \int_{z_{\min}}^{1} dz\, [1 + H_{ij}(z, \alpha)]\, \frac{d}{dz} \hat{\sigma}^{(0)}(\eta, z)$$

$$H_{ij}(z, \alpha) = \int_{0}^{\ell n(\frac{1}{1-z})} dx\, e^{E_{ij}(x,\alpha)} \sum_{j=0}^{\infty} Q_j(x, \alpha).$$

$E_{ij}(x, \alpha)$: polynomial in x

Q_j: functions produced by analytical inversion of Mellin transform, expressed in terms of derivatives of E, $P_k(x, \alpha)$

P_k: $\partial^k E(x, \alpha)/k!\partial^k x$

Q_j: starts from order α; Q_j contributes j more powers of α than of $\ell n(1 - z)$

- Domain of applicability of perturbative resummation in z and η
  - In moment space (n): perturbative truncation is valid for $1 < \ln n < t = 1/(2\alpha b_2)$; this interval agrees with the intuitive definition of the perturbative region, where inverse power terms are unimportant: $\frac{\Lambda}{(1-z)m} \leq 1$ $\Rightarrow 2b_2\,\alpha \ln n < 1$.
  - In moment space, we also keep only the leading logarithmic terms, $\alpha^\delta \ln^{\delta+1} n$, – those known to be universal.
  - In momentum space, after the Mellin transform, a descending series of subleading logarithms is generated, $Q_j \sim \alpha^{j+k} \ln^k(1-z)$.
  - To be consistent with leading log resummation, all $Q_j, j \geq 1$ must be negligible compared to Q_o. $\Rightarrow$ restriction on z.
  - Another way to motivate this restriction: the integral over z should not extend into the region in which unknown subleading terms could be important quantitatively.
  - Upshot: calculable threshold cutoffs on the z and η integrals.

Cutoffs restrict perturbative resummation to

$$z \leq z_{max} < 1 \text{ and } \eta \geq \eta_{pert}$$

determined by $(\partial E(x, \alpha)/\partial x) \leq 1$.

- Perturbative domain is smaller in gg than $q\bar{q}$ case because $C_{gg} > C_{q\bar{q}}$

@m = 175 GeV

$\eta^{q\bar{q}}_{pert} \simeq 0.007; \qquad \eta^{gg}_{pert} \simeq 0.05$

$\left(\sqrt{\hat{s}} - \sqrt{\hat{s}_{thresh}}\right) = \overset{1.22}{q\bar{q}}, \overset{8.64}{gg}$ GeV

NOTE: 1.2 GeV IS REASONABLE PHYSICAL PERTURBATIVE BOUNDARY, COMPARABLE TO Γ_t.

4. Calculations at $\sqrt{s} = 1.8$ TeV and 2.0 TeV

Physical

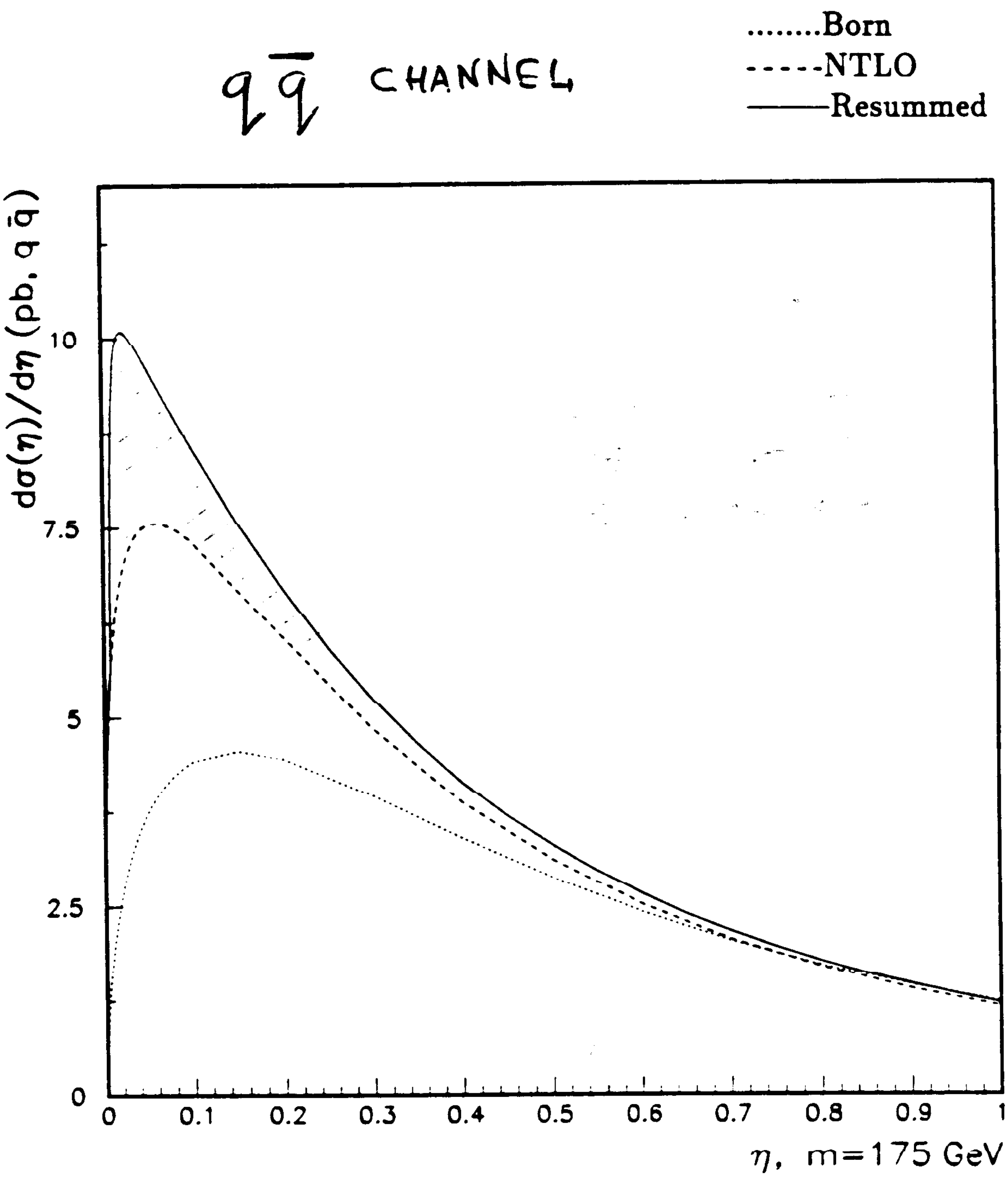
a. ~~Partonic~~ cross sections: $q\bar{q}$, gg vs. η

b. Physical cross sections for two channels vs. m and vs. QCD renormalization/factorization scale μ

c. Calculated cross section at $m = 175$ GeV with range of theoretical uncertainty

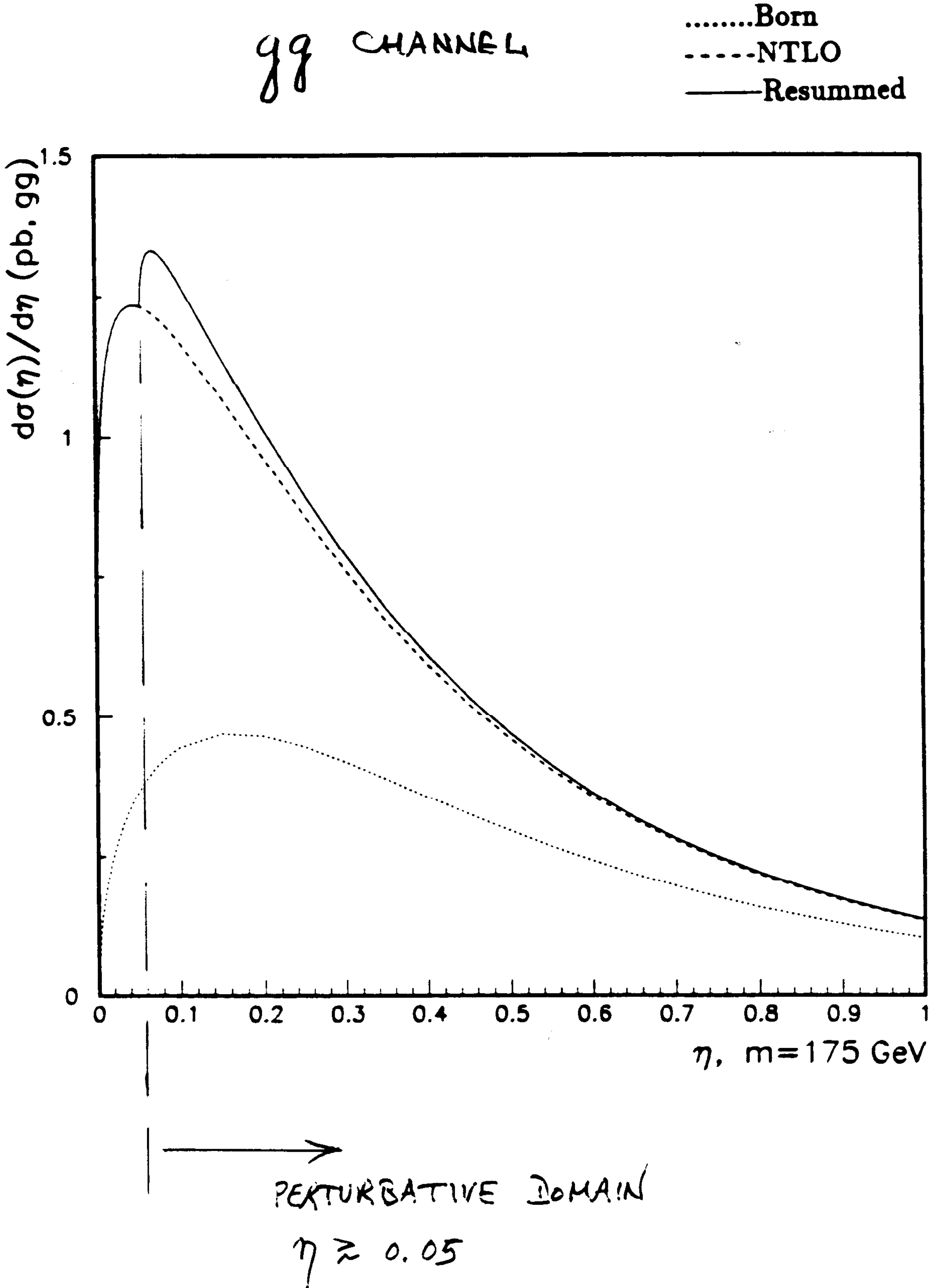
Differential physical cross sections $\frac{d\sigma_{q\bar{q}}(\eta)}{d\eta}$; $\frac{d\sigma_{gg}(\eta)}{d\eta}$

$q\bar{q}$ CHANNEL

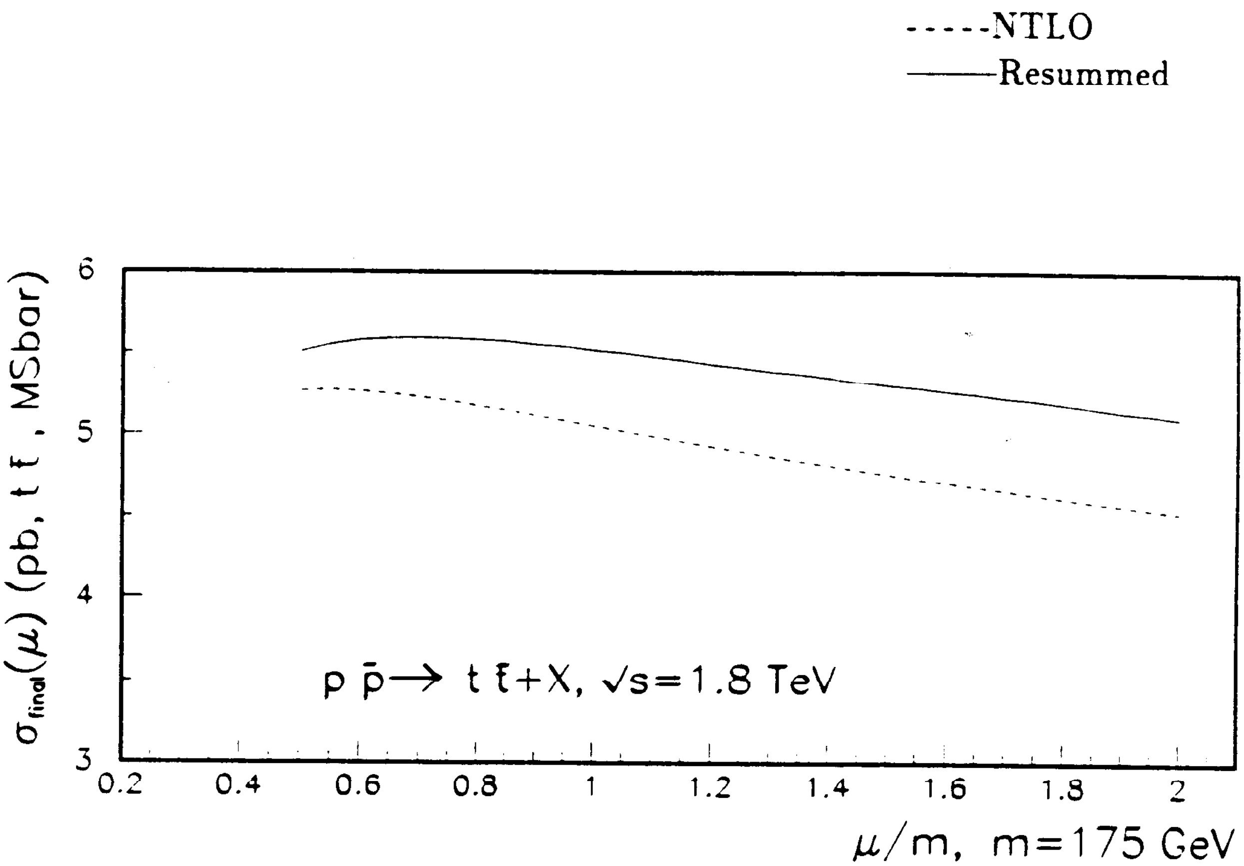


(PERTURBATIVE DOMAIN $\eta \gtrsim 0.007$)

Differential cross sections $\frac{d\sigma_{q\bar{q}}(\eta)}{d\eta}$; $\frac{d\sigma_{gg}(\eta)}{d\eta}$



Factorization/renormalization scale dependence, after addition of $q\bar{q}$ and gg contributions; $\sqrt{s} = 1.8$ TeV



Take variation of $\sigma_{t\bar{t}}$ in range $0.5 < \frac{\mu}{m} < 2.0$ as measure of perturbative uncertainty.

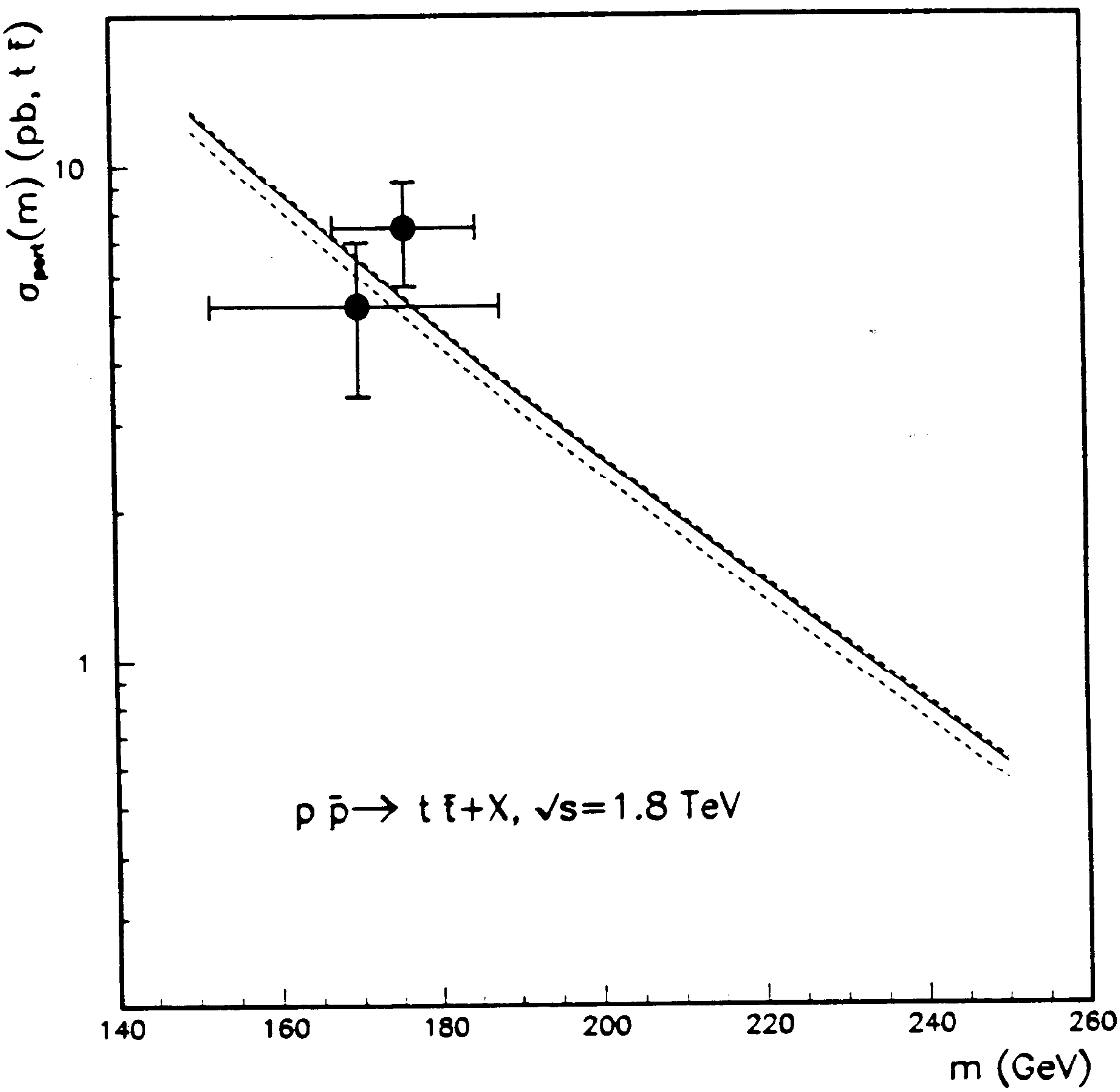
~ 10 % SPREAD

$\sigma^{res} \simeq 10\%$ GREATER THAN σ^{NLO}

AT M = 175 GeV

Comparison with 1996 data.
Curves for 3 values of μ

—— $\mu = m$



$$\sigma^{t\bar{t}}(m = 175\ \mathrm{GeV}) = 5.52^{+0.07}_{-0.42}\ pb$$

$\sigma_{CDF} = 7.5 \pm 1.8\ pb$ $\quad$ $m = 176 \pm 9$ GeV

$\sigma_{D0} = 5.2 \pm 1.8\ pb$ $\quad$ $m = 170 \pm 15 \pm 10$ GeV

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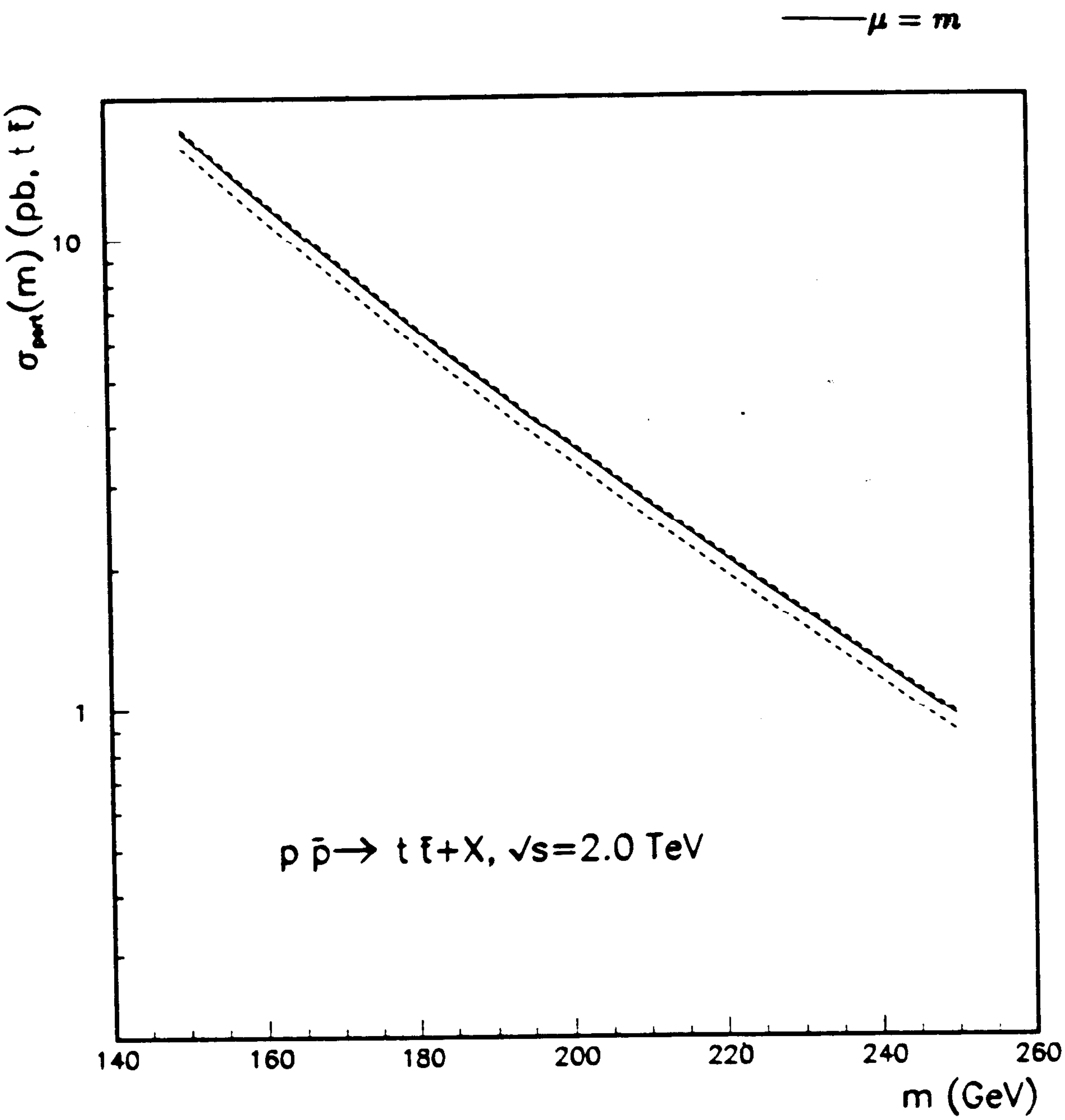
Prediction at $\sqrt{s} = 2.0$ TeV



$$\sigma^{t\bar{t}}(m = 175 \text{ GeV}) = 7.56^{+0.10}_{-0.55}\ pb$$

– Extend calculation to very large m at $\sqrt{s} = 1.8$ TeV:

m(GeV)	$\sigma^{res}/\sigma^{NLO}$	
500	1.21	
600	1.26	hep-ph/9606421
700	1.34	

The numbers reflect the increasing importance of the near-threshold region as m increases.

$\Rightarrow\sim$ 25% of "excess" jet cross section at large p_T seen by CDF may be explained

– Growth of the top cross section with $\sqrt{s}$ at $m = 175$ GeV ($p\bar{p}$ collisions):

$\sqrt{s}$ (TeV)	$\sigma(pb)$
1.8	5.52
2.0	7.56
3.0	22.40
4.0	46.0

Numerical comparison with other calculations

$$\sigma^{t\bar{t}}(m = 175\ GeV)@\sqrt{s} = 1.8\ TeV$$

BC [1]	$5.52^{+0.07}_{-0.42}\ pb$
LSvN [2]	$4.95^{+0.70}_{-0.42}\ pb$
CMNT [3]	$4.75^{+0.63}_{-0.68}\ pb$

- All agree within their estimated uncertainties, but

- Resummation methods differ, methods for estimating uncertainties differ, and parton sets differ

[1] Berger & Contopanagos, Phys. Rev. D54, 3085 (1996).

[2] Laenen, Smith & vanNeerven, Phys. Lett. **B321**, 254 (1994).

[3] Catani, Mangano, Nason, & Trentadue, Phys. Lett. B378, 329 (1996).

BC *vs.* CMNT Similarities and Differences

- Both use the same universal leading log expression in moment space
- Differences occur in the transformation to momentum space
- The Mellin transform to momentum space generates subleading logs – CMNT keep all of these. BC keep only the universal leading logs and restrict phase space to the region where they would not be numerically significant regardless.
  - The subleading terms are negative and numerically significant in the CMNT approach ⇒ explains smaller effects of resummation in their case.

BC: Berger & Contopanagos, Phys. Rev. D54, 3085 (1996).
CMNT: Catani, Mangano, Nason, & Trentadue, Phys. Lett. B378, 329 (1996).

- Examine the expansion in α_s in momentum space, after transformation:

$$1 + \alpha\left(g_1\, x^2 \; + 2\gamma_E\, g_1\, x\right) + O(\alpha^2)$$

$$x = \ell n(1-z); \quad g_1 = 2C_{ij}$$

  - BC retain only the leading log, $g_1\, x^2$
  - The subleading term, $2\gamma_E\, g_1\, x$, is not universal. It is not the same as the exact $O(\alpha_s)$ answer. It can be changed, arbitrarily, if one keeps non-leading terms in moment space. CMNT keep the $2\gamma_E\, g_1\, x$ term.
  - The subleading term is negative and numerically significant ($2\gamma_E\, g_1\, x$ kills $\geq$ half of $g_1\, x^2$).
  - The influence of subleading logs is amplified at $O(\alpha^2)$.
  - Numerically significant subleading log contributions mean that non-universal structures are not under control.

- Further justification for retaining only the $\alpha g_1\, x^2$ term is that it approximates the exact NLO result very well. The CMNT choice does not ($\sim 1/3$).

BC: **Berger & Contopanagos, Phys. Rev. D54, 3085 (1996).**
CMNT: **Catani, Mangano, Nason, & Trentadue, Phys. Lett. B378, 329 (1996).**

– Threshold resummation is applicable to other processes in which the mass or p_T produced is large relative to $\sqrt{s}$. Examples: hadronic jets at very large p_T at collider energies; squark and gluino production

– All are "two-scale" problems

- large perturbative scale: m or p_T
- ratio of two large scales: $m, \sqrt{s}$

$$\eta \simeq \frac{s}{4m^2} - 1 \simeq \frac{1}{2}(1-z) \to 0$$

- large logarithm: $\ell n(1-z)$

– Complementary region, $\frac{m}{\sqrt{s}}$ or $\frac{p_T}{\sqrt{s}}$ small, is important for, e.g., b quark production at the Tevatron. Fixed-order QCD is not adequate for the gg subprocess at large η. (Collins & Ellis)

5. Summary

- Calculation of top quark production in p QCD including resummation of initial-state gluon radiation to all orders in α_s. Should be more reliable than a fixed-order calculation.

- Advantages of the perturbative resummation method are that there are no arbitrary infra-red cutoffs, and there is a well- defined perturbative region of applicability where nonuniversal subleading logs are numerically suppressed

- For $m \simeq 175$ GeV, cross section at $\sqrt{s} =$ 1.8 TeV is increased by $\sim$10% with respect to NTLO QCD calculation; greater increase for larger masses.

- $\sigma^{t\bar{t}}\,(m = 175 \text{ GeV}) = 5.52^{+0.07}_{-0.42}\, pb$ @ 1.8 TeV

- Agreement with data within experimental uncertainties

- $\sim$37% increase @ 2.0 TeV