STEFANO FRIXIONE
ETH ZÜRICH

INFRARED - SAFE QUANTITIES AT NLO IN QCD

CHICAGO, APRIL 17

RESULTS OBTAINED IN COLLABORATION WITH

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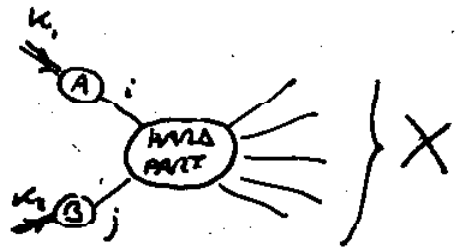
- JET PRODUCTION IS A BENCHMARK PROCESS FOR TESTING QCD
  - PERTURBATIVE EXPANSION IS EXPECTED TO BE RELIABLE
  - ENORMOUS AMOUNT OF DATA
- ALL THE PARTICLES INVOLVED IN THE HARD SCATTERING BEING MASSLESS, THE STRUCTURE OF THE INFRARED DIVERGENCIES IS QUITE COMPLICATED
- TO PROVE THE CANCELLATION OF THE DIVERGENCIES IS AN ESSENTIAL PART OF THE CALCULATION, TECHNICALLY RATHER INVOLVED
- SEVERAL METHODS HAVE BEEN DEVISED IN THE PAST TO DEAL WITH THIS PROBLEM

$\boxed{\text{IS IT POSSIBLE TO IMPROVE?}}$ (IN WHAT SENSE?)

USING FACTORIZATION THEOREMS OF QCD



$$d\sigma_{AB}(k_1,k_2) = \sum_{ij} \int dx\, dy\, f_i^{(A)}(x)\, f_j^{(B)}(y)\, d\hat{\sigma}_{ij}(xk_1, yk_2)$$

= [SMALL SCALES] ⊗ [LARGE SCALES]

= [LARGE DISTANCES] ⊗ [SHORT DISTANCES]

$f_i^{(A)}(x) =$ $\delta_{Ai}\,\delta(1-x)$; $f_\gamma^{(e)}(x)$

NON CALCULABLE IN PERTURBATIVE QCD BUT UNIVERSAL

$d\hat{\sigma}_{ij}$ = PROCESS DEPENDENT BUT CALCULABLE IN PERTURBATION THEORY (PROVIDED INFRARED SAFENESS AND $Q^2 \gg \Lambda^2_{QCD}$)

IN GENERAL, AT NLO,

$$d\hat{\sigma}_{ij}^{(NLO)} = d\sigma_{ij}(m+1) + d\sigma_{ij}(m) + d\sigma_{ij}(c)$$

WHERE (BORN TERM IS A M-PARTICLE PROCESS)

$$d\sigma_{ij}(m+1) = \left|\text{REAL DIAGRAMS}\right|^2 d\phi$$

$$= \frac{A}{\varepsilon^2} + \frac{B}{\varepsilon} + \text{FINITE}$$

$$d\sigma_{ij}(m) = \left(\text{VIRTUAL} \times \text{BORN}^* + \text{C.C}\right) d\phi$$

$$= \frac{A'}{\varepsilon^2} + \frac{B'}{\varepsilon} + \text{FINITE}$$

$$d\sigma_{ij}(c) = \text{COLLINEAR COUNTERTERMS}$$

$$= \frac{\alpha_s}{2\pi}\left(\frac{1}{\varepsilon}P_{ki} - K_{ki}\right) \otimes d\sigma_{kj}^{(LO)}$$

$$+ \frac{\alpha_s}{2\pi}\left(\frac{1}{\varepsilon}P_{kj} - K_{kj}\right) \otimes d\sigma_{ik}^{(LO)} = \frac{C}{\varepsilon} + \text{FINITE}$$

FOR INFRARED-SAFE QUANTITIES

$$\left.\begin{aligned} A + A' &= 0 \\ B + B' + C &= 0. \end{aligned}\right\} \text{THE CROSS SECTION IS FINITE}$$

- AN OBSERVABLE IS INFRARED-SAFE IF
  - IT DOES NOT CHANGE ABRUPTLY WHEN A SOFT PARTON IS EMITTED
  - IT DOES NOT CHANGE ABRUPTLY WHEN A PARTON EMITS COLLINEARLY A PAIR OF PARTONS

$$\frac{d\sigma(m)}{dX_1 \cdots dX_m} = S_m\left(\{X_i\}_{i=1}^{m}; \{p_i\}_{i=1}^{m}\right) M\left(p_a, p_b \to \{p_i\}_{i=1}^{m}\right) d\phi_m$$

EXAMPLES

THRUST: $S_m(\tau; \{p_i\}) = \delta\left(\tau - \max_{\{\vec{e}\}} \frac{\sum_{i=1}^{m} |\vec{p}_i \cdot \vec{e}|}{\sum_{i=1}^{m} |p_i|}\right)$

2 PARTONS $\to$ 2 JETS: $S_2(k_{J_1}, k_{J_2}; p_1, p_2) =$

$$= \sum_{\text{perm}} \delta(k_{J_1} - p_1)\,\delta(k_{J_2} - p_2)$$

IN GENERAL, S IS THE SUM OF TERMS CONTAINING STRINGS OF Θ (SELECTING KINEMATICAL CONFIGURATIONS) AND δ (DEFINING THE PHYSICAL OBSERVABLES)

$$S_m(X_1, \ldots, X_m; p_1, \ldots, p_m) =$$

$$= \sum_i \left(\prod_j \Theta\left(f_j^{(i)}(\{p\})\right)\right)\left(\prod_k \delta\left(X_k - g_k^{(i)}(\{p\})\right)\right)$$

THE INFRARED SAFENESS REQUIREMENT CAN BE EASILY EXPRESSED IN TERMS OF THE S-FUNCTIONS

$$S_m(\{X\};\dots,p_i,\dots) \xrightarrow[p_i^0\to 0]{} S_{m-1}(\{X\};\dots\dots)$$

$$S_m(\{X\};\dots,p_i,\dots,p_j,\dots) \xrightarrow[i/\!/j]{} S_{m-1}(\{X\};\dots,p_k,\dots) \qquad p_i+p_j\to p_k$$

$$S_m(\{X\};\dots,p_i,\dots) \xrightarrow[\substack{i/\!/1\\ i/\!/2}]{} S_{m-1}(\{X\};\dots\dots)$$

A VERY TRIVIAL EXAMPLE OF NON-INFRARED SAFE S-FUNCTION

$$S_m = \sum_{i=1}^{m} \theta(p_i^0) \quad , \quad \theta(0)=0$$

$$= m - \#\text{ OF SOFT PARTONS}$$

WHICH CORRESPONDS TO A DETECTOR WITH INFINITE RESOLUTION

- CALCULATE ANALYTICALLY THE DIVERGENCES
- CANCEL THE DIVERGENCES AND (NUMERICALLY) INTEGRATE THE FINITE PART

EXAMPLE: COLLINEAR SINGULARITY $(y = \cos\theta)$

$$f(y)/(1-y) \xrightarrow{y\to 1} \text{CONSTANT}\,; \quad d\phi \propto (1-y)^{-\varepsilon} dy$$

$$I = \int_{-1}^{1} dy\,(1-y)^{-1-\varepsilon} f(y) \qquad f(y) = f(1) - f'(1)(1-y) + O((1-y)^2)$$

SLICING:

$$I = \int_{-1}^{1-\delta} dy\,(1-y)^{-1-\varepsilon} f(y) + \int_{1-\delta}^{1} dy\,(1-y)^{-1-\varepsilon} f(y)$$

$$I = \int_{-1}^{1-\delta} dy\,\frac{f(y)}{1-y} + \int_{1-\delta}^{1} dy\,(1-y)^{-1-\varepsilon}\left(f(1) - f'(1)(1-y) + \cdots\right)$$

$$I = \int_{-1}^{1-\delta} dy\,\frac{f(y)}{1-y} - f(1)\left(\frac{1}{\varepsilon} - \log\delta\right) - f'(1)\,\delta + O(\delta^2) + O(\varepsilon)$$

MAIN DRAWBACK: THE FORMULA IS AN APPROXIMATION, AND δ MUST BE SMALL $\Rightarrow$ CANCELLATION BETWEEN LARGE NUMBERS (I.E., $\frac{\partial I}{\partial \delta} = 0$ MIGHT BE DIFFICULT TO PROVE)

SUBTRACTION:

$$I = \int_{-1}^{1} dy\,(1-y)^{-1-\varepsilon}\,\rho(y)$$

$$I = \int_{-1}^{1} dy\,\frac{\rho(y)-\rho(1)\theta(y-y_0)}{(1-y)^{1+\varepsilon}} + \rho(1)\int_{-1}^{1} dy\,\frac{\theta(y-y_0)}{(1-y)^{1+\varepsilon}}$$

$$I = \int_{-1}^{1} dy\,\frac{\rho(y)-\rho(1)\theta(y-y_0)}{1-y} - \rho(1)\left(\frac{1}{\varepsilon} - \log(1-y_0)\right) + O(\varepsilon)$$

MAIN DRAWBACK: $\rho(y)$ AND $\rho(1)$ HAVE DIFFERENT KINEMATICS, WHICH CAN GIVE INSTABILITY WHEN PERFORMING NUMERICALLY THE INTEGRAL

NEVERTHELESS

- THERE'S NO APPROXIMATION
- INSTABILITY CAN BE REDUCED BY PROPERLY CHOOSING THE ARBITRARY PARAMETER y_0 (THERE'S NO NEED TO CHECK THAT $\frac{\delta I}{\delta y_0} = 0$)
- THE SLICING METHOD IS RECOVERED FOR $y_0 \to 1$

THE SLICING AND THE SUBTRACTION METHODS HAVE BEEN SUCCESSFULLY APPLIED TO DESCRIBE 1-JET AND 2-JET QUANTITIES (MAINLY) IN HADRONIC COLLISIONS

NIERSA ET AL
ELLIS, KUNSZT, SOPER
GIELE, GLOVER, KOSOWER

WITH SUBTRACTION METHOD

- USE LAB FRAME TO PERFORM THE CALCULATIONS (LUMINOSITY ENTERS DIRECTLY THE INTEGRALS WHERE THE SUBTRACTION IS PERFORMED)
- DECOMPOSE MATRIX ELEMENTS INTO SINGLE-SINGULAR FACTORS (AT MOST ONE SOFT AND ONE COLLINEAR SINGULARITY IN EACH TERM)

$\Downarrow$

POSSIBLE PROBLEMS IN

a) EXTENSION TO LARGER NUMBER OF PARTONS

b) EXTENSION TO DIFFERENT TYPES OF COLLIDING PARTICLES

I.E. NO MAJOR PRINCIPLE PROBLEM, BUT THE CALCULATIONS CAN GET RAPIDLY SO CUMBERSOME TO MOTIVATE A NEW APPROACH

$\boxed{\text{MAIN IDEAS}}$. TO IMPROVE THE CALCULATION WITH THE SUBTRACTION METHOD AND GET A GENERAL FORMALISM

S.F., KUNSZT, SIGNER

a) PERFORM THE CALCULATIONS IN THE PARTONIC CENTER OF MASS FRAME

b) EXPLOIT THE PROPERTIES OF THE δ-FUNCTIONS (EVERY INFRARED-SAFE CROSS SECTION CAN BE WRITTEN USING δ-FUNCTIONS)

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a) $\Rightarrow$ • CROSS SECTIONS FOR e^+e^-, γH, H_1H_2 COLLISIONS ARE DEALT WITH EXACTLY IN THE SAME WAY ($\delta(1-x)$ IS THE PDF IF THE INCOMING PARTICLE IS NOT A HADRON; DEFINE COLOR FACTORS FOR THE "PARTONS" e AND γ)

• AS A BY-PRODUCT, THE INTEGRATION OVER THE EIKONAL FACTORS IS SIMPLIFIED

POINT 5) IS BASED UPON THE FOLLOWING FACT

$\forall S_m$ ONE CAN ALWAYS WRITE THE DECOMPOSITION

$$S_m = S_m^{(FIN)} + \sum_{i=3}^{m+2} S_i^{(SING)}$$

$$S_i^{(SING)} = S_i^{(0)} + \sum_{j \neq i} S_{ij}^{(1)} \theta(E_j - E_i)$$

$\longrightarrow$ "WIDE SLICING" OF THE PHASE SPACE

$S_m^{(FIN)} = 0$ IN ALL SINGULAR REGIONS

$S_i^{(0)} \neq 0 \iff E_i \to 0 \; ; \; i \neq 1 \; ; \; i \neq 2$

$S_{ij}^{(1)} \neq 0 \iff E_i \to 0 \; ; \; E_j \to 0 \; ; \; i \neq j$

THEREFORE

$$d\sigma(m) = S_m M^{(m)} d\phi_m$$

$$= \left(S_m^{(FIN)} + \sum_i S_i^{(0)} + \sum_{i,j}^{i \neq j} S_{ij}^{(1)} \theta(E_j - E_i) \right) M_m^{(m)} d\phi_m$$

$$d\sigma^{(FIN)} + \sum_i d\sigma_i^{(0)} + \sum_{i,j}^{i \neq j} d\sigma_{ij}^{(1)}$$

WHICH IS EQUIVALENT TO A SINGLE-SINGULAR FACTOR DECOMPOSITION, BUT IT IS ACHIEVED WITHOUT ANY ALGEBRA

$$d\sigma^{(fin)}_{a_1 a_2, i} = \frac{1}{n!}\mathcal{M}^{(n)}\mathcal{S}_i^{(fin)} d\phi_n$$

$$\begin{aligned} d\sigma^{(in,f)}_{a_1 a_2, i} &= \frac{1}{n!}\frac{1}{2}\left(\frac{1}{\xi_i}\right)_c \left[\left(\frac{1}{1-y_i}\right)_{\delta_I} + \left(\frac{1}{1+y_i}\right)_{\delta_I}\right] \\ &\times \left((1-y_i^2)\xi_i^2\mathcal{M}^{(n)}\right)\mathcal{S}_i^{(0)} \\ &\times \frac{1}{2(2\pi)^3}\left(\frac{\sqrt{S}}{2}\right)^2 d\tilde{\phi}_{n-1} d\xi_i dy_i d\varphi_i \end{aligned}$$

$$\begin{aligned} d\sigma^{(out,f)}_{a_1 a_2, ij} &= \frac{1}{n!}\left(\frac{1}{\xi_i}\right)_c \left(\frac{1}{1-y_j}\right)_{\delta_o} \\ &\times \left((1-y_j)\xi_i^2\xi_j\mathcal{M}^{(n)}\right)\mathcal{S}_{ij}^{(1)}\theta(\xi_j-\xi_i) \\ &\times \left(\frac{1}{2(2\pi)^3}\left(\frac{\sqrt{S}}{2}\right)^2\right)^2 \\ &\times d\tilde{\phi}_{n-2} d\xi_i d\xi_j dy_i dy_j d\varphi_i d\varphi_j \end{aligned}$$

$$
\begin{aligned}
d\hat{\sigma}^{(1,n-1v)}_{a_1 a_2} &= \frac{\alpha_S}{2\pi} Q \; d\sigma^{(0)} \\
&+ \frac{\alpha_S}{2\pi} \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^{n+1} \mathcal{I}^{(reg)}_{ij} \, d\sigma^{(0)}_{ij} \\
&+ \frac{\alpha_S}{2\pi} \frac{1}{(n-1)!} \mathcal{M}^{(v)}_{NS} \, \mathcal{S}_{n-1} \, d\phi_{n-1}
\end{aligned}
$$

$$
d\hat{\sigma}^{(1,n-1r)}_{a_1 a_2} = d\xi \, d\sigma^{(0)}_{da_2}((1-\xi)k_1, k_2) \; \times
$$

$$
\frac{\alpha_S}{2\pi} \sum_d \Bigg\{ \xi P^{<}_{da_1}(1-\xi, 0) \left[\left(\frac{1}{\xi}\right)_c \log \frac{S\delta_I}{2\mu^2} + 2 \left(\frac{\ln}{}\right.\right.
$$

$$
- \xi P'^{<}_{da_1}(1-\xi, 0) \left(\frac{1}{\xi}\right)_c - K_{da_1}(1-\xi) \Bigg]
$$

$$
+ (1 \leftrightarrow 2)
$$

$$p_1 + p_2 \to \overbrace{p_3 + \cdots + p_{n+2}}^{n}$$

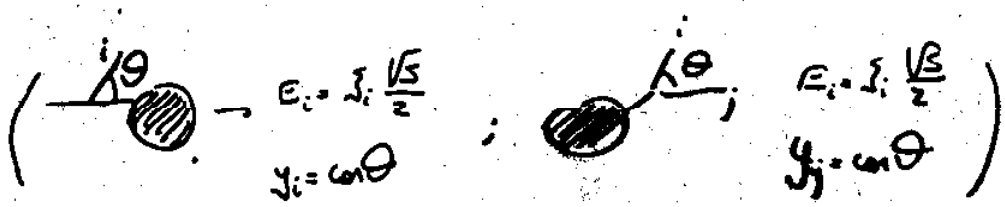
$$d\hat{\sigma}^{(1)}_{a_1 a_2} = d\hat{\sigma}^{(1,n)}_{a_1 a_2} + d\hat{\sigma}^{(1,n-1)}_{a_1 a_2}$$

$$d\hat{\sigma}^{(1,n-1)}_{a_1 a_2} = d\hat{\sigma}^{(1,n-1v)}_{a_1 a_2} + d\hat{\sigma}^{(1,n-1r)}_{a_1 a_2}$$

$$d\hat{\sigma}^{(1,n)}_{a_1 a_2} = \sum_{i=3}^{n+2} \left(d\sigma^{(fin)}_{a_1 a_2, i} + d\sigma^{(in,f)}_{a_1 a_2, i} + \sum_{\substack{j=3 \\ j \neq i}}^{n+2} d\sigma^{(out,f)}_{a_1 a_2, ij} \right)$$

AT THIS POINT, THE IMPLEMENTATION OF THE SUBTRACTION ALGORITHM BECOMES QUITE SIMPLE

- FOR EVERY TERM, ADOPT A PHASE SPACE PARAMETERIZATION SUITABLE TO DEAL WITH THE SINGULARITY



- ADD AND SUBTRACT THE LEADING SINGULARITY UNDER THE INTEGRAL SIGN

- THE RESULT IS FINITE AND INDEPENDENT ON NON-PHYSICAL PARAMETERS. FOLD IT WITH THE LUMINOSITY

THE FORMALISM HAS BEEN APPLIED TO:

$e^+e^- \longrightarrow$ 4 JETS SIGNER, DIXON

$H_1H_2 \longrightarrow$ 3-PARTON INFRARED-SAFE QUANTITIES S.F.

$eH, \gamma H \longrightarrow$ 3-PARTON INFRARED-SAFE QUANTITIES S.F., RIDOLFI

$H_1H_2 \longrightarrow$ JETS CONTAINING A HEAVY QUARK S.F., MANGANO

(OTHER RECENT APPROACHES: CATANI, SEYMOUR
KLASEN, KRAMER
MERKES, ZEPPENFELD)

$$k_j = (1-z)k_1\,, \quad k_p = zk_1$$

$$\mathcal{M}^{(n)}\left(k_1, k_2; \ldots, k_j, \ldots\right) \xrightarrow{1\|j}$$
$$\frac{4\pi\alpha_S}{k_1\cdot k_j} P^{<}_{a_p a_1}(z)\mathcal{M}^{(n-1)}\left(zk_1, k_2; \ldots\right)$$
$$+\frac{4\pi\alpha_S}{k_1\cdot k_j} Q_{a_p^\star a_1}(z)\tilde{\mathcal{M}}^{(n-1)}\left(zk_1, k_2; \ldots, k_j, \ldots\right)$$

$$\begin{aligned}
Q_{g^\star g}(z) &= -4C_A\frac{1-z}{z} \\
Q_{g^\star q}(z) &= -4C_F\frac{1-z}{z} \\
Q_{gg^\star}(z) &= -4C_A\, z(1-z) \\
Q_{qg^\star}(z) &= 4T_F\, z(1-z)
\end{aligned}$$

GIVEN THE OBSERVABLE O, WE HAVE

$$S_i^{(0)}(O) = \delta(O - O_i^{(0)})\, \mathcal{F}_i^{(0)}$$

$$S_{ij}^{(1)}(O) = \delta(O - O_{ij}^{(1)})\, \mathcal{F}_{ij}^{(1)}$$

$$S^{(FIN)}(O) = \delta(O - O^{(FIN)})\, \mathcal{F}^{(FIN)}$$

WITH $$\boxed{\mathcal{F}^{(FIN)} + \sum_i \mathcal{F}_i^{(0)} + \sum_{ij} \mathcal{F}_{ij}^{(1)}\, \theta(E_j - E_i) = 1}$$

THE δ TERMS PLAY NO RÔLE IN SEPARATING THE SINGULAR REGIONS $\Rightarrow$ IN A COMPUTER PROGRAM, THE DEFINITION OF THE OBSERVABLE CAN BE DONE IN THE VERY LAST STEP

$$\Downarrow$$

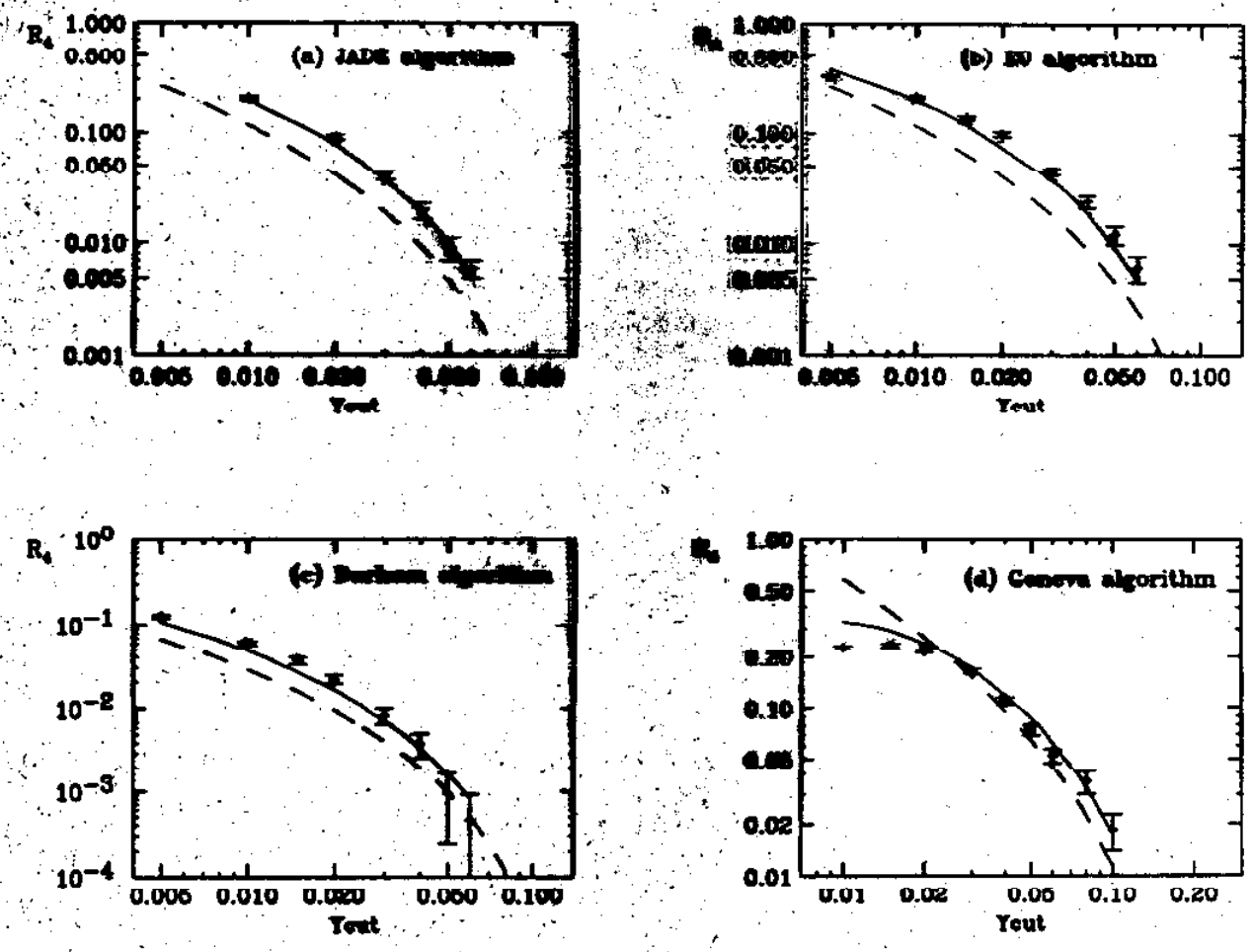
- SELECT A SET $\{\mathcal{F}\}$ OF FUNCTION FULFILLING THE CONDITION ABOVE. USE THEM TO PERFORM THE CALCULATION AS PREVIOUSLY DESCRIBED

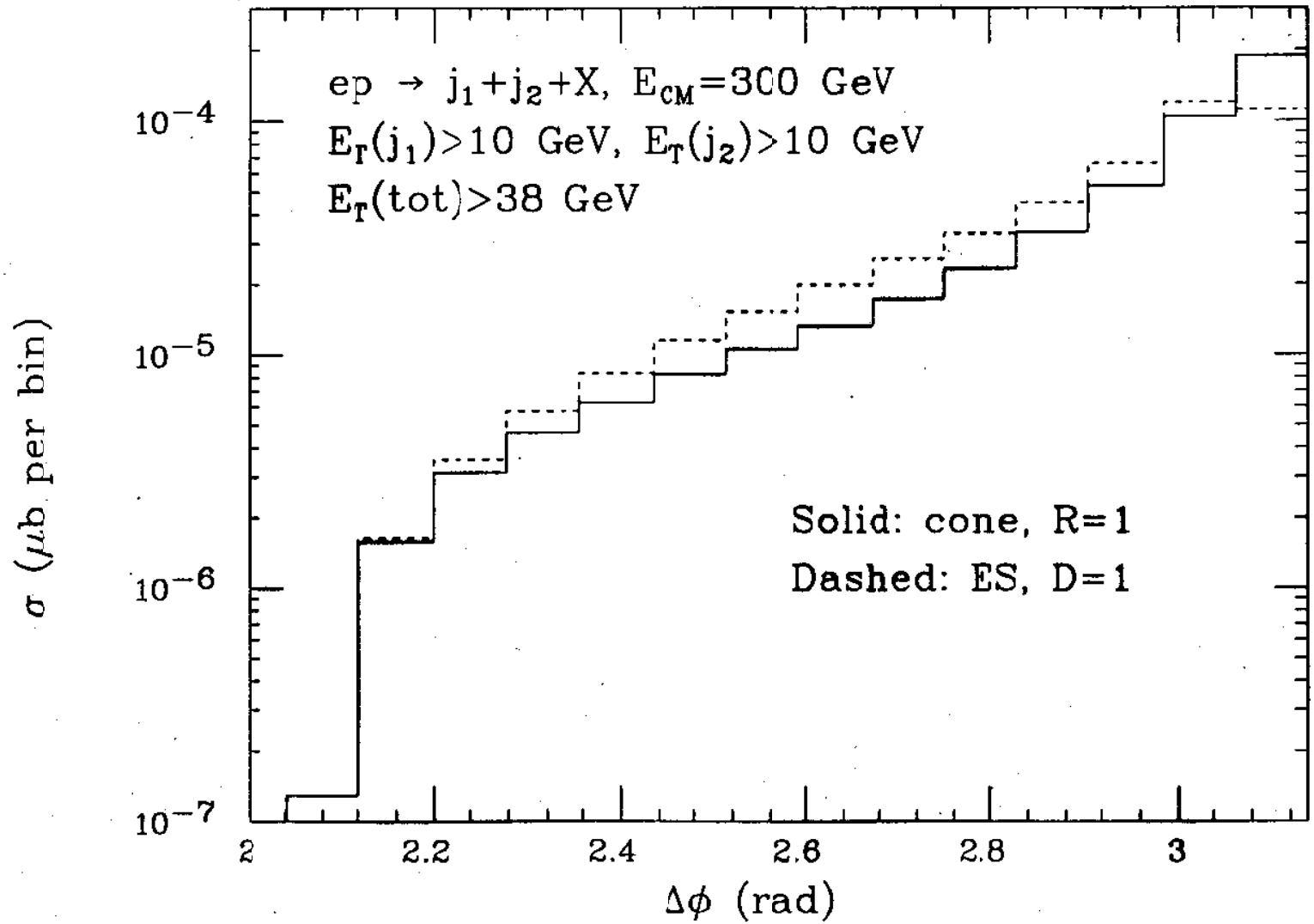
- IMPLEMENT THE JET-DEFINING ALGORITHM (OR ANY OTHER INFRARED-SAFE DEFINITION) IN THE FINAL STEP OF THE CALCULATION

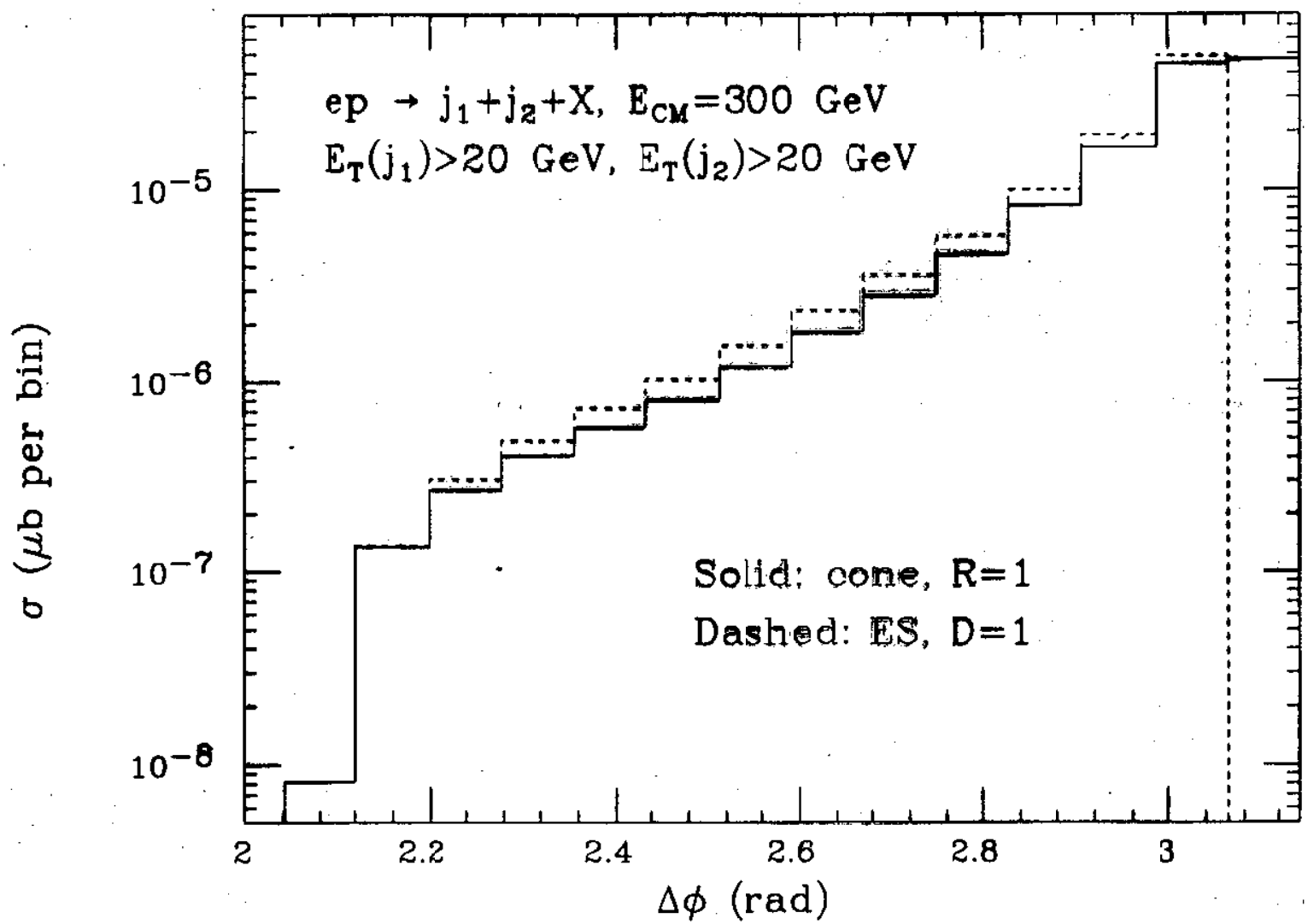
DIXON, SIGNER

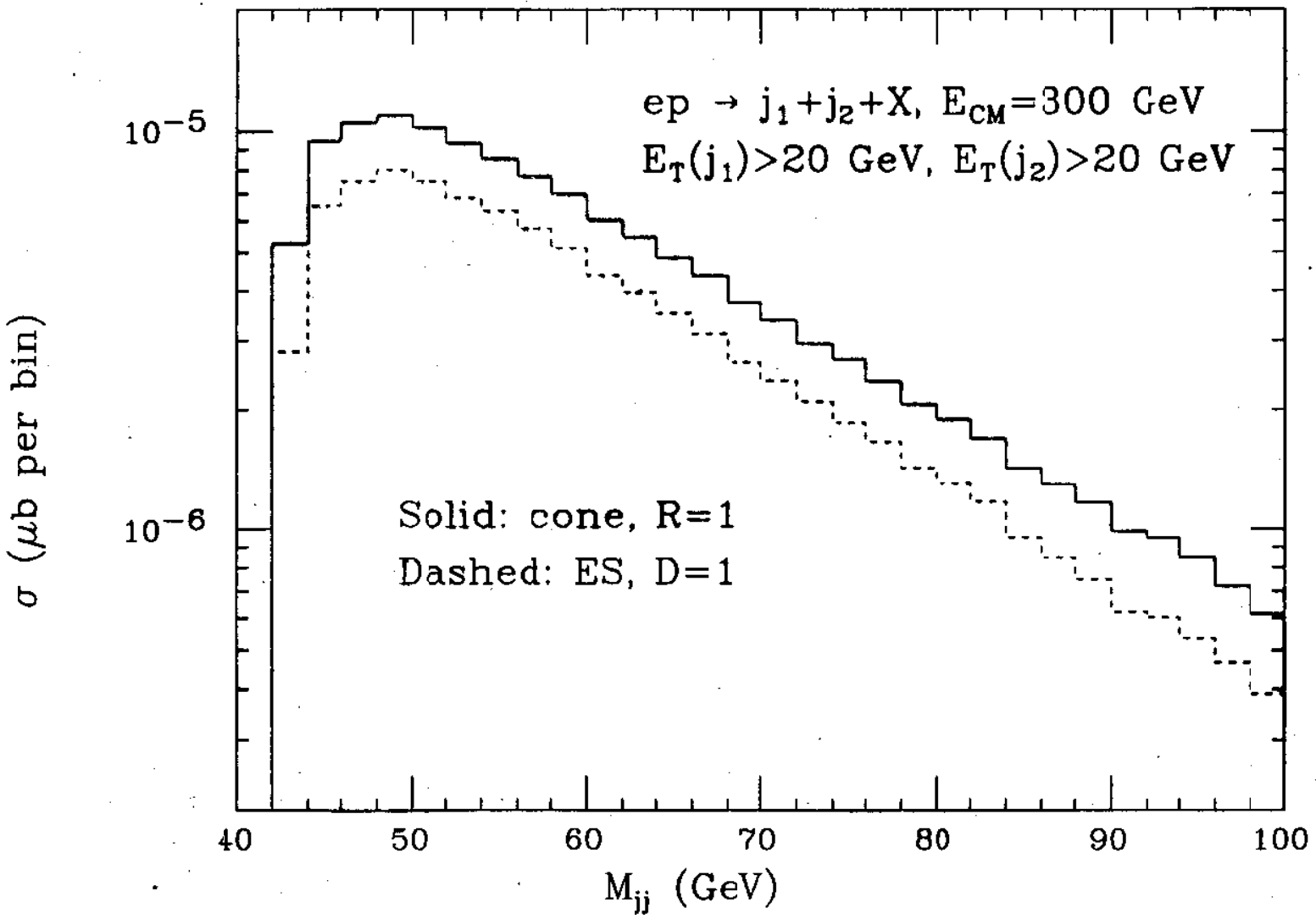
HEP-PH/9609460

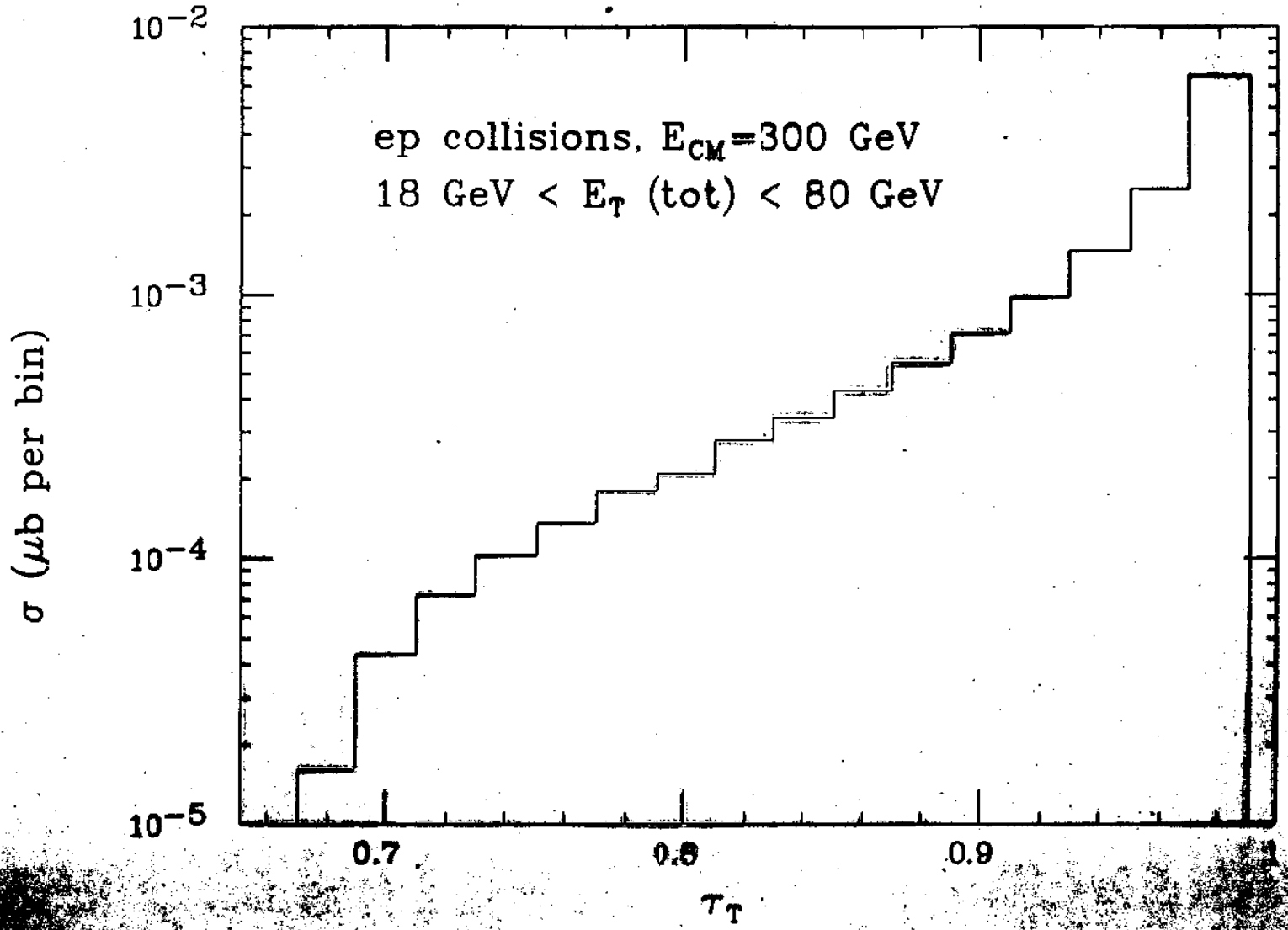
PHYS REV LETT 78(97)811

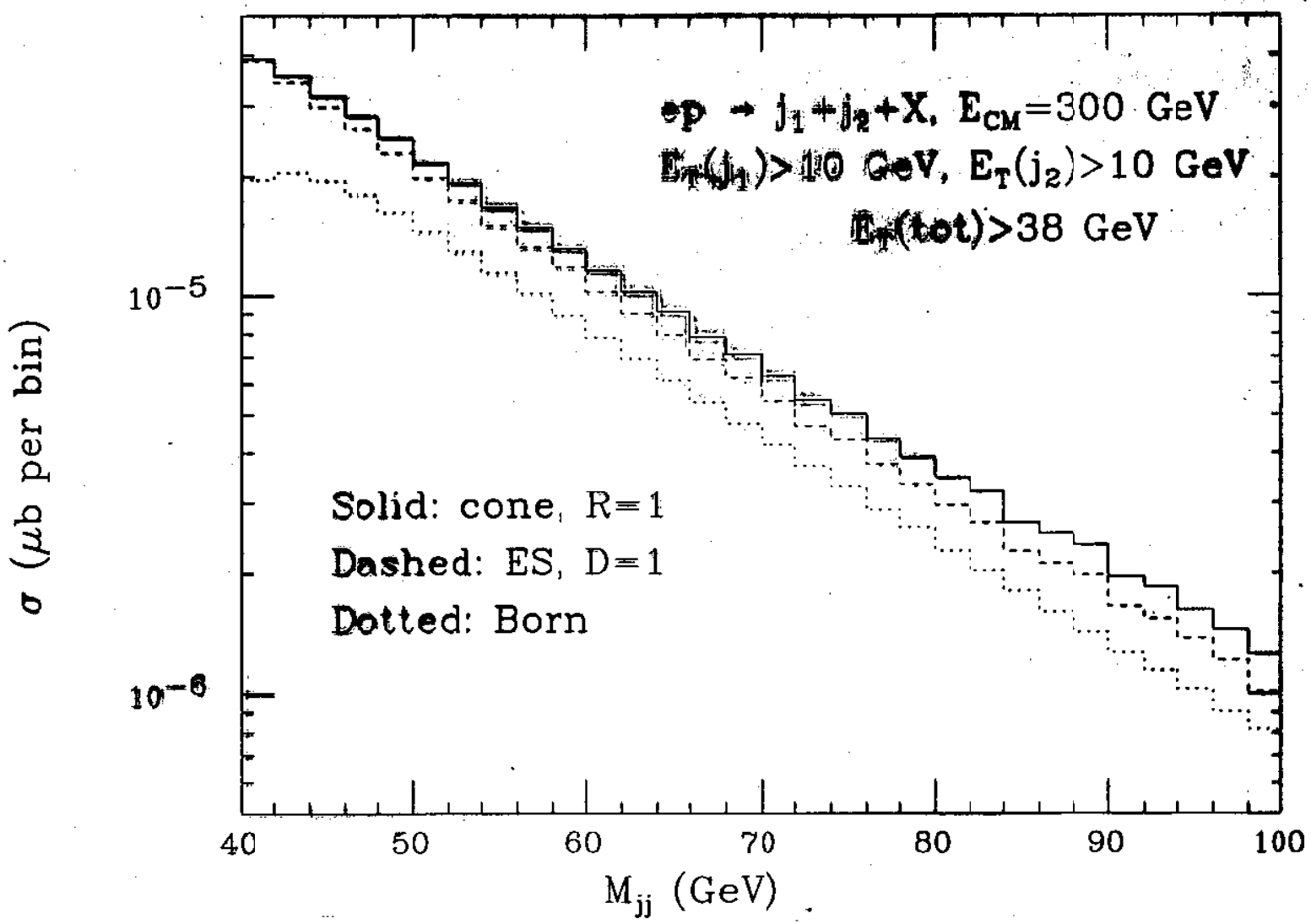


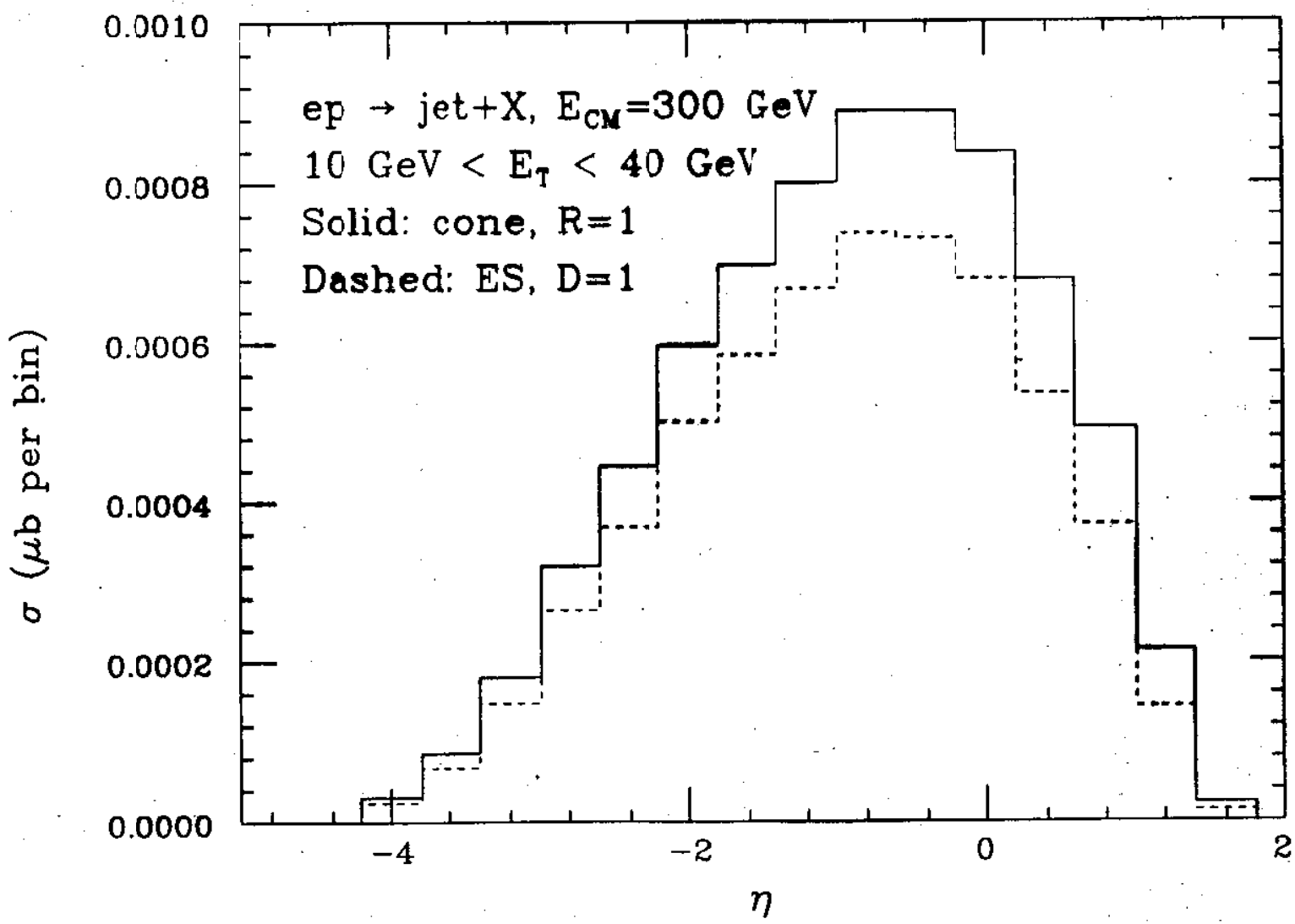

ep → j1+j2+X, ECM=300 GeV
ET(j1)>10 GeV, ET(j2)>10 GeV
ET(tot)>38 GeV
Solid: cone, R=1
Dashed: ES, D=1
σ (μb per bin)
10−4
10−5
10−6
10−7
2
2.2
2.4
2.6
2.8
3
Δφ (rad)

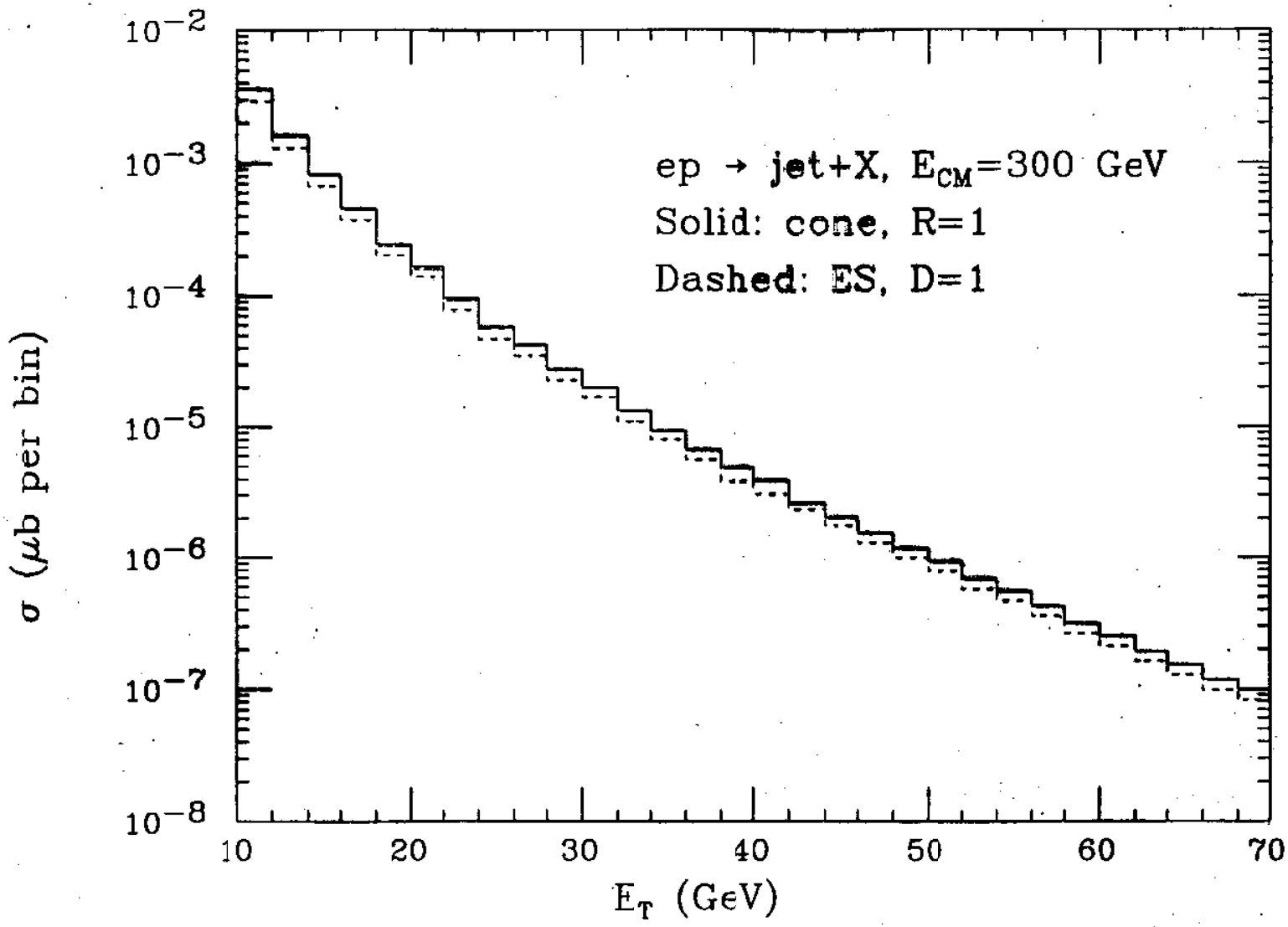

ep → j1+j2+X, ECM=300 GeV
ET(j1)>20 GeV, ET(j2)>20 GeV
Solid: cone, R=1
Dashed: ES, D=1
σ (μb per bin)
10−5
10−6
10−7
10−8
2
2.2
2.4
2.6
2.8
3
Δφ (rad)

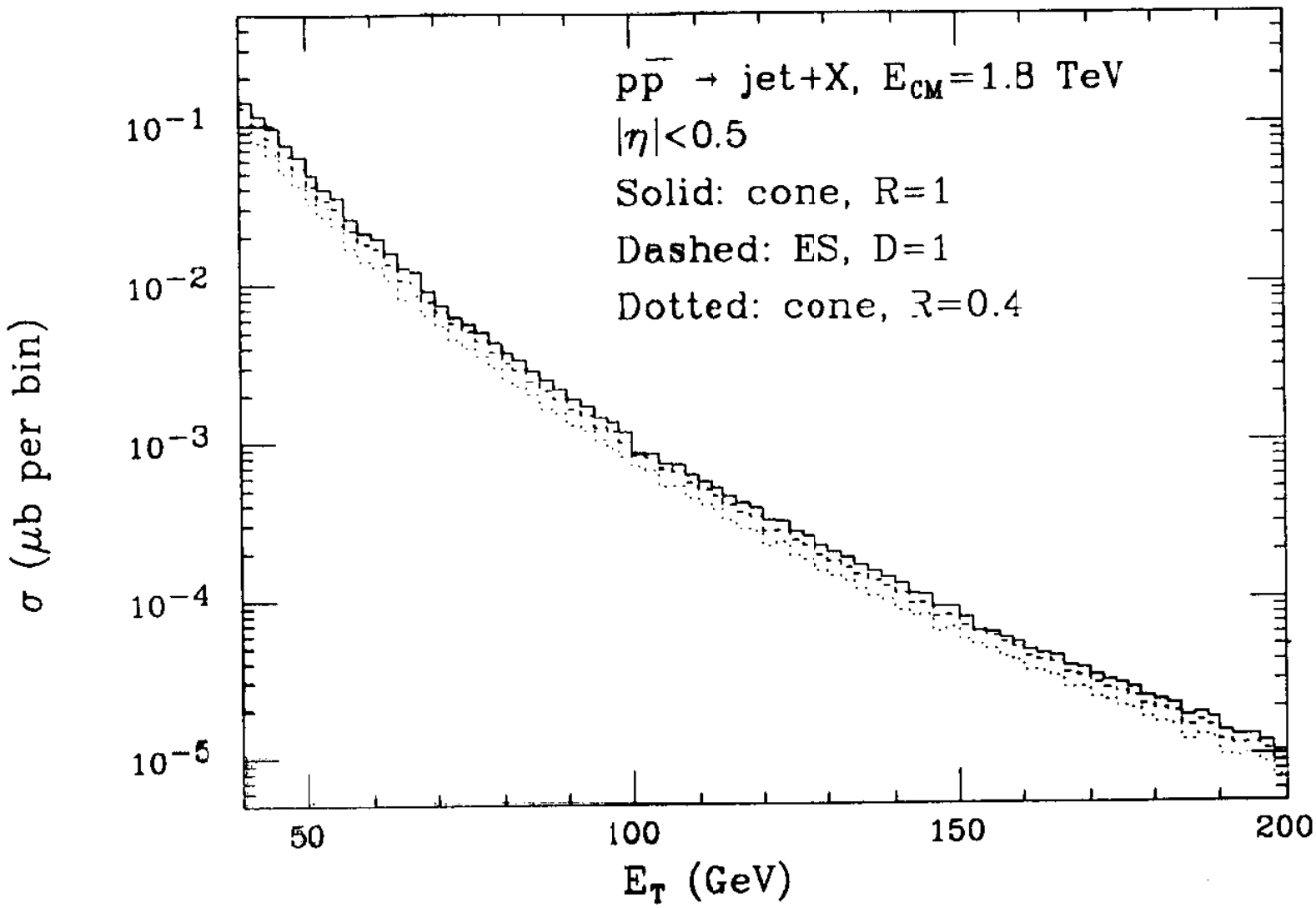

ep → j1+j2+X, ECM=300 GeV
ET(j1)>20 GeV, ET(j2)>20 GeV
Solid: cone, R=1
Dashed: ES, D=1
10−5
10−6
σ (μb per bin)
40
50
60
70
80
90
100
Mjj (GeV)

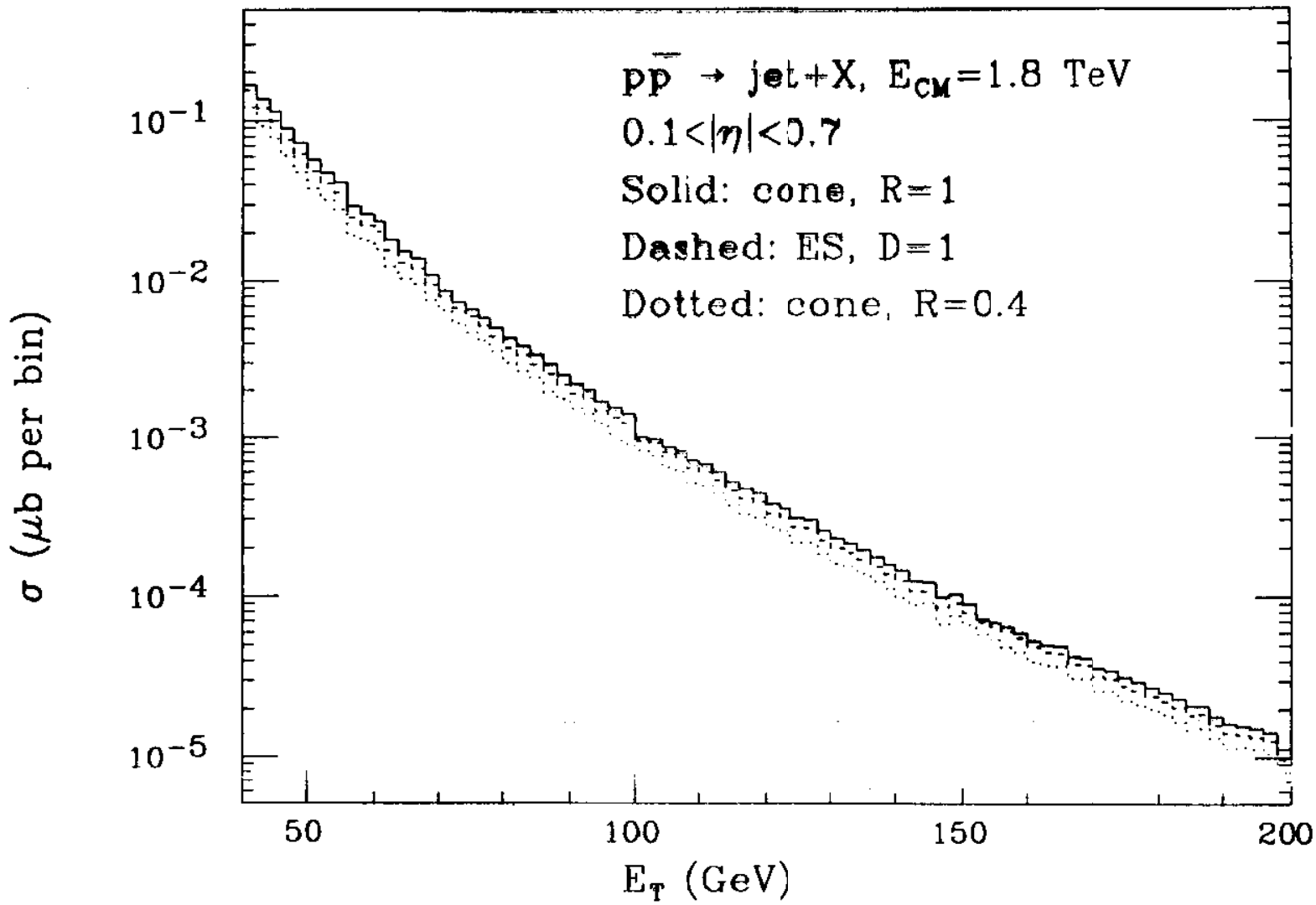

ep collisions, E_{CM}=300 GeV
18 GeV < E_T (tot) < 80 GeV
10^{-2}
10^{-3}
10^{-4}
10^{-5}
σ (μb per bin)
0.7
0.8
0.9
1
τ_T


ep → j1+j2+X, ECM=300 GeV
ET(j1)>10 GeV, ET(j2)>10 GeV
ET(tot)>38 GeV
Solid: cone, R=1
Dashed: ES, D=1
Dotted: Born
σ (μb per bin)
10−5
10−6
40
50
60
70
80
90
100
Mjj (GeV)


ep → jet+X, E_{CM}=300 GeV
10 GeV < E_T < 40 GeV
Solid: cone, R=1
Dashed: ES, D=1
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
σ (μb per bin)
−4
−2
0
2
η


ep → jet+X, E_{CM}=300 GeV
Solid: cone, R=1
Dashed: ES, D=1
σ (μb per bin)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10
20
30
40
50
60
70
E_T (GeV)


$p\bar{p} \rightarrow$ jet+X, E_{CM}=1.8 TeV
$|\eta|$<0.5
Solid: cone, R=1
Dashed: ES, D=1
Dotted: cone, R=0.4
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
σ (μb per bin)
50
100
150
200
E_T (GeV)


$p\bar{p} \rightarrow$ jet+X, E_{CM}=1.8 TeV
$0.1<|\eta|<0.7$
Solid: cone, R=1
Dashed: ES, D=1
Dotted: cone, R=0.4
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
σ (μb per bin)
50
100
150
200
E_T (GeV)

AT THE PARTON LEVEL AND AS FAR AS THE JET DEFINITION IS CONCERNED, THE CASE OF HEAVY QUARKS IS NOT AT ALL SPECIAL. FOR EXAMPLE

$$i \in J \iff \sqrt{(\eta_i-\eta_J)^2+(\varphi_i-\varphi_J)^2} \le R \qquad (i = Q, \bar{Q}, q, \bar{q}, g)$$

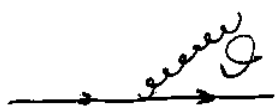
$$E_{TJ} = \sum_i E_{Ti} \; ; \; \eta_J = \frac{1}{E_{TJ}} \sum_i E_{Ti}\eta_i \; ; \; \varphi_J = \frac{1}{E_{TJ}} \sum_i E_{Ti}\varphi_i$$

- THE KEY DIFFERENCE BETWEEN HEAVY-QUARK JETS AND ORDINARY JETS ARISES IN THE CASE OF FINAL STATE COLLINEAR EMISSION

$\propto \frac{1}{\rho - \cos\theta} \quad (\rho > 1)$

$\downarrow$

$\boxed{\text{NO}}$ FINAL STATE DIVERGENCES



$\propto \frac{1}{1-\cos\theta}$

$\downarrow$

FINAL STATE DIVERGENCES

$\Downarrow$

THE STRUCTURE OF THE SINGULARITIES IN THE HEAVY-QUARK JET CROSS SECTION IS IDENTICAL TO THE STRUCTURE OF THE SINGULARITIES IN THE OPEN HEAVY QUARK ONE

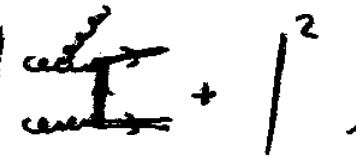
AT NLO, THE CROSS SECTION FOR HEAVY-QUARK JET PRODUCTION IS INDEED VERY SIMPLE

$$d\hat{\sigma}_{ij} = d\hat{\sigma}_{ij}^{(OPEN)} + d\Delta_{ij}$$

$d\hat{\sigma}_{ij}^{(OPEN)}$ OPEN HEAVY QUARK CROSS SECTION KNOWN FROM THE LITERATURE

$d\Delta_{ij}$ IS THE NEW FEATURE OF THE GAME, DUE TO THE JET DEFINITION

$$d\Delta_{ij} = \tilde{S}_3 \, M_{ij}^{(R)} \, d\phi_3 \, dE_J \, d\eta_J \, d\varphi_J$$

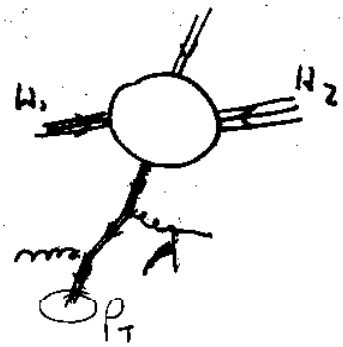
$M_{qg}^{(R)} =$  ; $M_{q\bar{q}}^{(R)}$ ....

$$\tilde{S}_3 = \begin{cases} \text{JET} = Q & \text{(WITH NEGATIVE WEIGHT)} \\ \text{JET} = Qg \\ \text{JET} = Q\bar{Q} \end{cases}$$

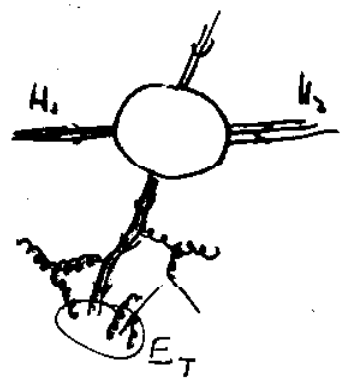
AT TEVATRON, THE COMPARISON DATA VS THEORY FOR b PRODUCTION IS SIGNIFICANTLY BETTER FOR HEAVY QUARK JETS THAN FOR OPEN b PRODUCTION

A VERY SPECIAL CASE

TO UNDERSTAND THE PRODUCTION MECHANISM OF THE b QUARK, WE CAN LOOK FOR THOSE JETS WHICH CONTAIN A HEAVY FLAVOUR



OPEN HEAVY QUARK



HEAVY-QUARK JET

- LOGARITHMS $\log \frac{p_T}{m}$ $(Q \to gQ \; ; \; g \to Q\bar{Q})$ WHICH AFFECT OPEN HEAVY QUARK CROSS SECTION, ARE EXPECTED NOT TO BE IMPORTANT FOR HEAVY-QUARK JETS
- HEAVY-QUARK FRAGMENTATION FUNCTION IS NOT NEEDED FOR THE THEORETICAL PREDICTION OF THE HEAVY-QUARK JET CROSS SECTION
- BRANCHING RATIOS AND DECAY SPECTRA DEPENDENCE SHOULD ONLY MARGINALLY AFFECT THE EXPERIMENTAL ANALYSIS

CONCLUSIONS

- IMPLEMENTATIONS OF THE SUBTRACTION METHOD IN THE PAST SOMEWHAT LACKED IN FLEXIBILITY
- IT HAS BEEN SHOWN THAT A GENERAL FORMALISM CAN BE FORMULATED IN TERMS OF THE SUBTRACTION METHOD, AND STRAIGHTFORWARDLY APPLIED TO
  - ANY NUMBER OF FINAL STATE PARTONS
  - $H_1 H_2$, γH, e^+e^- PRODUCTION PROCESSES

  FINITE CROSS SECTIONS ARE OBTAINED AT THE PARTON LEVEL
- AT PRESENT, PHENOMENOLOGICAL APPLICATIONS TO
  - $e^+e^- \to 4$ JETS
  - $H_1 H_2, \gamma H \to$ 3-PARTON INFRARED-SAFE QUANTITIES
  - $H_1 H_2 \to$ HEAVY FLAVOUR JETS

FRIXIONE, MANGANO

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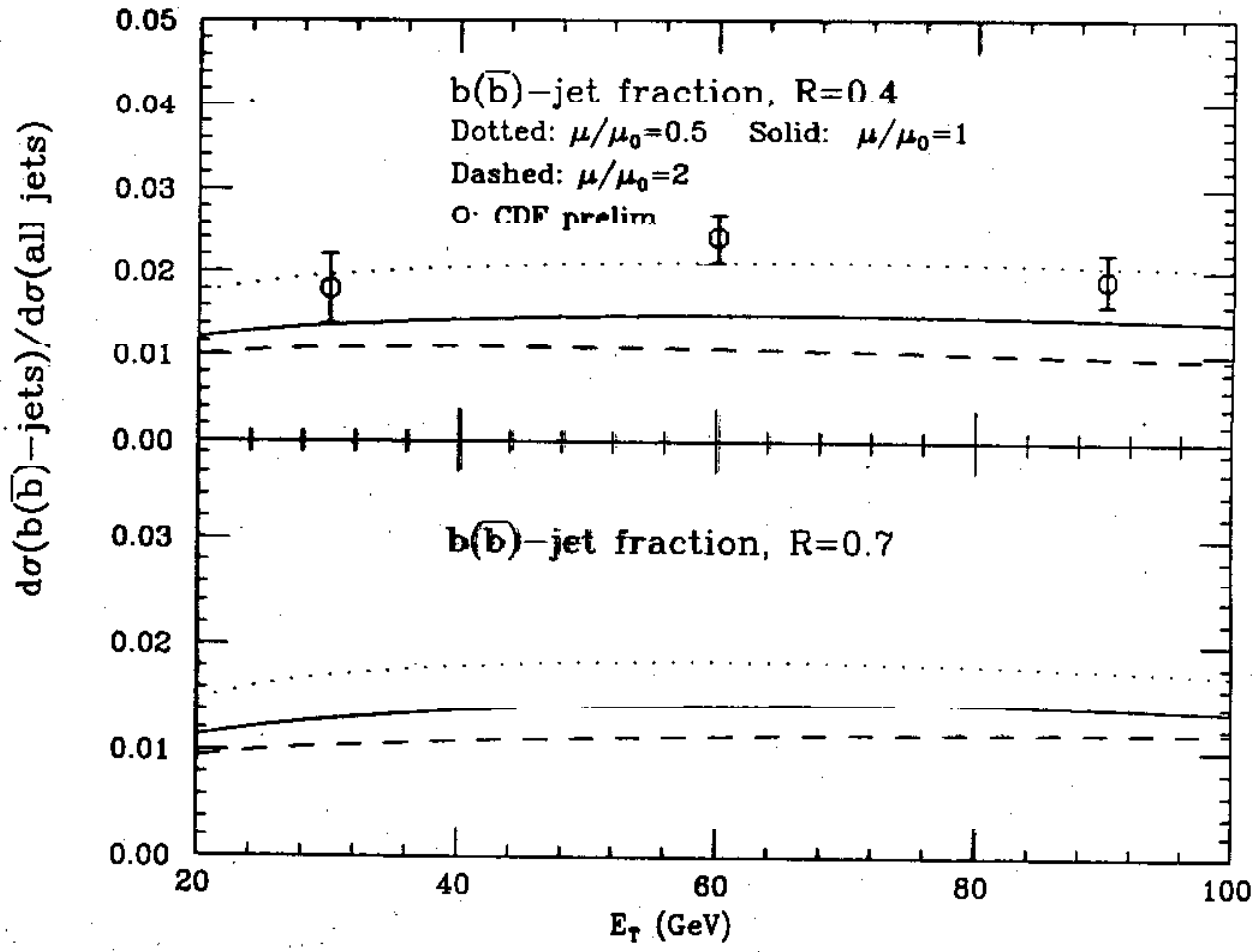

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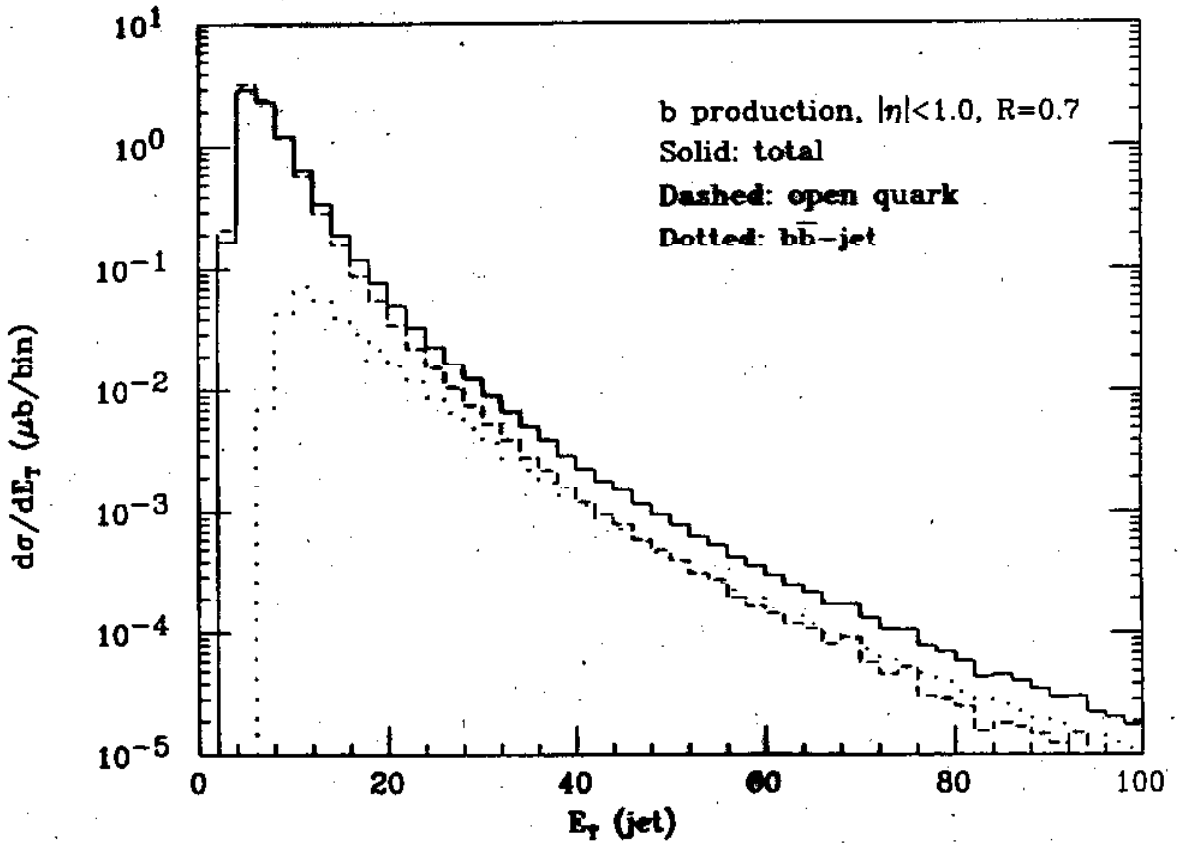

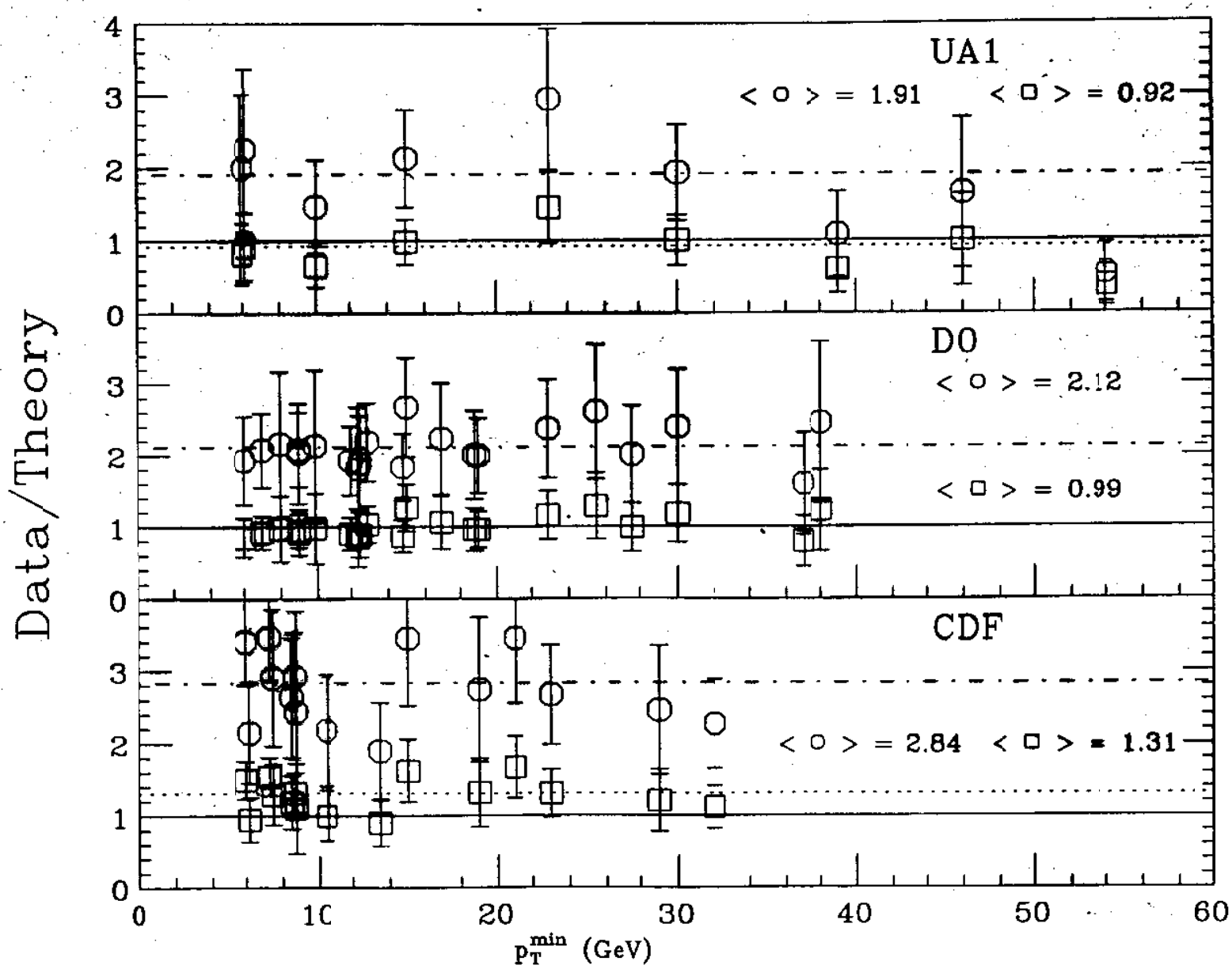
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D0
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CDF
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Data/Theory
p_T^{min} (GeV)
0
10
20
30
40
50
60
1
2
3
4