Features of SO(10) GUT Models

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Introduction

Many models in literature attempt to explain only neutrino masses and mixings.

More ambitious attempts construct SUSY GUT models to understand both lepton and quark sectors.

My talk will be restricted mainly to $SO(10)$ GUT models with 3 families in 4 dimensions, but some considerations will also be given to extending the models to 5 dimensions.

We'll see that several models are still quite successful in explaining the data.

SO(10) Model Structure

Essential Ingredients:

- 3 families of 16 LH q's and ℓ's $\rightarrow \mathbf{16}_i,\ i = 1,2,3$
- Higgs fields in $\{\mathbf{45}_H,\ \mathbf{16}_H,\ \overline{\mathbf{16}}_H\}$ or $\{\overline{\mathbf{126}}_H,\ \mathbf{45}_H,\ \mathbf{54}_H\}$ are needed to break $SO(10) \rightarrow$ SM.
- 2 Higgs doublets fit neatly into $\mathbf{10}_H \supset 5 + \bar{5}$ of $SU(5)$ or $\mathbf{10}_H \supset (6,1,1) + (1,2,2)$ of $SU(4) \times SU(2) \times SU(2)$.
- Doublet-triplet splitting can be achieved via Dimopoulos-Wilczek mechanism, if $\langle \mathbf{45}_H \rangle$ points in $B - L$ direction.
- With only one $\mathbf{10}_H$ effecting the electroweak breaking, $\tan\beta \equiv v_u/v_d \sim 55$.

Additional Higgs fields may be desirable:

- $\mathbf{16}'_H,\ \overline{\mathbf{16}}'_H$ can help to stabilize doublet-triplet splitting.
- If $\langle \bar{5}(\mathbf{16}'_H\rangle \neq 0$, then $H_d \sim \bar{5}(\mathbf{10}_H)\cos\gamma + \bar{5}(\mathbf{16}'_H)\sin\gamma$ and $\tan\beta \sim 1 - 55$ is possible.
- $\mathbf{126}_H$ and $\overline{\mathbf{126}}_H$ are other possibilities.

Additional Matter Fields may be desirable:

- $\mathbf{16},\ \overline{\mathbf{16}}$ pairs may get supermassive and can be integrated out in Froggatt-Nielsen type diagrams for the mass matrix elements.

Horizontal Flavor Symmetries

While $SO(10)$ relates q's and ℓ's of one family, it is necessary to invoke some horizontal flavor symmetry or some effective criterion to avoid the bad $SU(5)$ relations: $m_d = m_e,\ m_s = m_\mu$. This can be done with 4 different levels of model building:

- Level 1: Simply impose a certain texture such as a modified Fritzsch form for the mass matrices.
- Level 2: Introduce an effective λ expansion for each mass matrix. The prefactors typically are not precisely determined, however.
- Level 3: Assign effective operators for each matrix element possibly with some flavor symmetry imposed.
- Level 4: Introduce a horizontal flavor symmetry which assigns flavor charges to every Higgs and matter superfield. Higgs and Yukawa superpotentials are constructed in terms of renormalizable (and possibly some non-renormalizable) terms which obey that flavor symmetry. Matrix elements then follow from Froggatt-Nielsen diagrams.

General Observations

- $SO(10)$ models differ by their choice of Higgs structure, horizontal flavor symmetry and flavor charge assignments (if any).

- Desirable Georgi-Jarlskog relations, $m_s = m_\mu/3$, $m_d = 3m_e$, can be obtained if $\langle \mathbf{45}_H \rangle$ points in the $B-L$ direction, or if $\langle \overline{5}(\mathbf{126}_H) \rangle$ is involved, as Clebsch factors of 1 and -3 appear respectively for the quark and lepton Yukawa couplings.

- Most early models were easily able to accommodate SMA solar solution, while some could accommodate LOW or QVO solution as well.

- To obtain the LMA solution, some fine tuning is generally required.
  - Typically models which require special features of the Dirac and right-handed Majorana mass matrices, N and M_R, to get maximal atmospheric mixing have trouble getting LMA.
  - Easier if M_R can be independently adjusted to get LMA, while N and L conspire to give maximal atmospheric mixing.

Type I canonical seesaw mechanism involves only the Dirac and right-handed Majorana neutrino mass matrices:

$$M_\nu = -N^T M_R^{-1} N.$$

Type II seesaw also includes the left-handed Majorana mass matrix involving an induced triplet VEV which can arise if both parity and $B-L$ are broken at the same scale:

$$M_\nu = M_L - N^T M_R^{-1} N$$

Two special categories of models exist in literature:

(1) $SO(10)$ Models with Lopsided Textures

- $\mathbf{10}_H$, $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$, $\mathbf{45}_H$ ... Higgs fields are present and couple to matter fields.

- No Higgs representations with rank > 2 are required, but $B-L$ symmetry is broken by 1 unit with $\langle 1(\mathbf{16}_H)\rangle$ and $\langle 1(\overline{\mathbf{16}}_H)\rangle$ VEVs. Hence R-parity is not preserved after the breaking and matter parity must be introduced to retain a stable LSP.

- $H_u \sim \langle 5(\mathbf{10}_H)\rangle$, while H_d arises from a combination of $\langle \bar{5}(\mathbf{10}_H)\rangle$ and $\langle \bar{5}(\overline{\mathbf{16}'}_H)\rangle$.

- With a flavor symmetry present, lopsided D and L mass matrices result and moderate values of $\tan\beta$ are possible.

- Large $U_{\mu 3}$ atmospheric neutrino mixing but small V_{cb} quark mixing result from this lopsided structure.

- Somewhat enhanced leptonic flavor violation is also predicted.

(2) Minimal $SO(10)$ Models

- $\mathbf{10}_H$, $\overline{\mathbf{126}}_H$ are only Higgs coupled directly to the matter fields.

- $\mathbf{120}_H$ or $\mathbf{45}_H + \mathbf{54}_H$ needed to break $SO(10)$ to the SM.

- Tensor representations of rank > 2 are disfavored by string theory, but R-parity is preserved after $B - L$ symmetry is broken by 2 units with a $\langle(10, 1, 3)\overline{\mathbf{126}}_H\rangle$ VEV.

- H_u, H_d Higgses are combinations of doublets in $\mathbf{10}_H$, $\overline{\mathbf{126}}_H$, so moderate values of $\tan\beta$ are possible.

- Flavor symmetry is not required as linear combinations of $\mathbf{10}_H$, $\overline{\mathbf{126}}_H$ VEVs with known Clebsches for the mass matrix elements can be directly determined by mass and mixing data.

- With Type I seesaw, CP phases in linear combinations of Higgs doublets are required to fit data.

  Fukuyama, Kikuchi, Okada

- With Type II seesaw and induced left-handed Majorana term dominant, large atmospheric neutrino mixing follows from $b - \tau$ unification.

  Bajc, Senjanovic, Vissani
  Goh, Mohapatra, Ng

Comparison of Some Selected SO(10) Models

Model	Flavor Sym.	Texture	$\tan\beta$	Viable
AB	$U(1) \times Z_2 \times Z_2$	Lopsided	~ 5	Yes
BPW	effective operators	Sym/Asym	low	Yes
BR	$SU(3)$	Lopsided	1-10	?
BRT	$U(2) \times U(1)^n$	Sym/Asym	~ 55	No
BW	postulated	Sym	?	?
CM	$U(2) \times (Z_2)^3$	Sym	10	?
CW	$\Delta(48) \times U(1)$	Sym/Asym	~ 2	No
FKO	None	Minimal (I)	45	?
GMN	None	Minimal (II)	10	Yes
KM	$SU(3) \times U(1)$	Lopsided	small	?
M	$U(1)_A \times Z_2$	Lopsided	small	No
RV-S	$SU(3)$ and	Sym/Asym	?	Yes

AB	Albright, Barr	$\sin^2 2\theta_{atm} \simeq 0.99$
BPW	Babu, Pati, Wilczek	Type II seesaw
BR	Berezhiani, Rossi	LMA solution possible?
BRT	Blazey, Raby, Toby	$\rho < 0$, ν_s required
BW	Buchmüller, Wyler	appropriate LMA mixing?
CM	Chen, Mahanthappa	U_{e3} marginal
CW	Chou, Wu	ν_s required
FKO	Fukuyama, Kikuchi, Okada	$\Delta m^2_{sol}/\Delta m^2_{atm}$ too large
GMN	Goh, Mohapatra, Ng	$\sin^2 2\theta_{atm} \leq 0.90$
KM	Kitano, Mimura	appropriate LMA mixing?
M	Maekawa	U_{e3} violates bound
RV-S	Ross, Velasco-Sevilla	not fully developed

Future Tests of SO(10) Models

Of the models listed, some are already nearly ruled out by the more accurate quark and lepton mixing data, but several still survive. Of course I have assumed there are no sterile neutrinos. In addition, some models may be revived by their authors with further tweaking.

Critical tests to be made in the future with Off-Axis Beams, Superbeams and possibly Neutrino Factories:

- Value of θ_{13}. Presently the CHOOZ bound is

$$|U_{e3}| \simeq \sin\theta_{13} \lesssim 0.20 \text{ or } \sin^2 2\theta_{13} \lesssim 0.16$$

- Normal vs. inverted hierarchy.

- Test of CP violation in the leptonic sector involving the Dirac phase δ and the two Majorana phases χ_1, χ_2.

For the models which clearly appear to be still viable, the predictions are as follows.

Model	Hierarchy	$\lvert U_{e3}\rvert$	$\sin^2 2\theta_{13}$	CP Phase
AB	Normal	0.01-0.02	~ 0.001	$2^\circ - 5^\circ$
BPW	Normal	?	?	?
GMN	Normal	0.16	0.10	–
RV-S	Normal	~ 0.07	~ 0.02	?

Zee-type models with a conserved $L_e - L_\mu - L_\tau$ number require an inverted hierarchy.

Extension of the Models to Five Dimensions

There are several advantages to lifting the 4D SO(10) models to 5 dimensions:

- gauge coupling unification can be improved with

  $\alpha_s(M_Z) \simeq 0.125 \to 0.118$, Hall, Nomura

- doublet-triplet splitting achieved without Dimopoulos-Wilczek mechanism,
- dim-5 operator proton decay via colored higgsino exchange can be avoided.

- Procedure
  - Compactify the 5th dimension on an $S^1/(Z_2 \times Z_2)$ orbifold where

$$S_1: \quad y = y + 2n\pi R, \quad y' = y + \pi R/2,$$

  Z_2 maps $y \leftrightarrow -y$, Z'_2 maps $y' \leftrightarrow -y'$, so fundamental region is restricted to $-\pi R/2 \leq y \leq 0$, with physical brane at O $(y = 0)$, hidden brane at O' $(y = -\pi R/2)$.

  - Generic bulk fields $\phi(x^\mu, y)$ of definite (P, P') parity can be expanded in a Fourier series in y with 4D space-time coefficient functions.

    ϕ^{-+}, ϕ^{--} bulk fields vanish on visible brane,

    ϕ^{+-}, ϕ^{--} bulk fields vanish on hidden brane

- Assume all $SO(10)$ $\mathbf{45}_g$ gauge fields and the $\mathbf{10}_H$ and $\mathbf{45}_H$ Higgs fields live in the bulk, while all other Higgs and matter fields live on the physical brane.
- Assume the orbifold compactification breaks $N = 1$ 5D SUSY to $N = 1$ 4D SUSY under the action of Z_2, while $SO(10)$ is broken to $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$ under action of Z'_2. In 4D all bulk fields become massive except for the K-K zero modes of the $\phi^{++}(x^\mu, y)$ fields. Dermisek, Mafi
- Parities are assigned so that this occurs:

$$\begin{aligned} \mathbf{45}_H = \quad & \Phi^{++}_{(15,1,1)} + \Phi^{++}_{(1,3,1)} + \Phi^{++}_{(1,1,3)} + \Phi^{+-}_{(6,2,2)} \\ & + \Phi^{c--}_{(15,1,1)} + \Phi^{c--}_{(1,3,1)} + \Phi^{c--}_{(1,1,3)} + \Phi^{c-+}_{(6,2,2)} \\ \mathbf{10}_H = \quad & \Phi^{++}_{(1,2,2)} + \Phi^{+-}_{(6,1,1)} + \Phi^{c--}_{(1,2,2)} + \Phi^{c-+}_{(6,1,1)} \end{aligned}$$

- Findings
  - Superpotential on the hidden brane where the gauge symmetry is G_{PS} leads to $\langle \mathbf{45}_H \rangle$ pointing in the $B - L$ direction, which corresponds to one of the generators of G_{PS}.
  - On the visible brane, massless zero mode of $\Phi^{++}_{(1,2,2)}$ contains two Higgs doublets, while the colored Higgs triplets in $\Phi^{+-}_{(6,1,1)}$ are made superheavy by orbifold compactification.
  - Dim-5 proton decay operators are absent, since color-triplet Higgs do not have superheavy mass terms connecting them, so no exchange of colored higgsinos mediate proton decay.
  - Higgs superpotential is simplified, while Yukawa superpotential and mass matrices derived from it remain essentially unaltered. Albright, Barr

Summary

- A number of $SO(10)$ SUSY GUT models have been proposed in the literature. Some have been or are on the verge of being eliminated, while some still survive and are able to explain all the presently known quark and lepton mass and mixing data.

- Long baseline experiments which can determine whether the neutrino mass hierarchy is normal or inverted appear to have a direct bearing on the survival of $SO(10)$ vs conserved-lepton-number-type models.

- The observed value of $\sin^2 2\theta_{13}$ will further narrow down the list of viable models. Some predict that θ_{13} lies just below the CHOOZ bound and will be observable with off-axis beams and/or Superbeams. Others favor such low values of θ_{13} that a Neutrino Factory may be required to determine its value.

- The issue of proton decay via dim-5 operators is potentially a serious one with 4D models, if proton decay is not detected shortly. On the other hand, by formulating an $SO(10)$ model in 5 dimensions, one can eliminate the dim-5 operator contributions entirely. The dim-6 operators involving colored Higgs exchange will still be present, but the proton decay lifetime is then expected to be in the 10^{35-36} year range.