Cosmological Connections of Neutrino Physics

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Relic neutrino background

- Neutrinos decouple from other particles in the early universe when the temperature is a few MeV
- These neutrinos have a temperature of about 2K today -- which is slightly below the temperature of the cosmic microwave background photons. This corresponds to an energy of order 10^{-4} eV.

e^+e^- annihilation heats the photons but not the neutrinos

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$$

The number density per flavour is:

$$n_\nu + n_{\bar{\nu}} = 112\,\mathrm{cm}^{-3}$$

Unlike the CMB photons, the relic neutrino background has never been directly detected -- but we can infer its existence and some of its properties though indirect means:

Today, T_ν~2K, so :

If m > 5×10^{-4} eV, the relic neutrinos should be non-relativistic today.

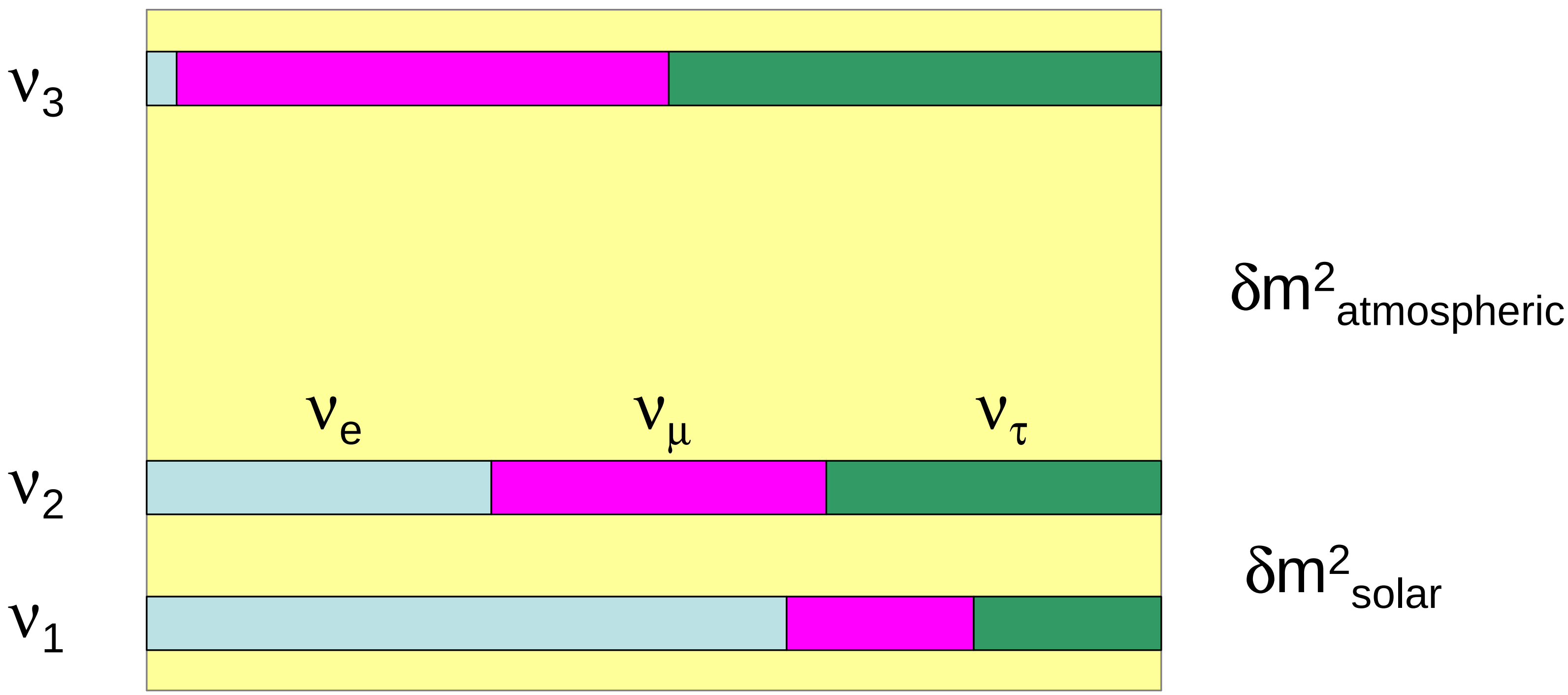
Given the solar and atmospheric mass squared differences:

$$\sqrt{\delta m^2_{atm}} > 0.04\,\mathrm{eV}$$
$$\sqrt{\delta m^2_{solar(LMA)}} > 0.004\,\mathrm{eV}$$

at least 2 of the 3 neutrinos states are non-relativistic today.

Flavour composition of the three neutrino mass eigenstates

(Ignoring LSND)



Central Idea:

What fraction of the total energy density of the universe is in the form of neutrinos?

Energy density = mass x number density

$$\rho_\nu = m_\nu n_\nu$$

- Large scale structure can measure ρ
- BBN can measure n

So, *provided the number density n is well constrained*, we can “weigh” neutrinos with cosmology.

Weighing neutrinos with cosmology

Oscillations tell us mass differences between the three neutrinos, not the masses themselves.

Relic neutrino density

$$\Omega_\nu = \frac{\sum m_\nu (\mathrm{eV})}{92.5 h^2} \qquad \sum m_\nu = m_1 + m_2 + m_3$$

In order for the neutrinos not to overclose the universe:

$$\sum m_\nu < 40 \ \mathrm{eV}$$

Gershtein & Zeldovich 1966.

Best laboratory limit: mass < 2.2 eV (Tritium beta decay)

Hot dark matter

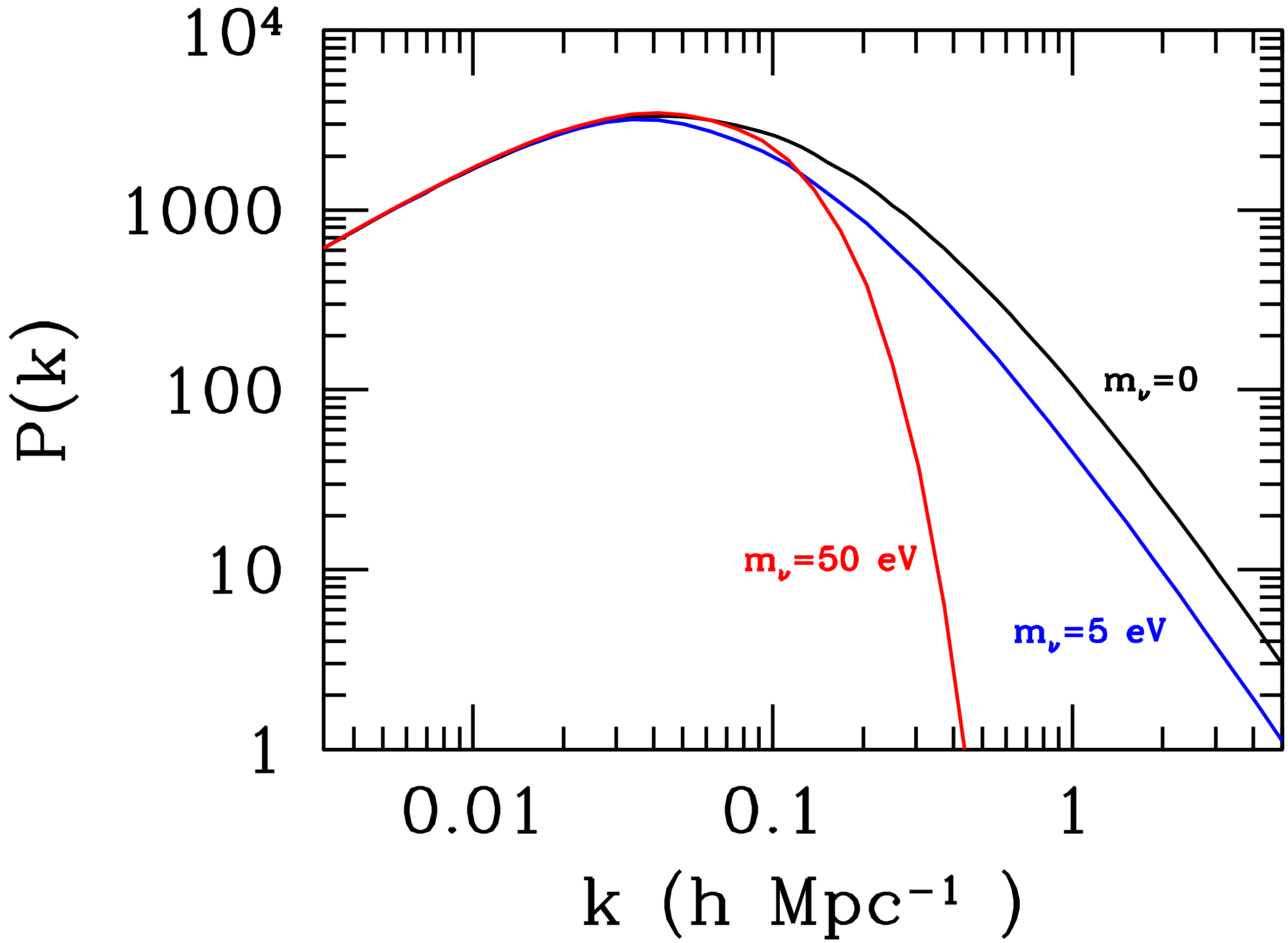
Neutrinos would be “hot” dark matter

Large scale structure considerations tell us that all the dark matter cannot be hot.

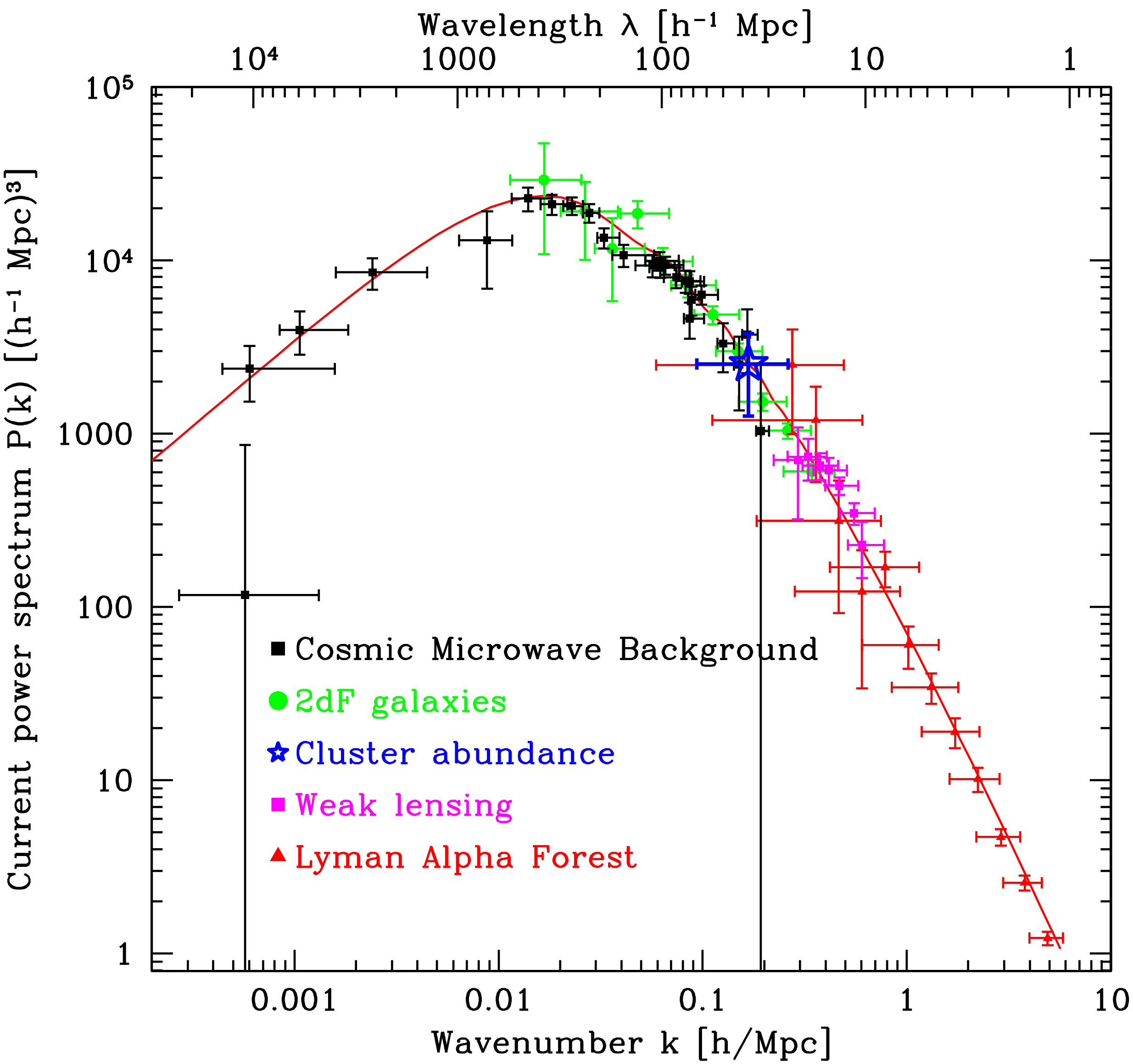
Observed structure in universe arises from primordial density fluctuations.

Characteristic feature of hot dark matter is the damping of density fluctuations on small scales .

Neutrinos “free stream” out of overdense regions – which tends to erase density fluctuations



S. Dodelson



Tegmark and Zaldarriaga

Mass limits from Large Scale Structure

2dF Galaxy survey → ***sum*** of the neutrino masses < 1.8 eV

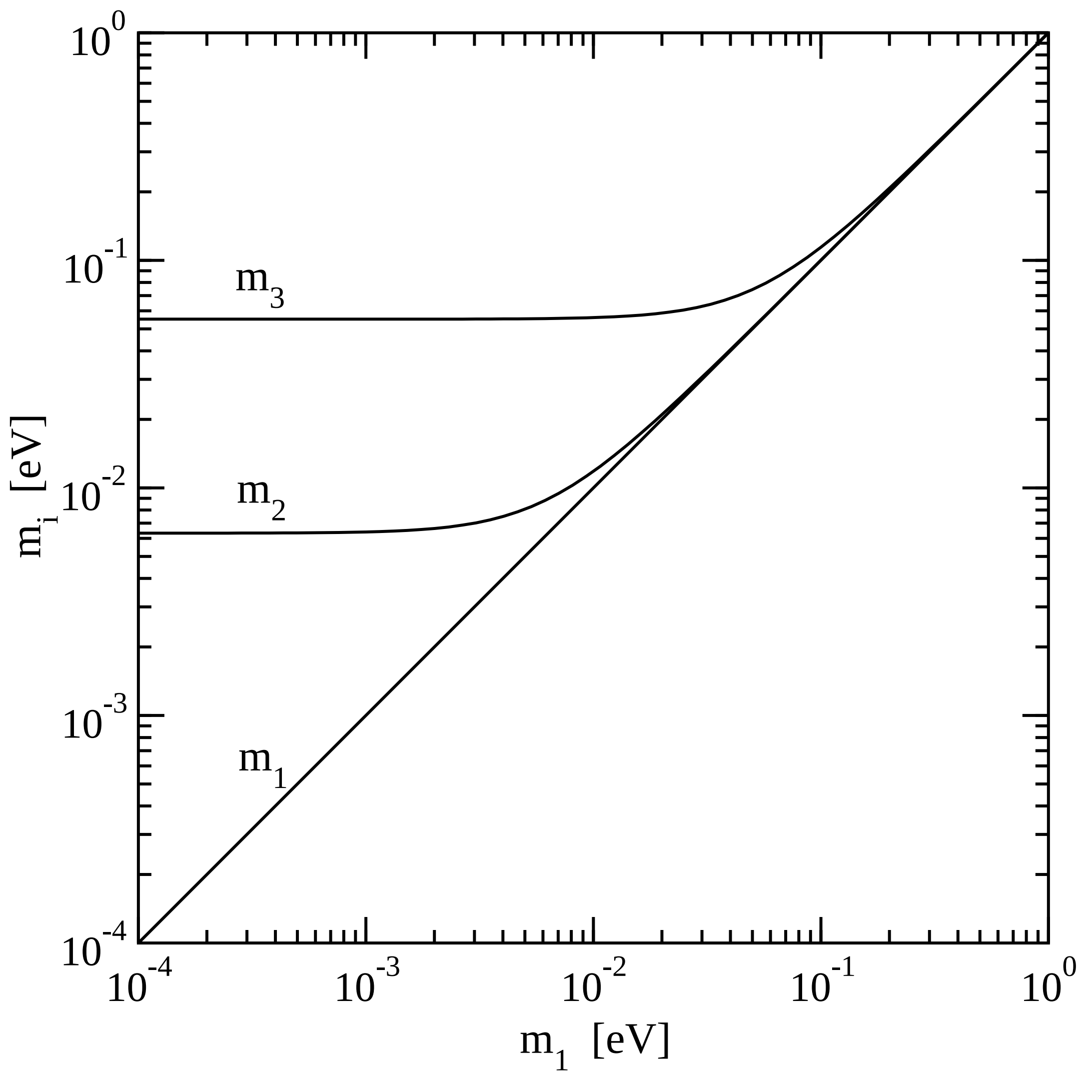
2dF + recent WMAP results → ***sum*** of the neutrino masses < 0.7 eV
Thus, each neutrino mass eigenstate < 0.23 eV

(Tritium beta decay limit: each neutrino mass < 2.2 eV)

Implications for sterile neutrinos and $\delta m^2_{lsdn} \sim O(eV^2)$ mass differences?

It depends whether the sterile neutrino was populated in the early universe – i.e. whether the sterile is brought into thermal equilibrium.

Neutrino mass degeneracy



Beacom & Bell (2002)

Models with a sterile neutrino to accommodate LSND:

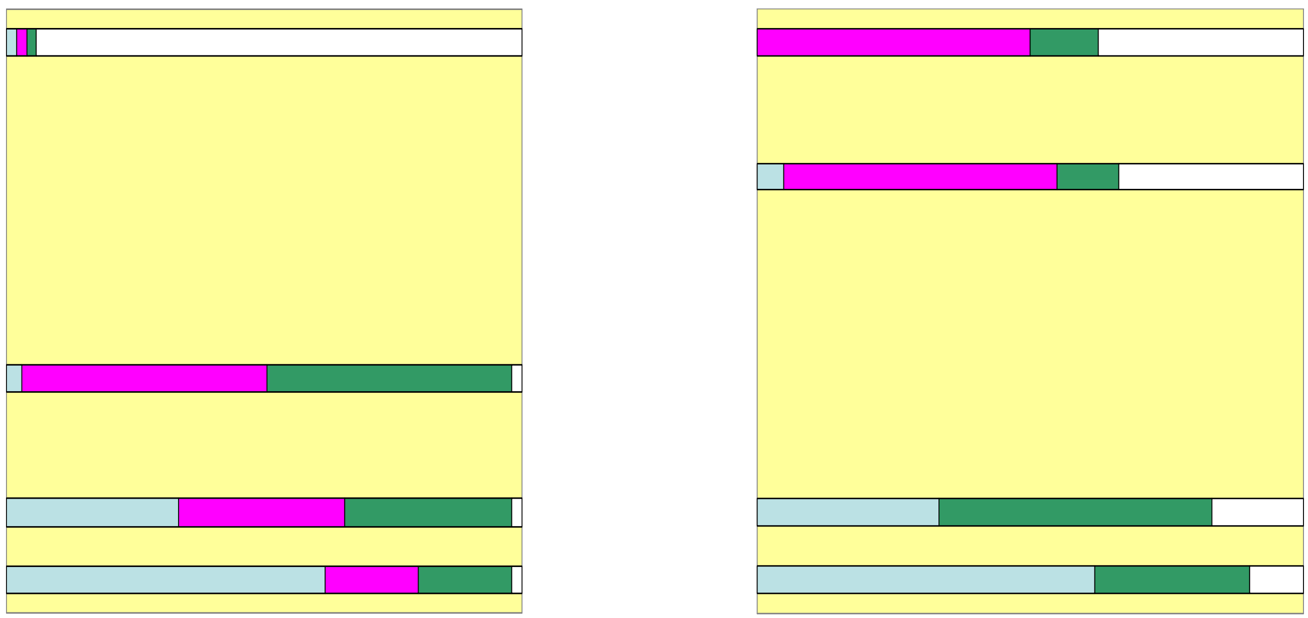
“3+1” scheme

“2+2” scheme

δm^2_{LSND}

$\delta m^2_{atmospheric}$

δm^2_{solar}



Neutrinos and Big Bang Nucleosynthesis

(the lepton number of the Universe)

Baryon asymmetry: $B = \frac{n_b - n_{\bar{b}}}{n_\gamma} \sim 10^{-10}$

Lepton asymmetry: → only very weak constraints.

Charge neutrality of the universe prevents a large asymmetry in the charged leptons, but a large lepton number could reside in the neutrino sector.

How would we know?

Theoretical predictions:

- L is related to B (e.g. Lepto/Baryogenesis scenarios in which the two are connected via spheralon transitions which freeze out near the electroweak phase transition). $L \sim 10^{-10}$
- L and B are unrelated if EW symmetry was never restored.
  (and large L can prevent EW sym from being restored – Linde)
- L is much bigger than B, e.g. Affleck-Dine baryogenesis scenarios in SUSY models.
- Large L is generated at temperatures below the EW scale , eg via active-sterile neutrino oscillations. $L \sim 0.1$
- Etc, etc..

Neutrino asymmetries

(or neutrino chemical potentials, or neutrino degeneracy)

In thermal equilibrium, the neutrinos will have Fermi-Dirac distributions:

$$f(p,\xi) = \frac{1}{1+\exp(p/T-\xi)}$$

Lepton asymmetries imply chemical potentials: ξ_ν

$e^+ + e^- \leftrightarrow \nu + \bar{\nu}$ maintains chemical equilibrium such that: $\xi_\nu = -\xi_{\bar{\nu}}$

Lepton asymmetries: $$L_\alpha = \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_\gamma} = \frac{\pi^2}{12\varsigma(3)}\left(\xi_\alpha + \frac{\xi_\alpha^3}{\pi^2}\right)$$

Such degeneracies increase the effective number of species in equilibrium:

$$\Delta N_\nu = \frac{30}{7}\left(\frac{\xi}{\pi}\right)^2 + \frac{15}{7}\left(\frac{\xi}{\pi}\right)^4$$

Measuring N_ν with the CMB

The CMB can be used to probe the relativistic energy density.

Nucleosynthesis and the CMB probe completely different epochs in time
→ possibility to make cross-checks of the consistency of the theory.

CMB is sensitive to the epoch of matter radiation equality – this can be shifted by increasing N_ν, which alters the height of the first peak.

Pre-WMAP: $N_\nu<13$ (Hannestad).
Post-WMAP: $N_\nu<5$ (Crotty et al, Hannestad).

Future precision with Planck: $\Delta N_\nu<0.1$ (Bowen et al; Lopez et al.)

At the moment, the best limits still come from BBN.

Constraints on relic neutrino asymmetries…

…in the absence of neutrino mixing

BBN+CMB set weak bounds on the lepton numbers:

e.g. Hansen et al. (2001)

$$\text{Very weak bound for } \nu_{\mu,\tau}: \quad \left|\xi_{\mu,\tau}\right| < 2.6$$

$$\text{Stronger bound for } \nu_e: \quad -0.01 < \xi_e < 0.22$$

Neutrinos and Big Bang Nucleosynthesis

Temperature ~ MeV

Neutron to proton ratio set by the processes:

$$n + \nu_e \leftrightarrow p + e^-$$
$$n + e^+ \leftrightarrow p + \overline{\nu}_e$$

Practically all the neutrons eventually end up in Helium nuclei. All the leftover protons form Hydrogen.

$$n / p \approx \exp[-(m_n - m_p)/T]$$

These process “freeze out” when:

Interaction rate < Expansion rate

Expansion rate ∝ energy density

$$H = \frac{\dot{R}}{R} \propto \rho_{\mathrm{radiation}}$$

$$\text{expansion rate} \rightarrow \text{contribution from} \quad \gamma, e, \nu_e, \nu_\mu, \nu_\tau + \text{antiparticles}$$

If there were extra energy density in neutrinos (or any other relativistic particles)

- The universe would expand faster
- Weak interaction rates would freeze out earlier
- Larger n/p ratio and hence more Helium

Successful nucleosynthesis puts constraints on the expansion rate…and therefore tells us how many relativistic particles species were in thermal equilibrium.

ν_e directly affects neutron-proton equilibrium:

$$n + \nu_e \leftrightarrow p + e^-$$
$$n + e^+ \leftrightarrow p + \overline{\nu}_e$$

$$n/p \approx \exp\left[-(m_n - m_p)/T - \xi_e\right]$$

If there were an electron neutrino asymmetry:

eg. $n(\nu_e) > n(\overline{\nu}_e)$

$\Rightarrow$ n/p $\downarrow$

Y_P $\downarrow$

BBN (+CMB) constraint:

$$\left|\xi_{\mu,\tau}\right| < 2.6$$

$$-0.01 < \xi_e < 0.22$$

Note that the upper limits can only be obtained in tandem.
This is the degenerate BBN scenario in which the effects of the asymmetry in ν_e is compensated by faster expansion rate due to extra energy density.

Without this compensation the limit for ν_e would be:

$$\left|\xi_e\right| \lesssim 0.04$$

How do neutrino oscillations change things?

- Active-sterile mixing

→ LSND inspired mixing schemes

- Active-active mixing

→ Oscillations with the solar and atmospheric parameters.

Sterile neutrinos?

Active -sterile oscillations in the early universe:

$$\nu_{\text{active}} \leftrightarrow \nu_{\text{sterile}}$$

would thermalise the sterile neutrinos….

…and we know that BBN works more or less OK with just three neutrinos

In fact, successful BBN sets stringent bounds on active-sterile oscillation parameters:

eg. Assuming a BBN bound of $\mathrm{N}_\nu < 3.4$

Naive constraint on $\nu_\mu \leftrightarrow \nu_s$ or $\nu_\tau \leftrightarrow \nu_s$ mixing :

$$\left|\delta m^2 (\sin^2 2\theta)^{1.6}\right| < 10^{-7}\ \mathrm{eV}^2$$

e.g. Dolgov; Enqvist, Kainulainen &Thomson.

BBN says N < 4 → Sterile neutrinos are cosmologically disfavoured if they mix significantly with active neutrinos.

ALL "3+1" and "2+2" models which accommodate LSND are problematic for BBN.

See di Bari(2001); Abazajian (2002) for recent analyses.

HOWEVER… there are ways out…

- Equilibration of the sterile is avoided if a lepton asymmetry is present
→the mixing angle is suppressed due to the refractive index

Foot & Volkas (1995)

- Some other much more exotic scenarios…
…low reheating scenarios, coherent majoron fields, etc …

If MiniBooNE were to confirm LSND, it would be of *enormous* cosmological significance.

Active-Active Oscillations

Oscillations between active neutrino species in the Early Universe have received much less attention than active sterile oscillations because:

1. Oscillations do nothing if we have equal numbers of each flavour
But there may very well be asymmetries between the flavours.

2. Its a much harder problem.
Neutrino-neutrino forward scattering makes things non-linear resulting in highly non-trivial dynamics.

Savage, Malaney & Fuller.   Kostelecky, Pantaleone & Samuel.

LMA mixing and BBN

The Large Mixing Angle solution has been confirmed as correct resolution of the solar neutrino anomaly → KamLAND & SNO

Best fit mixing parameters:

$$\delta m_0^2 \approx 4\times 10^{-5}\ \mathrm{eV}^2$$
$$\sin^2 2\theta_0 \approx 0.8$$

Large-angle mixing could potentially equilibrate the flavours.

Savage, Malaney & Fuller (1991); Lunardini and Smirnov (2001).

Matter effects are quite significant and must be included to determine if equilibration would take place before weak freeze-out.

Dolgov et al. (2002); Abazajian, Beacom and Bell (2002); Wong (2002).

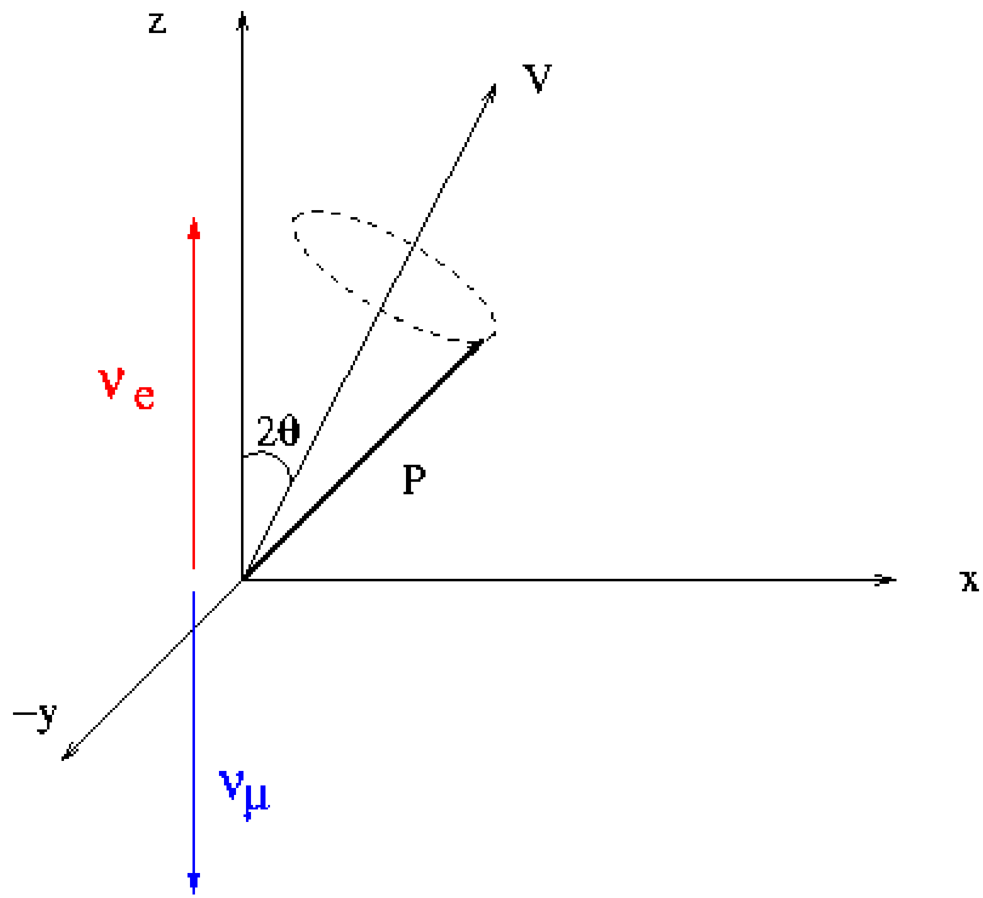
Density matrix parameterization:

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} \\ \rho_{\mu e} & \rho_{\mu\mu} \end{pmatrix} = \frac{1}{2}\left[1 + \boldsymbol{\sigma} \bullet \mathbf{P}\right]$$

$$= \frac{1}{2}\begin{pmatrix} 1+P_z & P_x + iP_y \\ P_x - iP_y & 1-P_z \end{pmatrix}$$

“Polarisation” vector:

$$\mathbf{P} = \left(P_x, P_y, P_z\right)$$



Oscillations are described by the precession of the P vector
-- just like a spin precessing in a magnetic field.

Evolution equations:

$$\partial_t \mathbf{P}_p = \left(\mathbf{A}_p + \alpha\,\mathbf{I}\right) \times \mathbf{P}_p$$
$$\partial_t \overline{\mathbf{P}}_p = \left(-\mathbf{A}_p + \alpha\,\mathbf{I}\right) \times \overline{\mathbf{P}}_p$$

A=vacuum mixing term + non-neutrino background

I= neutrino-neutrino forward scattering term

$$\mathbf{A}_p = \frac{\delta m_0^2}{2p}\left(\sin 2\theta_0 \hat{x} - \cos 2\theta_0 \hat{z}\right) + V_B \hat{z}$$

$$\mathbf{I} = \int \frac{d^3(p/T)}{(2\pi)^3}\left[\mathbf{P}_p - \overline{\mathbf{P}}_p\right]$$

Behaviour of single mode in absence of ν–ν forward scattering term:

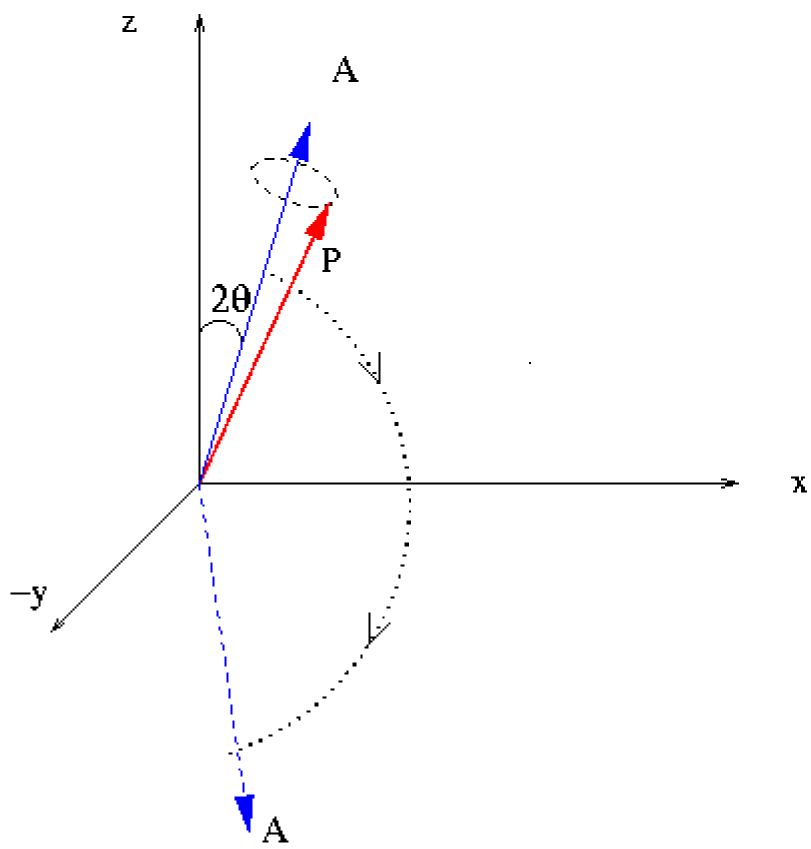
$$\partial_t \mathbf{P}_p \approx \mathbf{A}_p \times \mathbf{P}_p$$

$$\mathbf{A}_p = \frac{\delta m_0^2}{2p}\left(\sin 2\theta_0 \hat{x} - \cos 2\theta_0 \hat{z}\right) - \frac{8\sqrt{2} G_F p}{3M_W^2} E_e \hat{z}$$

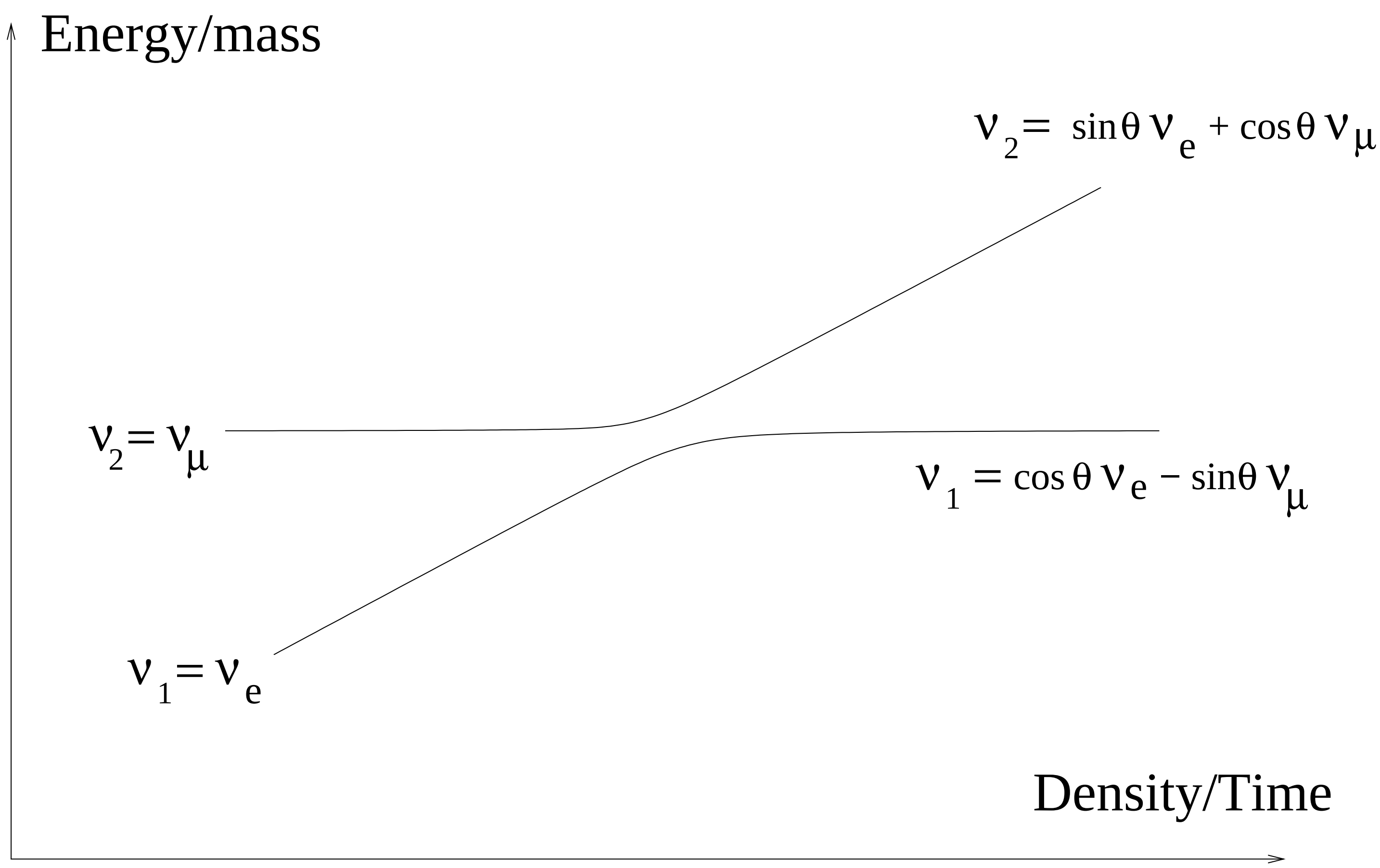
- The thermal potential is initially large and decreases slowly as the temperature falls

- A rotates from the Z-axis to a direction specified by the vacuum mixing parameters.

- The polarisation vector is initially aligned with A, and follows A as it makes this transition – this is just an adiabatic MSW transition


z
A
2θ
P
x
−y
A

MSW transitions



The neutrino-neutrino forward scattering term makes the problem highly non-linear.

Savage, Malaney & Fuller (1991)

Note that this term includes both diagonal and off-diagonal refractive indices, the off-diagonal contributions coming from forward scattering processes of the type:

$$\nu_\alpha(p)+\nu_\beta(k)\rightarrow\nu_\alpha(k)+\nu_\beta(p)$$

Pantaleone (1992).

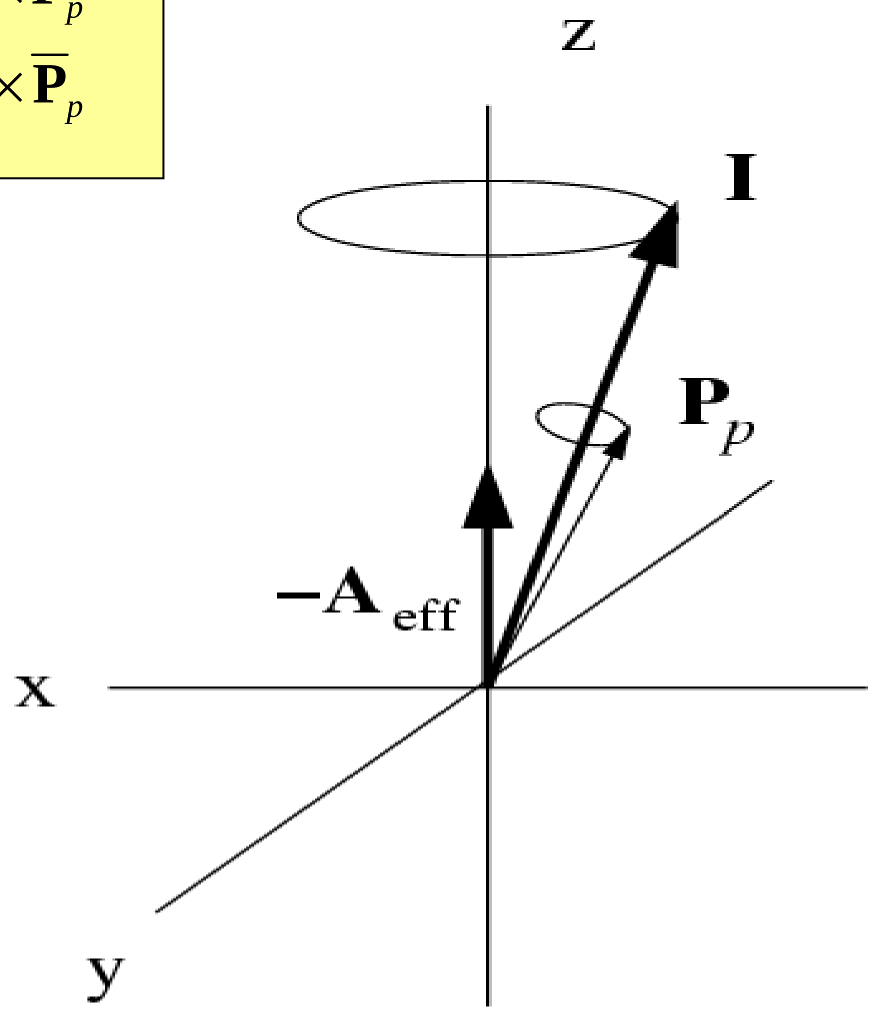
The non-linear term dominates in size as long as the initial asymmetry is larger than about:

$$L>10^{-5}$$

…that is, it dominates for all initial asymmetries of interest.

$$\partial_t \mathbf{P}_p = \left(\mathbf{A}_p + \alpha\,\mathbf{I}\right)\times \mathbf{P}_p$$
$$\partial_t \overline{\mathbf{P}}_p = \left(-\mathbf{A}_p + \alpha\,\mathbf{I}\right)\times \overline{\mathbf{P}}_p$$

$$\mathbf{I} = \int dp(\mathbf{P}_p - \overline{\mathbf{P}}_p)$$



Synchronisation

- In the absence of neutrino-neutrino forward scattering, each momentum mode has a different oscillation frequency.

- Including ν-ν forward scattering pins all the momentum modes together so they oscillate in sync.

Kostelecky, Pantaleone and Samuel.

The polarisation vector for each momentum mode is pinned to the collective polarisation vector I…like a collection of magnetic moments.

Pastor, Raffelt and Semikoz (2002)

The evolution of the collective polarisation is determined according to:

$$\partial_t \mathbf{I} \approx \mathbf{A}_{\text{eff}} \times \mathbf{I}$$

$$\mathbf{A}_{\text{eff}} \approx \frac{1}{\mathbf{I}^2} \int \mathbf{A}_p \left(\mathbf{P}_p + \overline{\mathbf{P}}_p \right) \bullet \mathbf{I}$$

When the neutrino self potential dominates, it synchronises the ensemble so that all neutrinos behave as though they have the same effective momentum.

Parameters describing the evolution of the collective polarisation:

$$\mathbf{A}_{\mathrm{eff}} \approx \Delta_{\mathrm{sync}}\left(\sin 2\theta_{\mathrm{sync}}\hat{x} - \cos 2\theta_{\mathrm{sync}}\hat{z}\right)$$

$$\Delta_{\mathrm{sync}} \propto \xi$$

Very sensitive to initial asymmetry.

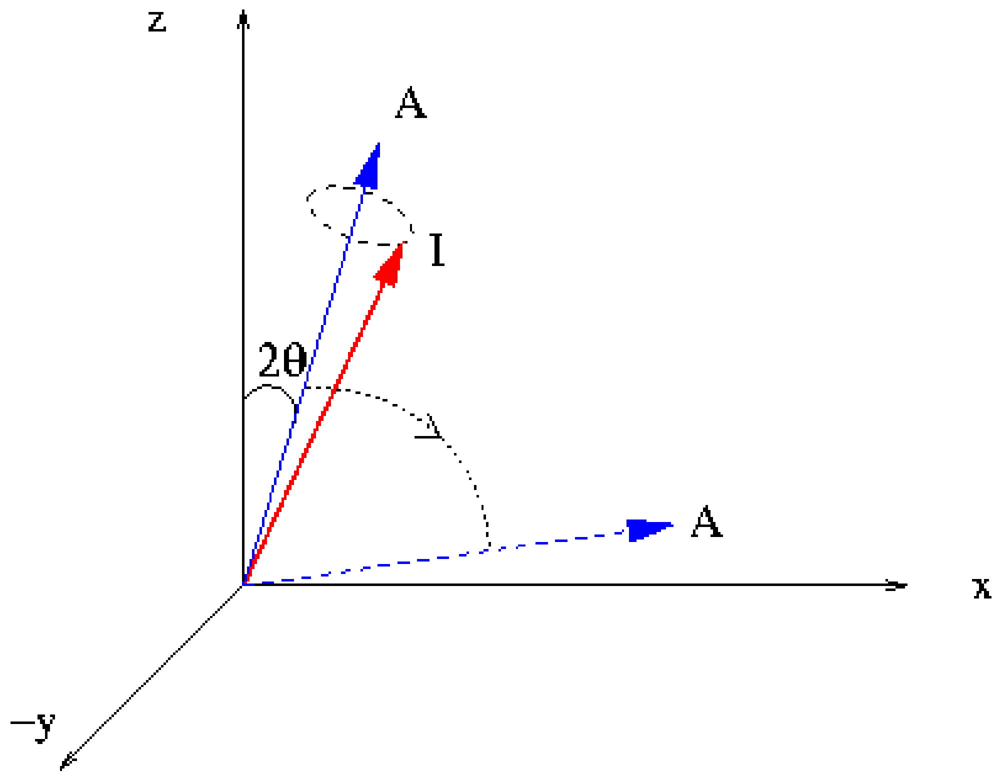
Effective mixing angle , however is insensitive to the initial asymm:

$$\sin^2 2\theta_{\mathrm{sync}} = \frac{\sin^2 2\theta_0}{\sin^2 2\theta_0 + \left[\cos 2\theta_0 - V_B(p_{\mathrm{sync}})\right]^2}$$

$$\frac{p_{\mathrm{sync}}}{T} = \pi\sqrt{1+\xi^2/2\pi^2} \approx \pi$$

Abazajian, Beacom and Bell (2002);
Wong (2002).

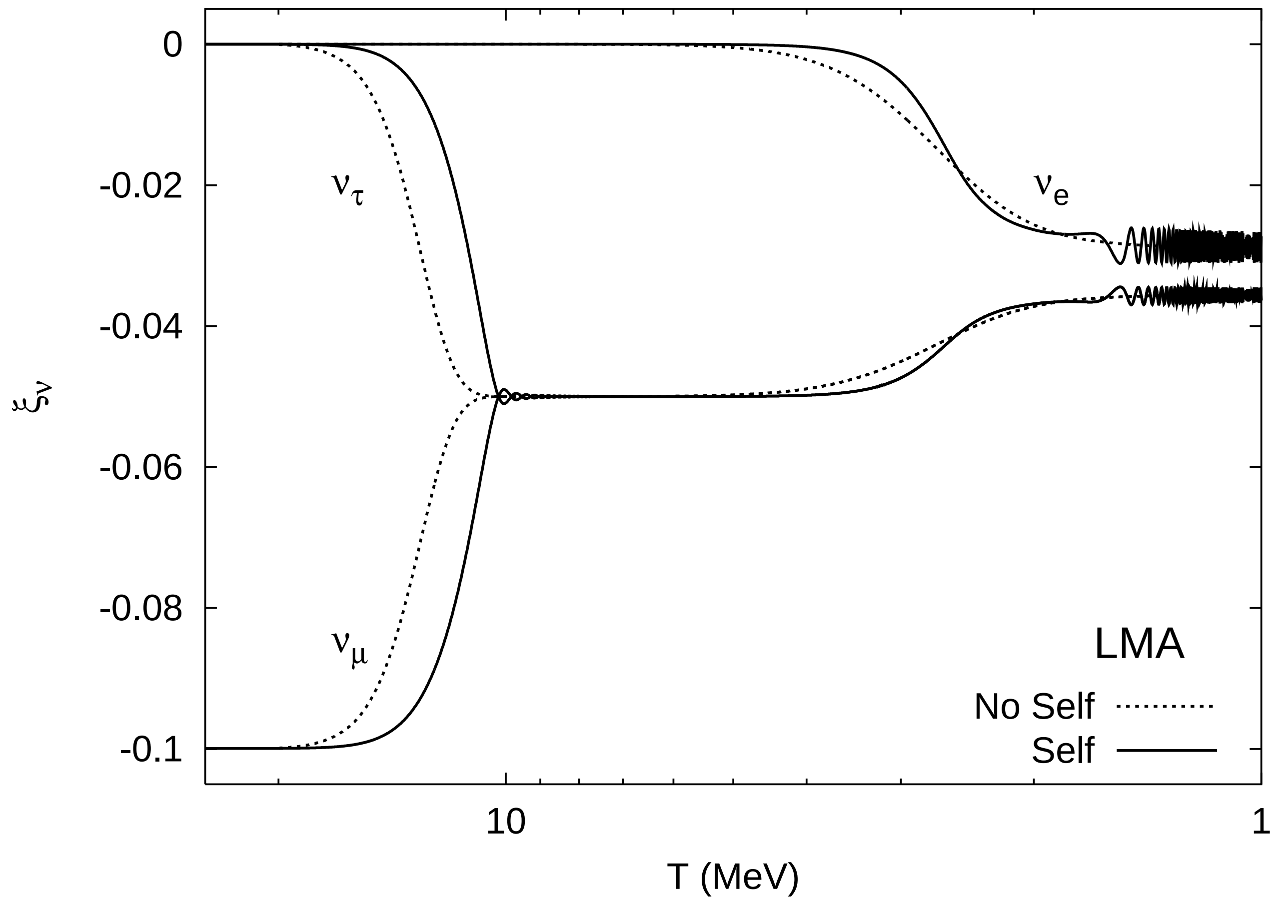
It is this “synchronised mixing angle” that determines when the flavour equilibration occurs.



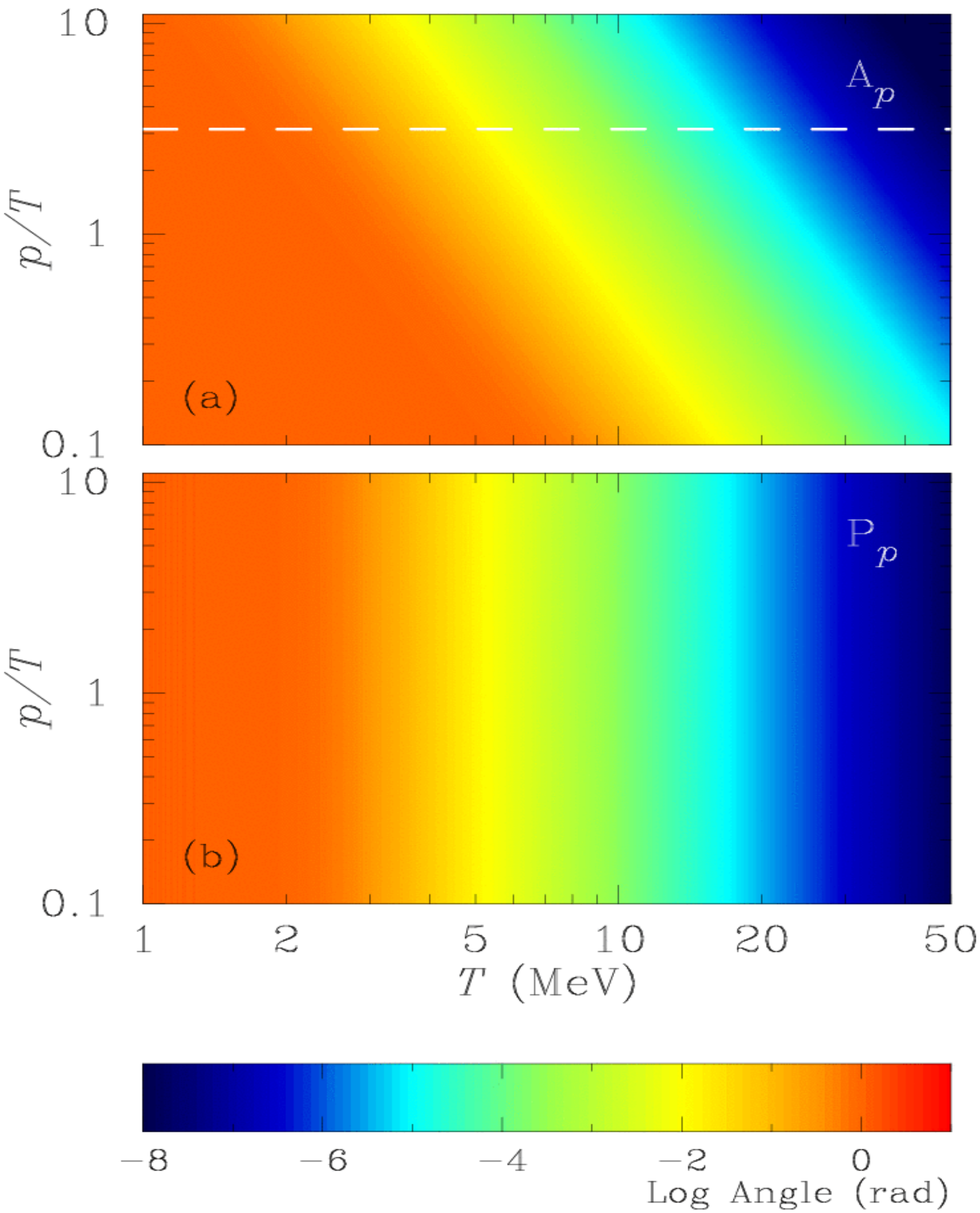
In the standard three-flavour picture of neutrino mixing:

$\nu_\mu - \nu_\tau$ equilibration takes place at T ~ 10 MeV

$\nu_e - \nu_\mu / \nu_\tau$ equilibration takes place at T ~ 2 MeV.



Dolgov, Hansen, Pastor, Petcov, Raffelt &Semikoz. (2002)


10
1
0.1
p/T
A_p
(a)
10
1
0.1
p/T
P_p
(b)
1
2
5
10
20
50
T (MeV)
−8
−6
−4
−2
0
Log Angle (rad)

New Constraints:

$$\xi_e^f \approx \left(\frac{1-\cos 2\theta_0}{2}\right)\xi_\mu^i$$

Using the best fit value of the LMA mixing angle $\sin^2 2\theta_0 \approx 0.8$

$$\xi_e^f < 0.04 \quad \Rightarrow \quad \xi_\mu^i < 0.3$$

Collisional processes will help make the equilibration more complete, as does non-zero U_{e3}.

Degenerate BBN is eliminated since chemical potentials in any flavour will effectively impact neutron-proton equilibrium.

What does this mean for the relativistic energy density?

LMA solar neutrino solution → close to complete flavour equilibration just before BBN, which sets the best limit on the lepton number of the universe:

Taking, *very conservatively*: $\xi_\mu < 0.3$

$\boxed{\text{the new limit is:}}$ $\Delta N_\nu < 0.04$

HUGE improvement over the old limit: $\Delta N_\nu < \text{ a few}$

Dolgov et al.; Wong; Abazajian, Beacom & Bell.

Implication: no uncertainty on n in cosmological determinations of mass via: $\rho_\nu = m_\nu n_\nu$

The end of the road for Degenerate Big Bang Nucleosynthesis?

Strictly speaking, we could still resurrect "degenerate" BBN with the presence of energy density in additional particle species.

i.e. it is possible that : $\xi_e \sim \xi_\mu \sim \xi_\tau \sim 0.2$

Provided another light particle species contributes the extra energy density required to compensate for the large electron-neutrino chemical potential.

This extra energy density can no longer consist of active- neutrinos….it would have to be something more exotic.

Such a non-standard contribution to the relativistic energy density would eventually be detectable via the CMB.

Other issues:

Neutrino-neutrino forward scattering is important in dense matter, such as the early universe, or just outside the core of a supernova.

Such interactions have generally been studied

within a single body framework.

A many-body framework may actually be called for, since the neutrinos will, in general, become entangled through the processes:

$$\nu_\alpha(p) + \nu_\beta(k) \to \nu_\alpha(k) + \nu_\beta(p)$$

(Bell, Rawlinson and Sawyer, 2003.)

In some circumstances, this may qualitatively change the flavour evolution in dense media.

Summary

•Cosmological limits on neutrino mass are competitive with the best laboratory limits

•Sterile neutrinos are cosmologically disfavoured by BBN…thought constraints can be avoided if a neutrino asymmetry exists.

•The LMA (large mixing angle) solar neutrino solution
$\rightarrow$equilibration of neutrino flavours just before weak freezeout

•This equilibration takes place via an MSW transition, synchronised across momentum modes due to neutrino-neutrino forward scattering.

•The stringent constraints on ν_e apply to all three flavours, eliminating DBBN.