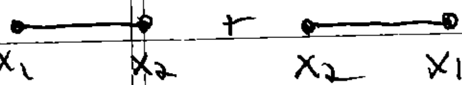
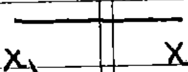


$= \frac{1}{2i}\Delta(x_1-x_2) + \frac{1}{2i}\Delta(x_2-x_1)$

+ higher order in g $\quad = \frac{1}{i}\Delta(x_1-x_2)$.

Symmetry Factor cancels. Draw as 1 diagram



. At higher orders,

$+ \mathcal{O}(g^3)$

Now look at $\langle 0 | T\{\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)\}|0\rangle$

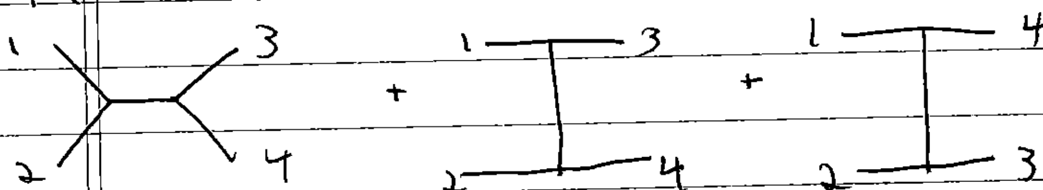
$$= \delta_1\delta_2\delta_3\delta_4\, e^{iW(J)}\Big|_{J=0} = \delta_1\delta_2\delta_3\delta_4\, iW(J)\Big|_{J=0}$$

$$+ \delta_1\delta_2\, iW(J)\Big|_{J=0}\, \delta_3\delta_4\, iW(J)\Big|_{J=0}$$

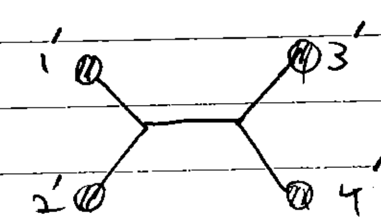
$$+ \delta_1\delta_3\, iW(J)\Big|_{J=0}\, \delta_2\delta_4\, iW(J)\Big|_{J=0}$$

$$+ \delta_1\delta_4\, iW(J)\Big|_{J=0}\, \delta_2\delta_3\, iW(J)\Big|_{J=0}$$

First term: what are the connected diagrams with $E=4$? $\quad E = 2P - 3V \Rightarrow V=2,\ P=5$



Note:



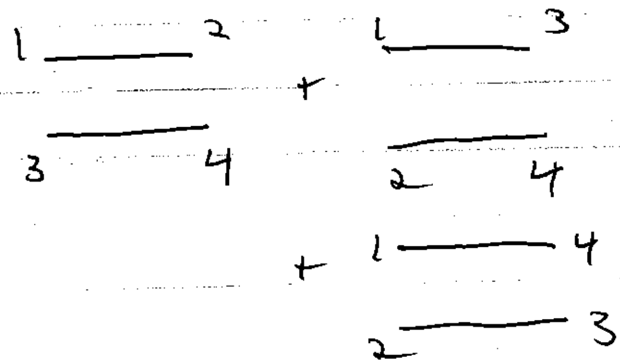
has $S = 2^3$ (Flip $1 \leftrightarrow 2$, $3 \leftrightarrow 4$, two vertices)

But, when $\partial_1\partial_2\partial_3\partial_4$ acts on this, δ_1 can act on any of the 4 sources $\Rightarrow$ δ_2 is then fixed to act on $J(x_2')$, & δ_3 has two possibilities (x_3' or x_4')

$\Rightarrow \frac{8}{8} = 1 \Rightarrow$ no symmetry factor.

This is a general result for <u>tree diagrams</u>

$V=0$, $P=2$ terms are

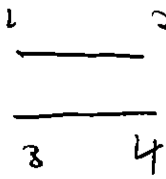


Look at $V=0$, $P=2$ terms first $\Rightarrow$ plug into LSZ

$\Rightarrow$ recall $\langle F|i\rangle = (+i)^4 \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \, e^{i(k_1\cdot x_1 + k_2\cdot x_2 + k_3\cdot x_3 + k_4\cdot x_4)}$

$$\{\partial_1^2+m^2\}\{\partial_2^2+m^2\}\{\partial_3^2+m^2\}\{\partial_4^2+m^2\}$$
$$\langle 0|T\{\phi_1\phi_2\phi_3\phi_4\}|0\rangle$$

Note: 1,2 incoming
3,4 outgoing

Study the  $= \frac{1}{i}\Delta(x_1-x_2)\frac{1}{i}\Delta(x_3-x_4)$ contribution

$\{\partial_1^2+m^2\}\Delta(x_1-x_2) = \delta^{(4)}(x_1-x_2)$, $\{\partial_3^2+m^2\}\Delta(x_3-x_4) = \delta^{(4)}(x_3-x_4)$

$$\langle F|i\rangle = \frac{(+i)^4}{i^2}\int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \, e^{i[-k_1\cdot x_1 - k_2\cdot x_2 + k_3\cdot x_3 + k_4\cdot x_4]}$$
$$\{\partial_2^2+m^2\}\delta^{(4)}(x_1-x_2)\{\partial_4^2+m^2\}\delta^{(4)}(x_3-x_4)$$

I.B.P on derivative terms

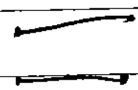
$$\Rightarrow \langle F|i\rangle = \frac{(+i)^4}{i^2}\int d^4x_1\, d^4x_2\, d^4x_3\, d^4x_4\, e^{i[-k_1\cdot x_1 - k_2\cdot x_2 + k_3\cdot x_3 + k_4\cdot x_4]}$$

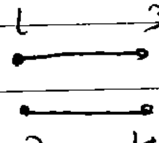
$$\{-k_2^2+m^2\}\{-k_4^2+m^2\}\,\delta^{(4)}(x_1-x_2)\,\delta^{(4)}(x_3-x_4)$$

$$= \frac{(+i)^4}{i^2}\int d^4x_1\, d^4x_3\, e^{-i(k_1+k_2)\cdot x_1}\, e^{+i(k_3+k_4)\cdot x_3}\,\{-k_2^2+m^2\}\{-k_4^2+m^2\}$$

$$= (2\pi)^2\frac{(+i)^4}{i^2}\{-k_2^2+m^2\}\{-k_4^2+m^2\}\,\delta^{(4)}(k_1+k_2)\,\delta^{(4)}(k_3+k_4)$$

$= 0$; $k_1 \neq -k_2$, since $k_1^0 + k_2^0 \geq 2m$

All other diagrams  vanish for this or similar reasons

For example, 

leads to $\delta^{(4)}(k_1-k_3)\,\delta^{(4)}(k_2-k_4)$

$\Rightarrow$ no scattering, particles go straight through.

Only contribution is from Fully connected diagrams; in this case, those from $\delta_1\delta_2\delta_3\delta_4\, iW(J)\big|_{J=0}$

In general, the fully connected diagrams are obtained from all derivatives acting on $W(J)$

$$\langle 0|T\{\phi_1\cdots\phi_n\}|0\rangle_c = \delta_1\cdots\delta_n\, iW(J)\big|_{J=0}$$

Look at our 3 contributions to $\langle 0|T\{\phi_1\phi_2\phi_3\phi_4\}|0\rangle_c$

$$= (ig)^2\left(\frac{1}{i}\right)^5 \int d^4y\, d^4z \Big\{ \Delta(x_1-y)\Delta(x_2-y)\Delta(y-z)\Delta(z-x_3)\Delta(z-x_4)$$
$$+ \Delta(x_1-y)\Delta(x_3-y)\Delta(y-z)\Delta(x_2-z)\Delta(x_4-z)$$
$$+ \Delta(x_1-y)\Delta(x_4-y)\Delta(y-z)\Delta(x_2-z)\Delta(x_3-z)\Big\}$$

Plug into LSZ $\Rightarrow$ the $(\partial_i^2+m^2)$ acting on each external coordinate replaces $\Delta(x_i-y) \to \delta^{(4)}(x_i-y)$

$$\langle F|i\rangle = (ig)^2\left(\frac{1}{i}\right)^5 (+i)^4 \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \int d^4y\, d^4z$$
$$e^{i(k_1\cdot x_1 + k_2\cdot x_2 + k_3\cdot x_3 + k_4\cdot x_4)}\, \Delta(y-z) \Big\{ \delta^{(4)}(x_1-y)\delta^{(4)}(x_2-y)\delta^{(4)}(x_3-z)\delta^{(4)}(x_4-z)$$
$$+ \delta^{(4)}(x_1-y)\delta^{(4)}(x_3-y)\delta^{(4)}(x_2-z)\delta^{(4)}(x_4-z)$$
$$+ \delta^{(4)}(x_1-y)\delta^{(4)}(x_4-y)\delta^{(4)}(x_2-z)\delta^{(4)}(x_3-z)\Big\}$$

$$= (ig)^2 \left(\frac{1}{i}\right)^5 (+i)^4 \int d^4y\, d^4z\, \Delta(y-z) \left\{ e^{-i(k_1+k_2)\cdot y}\, e^{+i(k_3+k_4)\cdot z} + e^{-i(k_1-k_3)\cdot y}\, e^{-i(k_2-k_4)\cdot z} + e^{-i(k_1-k_4)\cdot y}\, e^{-i(k_2-k_3)\cdot z} \right\}$$

Now set $\Delta(y-z) = -\int \frac{d^4k}{(2\pi)^4} \frac{e^{ik\cdot(y-z)}}{k^2-m^2+i\varepsilon}$

$$= -(ig)^2 \left(\frac{1}{i}\right)^5 (+i)^4 \int \frac{d^4k}{(2\pi)^4} \left\{ (2\pi)^4 \delta^{(4)}(k_1+k_2 \ast k)\, \delta^{(4)}(-k+k_3+k_4) + (2\pi)^4 \delta^{(4)}(k_1-k_3 \ast k)\, \delta^{(4)}(k_2-k_4 \ast k) + (2\pi)^4 \delta^{(4)}(k_1-k_4 \ast k)\, \delta^{(4)}(k_2-k_3 \ast k) \right\} \ast \frac{1}{k^2-m^2+i\varepsilon}$$

$$= -(2\pi)^4 (ig)^2 \left(\frac{1}{i}\right)^5 (+i)^4\, \delta^{(4)}(k_1+k_2-k_3-k_4) \left\{ \frac{1}{(k_1+k_2)^2-m^2} + \frac{1}{(k_1-k_3)^2-m^2}\ \frac{1}{(k_1-k_4)^2-m^2} \right\}$$

Define $\langle F|i\rangle = (2\pi)^4 \delta^{(4)}(k_1+k_2-k_3-k_4)\, iT$

$$\Rightarrow iT = (ig)^2 \left\{ \frac{i}{(k_1+k_2)^2-m^2} + \frac{i}{(k_1-k_3)^2-m^2} + \frac{i}{(k_1-k_4)^2-m^2} \right\}$$

T invariant matrix element

Leads to the momentum-space Feynman rules for computing the matrix element

Cross sections

We are considering $k_1 k_2 \to k_1' k_2'$ in ϕ^3. Begin with some discussion of kinematics. Useful to define the Mandelstam invariants to describe scattering.
~~For 2→2 scattering three of them~~

$$s = (k_1+k_2)^2 = (k_1'+k_2')^2 \Rightarrow \text{by momentum cons.,} \quad k_1^\mu + k_2^\mu = k_1'^\mu + k_2'^\mu$$

$$t = (k_1-k_1')^2 = (k_2-k_2')^2$$
$$u = (k_1-k_2')^2 = (k_2-k_1')^2$$

In terms of these, our ϕ^3 amplitude becomes

$$T = -g^2 \left\{ \frac{1}{s-m^2} + \frac{1}{t-m^2} + \frac{1}{u-m^2} \right\}$$

~~Note that s,t,u are clearly Lorentz invariant.~~
~~Let's work out some general relationships for these;~~
~~For generality, allow all particles to have different mass.~~

Useful to calculate in CM frame: $\vec{k}_1 + \vec{k}_2 = \vec{k}_1' + \vec{k}_2' = 0$

$$s = (k_1+k_2)^2 = (k_1^0+k_2^0)^2 = (E_1+E_2)^2 = (E_1'+E_2')^2 \Rightarrow s = \text{CM energy squared}$$

Using on-shell conditions $E_1 = \sqrt{\vec{k}_1^2 + m_1^2}$, $E_2 = \sqrt{\vec{k}_2^2 + m_2^2}$,

can derive $|\vec{k}_1|_{CM} = \dfrac{1}{2\sqrt{s}} \sqrt{s^2 - 2(m_1^2+m_2^2)s + (m_1^2-m_2^2)^2}$

~~Always useful to express quantities in L.I. form!~~

Can do same thing for final state momenta:

$$|\vec{k}_1'|_{cm} = \frac{1}{2\sqrt{s}}\sqrt{s^2 - 2(m_1'^2 + m_2'^2)s + (m_1'^2 - m_2'^2)}$$

Same expressions for $|\vec{k}_2|_{cm}, |\vec{k}_2'|_{cm}$

Now work on $t, u \Rightarrow \quad t = (k_1 - k_1')^2 = m_1^2 + m_1'^2 - 2k_1 \cdot k_1'$

$$= m_1^2 + m_1'^2 - 2E_1E_1' + 2|\vec{k}_1|_{cm}|\vec{k}_1'|_{cm}\cos\theta$$

↑ angle between;

$$\cos\theta = \frac{\vec{k}_1 \cdot \vec{k}_1'}{|\vec{k}_1||\vec{k}_1'|} \text{ in CM frame}$$

Now, $u = (k_1 - k_2')^2 = m_1^2 + m_2'^2 - 2E_1E_2' + 2\vec{k}_1 \cdot \vec{k}_2'$

use $E_2' = E_1 + E_2 - E_1'$; $\vec{k}_2' = \vec{k}_1 + \vec{k}_2 - \vec{k}_1' = -\vec{k}_1'$

$$\Rightarrow u = m_1^2 + m_2'^2 - 2E_1^2 - 2E_1E_2 + 2E_1E_1' - 2|\vec{k}_1|_{cm}|\vec{k}_1'|_{cm}\cos\theta$$

Note $s = (E_1 + E_2)^2 = \underbrace{|\vec{k}_1|_{cm}^2 + |\vec{k}_2|_{cm}^2}_{2|\vec{k}_1|_{cm}^2} + m_1^2 + m_2^2 + 2E_1E_2$

$$\Rightarrow \quad -2E_1E_2 = m_1^2 + m_2^2 - s + 2|\vec{k}_1|_{cm}^2$$

Plug into $u \Rightarrow u = m_1^2 + m_2'^2 - 2E_1^2 + m_1^2 + m_2^2 - s + 2|\vec{k}_1|_{cm}^2$
$$+ 2E_1E_1' - 2|\vec{k}_1|_{cm}|\vec{k}_1'|_{cm}\cos\theta$$

set $E_1^2 - |\vec{k}_1|_{cm}^2 = m_1^2$

$$\Rightarrow u = m_2^2 + m_2'^2 - s + 2E_1E_1' - 2|\vec{k}_1|_{cm}|\vec{k}_1'|_{cm}\cos\theta$$

Add $t+u = -s + m_1^2 + m_1'^2 + m_2'^2 + m_2^2 \Rightarrow s+t+u = \sum_i m_i$

$\Rightarrow$ general result.

Now let's define our cross section (warning! ugly derivation)

Probability for $i \to F$: $p = \dfrac{|\langle F|i\rangle|^2}{\langle F|F\rangle\langle i|i\rangle}$

We know $\langle F|i\rangle = (2\pi)^4 \delta^{(4)}(k_{in} - k_{out})\, i\, T$

$$\Rightarrow |\langle F|i\rangle|^2 = (2\pi)^4 \delta^{(4)}(k_{in} - k_{out})\, (2\pi)^4 \delta^{(4)}(0)\, |T|^2$$

Like earlier, write $(2\pi)^4 \delta^{(4)}(0) = \int d^4x = \underbrace{V}_{\text{space volume}} \underbrace{T_0}_{\text{total time}}$

Single particle normalization: $\langle k|k\rangle = (2\pi)^3 2E_k \delta^{(3)}(0)$
$= 2E_k V$

For two incoming & n' outgoing particles,

$$\langle i|i\rangle = 4E_1E_2V^2$$

$$\langle F|F\rangle = \prod_{j=1}^{n'} [2E_j' V]$$

$$\Rightarrow \dot{p} = \frac{p}{T_0} = |T|^2 (2\pi)^4 \delta^{(4)}(k_{in} - k_{out}) \frac{V}{4E_1E_2V^2 \prod_{j=1}^{n'} [2E_j' V]}$$

This is the probability for a specific set of final state momenta

Sum over momenta near each final state; instead of just $\vec{k}_j'$, allow $d\vec{k}_j' + \vec{k}_j'$. To see how this works, note the 1-particle completeness relation $\int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} |k\rangle\langle k| = 1$

{ Derive by showing that any single particle state can be written as $|p\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \langle k|p\rangle |k\rangle$ }

$$\Rightarrow \langle k_j'|k_j'\rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \langle k_j'|k\rangle\langle k|k_j'\rangle$$

$$\Rightarrow 1 = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} \frac{\langle k_j'|k\rangle\langle k|k_j'\rangle}{\langle k_j'|k_j'\rangle}$$

Integrand is [illegible] the measure for summing over states ⇒ differential probability to go from $k_j' \to k$

Now look at $k = k_j' + dk_j'$ ⇒ probability is

$$\frac{d^3k_j'}{(2\pi)^3} \frac{1}{2E_k} \frac{\langle k_j'|k\rangle\langle k|k_j'\rangle}{\langle k_j'|k_j'\rangle}$$

⇒ since $k \cong k_j'$, set $E_k = E_j'$

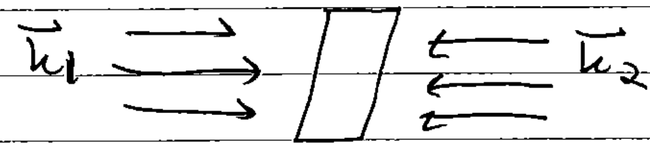
$$\frac{\langle k_j'|k\rangle\langle k|k_j'\rangle}{\langle k_j'|k_j'\rangle} \cong 2E_j' V$$

$\Rightarrow V \frac{d^3k_j'}{(2\pi)^3}$ ⇒ multiply $\dot{P}$ by this for each particle

$$\dot{P} = \frac{|T|^2 (2\pi)^4 \delta^{(4)}(K_{in}-K_{out})}{4E_1E_2V} \underbrace{\prod_{j=1}^{n'} \frac{d^3K_j'}{2E_j'(2\pi)^3}}_{\text{Lorentz invariant phase space for each particle}}$$

Now convert to differential cross section

$$d\sigma = \frac{\#\text{ of particles scattered per unit time}}{\#\text{ of particles passing per unit time per unit area}}$$



$$d\sigma = \frac{\dot{P}\, N_{tot}}{\{N_{tot} \times \Delta\text{Velocity}/V\}} = \frac{V\dot{P}}{|\vec{v}_1 - \vec{v}_2|}$$

$$\Rightarrow d\sigma = \underbrace{\frac{1}{4E_1E_2|\vec{v}_1-\vec{v}_2|}}_{\text{Flux factor}} \underbrace{|T|^2}_{\substack{\text{scattering}\\\text{amplitude}\\\text{squared}}} (2\pi)^4 \overset{\text{momentum conservation}}{\delta^{(4)}}(K_1+K_2-K_{out}) \underbrace{\prod_{j=1}^{n'} \frac{d^3K_j'}{2E_j'(2\pi)^3}}_{\text{phase space}}.$$

Can show that $[\sigma] = \frac{1}{\text{Energy}^2}$

Many special cases of this formula

It is often convenient to rewrite

$$\int \frac{d^3k_j'}{(2\pi)^3 2E_j'} = \int \frac{d^4k_j'}{(2\pi)^4}\,\theta(k_j'^0)\, 2\pi\delta(k_j'^2 - m^2)$$

Consider a 2-particle Final state. Phase-space + momentum conservation become

$$\frac{1}{4\pi^2}\int d^4k_1'\, d^4k_2'\, \delta(k_1'^2 - m_1'^2)\,\delta(k_2'^2 - m_2'^2)\,\delta^{(4)}(k_1 + k_2 - k_1' - k_2')\ \theta(k_1^{0\prime})\theta(k_2^{0\prime})$$

$$= \frac{1}{4\pi^2}\int d^4k_1'\, \delta(k_1'^2 - m_1'^2)\,\theta(k_1^{0\prime})\cdot \delta[(k_1 + k_2 - k_1')^2 - m_2'^2]$$

$$= \frac{1}{4\pi^2}\int \frac{d^3k_1'}{2E_1'}\, \delta[s - 2k_1\cdot k_1' - 2k_2\cdot k_1' + m_1'^2 - m_2'^2]$$

Go to CM Frame: $k_1^\mu = (E_1, 0, 0, |\vec{k}_1|)$ with $E_1 + E_2 = \sqrt{s}$

$t = (k_1 - k_1')^2$ $\quad k_2^\mu = (E_2, 0, 0, -|\vec{k}_1|)$ $\quad |\vec{k}_1| = \beta_1 E_1 = \beta_2 E_2$

$u = (k_2 - k_1')^2$ $\quad k_1'^\mu = E_1'(1, 0, \beta_1'\sin\theta, \beta_1'\cos\theta)$

$\beta_1' = \sqrt{1 - \frac{m_1'^2}{E_1'^2}}$ and $\beta E_1' = |\vec{k}_1'|$

δ-Function becomes $\delta[s + t + u - m_1^2 - m_2^2 - m_1'^2 - m_2'^2]$

set $t = (k_1 - k_1')^2 = m_1^2 + m_1'^2 - 2E_1E_1'(1 - \beta_1\beta_1'\cos\theta)$

$d^3k_1' = |\vec{k}_1'|^2\, d|\vec{k}_1'|\, d\cos\theta\, d\phi = \beta_1'(E_1')^2\, dE_1'\, d\cos\theta\, d\phi$

$$\Rightarrow \delta \Rightarrow \delta[s - 2E_1E_1'(1 - \beta_1\beta_1'\cos\theta) - 2E_2E_1'(1 - \beta_2\beta_1'\cos\theta) + m_1'^2 - m_2'^2]$$

$$= \delta[s + m_1'^2 - m_2'^2 - 2\sqrt{s}\,E_1']$$ using $\beta_1E_1 = \beta_2E_2$

Remove E_1' integration $\Rightarrow E_1' = \dfrac{s+m_1'^2-m_2'^2}{2\sqrt{s}}$

Get $\dfrac{1}{2\sqrt{s}}$ Jacobian

$\Rightarrow \dfrac{1}{4\pi}\,\dfrac{1}{2\sqrt{s}}\,\dfrac{1}{2}\,(\beta_1' E_1')\,d\Omega$ From Phase-space

$\Rightarrow$ so far, we have

$$\frac{d\sigma}{d\Omega_{CM}} = \frac{1}{4E_1E_2|\vec{v}_1-\vec{v}_2|}\,|T|^2\,\frac{1}{16\pi^2}\,\frac{\beta_1' E_1'}{\sqrt{s}}$$

In CM Frame, $E_1E_2|\vec{v}_1-\vec{v}_2| = E_2|\vec{k}_1| + E_1|\vec{k}_1| = \sqrt{s}\,|\vec{k}_1|$

$$\Rightarrow \frac{d\sigma}{d\Omega_{CM}} = \frac{1}{64\pi^2 s}\,|T|^2\,\underbrace{\frac{\beta_1' E_1'}{|\vec{k}_1|}}_{\frac{|\vec{k}_1'|}{|\vec{k}_1|}}$$

Can transform this by noting there is no ϕ dependence

$$\frac{d\sigma}{dc_\theta} = \frac{1}{32\pi s}\,|T|^2\,\frac{|\vec{k}_1'|}{|\vec{k}_1|}$$

$\Rightarrow t = m_1^2 + m_1'^2 - 2E_1E_1' + 2|\vec{k}_1||\vec{k}_1'|\cos\theta$

$\Rightarrow dt = 2|\vec{k}_1||\vec{k}_1'|\,dc_\theta$

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s}\,|T|^2\,\frac{1}{|\vec{k}_1|^2}$$

One more factor to include: when integrating over identical final-state particles, must not double-count phase-space should have $\int \frac{d^3k_1}{(2\pi)^3 2E_1} \frac{d^3k_2}{(2\pi)^3 2E_2} \; \Theta(k_{1x}-k_{2x})\,\Theta(k_{1y}-k_{2y})\,\Theta(k_{1z}-k_{2z})$

Equivalent to $\frac{1}{2!}\int \frac{d^3k_1}{(2\pi)^3 2E_1} \frac{d^3k_2}{(2\pi)^3 2E_2}$

Let's compute σ for our ϕ^3 theory

$$T = g^2\left\{\frac{1}{s-m^2} + \frac{1}{t-m^2} + \frac{1}{u-m^2}\right\}$$

$E_1 = E_2 = \frac{\sqrt{s}}{2}$; $|\vec{k}_1| = \beta\frac{\sqrt{s}}{2}$ with $\beta = \sqrt{1-\frac{4m^2}{s}}$

In the final state, $E_1' = E_2' = \frac{\sqrt{s}}{2}$; $\beta_1' = \beta_2' = \sqrt{1-\frac{4m^2}{s}}$

$$t = 2m^2 - \frac{s}{2}\left\{1-\left(1-\frac{4m^2}{s}\right)\cos\theta\right\}$$

$$= -\frac{1}{2}(s-4m^2)(1-\cos\theta) \;;$$

$$u = -\frac{1}{2}(s-4m^2)(1+\cos\theta) \qquad s+t+u = 4m^2$$

$t_{min} = -(s-4m^2)$; $t_{max} = 0$

$$\sigma = \frac{1}{2!}\,\frac{1}{64\pi s}\,\frac{4}{s-4m^2}\, g^4 \int_{t_{min}}^{t_{max}} dt \left\{\frac{1}{s-m^2} + \frac{1}{t-m^2} + \frac{1}{3m^2-s-t}\right\}$$

$$= \frac{g^4}{32\pi s(s-4m^2)}\left\{\frac{4m^2 \ln\left[\frac{s-3m^2}{m^2}\right]}{(s-m^2)(s-2m^2)} + \frac{(s-4m^2)(2s^2-3m^2s+m^4)}{m^2(s-m^2)^2(s-3m^2)}\right.$$