Two Reasons Why a talk is needed on Coherence

1. There may be some interesting new physics.
2. Confusion arises from misunderstanding simple QM.

First Reason - Possible New Physics?

We now know that after a weak interaction on the sun, neutrino waves with at least two different masses may leave the sun and arrive on earth.

Caveat - Maybe there is only one mass eigenstate? **MSW**?

Standard model says these two waves remain coherent after traversing over 100 million kilometers.

It is like a two-slit experiment where an electron goes through two slits and produces an interference pattern on a screen over 100 million kilometers away.

There is no other experiment showing preservation of quantum-mechanical correlations over such large distances.

Whether these correlations are preserved or whether there is some dephasing is worth investigation.

How can neutrinos with different masses be coherent?

Review experimentally known neutrino information

Neutrinos have several different mass eigenstates

Consider two different stable neutrino mass eigenstates

$\pi \to \mu\nu$; $\pi \to e\nu$ at rest "Missing Mass" **experiments.** $M_{\nu_\mu}^2 = (M_\pi - E_\mu)^2 - p_\mu^2$; $M_{\nu_e}^2 = (M_\pi - E_e)^2 - p_e^2$.

In initial Lederman-Schwartz-Steinberger experiment

Neutrinos emitted in $\pi \to \mu\nu$ produced no e, only μ. Simply described in with ν_μ and ν_e mass eigenstates.

Ruled out by subsequent experiments. Mass eigenstate neutrino incident on detector, can produce either e or μ

Amplitudes for electrons at the detector from both mass eigenstates must be coherent and exactly cancel.

Missing mass experiment was not performed

Sufficient information was not available to determine neutrino mass from energy and momentum conservation.

Missing information was not simple ignorance.

Ignorance alone cannot provide coherence.

Experimental set up and quantum mechanics must forbid knowledge necessary to determine the neutrino mass.

Instructive Example - Bragg scattering by a crystal

Coherence from incomplete momentum information on scattering from different atoms produces constructive interference at Bragg angles and peaks in angular distribution.

Single scattered photon transfers momentum to scattering atom. Detecting recoil momentum would identify scattering atom and destroy coherence.

QM prevents measurement of individual atom momenta

QM of Crystal dynamics and incident photon interactions allow elastic scattering. Photon scattered by single atom in crystal but crystal quantum state unchanged.

Purely quantum effect. Classical momentum transfer to an atom in classical crystal changes atom momentum and motion. Allows identification of scattering atom.

Simple toy model - each atom bound to equilibrium position by harmonic oscillator potential.

Atom scattering the photon initially in definite discrete energy level $|i\rangle$.

Cannot absorb the momentum transfer according to the energy and momentum kinematics of free particles.

Final state $|f\rangle$ must be allowed energy level.

Finite probability that $|f\rangle = |i\rangle$ (elastic scattering)

Which atom scattered photon? Information unavailable.

Coherent scattered amplitudes from all scattering atoms

Amplitudes arising from different processes which would be classiscally distinguishable can be coherent.

The quantum mechanics of localized states can conceal the information which would be classically available from energy-momentum conservation for free particles.

Same effect conceals neutrino mass in π decay

No problem in measuring decay muon momentum

Initial p_π information must be incomplete

Not strictly at rest; localized in some energy level $|i\rangle$ of the material where it stopped.

Initial state $|i\rangle$ has pion coherent linear combination of different momentum eigenstates with sharp energy. Muon energy determines neutrino energy but not momentum.

At neutrino detector, amplitudes with same energy and different momenta produced from the different coherent momentum components in the initial pion wave function can be coherent with a definite relative phase.

Amplitudes with different energies not coherent

This can explain why no electrons are observed at a short distance from the detector.

If neutrino amplitudes propagate as free particles, the relative phase is completely determined between the amplitudes for neutrinos having the same energy but different masses and different momenta

This produces neutrino oscillations with the same relation between mass differences and phase differences given by the standard treatments.

The Right Way to Treat Flavor Oscillations

WHAT IS THE PROBLEM?

An amplitude with definite flavor is created at a source

A coherent mixture of amplitudes from mass eigenstates

Neutrinos propagate freely from source to detector

Mass eigenstates propagate independently - no interactions

Relative phases of mass eigenstates change during propagatioı

The amplitude flavor is measured at a remote detector

WHAT IS THE SOLUTION? TRIVIAL QM EXERCIS

1. Solve the free Schroedinger or Dirac Equation
2. Introduce the proper initial conditions at the source
3. Introduce the proper QM description of the detector
4. Calculate the transition matrix element at the detector

The free Dirac or Schroedinger Equation is trivial

No need for fancy field theory or Feynman diagrams

No need for Lorentz transformations

Mixtures of noninteracting mass amplitudes - no problem

It is obvious that
Is it obvious?

Yes - it is obvious!

WHY DOESN'T EVERYONE DO THIS?

The Textbooks are misleading!

The textbook neutrino oscillation occurs in time

No experiment measures time!

The textbook neutrino is mixture of mass eigenstates

Same momentum and different energies

No experiment knows how to make such a neutrino!

The textbook calculates a neutrino oscillation frequency

Gedanken result - No experiment measures a frequency!

Experiments measure wave length!

Textbook converts frequency to wave length $\lambda\nu = v$

Neutrinos with different masses have different velocities

Different transit times between source and detector

Textbooks ignore these differences - get right answers

People worrying about these differences get wrong answers

Avoid confusion - calculate real experiment!

Emitted neutrinos carry energy and momentum

Neutrino source S is a macaroscopic object which must follow the rules of quantum mechanics. The source recoils with conservation of energy and momentum

Emission and propagation of neutrinos follow QM

Examine transition $|i\rangle \equiv |S(E,p)\rangle \rightarrow |f\rangle \rightarrow |SD\rangle$,

$$|f\rangle \equiv c_1 \cdot |S(E-E_1, p-p_1)\rangle \cdot |\nu(E_1, p_1)\rangle +$$

$$+c_2 \cdot |S(E-E_2, p-p_2)\rangle \cdot |\nu(E_2, p_2)\rangle$$

$$|\langle SD|\, T(\nu)\, |f\rangle\,|^2 = |c_1 \cdot \langle D|\, T_1(\nu)\, |\nu\rangle\,|^2 + |c_2 \cdot \langle D|\, T_2(\nu)\, |\nu\rangle\,|^2$$

Source states drop out of the relation because of orthogonality.

$$\langle S(E-E_1, p-p_1)|\, S(E-E_2, p-p_2)\rangle = 0$$

Interference term vanishes. Missing mass experiment.

No coherence beween two mass eigenstates.

CORRECTION

Source is wave packet in momentum space.

$$|i\rangle = \int g(p)dp\,|S(E,p)\rangle \equiv \Psi_S(X)$$

X denotes the center of mass co-ordinate of the source.

$$|f\rangle \equiv c_1 \cdot e^{-ip_1 X}\Psi_S(X) \cdot e^{ip_1 x_1} + c_2 \cdot e^{-ip_2 X}\Psi_S(X) \cdot e^{ip_2 x_2}$$

x_1 and x_2 co-ordinates of neutrinos - masses m_1 and m_2

Consider two neutrino states with same energy.

$$|\langle SD|\,T(\nu)\,|f\rangle\,|^2 = |c_1\,\langle D|\,T_1(\nu)\,|\nu\rangle\,|^2 + |c_2\,\langle D|\,T_2(\nu)\,|\nu\rangle\,|^2 +$$

$$+\{c_1^* c_2 F(\delta p)\,\langle \nu|\,T_1(\nu)\,|D\rangle\,\langle D|\,T_2(\nu)\,|\nu\rangle + c.c.\}$$

$$F(\delta p) = \int dX \Psi_S^*(X) e^{i\delta p X}\Psi_S(X) \approx 1 - (1/2)\cdot \delta p^2 \langle X^2\rangle$$

Interference term no longer vanishes

Interference term - proportional to "source form factor"

$$F(\delta p) = \int dX \Psi_S^*(X) e^{i\delta p X} \Psi_S(X) \approx 1 - \frac{2\pi^2 \langle X^2 \rangle}{\lambda^2} \approx 1$$

$\lambda = 2\pi/\delta p$ wave length of the neutrino oscillation produced by momentum difference δp

Departure from coherence in interference proportional to the ratio of the mean square quantum fluctuation in the position of the source to the square of the oscillation wave length and is clearly negligible for wave lengths of the order to the source-detector distance.

Lipkin's Principle for oscillation coherence

If you can measure it you can measure it!

PROOF

Any sensible experiment must have $x_s << \lambda$

$$\lambda >> x_s; \quad \delta p_s \approx \frac{\hbar}{x_s} >> \frac{\hbar}{\lambda} \approx \delta p_{osc}$$

Any sensible experiment will have δp coherence

Emitted neutrinos carry energy and momentum

Sun recoils with conservation of energy and momentum

Emission and propagation of neutrinos follow QM

Examine transition $|i\rangle \equiv |S(E,p)\rangle \rightarrow |f\rangle \rightarrow |SD\rangle$,

$$|f\rangle \equiv c_1 \cdot |S(E - E_1, p - p_1)\rangle \cdot |\nu(E_1, p_1)\rangle +$$

$$+c_2 \cdot |S(E - E_2, p - p_2)\rangle \cdot |\nu(E_2, p_2)\rangle$$

$$|\langle SD|\, T(\nu)\, |f\rangle\,|^2 = |c_1 \cdot \langle D|\, T_1(\nu)\, |\nu\rangle\,|^2 + |c_2 \cdot \langle D|\, T_2(\nu)\, |\nu\rangle\,|^2$$

Sun states drop out of the relation because of **orthogonality**.

$$\langle S(E - E_1, p - p_1)|\, S(E - E_2, p - p_2)\rangle = 0$$

Interference term vanishes. Missing mass experiment.

No coherence beween two mass eigenstates.

CORRECTION

Sun is wave packet in momentum space.

$$|i\rangle = \int g(p)dp\,|S(E,p)\rangle \equiv \Psi_S(X)$$

X denotes the center of mass co-ordinate of the sun.

$$|f\rangle \equiv c_1 \cdot e^{-ip_1X}\Psi_S(X) \cdot e^{ip_1x_1} + c_2 \cdot e^{-ip_2X}\Psi_S(X) \cdot e^{ip_2x_2}$$

x_1 and x_2 co-ordinates of neutrinos - masses m_1 and m_2

Consider two neutrino states with same energy.

$$|\langle SD|\,T(\nu)\,|f\rangle\,|^2 = |c_1\,\langle D|\,T_1(\nu)\,|\nu\rangle\,|^2 + |c_2\,\langle D|\,T_2(\nu)\,|\nu\rangle\,|^2 +$$

$$+\{c_1^* c_2 F(\delta p)\,\langle\nu|\,T_1(\nu)\,|D\rangle\,\langle D|\,T_2(\nu)\,|\nu\rangle + c.c.\}$$

$$F(\delta p) = \int dX\,\Psi_S^*(X) e^{i\delta p X}\Psi_S(X) \approx 1 - (1/2)\cdot\delta p^2\langle X^2\rangle$$

Interference term no longer vanishes

Interference term - proportional to "solar form factor"

$$F(\delta p) = \int dX \Psi_S^*(X) e^{i\delta p X} \Psi_S(X) \approx 1 - \frac{2\pi^2 \langle X^2 \rangle}{\lambda^2} \approx 1$$

$\lambda = 2\pi/\delta p$ wave length of the neutrino oscillation produced by momentum difference δp

Departure from coherence in interference proportional to the ratio of the mean square quantum fluctuation in the position of the sun to the square of the oscillation wave length and is clearly negligible for wave lengths of the order to the sun-earth distance.

Lipkin's Principle for oscillation coherence

If you can measure it you can measure it!

PROOF

Any sensible experiment must have $x_s << \lambda$

$$\lambda >> x_s; \quad \delta p_s \approx \frac{\hbar}{x_s} >> \frac{\hbar}{\lambda} \approx \delta p_{osc}$$

Any sensible experiment will have δp coherence

Detector destroys
all relevant
ignorance

All coherence between
different energies
DESTROYED!

Detector Preserves
all relevant ignorance

All Coherence between
different energies
preserved!

Components with same E, different p coherent
Components with differen E - incoherent

Here it ain't

Here it ain't

Coherent Components with
Different energies cancel

Here it
ain't

Here it
ain't

Coherent components with
different momenta cancel

FIGURES

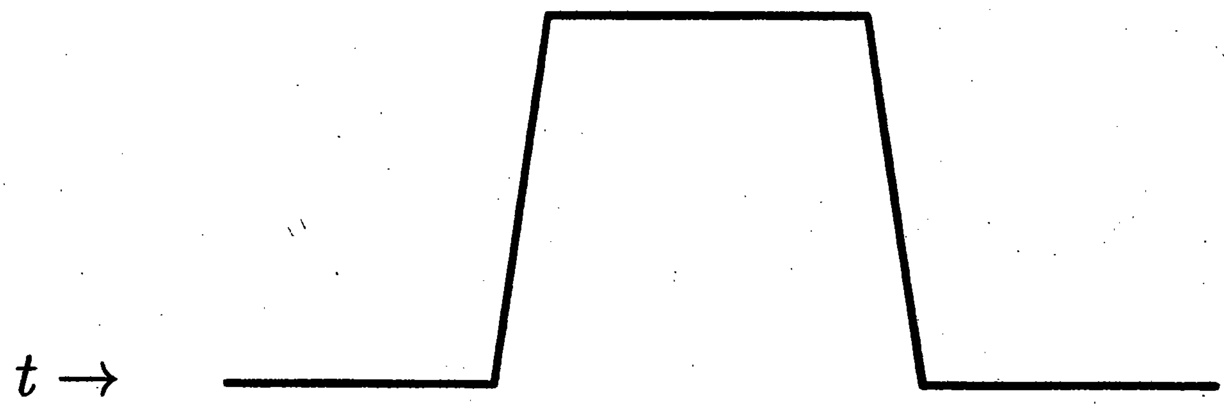

FIG. 1.
Neutrino Wave Packet at fixed point in space

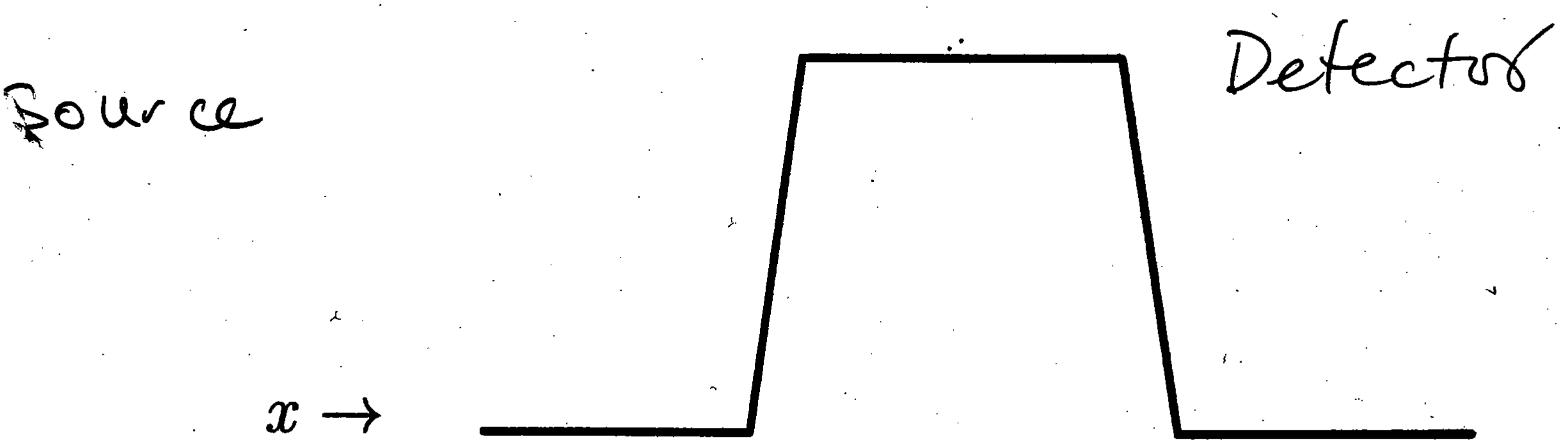

FIG. 2.
Neutrino Wave Packet in space at fixed time

The Role of the Neutrino Detector

Crucial in all discussions of coherence

Neutrino described by wave packet which must vanish

1. Outside of a finite region in space at any given time

   Must have coherent components with different momenta

2. Outside of a finite interval in time at any point in space

   Must have coherent components with different energies

Neutrino wave packet incident on realistic neutrino detector

Loses all coherence between components with different energies.

All coherence between components having the same energy and slightly different momenta is preserved.

Completely independent of the neutrino source

Original Lederman-Schwartz-Steinberger experiment proves:

Such preserved coherence must exist in neutrinos emitted in $\pi - \mu$ decay.

Original Lederman-Schwartz-Steinberger experiment proves:

Such preserved coherence must exist in neutrinos emitted in $\pi - \mu$ decay.

Experiment saw only muons and no electrons

At least two different neutrino mass eigenstates emitted $\pi - \mu$ decay

At least one must couple to electrons.

Only explanation for absence of electrons at detector

Destructive interference from amplitudes produced by different mass eigenstates.

All experiments detect neutrinos with detectors

1. At rest in the laboratory system

   A. Forget about Lorentz Invariance

   B. Nobody needs Lorentz frame with moving detector

2. In thermal equilibrium with their environment

   A. Described by a density matrix diagonal in **energy**

   B. Unable to observe relative phases

   Between states with different energies

3. Localized in space in a region

   Tiny compared with the distance to the source

   A. Described by a wave function or density matrix **not** diagonal in momentum

   B. Well defined relative phases between eigenstates **with** different momenta

   C. Able to observe coherence between neutrinos

With same energy and different momenta

Many papers do not correctly describe the detector

"Which Path" - With a Quantum Detector

When are amplitudes for two paths coherent?

Quantum Mechanics gives the answer!

Two amplitudes $|L(x)\rangle$ and $|R(x)\rangle$ for two paths

With no detector, wave function at point x on screen

$$\Psi(x) = |L(x)\rangle + |R(x)\rangle$$

With no detector, intensity at point x on screen

$$I(x) = |\Psi(x)|^2 = |\,|L(x)\rangle\,|^2 + |\,|R(x)\rangle\,|^2 + 2Re[\langle L(x)\,|R(x)\rangle]$$

With quantum detector in "R" path; $D_i \to D_f$

Wave function at point x on screen

$$\Psi(x, D) = |L(x), D_i\rangle + \big|R(x), D_f\big\rangle$$

With quantum detector, intensity at point x

$$I(x) = |\,|L(x)\rangle\,|^2 + |\,|R(x)\rangle\,|^2 + 2Re[\langle L(x)\,|R(x)\rangle \cdot \langle D_i\,\big|D_f\big\rangle]$$

Interference term with quantum detector

Additional factor - detector overlap $\langle D_i\,\big|D_f\big\rangle$

Can add phase of $\langle D_i\,\big|D_f\big\rangle$

A Toy Model for a Quantum Detector

A spin 1/2 nucleus rotated 180^o about z axis

$$\left|D_f\right\rangle = e^{i\pi s_z}\left|D_i\right\rangle = e^{i\pi\sigma_z/2}\left|D_i\right\rangle$$

$$\left\langle D_i \left| D_f\right.\right\rangle = \left\langle D_i\right| e^{i\pi\sigma_z/2}\left|D_i\right\rangle = \left\langle D_i\right| i\sigma_z\left|D_i\right\rangle = i\langle\sigma_z\rangle_i$$

With no detector, intensity at point x on screen

$$I(x) = |\,|L(x)\rangle\,|^2 + |\,|R(x)\rangle\,|^2 + 2Re[|\langle L(x)\,|R(x)\rangle\,|\cdot e^{i\theta(x)}]$$

$\theta(x)$ **is relative phase of** $|L(x)\rangle$ **and** $|R(x)\rangle$

$$I(x) = |\,|L(x)\rangle\,|^2 + |\,|R(x)\rangle\,|^2 + 2|\langle L(x)\,|R(x)\rangle\,|\cos\theta(x)$$

With quantum detector in "R" path

Wave function at point x on screen

$$\Psi(x,D) = [|L(x),D_i\rangle + i\sigma_z\,|R(x),D_i\rangle]$$

With quantum detector, intensity at point x

$$I(x) = |\,|L(x)\rangle\,|^2 + |\,|R(x)\rangle\,|^2 - 2|\langle L(x)\,|R(x)\rangle\,|\sin\theta(x)]\cdot\langle\sigma_z\rangle_i$$

Interference term with quantum detector

Additional factor $\langle D_i|\,\sigma_z\,|D_i\rangle = \langle\sigma_z\rangle_i$

With extra 90^o phase.

Detailed Quantum Mechanics of Neutrino Detector

Initial state of neutrino and detector

$$\Psi_i(\nu, D) = \sum_{k=1}^{N_\nu} \sum_{\vec{P}_k} \left| \nu(E_\nu, m_k, \vec{P}_k), D_i(E_i) \right\rangle$$

N_ν **neutrino mass states**

$E_\nu, m_k, \vec{P}_k$ **neutrino energy, mass and momentum**

$D_i(E_i)$ **initial state of the detector - energy** E_i**.**

Final muon detector state after absorption of neutrino with mass m_k**; emission of a** $\mu^\pm$ **with energy and momentum** E_μ **and** $\vec{P}_\mu$

$$\Psi_f(\mu^\pm, D) = \sum_{k=1}^{N_\nu} \sum_{\vec{P}_k} \left| \mu^\pm(E_\mu, \vec{P}_\mu), D_{kf}^\mp(E - E_\mu) \right\rangle$$

$D_{kf}^\mp$ **is final detector state produced in "path** k**"**

$E = E_\nu + E_i$ **is total conserved energy**

Transition in detector on nucleon, co-ordinate $\vec{X}$, charge exchange $I_{\mp}$; momentum transfer $\vec{P}_k - \vec{P}_\mu$.

$$\left\langle D^{\mp}_{kf}\right| T^{\mp} \left|D_i\right\rangle = \left\langle D^{\mp}_{kf}\right| I_{\mp} e^{i(\vec{P}_k - \vec{P}_\mu)\cdot\vec{X}} \left|D_i\right\rangle$$

Detector overlap between absorbing m_k and m_j

$$\left\langle D^{\mp}_{kf} \middle| D^{\mp}_{jf}\right\rangle = \left\langle D_i\right| e^{i(\vec{P}_j - \vec{P}_k)\cdot\vec{X}} \left|D_i\right\rangle$$

If quantum fluctuations in active nucleon position in detector initial state small in comparison with oscillation wave length, $\hbar/(\vec{P}_j - \vec{P}_k)$

$$|\vec{P}_j - \vec{P}_k|^2 \cdot \langle D_i|\,|\vec{X}^2|\,|D_i\rangle \ll 1$$

$$\left\langle D^{\mp}_{kf} \middle| D^{\mp}_{jf}\right\rangle \approx 1 - (1/2)\cdot|\vec{P}_j - \vec{P}_k|^2 \cdot \langle D_i|\,|\vec{X}^2|\,|D_i\rangle \approx 1$$

Full overlap after absorbing neutrinos with same energy and different momenta

Neutrinos with different energies - no coherence

Interpreting the Standard Textbook Wave Function

Real & Gedanken ν-oscillation Experiments

Source creates particle mixture - two or more mass eigenstates

Different mixture observed in detector

Flavor eigenstate with sharp momentumm - different **energies**

Oscillates in time with well-defined oscillation **period**

Flavor eigenstate with sharp energy - different **momenta**

Oscillates in space with well-defined oscillation wave **length**

Confusion in Description of Flavor Oscillations

Sharp momentum or sharp energy - "Gedanken" experiments

Conventional Wisdom - Oscillations in Time

For simplicity assume 45^o mixing angle

$$|\nu_e\rangle = (1/\sqrt{2})(|\nu_1\rangle + |\nu_2\rangle); \qquad |\nu_\mu\rangle = (1/\sqrt{2})(|\nu_1\rangle - |\nu_2\rangle)$$

ν_e produced at t=0 with momentum p and energies

$$E_1^2 = p^2 + m_1^2; \qquad E_2^2 = p^2 + m_2^2$$

$|\nu_e\rangle$ and $|\nu_\mu\rangle$ components oscillate in time

$$\left|\frac{\langle\nu_\mu\,|\nu_e(t)\rangle}{\langle\nu_e\,|\nu_e(t)\rangle}\right| = \left|\frac{e^{iE_1t} - e^{iE_2t}}{e^{iE_1t} + e^{iE_2t}}\right| = \tan\left(\frac{(E_1 - E_2)t}{2}\right) =$$

$$= \tan\left(\frac{(m_1^2 - m_2^2)t}{2(E_1 + E_2)}\right)$$

This is a "non-experiment". Real experiment measures **space**

Now Comes the Hand Waving - Method A

Convert time into distance

$$x = vt = \tfrac{p}{E}\cdot t$$

$$\left|\frac{\langle\nu_\mu\,|\nu_e(t)\rangle}{\langle\nu_e\,|\nu_e(t)\rangle}\right| = \tan\left(\frac{(m_1^2 - m_2^2)t}{2(E_1 + E_2)}\right) \approx \tan\left(\frac{(m_1^2 - m_2^2)\boldsymbol{x}}{\boldsymbol{4p}}\right)$$

Problems with Hand Waving - Method A again

$$x = vt = \frac{p}{E} \cdot t$$

$$\left| \frac{\langle \nu_\mu | \nu_e(t) \rangle}{\langle \nu_e | \nu_e(t) \rangle} \right| = \tan \left(\frac{(m_1^2 - m_2^2)t}{2(E_1 + E_2)} \right) \approx \tan \left(\frac{(m_1^2 - m_2^2)x}{4p} \right)$$

A Different Hand Waving - Method B

But ν_1 and ν_2 states have different velocities

$$x = v_1 t_1 = \frac{p}{E_1} \cdot t_1 = v_2 t_2 = \frac{p}{E_2} \cdot t_2$$

$$\left| \frac{\langle \nu_\mu | \nu_e(x) \rangle}{\langle \nu_e | \nu_e(x) \rangle} \right| = \left| \frac{e^{iE_1 t_1} - e^{iE_2 t_2}}{e^{iE_1 t_1} + e^{iE_2 t_1}} \right| = \tan \left(\frac{(E_1 t_1 - E_2 t_2)}{2} \right) =$$

$$= \tan \left(\frac{(m_1^2 - m_2^2)x}{2p} \right)$$

Differs by factor of 2 in oscillation wave length. Which is correct?

A Real Calculation Without Hand Waving (?)

All confusion avoided by direct use of real experiment

ν_e produced at x=0 with energy E

)nly neutrinos with SAME ENERGY can be coherent **at detecto**

$|\nu_e\rangle$ and $|\nu_\mu\rangle$ components oscillate in **space**

$$\left|\frac{\langle\nu_\mu|\nu_e(x)\rangle}{\langle\nu_e|\nu_e(x)\rangle}\right| = \left|\frac{e^{ip_1x} - e^{ip_2x}}{e^{ip_1x} + e^{ip_2x}}\right| = \tan\left(\frac{(p_1 - p_2)x}{2}\right) =$$

$$= \tan\left(\frac{(m_1^2 - m_2^2)x}{2(p_1 + p_2)}\right)$$

Simple argument is right

Treatment is completely relativistic

Needs no discussion of time dependence or "**proper times**"