RADIATIVE AMPLIFICATION OF MIXING ANGLES

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Collab. with

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PRL 89 (2002) 161801

& ...to appear....

SCHEME

1. Introduction
2. Rad. corrections in SM, MSSM [~ Review]
3. Impact of rad. corrections
4. Rad. Amplification of mixing angles
5. A new source of ν-masses in SUSY
6. Radiative effects
7. Conclusions

In the leptonic sector

$\sin^2 2\theta_{23}$, $\sin^2 2\theta_{12}$ large

$\sin^2 \theta_{13}$ small

⇒ ~ bimaximal structure

very different from the quark sector, where all mixings are small

Quite difficult to implement in particular models

Where do the large mixings come from?

Attractive possibility:

Radiative amplification of mixing angles

i.e. they start **small** at Λ

and finish **large** at M_Z

The analysis is quite

model-independent

MIXING MATRIX

$\mathcal{M}_\nu$ is diagonalized by an (MNS) unitary matrix, U

$$U^T \mathcal{M}_\nu U = \mathrm{Diag}(m_1, m_2, m_3)$$

Ignoring ~~CP~~ phases in this talk:

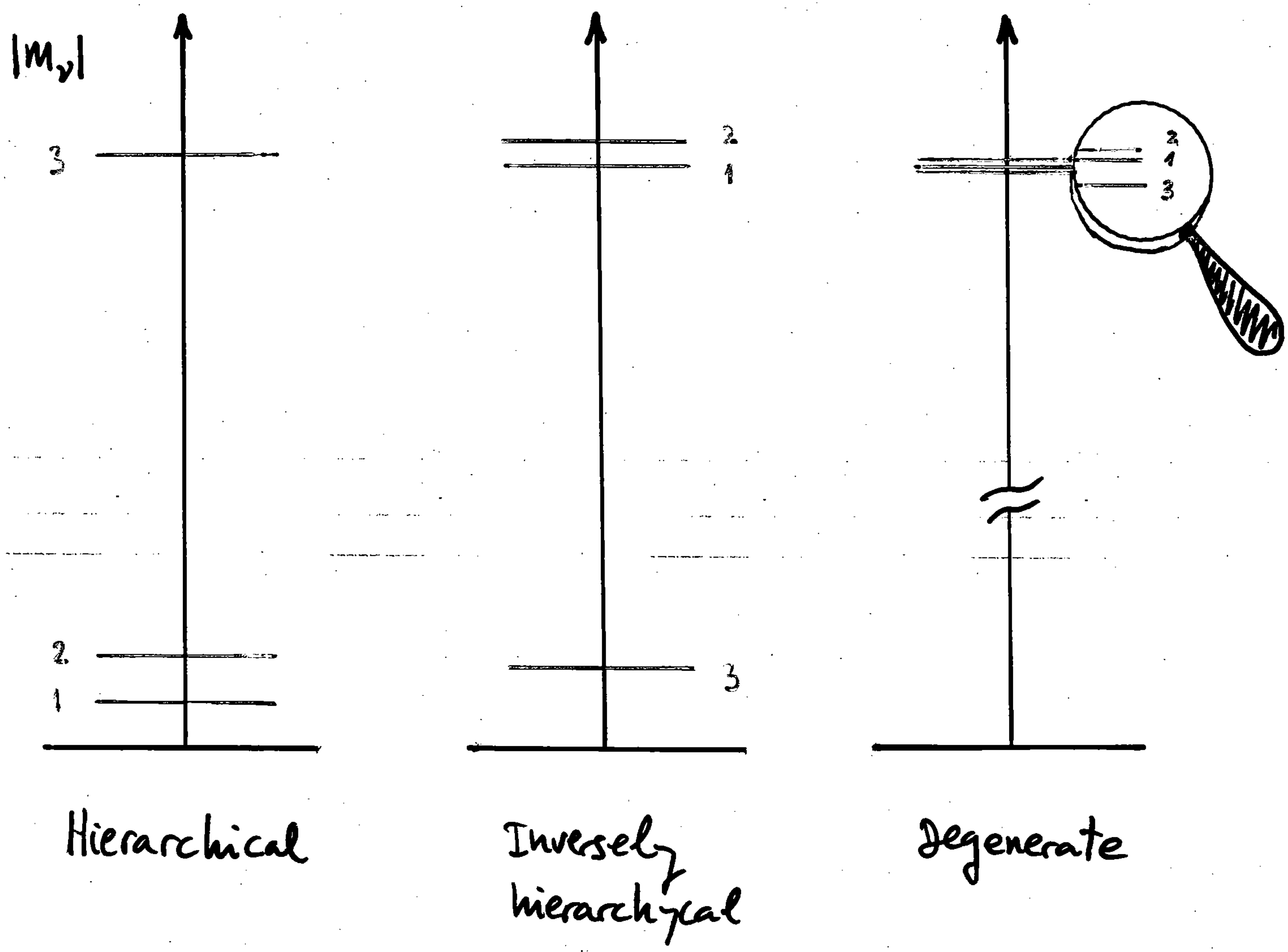
$$\underset{\text{flavor eigenstates}}{\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}} = \underbrace{\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12} & -s_{23}c_{12} - c_{23}s_{13}s_{12} & c_{23}c_{13} \end{pmatrix}}_{U} \underset{\text{mass eigenstates}}{\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}}$$

$s_{12} \equiv \sin\theta_{12}$, etc.

θ_{13} small

$\theta_{12} \equiv \theta_{sol}$	$\Delta m^2_{21} \equiv \Delta m^2_{sol}$	$\Delta m^2_{ij} \equiv m_i^2 - m_j^2$
$\theta_{23} \equiv \theta_{atm}$	$\Delta m^2_{31} \equiv \Delta m^2_{atm}$	

Possible Mass patterns



(label 3 : most split eigenstate)

RADIATIVE CORRECTIONS IN THE SM AND THE MSSM

SM: $$\delta\mathcal{L} = -\frac{1}{2M}\lambda_{ij}\left(H L_i\right)\left(H L_j\right) + h.c.$$

$$\rightarrow \quad \mathcal{M}_\nu = \lambda \frac{v^2}{M}$$

MSSM: $$\delta W = \frac{1}{2M}\lambda_{ij}\left(H_2 L_i\right)\left(H_2 L_j\right)$$

$$\rightarrow \quad \mathcal{M}_\nu = \lambda \frac{v^2}{M} \sin^2\beta$$

$v = 174$ GeV

$M \approx 10^{14}$ GeV

$$\tan\beta = \frac{\langle H_2 \rangle}{\langle H_1 \rangle}$$

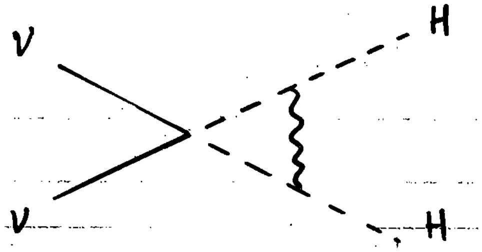
In both cases:

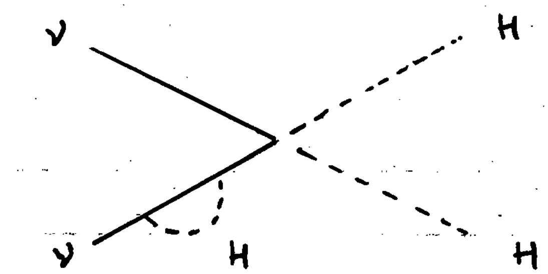
$$\frac{d\mathcal{M}_\nu}{dt} = -u\,\mathcal{M}_\nu + \mathcal{M}_\nu \mathbf{P} + \mathbf{P}^T \mathcal{M}_\nu$$

$(t \equiv \log Q)$

Universal part (flavour-blind)

modifies flavour-structure





$$u \propto \frac{h_t^2}{16\pi^2}$$

$$\mathbf{P}_{SM} = \frac{3}{32\pi^2} Y_e^+ Y_e \simeq \frac{3}{32\pi^2} h_\tau^2 \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}$$

$$\mathbf{P}_{MSSM} = \frac{-1}{16\pi^2} Y_e^+ Y_e \simeq -\frac{1}{16\pi^2} \frac{h_\tau^2}{\cos^2\beta} \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}$$

Babu, Leung & Pantaleone '93
Chankowski, Pluciennik '93
Antusch, Drees et al. '01

IMPACT OF RAD. CORRECTIONS

JAC, Espinosa, Ibarra & Navarro '99

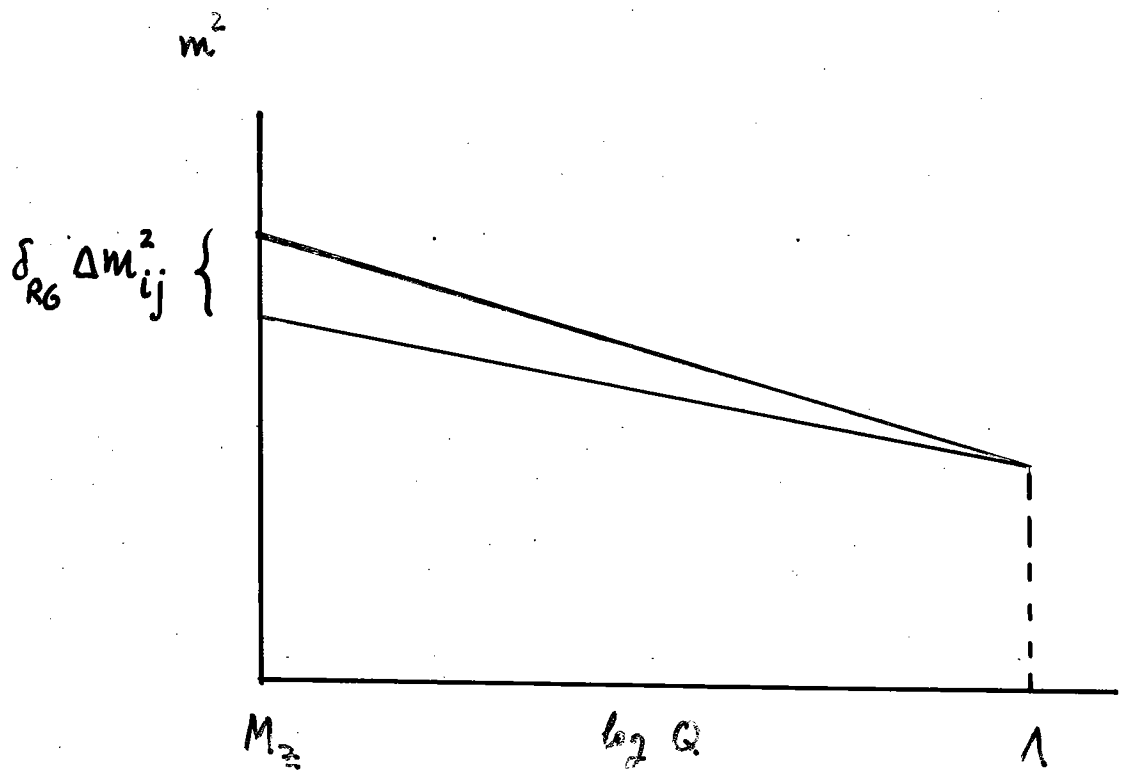
Ellis & Lola '99
Chankowski et al. '99
Barbieri, Ross & Strumia
Antusch et al. '01
⋮

Masses

$$\frac{d m_i}{dt} = - u\, m_i \underbrace{- 2 m_i \left[U^T P\, U \right]_{ii}}_{-2 K_\tau |U_{\tau i}|^2 m_i}$$

SM: $K_\tau = \frac{3}{32\pi^2} h_\tau^2 \sim 10^{-6}$

MSSM: $K_\tau = \frac{-1}{16\pi^2} \frac{h_\tau^2}{\cos^2\beta} \sim -\frac{1}{\cos^2\beta}\, 7\times 10^{-7}$



$$\delta_{RG}\, \Delta m^2_{21} = 4 K_\tau\, m_\nu^2 \left(U_{\tau i}^2 - U_{\tau j}^2 \right) \log \frac{\Lambda}{M_Z} \qquad (\text{L.L.})$$

SM: $$\delta_{RG}\, \Delta m^2_{21} \simeq \underbrace{2.5 \times 10^{-6}}_{\text{too small}} \left(\frac{m}{0.2\,\text{eV}} \right)^2 \text{eV}^2$$

MSSM: $$\delta_{RG}\, \Delta m^2_{21} \simeq \underbrace{1.5 \times 10^{-6} \frac{1}{\cos^2\beta} \left(\frac{m}{0.2\,\text{eV}} \right)^2 \text{eV}^2}$$

$= 7 \times 10^{-5}$ for $\tan\beta = 7$

(the right size)

$\Rightarrow$ [In the MSSM Δm^2_{sol} could be well a radiative effect]

ANGLES

$$\frac{dU}{dt} = UT \qquad \begin{cases} T_{ii} = 0 \\ T_{ij} = \left(\dfrac{m_i + m_j}{m_i - m_j}\right) \left[U^T P U\right]_{ij} \end{cases}$$

For $m_i \approx m_j$: $\left(\dfrac{m_i + m_j}{m_i - m_j}\right) \gg 1 \Rightarrow$ U changes rapidly until $\left[U^T P U\right]_{ij} = 0$

Infrared Fixed Point (IRFP)

More precisely:

$$\left[U^T P U\right]_{ij} = K_\tau U_{\tau i} U_{\tau j}$$

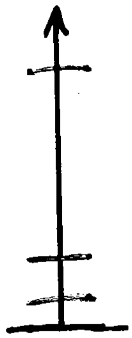
If $\left(\dfrac{m_i + m_j}{m_i - m_j}\right) \gtrsim K_\tau^{-1} \Rightarrow U_{\tau i} U_{\tau j} \to 0$.

$$\Rightarrow \quad U_{\tau i} \to 0 \quad \text{or} \quad U_{\tau j} \to 0$$

(depending on the sign of $m_i - m_j$)

IRFP is interesting, but ... not acceptable from phen.

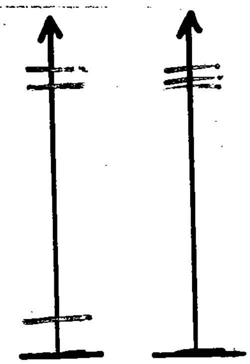
Implications (summary)



The hierarchical scenario is not sensitive to rad. corrections

$$\left\{ \begin{array}{ll} \dfrac{m_i - m_j}{m} & \text{stable} \\ U_{ij} & \text{stable} \end{array} \right.$$

(Possible exception: MSSM with (very) large $\tan\beta$)



Total or partial degeneracy

$$\Delta_{RG} m_{ij}^2 \sim O(\Delta_{sol} m^2)$$

(for MSSM) ✓

$m_i \simeq -m_j$ U_{ij} stable ✓

$m_i \simeq m_j$ $\left\{ \begin{array}{l} U_{3i} U_{3j} \to 0 \\ U_{3i}^2 + U_{3j}^2 \sim \text{const.} \end{array} \right.$

⇒ zeroes at the last row of U

→ INCONSISTENT

if U gets the IRFP regime

RADIATIVE AMPLIFICATION OF MIXING ANGLES

Can we start with **small** mixing at Λ

and finish with **large** mixing at M_Z ?

Babu, Leung & Pantaleone '93
Ellis et al. '99
JAC, Espinosa, Ibarra & Navarro '99
Balaji et al. '00
Pantaleone et al. '01
Antusch, Lindner et al. '02
Dutta '02
Bhattacharyya et al. '02
Frigerio & Smirnov '02
Mohapatra, Parida & Rajasekaran '03
⋮

⊛ This requires (SM & MSSM) that the running stops before reaching the IR.FP

⊛ In principle it is possible, but in general requires **fine-tuning**

- JAC, Espinosa, Ibarra & Navarro '99
- JAC, Espinosa & Navarro (to appear)

The 2-flavour approximation

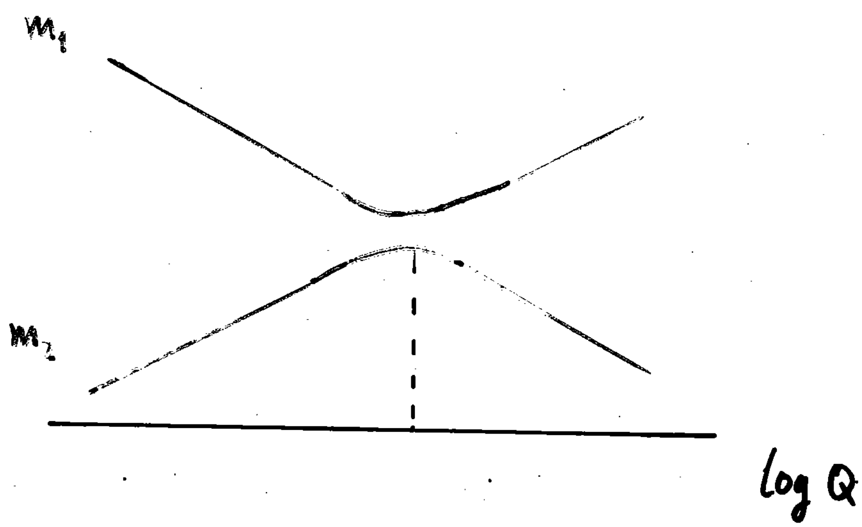
$$\left\{ \begin{array}{l} \dfrac{d\theta}{dt} = \dfrac{1}{2}\, \kappa_\tau \sin 2\theta \, \dfrac{m_1+m_2}{m_1-m_2} \\[2ex] \dfrac{d\Delta_{21}}{dt} \simeq -\kappa_\tau \cos 2\theta \end{array} \right.$$

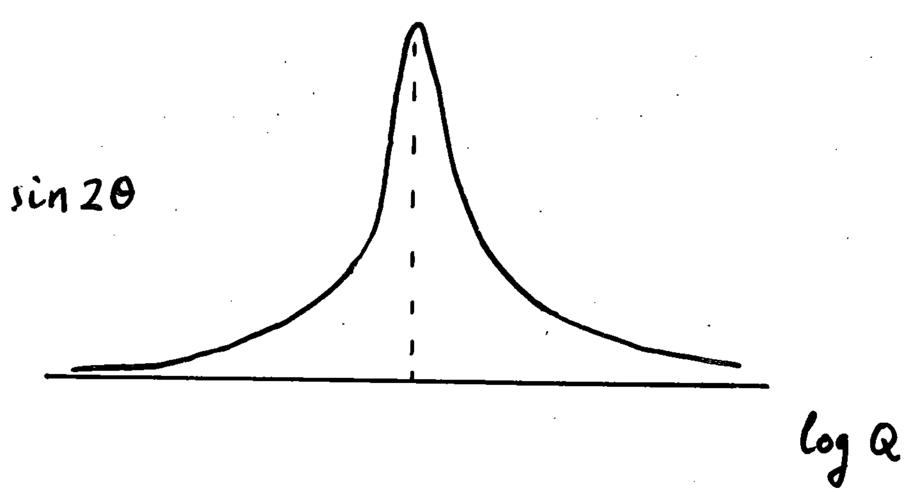
$$Y \sim \begin{pmatrix} 0 & \\ & h_\tau \end{pmatrix}$$

$$\Delta_{21} \equiv \frac{m_1-m_2}{m_1+m_2} \simeq \frac{\Delta m^2_{21}}{4m^2}$$

$$\Rightarrow \boxed{\Delta_{21} \cdot \sin 2\theta \simeq \text{RG-invariant}}$$

(for approx. deg.)





... if you wish

$$\sin 2\theta\,(M_Z) = \underset{\substack{\uparrow \\ \text{Amplification} \\ \text{factor}}}{\mathbf{F}} \sin 2\theta\,(\Lambda)$$

you need:

⊛ Appropriate sign of $m_2 - m_1$

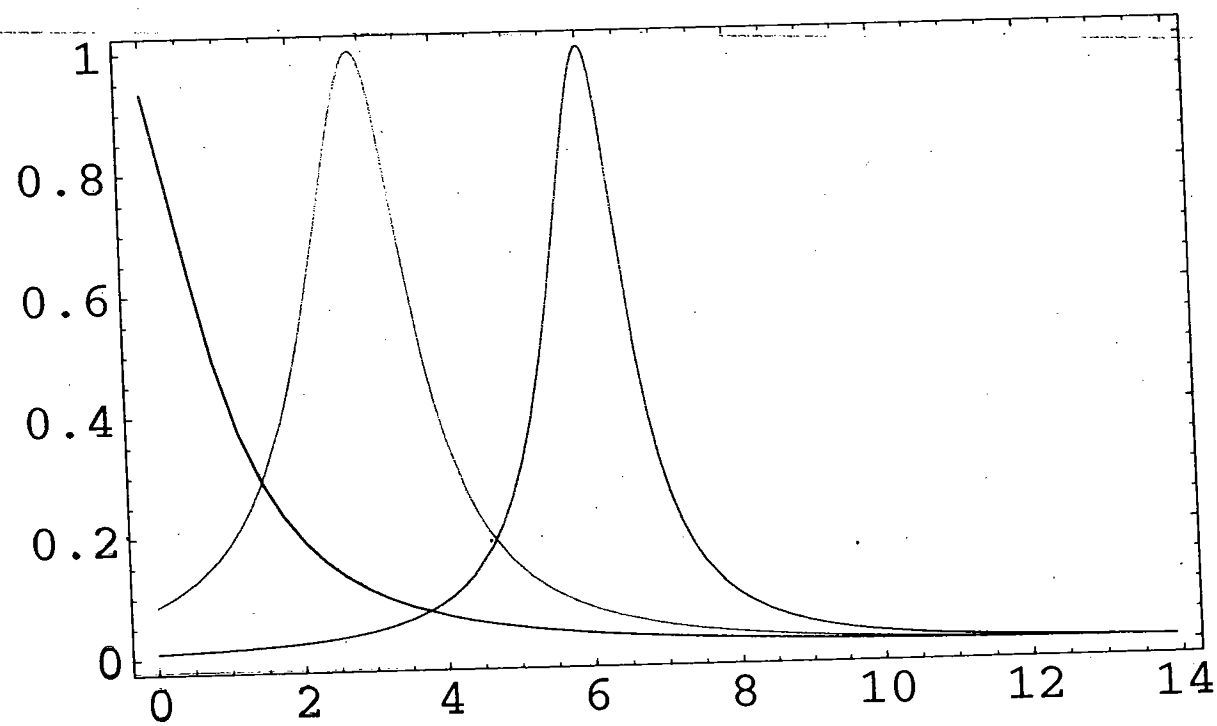
⊛ $\Delta_{21}(M_Z) = \frac{1}{F}\,\Delta_{21}(\Lambda)$

$$\Rightarrow \quad \Delta_{21}(\Lambda) + \delta_{RG}\Delta_{21} = \frac{1}{F}\,\Delta_{21}(\Lambda)$$

<u>**fine-tuning**</u> of one part in **F**

⊛ $\delta_{RG}\Delta_{21} \simeq \underbrace{\kappa_\tau \log \frac{\Lambda}{M_Z}}_{\sim\ \text{fixed}} = (F-1)\,\Delta_{21}(M_Z)$

(not always possible)



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14

Three flavors

$$\frac{d\theta_{23}}{dt} = -\kappa_\tau c_{23}\left(-s_{12} U_{\tau 1} \nabla_{31} + c_{12} U_{\tau 2} \nabla_{32}\right)$$

$$\frac{d\theta_{13}}{dt} = -\kappa_\tau c_{23} c_{13}\left(c_{12} U_{\tau 1} \nabla_{31} + s_{12} U_{\tau 2} \nabla_{32}\right)$$

$$\frac{d\theta_{12}}{dt} = -\kappa_\tau\left(c_{23} s_{13} s_{12} U_{\tau 1} \nabla_{31} - c_{23} s_{13} c_{12} U_{\tau 2} \nabla_{32} + U_{\tau 1} U_{\tau 2} \nabla_{21}\right)$$

with $$\nabla_{ij} \equiv \frac{m_i + m_j}{m_i - m_j} \equiv \left(\Delta_{ij}\right)^{-1}$$

JAC, Espinosa, Ibarra & Navarro
'99

Solar angle, θ_{12}

$\rightarrow \quad \nabla_{21} := \dfrac{m_2 + m_1}{m_2 - m_1} \gg 1$ dominates the R.G.E.'s

$\rightsquigarrow$
- $m_1 \simeq m_2 \simeq |m_3|$ (quasi-degeneracy)
- $m_1 \simeq m_2 \gg |m_3|$ (inverse hierarchy)

- θ_{12} runs as in a 2-flavor model
- θ_{13}, θ_{23} are stable

$\rightarrow$

* To amplify $\sin 2\theta_{12}$ a factor F requires to fine-tune $\Delta_{12}(\Lambda)$ with 1 part in F accuracy

* $$\delta_{RG}\Delta_{21} \simeq \frac{1}{2}\kappa_\tau \log\frac{\Lambda}{M_Z} = (F-1)\,\Delta_{21}(M_Z)$$

- **SM** cannot work:

$$\kappa_\tau = \frac{3}{32\pi^2} h_\tau^2 \approx 10^{-6}$$

$$\Rightarrow \quad F-1 = \left(\frac{\Delta m^2_{sol}}{7\times10^{-5}\,eV^2}\right)^{-1} \left(\frac{m}{0.2\,eV}\right)^2 \times 3\times10^{-2}$$

$\Rightarrow$ no amplification

- **MSSM** could work:

$$\kappa_\tau = \frac{-1}{16\pi^2} \frac{h_\tau^2}{\cos^2\beta} \approx 7\times10^{-7}/\cos^2\beta$$

$$\Rightarrow \quad F-1 = \left(\frac{\Delta m^2_{sol}}{7\times10^{-5}\,eV^2}\right)^{-1} \left(\frac{m}{0.2\,eV}\right)^2 \times 2\times10^{-2} \frac{1}{\cos^2\beta}$$

$$\Rightarrow \quad \tan\beta = 7\sqrt{F-1} \left(\frac{\Delta m^2_{sol}}{7\times10^{-5}\,eV^2}\right)^{1/2} \left(\frac{m}{0.2\,eV}\right)^{-1}$$

E.g. $F \gtrsim 10$ requires $\tan\beta \gtrsim 21 \left(\frac{m}{0.2\,eV}\right)^{-1}$

Atm. angle, θ_{23}

θ_{23} can change if ∇_{31} and/or $\nabla_{32} \gg 1$

Suppose

$\nabla_{31} \gg 1, \nabla_{32}, \nabla_{21}$

$$\Rightarrow \quad m_1 \simeq -m_2 \simeq \pm m_3$$

- The 2-flavor appr. does not work

- $U_{\tau 3} U_{\tau 1} \Delta_{31} \simeq$ RG-invariant

  $U_{k2} \simeq$ RG-invariant

➡ To amplify $U_{\tau 3} U_{\tau 1}$ a factor F requires to fine-tune $\Delta_{31}(\Lambda)$

- Schematically:

$$\begin{pmatrix} -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} & \frac{1}{2} & \frac{1}{2\sqrt{3}} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{2\sqrt{3}} & \frac{1}{2} & \frac{2}{\sqrt{6}} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \end{pmatrix}$$

A more serious problem

$$\delta_{RG}\,\Delta_{31} \simeq \kappa_\tau \underbrace{\langle U_{\tau 3}^2 - U_{\tau 1}^2 \rangle}_{\neq 0} \log\frac{\Lambda}{M_Z} = (F-1)\,\Delta_{31}^{exp}$$

$-\frac{1}{16\pi^2} h_\tau^2/\cos^2\beta \rightarrow$ must be adjusted to get

$$\delta_{RG}\,\Delta_{21} = \underbrace{\frac{\langle U_{\tau 2}^2 - U_{\tau 1}^2 \rangle}{\langle U_{\tau 3}^2 - U_{\tau 1}^2 \rangle}}_{O(1)} \underbrace{(F-1)}_{>1} \underbrace{\Delta_{31}^{exp}}_{\Delta_{atm}}$$

$$\gtrsim \Delta_{atm} \qquad \text{(not acceptable)}$$

$\nabla_{31}\,,\ \nabla_{32} \gg 1$ $\quad\left(\Rightarrow \ \nabla_{21} \gg \nabla_{31} \simeq \nabla_{32}\right)$

⊛ If $\kappa_\tau \nabla_{31} \log \frac{\Lambda}{M_Z} \sim O(1) \Rightarrow \kappa_\tau \nabla_{21} \log \frac{\Lambda}{M_Z} \gg 1$

$\Rightarrow \quad U \to$ IRFP (with $\theta_{12}=0$), unrealistic

unless $\quad U_{\tau 1} U_{\tau 2} \simeq 0 \Rightarrow \left[U_{\tau 1} \text{ and/or } U_{\tau 2} \simeq 0\right]$

(along most of the running)

⊛ Since $\kappa_\tau \nabla_{31} \log \frac{\Lambda}{M_Z} \sim O(1)$

$$\Rightarrow \quad \delta_{RG}\Delta_{21} \simeq \kappa_\tau \langle U_{\tau 2}^2 - U_{\tau 1}^2 \rangle \log \frac{\Lambda}{M_Z} = O(\Delta_{31})$$

(unrealistic)

unless $\left[U_{\tau 2}^2 - U_{\tau 1}^2 \simeq 0\right]$ (along most of the running)

$$\Rightarrow \quad U_{\tau 2} \simeq U_{\tau 1} \simeq 0 \ \to \ \boxed{\sin\theta_{23} \simeq \sin\theta_{13} \simeq 0}$$

(along most of the running)

$\Rightarrow$

- θ_{23} must be amplified

- θ_{23} runs as in a 2-flavor approx.

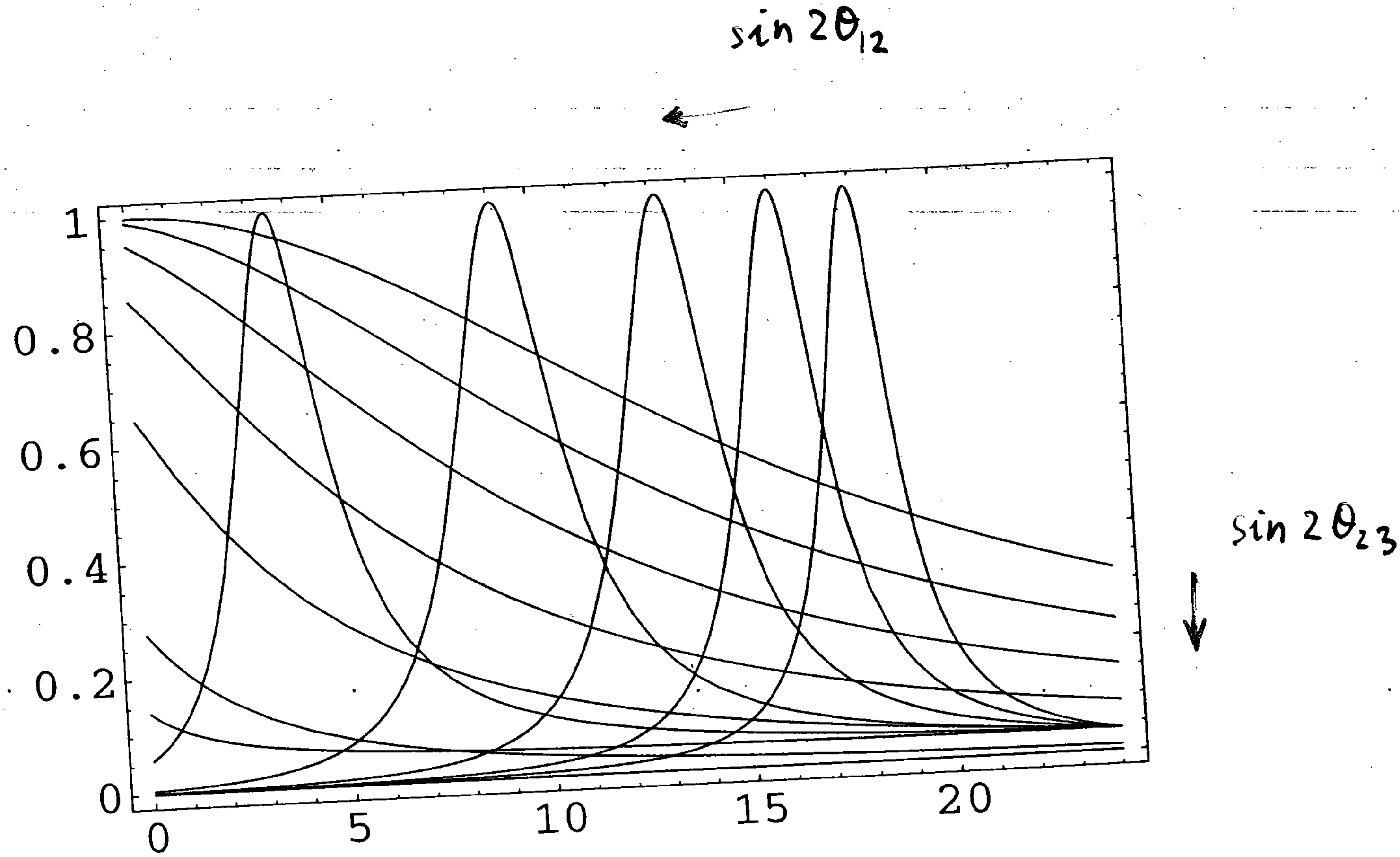
  $\Rightarrow$ fine-tuning of a part in F

- $\theta_{23}(\Lambda)$ must be very small

  (i.e. its amplification must be very important)

Otherwise θ_{12} starts to run (and $\sin 2\theta_{12} \to 0$) too soon:

$$\frac{d\,\theta_{12}}{dt} \simeq -\frac{1}{2}\,\kappa_\tau\,\underbrace{\sin^2\theta_{23}}\,\sin 2\theta_{12}\;\nabla_{12}$$


sin 2θ12
sin 2θ23
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A NEW SOURCE OF ν- MASSES IN SUSY

(JAC, Espinosa & Navarro '02)

Conventionally

$$\Delta W = \frac{1}{2\Lambda} \lambda_{\alpha\beta} (L_\alpha H_2)(L_\beta H_2)$$

$$\Rightarrow \Delta \mathcal{L} = -\frac{1}{2\Lambda} \lambda_{\alpha\beta} (L_\alpha H_2)(L_\beta H_2) + h.c.$$

But

It may happen that the

$\cancel{L}$ - operators are not in W,

but in K

SUPERSYMMETRY (GLOBAL)

The theory is determined by three functions of Φ :

Chiral superfields (contain matter fermions + scalars)

$K(\Phi, \Phi^\dagger)$ real, Dim = 2

$W(\Phi)$ holomorphic, Dim = 3

$f_{ab}(\Phi)$ holomorphic, Dim = 0

$$K = \sum_\alpha |\Phi_\alpha|^2 + \text{non-ren. terms}$$

$$W = Y_E H_1 L e_R + \text{other Yukawas}$$
$$+ \mu H_1 H_2 + \text{non-ren. terms}$$

$$f_{ab} = \frac{\delta_{ab}}{g_a^2} + \text{non-ren. terms}$$

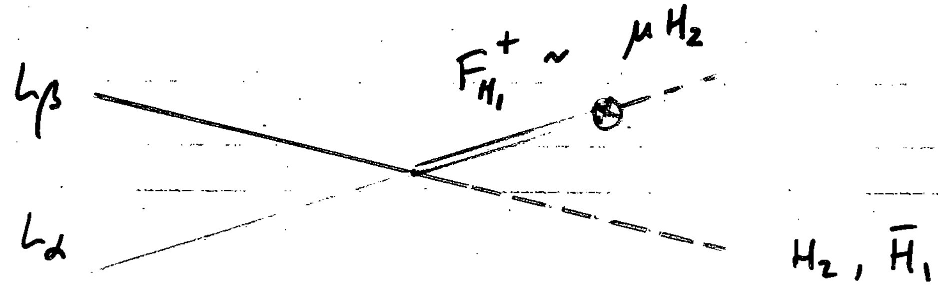
$$\mathcal{L} = \int d^2\theta d^2\bar\theta\, K(\Phi,\Phi^\dagger) + \left(\int d^2\theta\, W(\Phi) + \text{h.c.}\right) + \underbrace{\left(\int d^2\theta\, f_{ab} W^{\alpha a} W^b_\alpha + \text{h.c.}\right)}_{\text{super YM (flavor blind)}}$$

NEW SOURCE OF ν-MASSES

JAC, Espinosa & Navarro '02

L - violation in K

$$\Delta K = \frac{1}{2\Lambda^2} \underset{\substack{\uparrow \\ K = K_S + K_A}}{K_{\alpha\beta}} (L_\alpha H_2)(L_\beta \bar{H}_1) + \frac{1}{4\Lambda^2} \underset{\substack{\uparrow \\ \text{symmetric}}}{K'_{\alpha\beta}} (L_\alpha \bar{H}_1)(L_\beta \bar{H}_1) + \text{h.c.}$$



$$\Delta \mathcal{L} = \frac{1}{2} \frac{\partial^3 K(\phi)}{\partial\phi_i \, \partial\phi_j \, \partial\phi_\ell^*} \left(\frac{\partial^2 K}{\partial\phi_\ell \, \partial\phi_k^*} \right) \frac{\partial W}{\partial\phi_k} \psi_i \psi_j + \text{h.c.}$$

$$\Delta \mathcal{L} = \frac{\mu}{\Lambda^2} \left[(K_S)_{\alpha\beta} (L_\alpha H_2)(L_\beta H_2) + K'_{\alpha\beta} (L_\alpha \bar{H}_1)(L_\beta H_2) \right] + \text{h.c.}$$

$$M_\nu = \frac{\mu v^2}{\Lambda^2} \left[K_S \sin^2\beta + K' \sin\beta \cos\beta \right]$$

(Only the symmetric part of K contributes to M_ν)

$$\mathcal{M}_\nu = \frac{\mu v^2}{\Lambda^2}\left[\kappa_S \sin^2\beta + \kappa' \sin\beta\cos\beta\right]$$

① Additional μ/Λ suppression of m_ν

Taking $\mu \lesssim 1$ TeV :

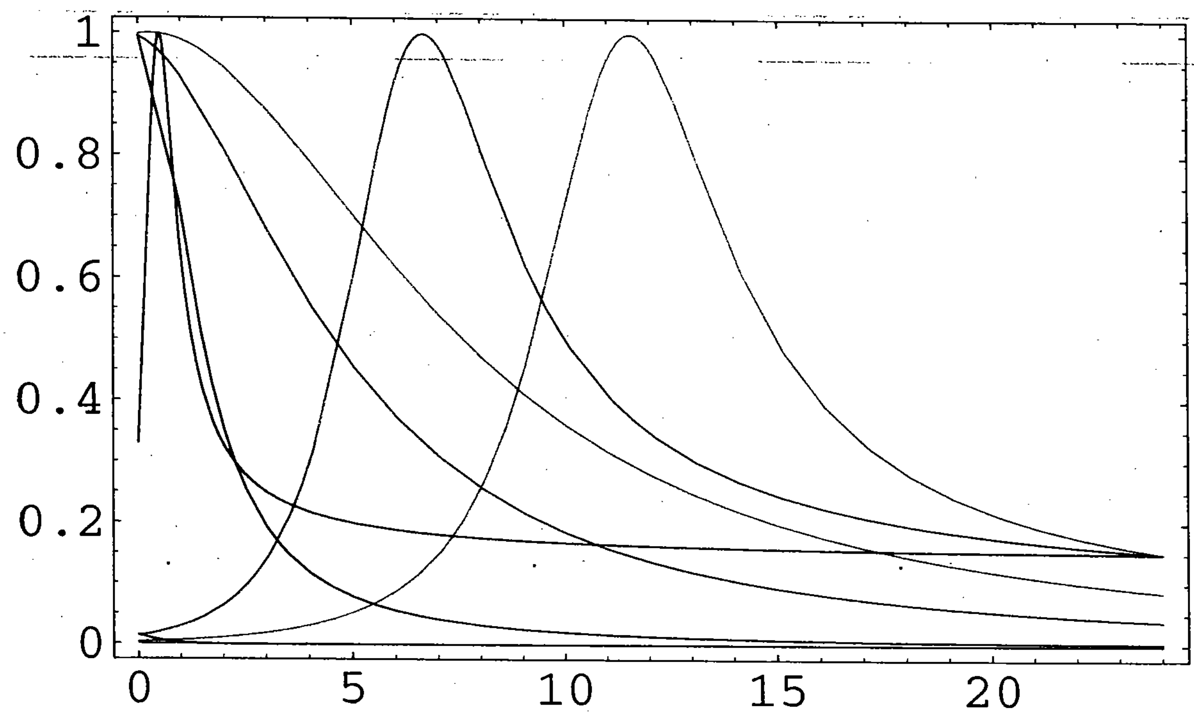
$$\Lambda \sim 10^{[illegible]}\ \text{GeV} \xrightarrow[\text{down to}]{} \Lambda \sim 10^{7-9}\ \text{GeV}$$

② Different dependence on $\tan\beta$:

If $\kappa = 0$, $\tan\beta \gg 1$, extra suppression: $\sin\beta\cos\beta \to 0$

③ Easier to make contact experimentally with $\not{L}$ physics, or theoretically with scenarios that have a low fundamental scale

④ If $\Delta W = \frac{1}{\Lambda}(LH_2)^2$ is present, the latter dominates as a source of $\mathcal{M}_\nu$



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WHY SHOULD $\not{L}$ APPEAR IN K RATHER THAN IN W ?

* R-symmetries

$$K : \int \underbrace{d^2\theta}_{1} \; \underbrace{d^2\bar{\theta}}_{-1} \; \kappa' \, (\underbrace{L}_{1}\underbrace{\bar{H}_1}_{-1})^2 \quad \leftarrow \text{R-charges}$$

$$W : \int \underbrace{d^2\theta}_{1} \left[\mu \underbrace{H_1}_{1} \underbrace{H_2}_{-2} + \cdots + \underbrace{(\underbrace{L}_{1}\underbrace{H_2}_{-2})^2}_{2} \right] \longrightarrow \text{FORBIDDEN}$$

* Spont. breaking of L by $\langle \Phi \rangle \neq 0$

$L = 2$ field

$$K \supset \int d^2\theta \, d^2\bar{\theta} \; \bar{\Phi} \left[\kappa_5 (LH_2)(L\bar{H}_1) + \kappa'(L\bar{H}_1)(L\bar{H}_1) + \text{h.c.} \right] \quad \checkmark$$

$$W \supset \int d^2\theta \; \Phi^n (LH_2)^2 \quad \text{forbidden}$$

* String selection rule

$(LH_2)^2$ may be absent from W with no QFT reason

* K is less protected than W

against (non-)perturbative corrections. It makes sense to suppose that $\not{L}$ is a grav. non-pert. effect that appears in K

RADIATIVE EFFECTS : RGEs FOR κ, κ'

$\underline{\underline{\kappa'}}$

$$\frac{d\kappa'}{dt} = -\left[u'\kappa' + \underbrace{P_E}_{Y_E^\dagger Y_E}\kappa' + \kappa' P_E^T\right]$$

$\Rightarrow$ It has a "standard" form

$\underline{\underline{\kappa}}$:

$$\frac{d\kappa}{dt} = u\kappa + P_E\kappa - \kappa P_E^T + 2\left(P_E\kappa - \kappa^T P_E^T\right)$$

$$\Downarrow \qquad \kappa = \kappa_S + \kappa_A$$

$$\frac{d\kappa_A}{dt} = u\kappa_A + P_E\kappa_S - \kappa_S P_E^T + 2\left(P_E\kappa - \kappa^T P_E^T\right)$$

$$\boxed{\frac{d\kappa_S}{dt} = u\kappa_S + P_E\kappa_A - \kappa_A P_E^T}$$

New and very interesting effects

$$\boxed{\frac{d\kappa_S}{dt} = u\,\kappa_S + P_E\,\kappa_A - \kappa_A\,P_E^T}$$

- If initially $\kappa_A = 0$ $\rightarrow$ $\boxed{\text{stable texture}}$

- If initially $\kappa_S = 0$ $\rightarrow$ $\boxed{\text{rad. generated mass matrix}}$

  (with $2ee$-texture)

- 2-flavor context:

$$\begin{pmatrix} s_{11} & s_{12} \\ s_{12} & s_{22} \end{pmatrix} \longrightarrow \alpha \begin{pmatrix} s_{11} & s_{12} + a\,\epsilon_\tau \\ s_{12} + a\,\epsilon_\tau & s_{22} \end{pmatrix}$$

$$\epsilon_\tau \equiv \kappa_\tau \log \frac{\Lambda}{M_Z}$$

$$\Rightarrow \boxed{\begin{array}{l} \sin 2\theta \rightarrow 1 \\[1em] \dfrac{\Delta m^2}{m^2} \rightarrow 0 \end{array}}$$

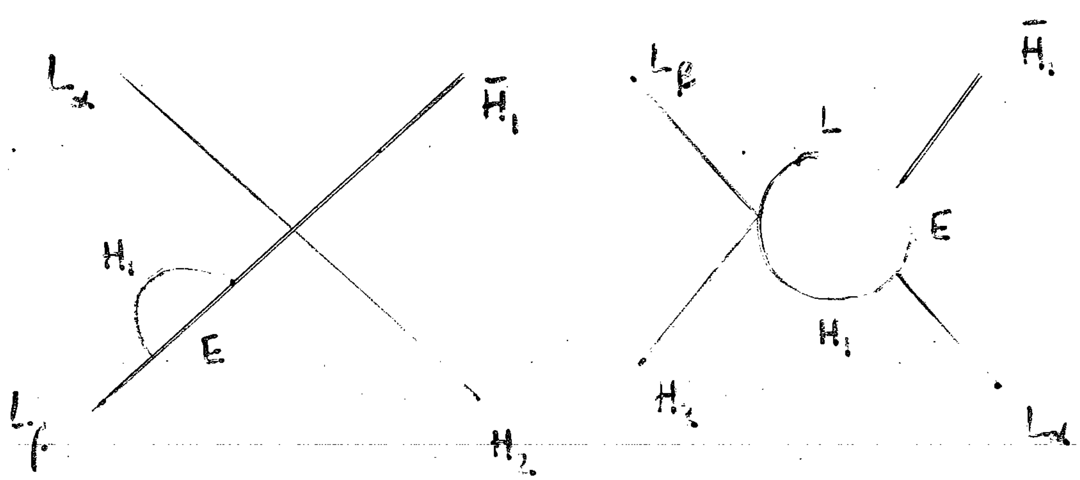
Stable Mixing Angles

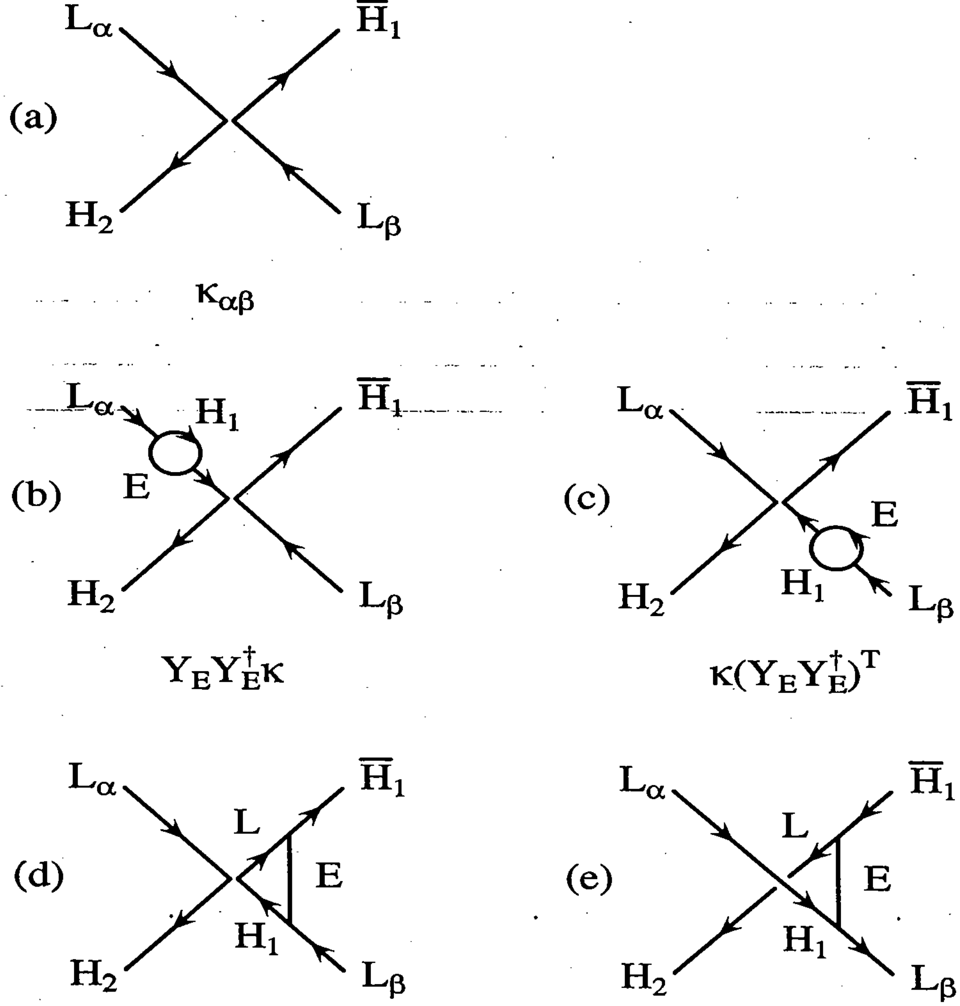
Suppose $\kappa' = \kappa_A = 0$ (i.e. M_ν from κ_S)

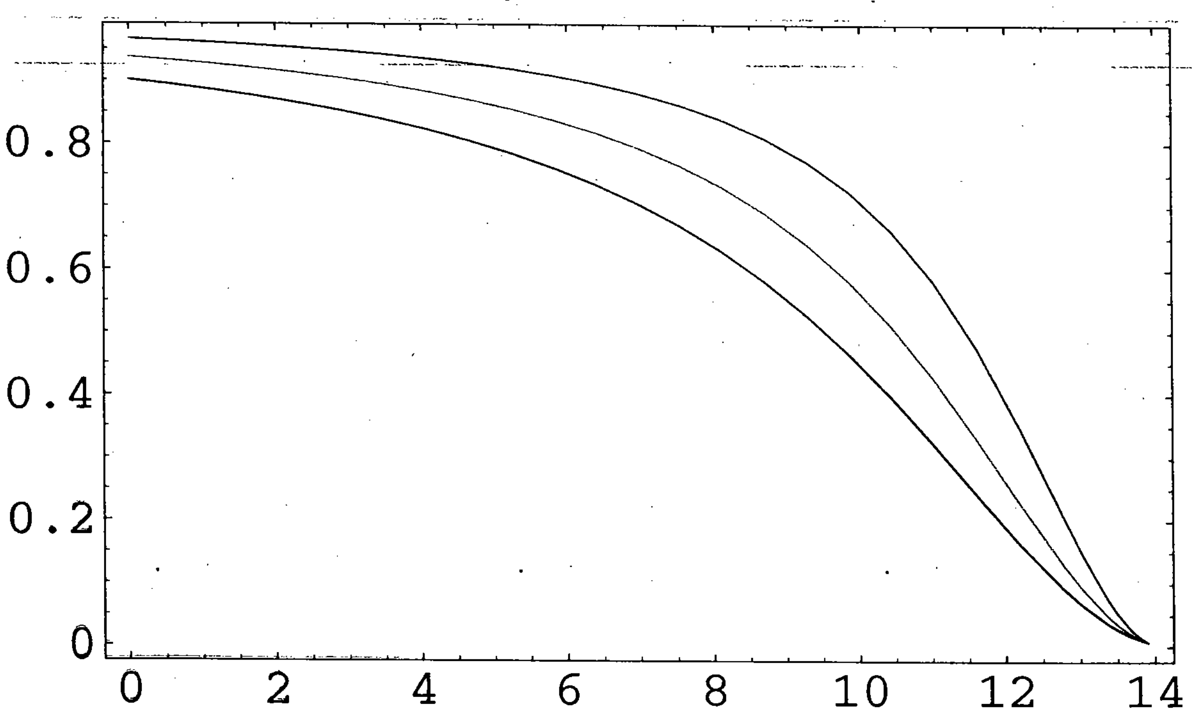
$$\rightarrow \quad \frac{d\kappa_S}{dt} = u\,\kappa_S$$

$\Rightarrow$ ν-mixing angles and relative splittings do not run at 1-loop

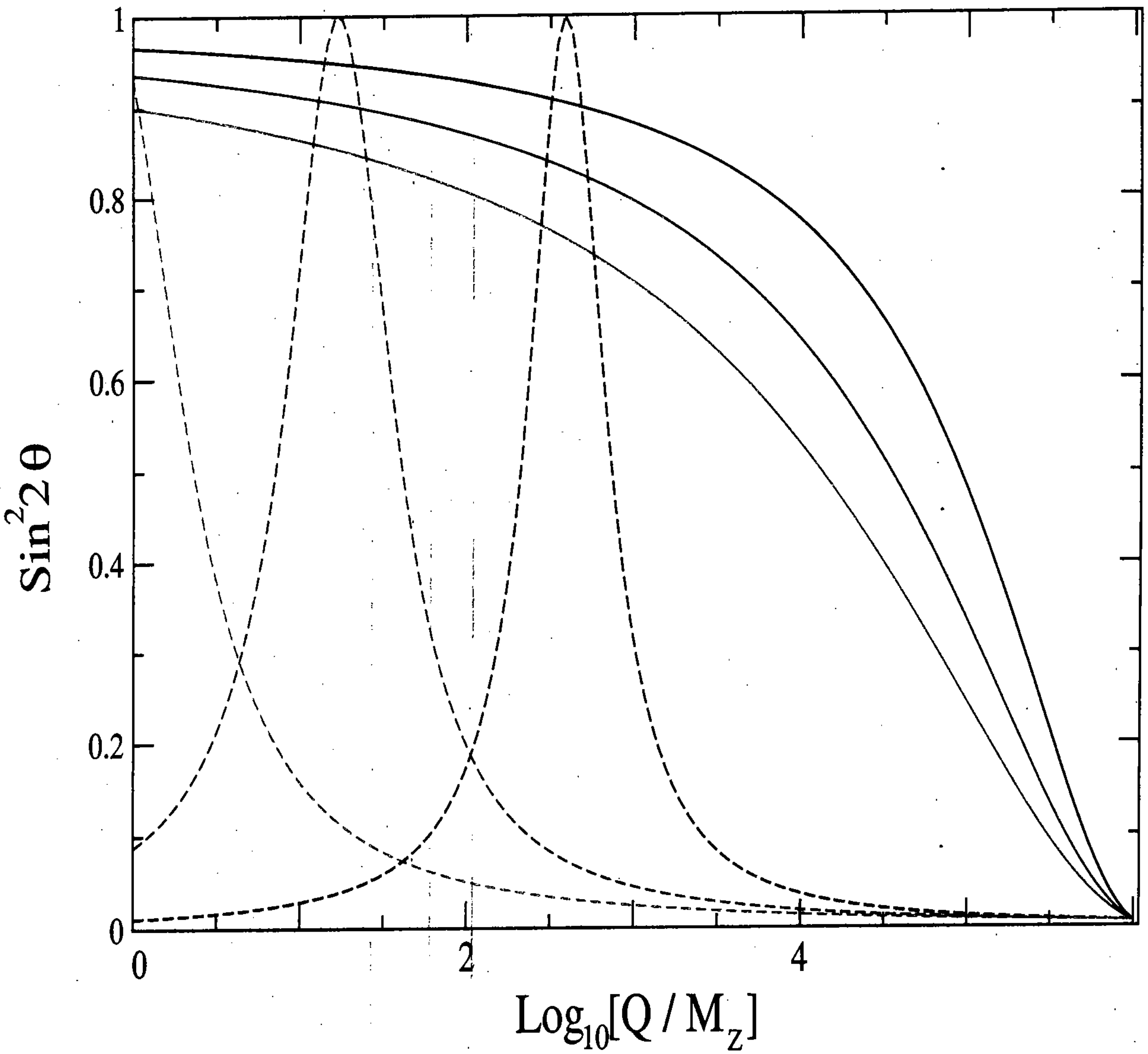
- It protects textures generated at Λ against rad. corrections
- This results from a cancellation (accidental?) of $P_E \kappa_S + \kappa_S P_E^T$ terms




(a)
L_α
$\overline{H}_1$
H_2
L_β
$\kappa_{\alpha\beta}$
(b)
L_α
H_1
E
$\overline{H}_1$
H_2
L_β
$Y_E Y_E^\dagger \kappa$
(c)
L_α
$\overline{H}_1$
E
H_1
H_2
L_β
$\kappa (Y_E Y_E^\dagger)^T$
(d)
L_α
$\overline{H}_1$
L
E
H_1
H_2
L_β
$\kappa (Y_E Y_E^\dagger)^T$
(e)
L_α
$\overline{H}_1$
L
E
H_1
H_2
L_β
$\kappa^T (Y_E Y_E^\dagger)^T$



0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14



Sin²2θ
1
0.8
0.6
0.4
0.2
0
0
2
4
Log10[Q / MZ]

2 - flavor approximation

$$\frac{d\theta}{dt} = 2\,\kappa_\tau \left[\frac{(\kappa_A)_{12}}{m_2 - m_1} \right] \cos 2\theta$$

$$\left(\text{SM, MSSM:} \quad \frac{d\theta}{dt} \sim \kappa_\tau \left[\frac{m_2 + m_1}{m_2 - m_1} \right] \sin 2\theta \right)$$

Now, the IRFP is $\theta \to \pm \pi/4$

$\Rightarrow$ **θ gets amplified with no fine-tuning!**

& no need of mass (quasi) degeneracy

In general

$$\frac{dU}{dt} = U\,T\,, \qquad \begin{cases} T_{ii} = 0 \\ T_{ij} \simeq 2K_\tau \dfrac{(K_A)_{\alpha\tau}}{m_i - m_j} U_{\alpha i} U_{\tau j} \end{cases}$$

- Ampl. of solar angle, θ_{12} requires T_{21} dominant
- " atm. " θ_{23} " T_{31} or T_{32} "

- The exact form of the IRFP is more complicated and related to the value of θ_{13}

CONCLUSIONS

- Rad. effects play an important role in ν - physics
- In the SM θ_{12} & θ_{23} cannot be efficiently amplified, nor the splittings can be a rad. effect
- In the MSSM they can, but with a fine-tuning price
- In SUSY ν-masses can take place in K rather than in W
- This implies a different scale for $\not{L}$ - physics and new interesting effects
- The mixings get amplified in a natural way (no fine-tuning)