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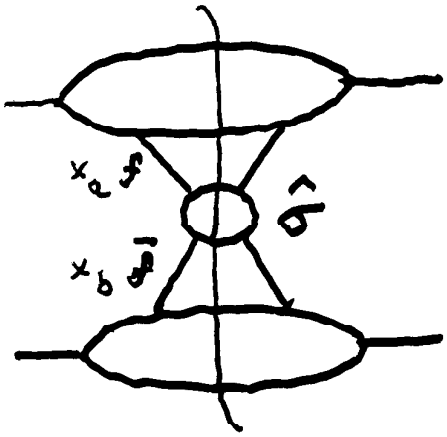
THRESHOLD RESUMMATION FOR QCD HARD SCATTERING

$$\frac{d\sigma_{h_1 h_2 \to Q\bar{Q}}}{dQ^2\, dy\, d\cos\theta} = \phi_{f/h_1} \otimes \hat{\sigma}_{f\bar{f}\to Q\bar{Q}} \otimes \phi_{\bar{f}/h_2}$$

Dawson, Nason, Ellis

Beenaker et al 1 loop



$$\hat{\sigma} = \hat{\sigma}\left(1 - \frac{Q^2}{x_a x_b s}\right) \equiv \hat{\sigma}(1-z)$$

$$\alpha_s^n \left[\frac{\ln^{2n-1}(1-z)}{1-z}\right]_+ + \cdots$$

resum?

DY – S., Catani-Trentadue exponentiation in moment space

$\hat{\sigma}$ from

$$d\sigma_{q\bar{q}\to Q\bar{Q}} = \phi_{q/q} \otimes \hat{\sigma}_{qq\to Q\bar{Q}} \otimes \phi_{\bar{q}/\bar{q}}$$

Moments

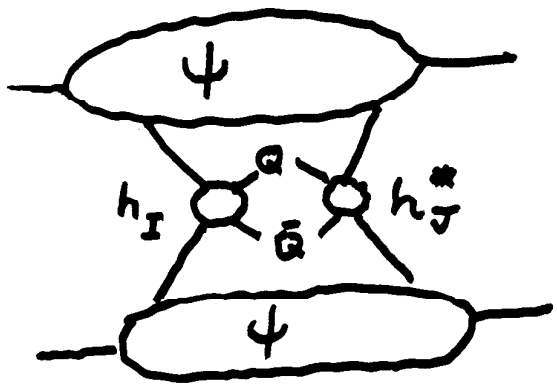
$$\hat{\sigma}_{q\bar{q}}(N) = \frac{d\sigma(N)}{[\phi_{q/q}(N)]^2_{AB}} \quad \text{(etc. for G(x))}$$

Alternate Factorization near $z=1$

(all $\left(\frac{1}{1-z}\right)_+$)

(for $\sigma_{f\bar{f}} \to Q\bar{Q}$)

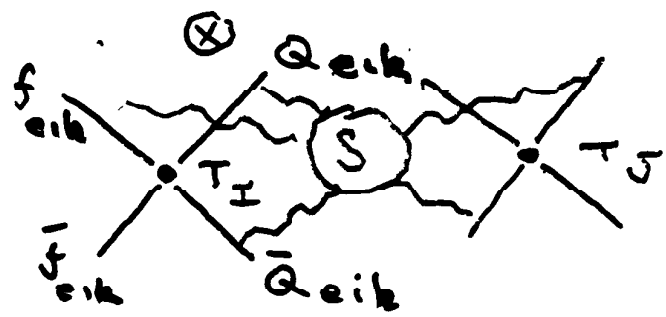
$$\sigma_{f\bar{f}} = \psi_{f/f} \otimes h_I h_J^* \otimes S_{IJ} \otimes \psi_{\bar{f}/\bar{f}} \qquad (3)$$



e.g.

$T_I =$

$T_2 =$



$$\sigma_{f\bar{f}}(N) = h_I h_J^* S_{IJ}(N) \psi^2(N) \qquad (C)$$

equate (A) = (C)

$$\omega_{f\bar{f}}(N) = \underbrace{\left[\frac{\psi_f(N)}{\phi_f(N)}\right]^2}_{\text{known from DY}} \overset{\text{new}}{S_{IJ}(N)} h_I h_J^*$$

NNL @ each order from

$$S_{IJ} = S_{IJ}\left(\frac{Q}{N\mu}, \alpha_s(\mu)\right)$$

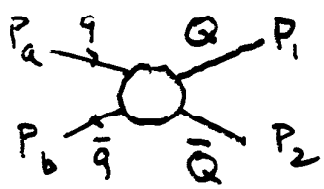
satisfies RGE:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g}\right) S_{IJ} = -(\Gamma_S^{+})_{JB} S_{BI} - S_{JA}(\Gamma_S)_{AI}$$

(Botts, Sotiropoulos)

typical

$$\Gamma = \frac{\alpha_s}{\pi}\begin{pmatrix} C_F\left(\ln\frac{u_1^2}{t_1^2} - L_\beta - 1 - i\pi\right) - \frac{C_A}{2}\ln\frac{u_1^2}{t_1^2} & \frac{1}{2}\ln\frac{u_1^2}{t_1^2} \\ \frac{1}{2}\ln\frac{u_1^2}{t_1^2} & C_F\left(\ln\frac{u_1^2}{t_1^2} - L_\beta - 1 - i\pi\right) + \frac{C_A}{2}\left[-\ln\frac{m_S^2}{t_1^2} + L_\beta + \pi i\right] \end{pmatrix}$$



$$u_1 = (P_a - P_1)^2 - M^2$$
$$t_1 = (P_a - P_1)^2 - M^2$$

Laenen
Smith
van Neerven
Meng
Beenakker

$$L_\beta = \frac{1 - 2m^2/s}{\beta}\left[\ln\frac{1-\beta}{1+\beta} + i\pi\right]$$

The Resummed Cross Section
(NLL in exponent)

$$\hat{\sigma}_{f\bar{f}\to Q\bar{Q}}(N) = \int_0^1 dz\, z^{N-1}\, \hat{\sigma}^{(z)}_{f\bar{f}\to Q\bar{Q}}$$

$f \simeq q, \bar{q}, g$

$$= A'(\alpha_s)\, h_J^*\left(\frac{Q}{\mu}, \alpha_s(\mu)\right) h_I\left(\frac{Q}{\mu}, \alpha_s(\mu)\right)$$
$$\cdot\, S_{IJ}(1, \alpha_s(Q/N))$$
$$\cdot\, \exp E_{IJ}(N, \theta, Q^2)$$

$$E_{IJ} = \int_0^1 dz\, \frac{z^{N-1}-1}{1-z}\left[\int_0^z \frac{dy}{1-y}\, g_1^{(f\bar{f})}[\alpha_s((1-x)(1-y)Q^2)]\right.$$
$$+ g_2^{(f\bar{f})}[\alpha_s((1-x)Q^2)]$$
$$\left\{\begin{array}{l} + g_3^{I}[\alpha_s[(1-x)^2 Q^2, \underline{\theta}]] \\ + g_3^{J}[\alpha_s[(1-x)^2 Q^2, \underline{\theta}]] \end{array}\right.\Bigg]$$

$$g_3^{I}(\alpha_s, \theta) = -\lambda_I[\alpha_s, \theta] + C_F \frac{\alpha}{\pi}$$

C_A for $f = g$

λ_I eigenvalues of Γ_{IJ}

Outlook, Ongoing Progress

- Inversion of moments
  - Laenen et al
  - Berger Contopangos
  - Catani et al
  - (Appel, Mackenzie G.S.)
- Extension to $GG \to Q\bar{Q}$ done
- Extension to $GG \to GG$
  - G. Oderda, N.K., GS
- Jet Cross Sections
- Preliminary estimates in $Q\bar{Q}$: Kidonakis
  Smith
  R. Vogt