JHF / NuMI Complementarity

Stephen Parke
Fermilab

- $P(\nu_\mu \to \nu_e)$:
- Bi-probability Plots:
- JHF / NuMI:
- Summary:

Trends in Neutrino Physics
Argonne, May 15, 2003

Parameters:

PDG MNS matrix.

ATMOSPHERIC:

$|\delta m^2_{atm}| = 2.5 \times 10^{-3} eV^2$
$\sin^2 2\theta_{23} = 0.96,\ 1.0$ > 0.91

SOLAR:

$\delta m^2_{\odot} = +7 \times 10^{-5} eV^2$ $\sin^2 2\theta_{12} = 0.85$

$\delta m^2_{\odot} \sin 2\theta_{12}$

AND

$\sin^2 2\theta_{13} = 0.02,\ 0.05,\ 0.09.$

OM ≡ Oscillation Maximum

ENERGIES AND BASELINES:

$E^{JHF}_{OM} = 0.6 \text{ GeV} \left(\frac{L}{295 \text{ km}}\right) \left(\frac{\delta m^2_{atm}}{2.5\times10^{-3} \text{ eV}^2}\right)$ $\rho = 2.3 \text{ g cm}^{-3}$

$E^{NuMI}_{OM} = 1.5 \text{ GeV} \left(\frac{L}{732 \text{ km}}\right) \left(\frac{\delta m^2_{atm}}{2.5\times10^{-3} \text{ eV}^2}\right)$ $\rho = 2.8 \text{ g cm}^{-3}$

Event Rates "Peak" at energies ~30% higher (if same flux) i.e. 0.8 and 2.0 GeV respectively.

	LBL	"Competition"
θ_{13}	$\sin\theta_{23}\ \sin\theta_{13}$ + possible degeneracies θ_{13} and δ	Reactors θ_{13} indep of all degenaries
$\delta m^2_{31} \lesseqgtr 0$	OK if sufficient matter effect	- possible $0\nu\beta\beta$ decay - supernova - cosmology
$\not{C}\not{P}(\delta)$	$\sin\theta_{23}\ \sin\delta$ $\cos\theta_{23}\ \cos\delta$	—
$\theta_{23} \lesseqgtr \pi/4$ $\sin^2 2\theta_{23} \neq 1$	$\sin^2 2\theta_{23} = 1-\epsilon^2 = 0.96$ $\sin^2\theta_{23} = \frac{1\mp\epsilon}{2} = 0.4$ $\cos^2\theta_{23} = \frac{1\pm\epsilon}{2} = 0.6$ two sol^n	Reactors + $\sin\theta_{23}\sin\theta_{13}$ from LBL

$$a_{\mu\to e} = \sum_{i=1,2,3} U_{\mu i} U_{ei}^{*} \, e^{-\frac{i m_i^2 L}{2E}}$$

eliminate $\quad U_{\mu 1} U_{e1}^{*} = - U_{\mu 3} U_{e3}^{*} - U_{\mu 2} U_{e2}^{*}$

$$\Longrightarrow$$

$$\boxed{\nu_\mu \to \nu_e}$$

$$a_{\mu\to e} = \Big[\underbrace{e^{-i\Delta_{32}} 2 U_{\mu 3} U_{e3}^{*} \sin\Delta_{31}}_{a_\otimes} + \underbrace{2 U_{\mu 2} U_{e2}^{*} \sin\Delta_{21}}_{a_\odot} \Big]$$

$$\Delta_{ij} = \frac{\delta m^2_{ij} L}{4E} = 1.2669\ldots \frac{\delta m^2 L}{E}$$

and $\sum U_{\mu i} U_{ei}^{*} = 0$

$$2 U_{\mu 3} U_{e3}^{*} = e^{i\delta} \sin\theta_{23} \sin 2\theta_{13}$$

$$2 U_{\mu 2} U_{e2}^{*} \approx \cos\theta_{13} \cos\theta_{23} \sin 2\theta_{12}$$ ← green

$U_{\mu 2} = c_{23} c_{12} - \cancel{s_{13} s_{23} s_{12} e^{i\delta}}$

$$P_{\mu\to e} = \left| e^{-i(\Delta_{32}-\delta)} \sqrt{P_\otimes} + \sqrt{P_\odot} \right|^2$$

interference term

$$= P_\otimes + 2\sqrt{P_\otimes P_\odot} \cos(\Delta_{32} - \delta) + P_\odot$$

$$P_\otimes = \left(\sin\theta_{23} \sin 2\theta_{13} \sin\Delta_{31} \right)^2$$

$\nu \to \bar{\nu}$
$\delta \to -\delta$

$$P_\odot = \left(\cos\theta_{13} \cos\theta_{23} \sin 2\theta_{12} \sin\Delta_{21} \right)^2$$

3 active flavors

(but can be easily modified to accommodate 3+1)

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$$

The parameterization used for the unitary MNS matrix, U, is

PDG

$$\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

where $c_{jk} \equiv \cos\theta_{jk}$ and $s_{jk} \equiv \sin\theta_{jk}$.

The primary element of interest here is

$$|U_{e3}|^2 \quad \text{or} \quad \sin^2 2\theta_{13}$$

and δ.

$$2\sqrt{P_{\otimes} P_{\odot}} \cos\left(\Delta_{32} - \delta\right)$$

$$= 8 J_r \sin\Delta_{21} \sin\Delta_{31} \sin\Delta_{32} \sin\delta$$

$$+ 8 J_r \sin\Delta_{21} \sin\Delta_{31} \cos\Delta_{32} \cos\delta$$

$$J_r = s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2$$

$P_{\mu\to e}$ differs for $\left\{ \begin{array}{c} \text{neutrinos} \\ \text{verses} \\ \text{anti-neutrinos} \end{array} \right.$

because of a CHANGE in phase of interference term between

$a^{\otimes}_{\mu\to e}$ and $a^{\odot}_{\mu\to e}$

$P(\nu_\mu \to \nu_e)$

Why Everybody is Excited!

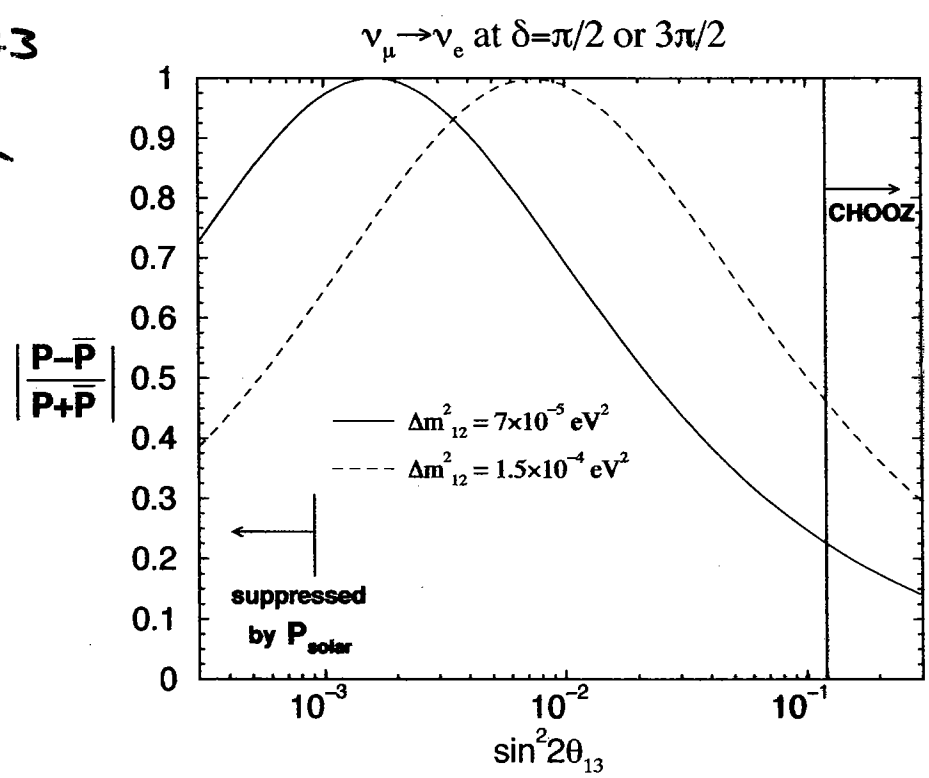
- Maximum Allowed Asymmetry ($\delta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$) for $\nu_\mu \to \nu_e$ at first Oscillation Maximum in vac:

- $P,\ \bar{P} = |a^{atm}_{\mu\to e} + a^{\odot}_{\mu\to e}|^2 \approx (\sin\theta_{23}\sin 2\theta_{13} \pm \sqrt{P_\odot})^2$
  - $|P - \bar{P}| \approx 4\sqrt{P_\odot}\sin\theta_{23}\ \sin 2\theta_{13}$
  - $P + \bar{P} \approx 2\sin^2\theta_{23}\ \sin^2 2\theta_{13} + 2P_\odot$

easily generalized
- $\delta \neq \frac{\pi}{2}$
- off O.M.
- matter

$a^{atm}_{\mu\to e} \propto U_{e3}$

$a^{\odot}_{\mu\to e} \propto U_{e2}$



Asymmetry LARGE: !

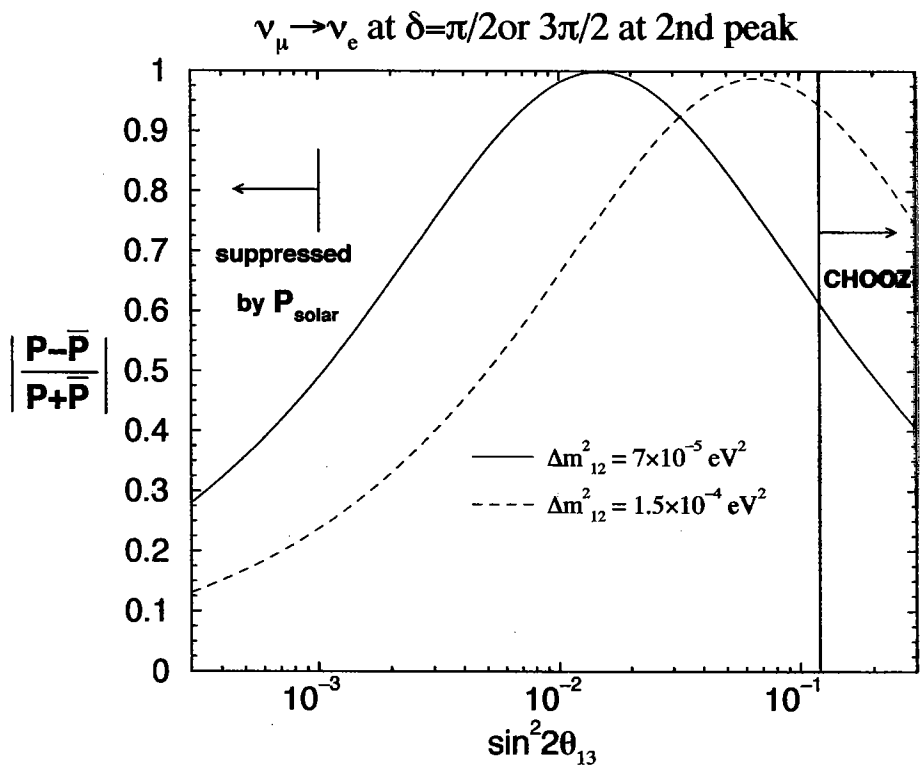
- Peak occurs at

$$\sin^2 2\theta_{13} \approx \frac{\sin^2 2\theta_{12}}{\tan^2\theta_{23}}\left[\frac{\pi}{2}\ \frac{\delta m^2_{12}}{\delta m^2_{13}}\right]^2$$

at OM $\sqrt{P_\odot} = \cos\theta_{13}\cos\theta_{23}\sin 2\theta_{12}\sin\left(\frac{\pi}{2}\ \frac{\delta m^2_{12}}{\delta m^2_{13}}\right)$

- For BK

2nd Peak



- Peak occurs at $\sin^2 2\theta_{13} \approx \frac{\sin^2 2\theta_{12}}{\tan^2 \theta_{23}} \left[\frac{3\pi}{2} \frac{\delta m^2_{12}}{\delta m^2_{13}}\right]^2$

FULL EXPRESSION:

$$P_{\mu\to e} = P_{\otimes} - 2\, s_{23}^2\, s_{2(13)}^2\, s_{12}^2 \sin\Delta_{31} \cos\Delta_{32} \underbrace{\sin\Delta_{21}}_{\text{small}}$$

$$+ P_{\odot}\left(1 - 2 s_{13} \tan\theta_{23} \tan\theta_{12} \cos\delta + s_{13}^2 \tan^2\theta_{23} \tan^2\theta_{12}\right)$$

$$+ 2\sqrt{P_{\otimes} P_{\odot}} \cos\left(\Delta_{32} \mp \delta\right)$$

$$P_{\otimes} = s_{23}^2\, s_{2(13)}^2 \sin^2\Delta_{31}$$

$$P_{\odot} = c_{13}^2\, c_{23}^2\, s_{2(12)}^2 \sin^2\Delta_{21}$$

$P_{\odot}$ is irrelevant unless s_{13} is small

where $(1 - 2 s_{13} \cdots) \simeq 1$

$$\frac{\Delta_{21}}{\Delta_{31}} = \frac{\delta m_{21}^2}{\delta m_{31}^2} \approx \frac{1}{30}$$

also $\sin\Delta_{31} \approx 1$

$\cos\Delta_{32} \approx 0$

for small θ_{13}

$P_{\odot}$

$$P_{\mu\to e} = 4\left(\sin\theta_{23}\sin\theta_{13}\right)^2 \sin^2\Delta_{31}$$

$$+ 8\sin\theta_{12}\cos\theta_{12}\left(\sin\theta_{23}\sin\theta_{13}\right)\sin\Delta_{31}\sin\Delta_{21}$$

$$\times\left\{\left(\cos\theta_{23}\cos\delta\right)\cos\Delta_{32} + \left(\cos\theta_{23}\sin\delta\right)\sin\Delta_{32}\right\}$$

$2\sqrt{P_{\odot}P_{\bullet}}\cos(\Delta_{32}-\delta)$

$$+ P_{\odot}$$

LBL's naturally measure the 3 variables

$$\left(\sin\theta_{23}\sin\theta_{13}\right)^2$$

$$\left(\cos\theta_{23}\sin\delta\right)$$

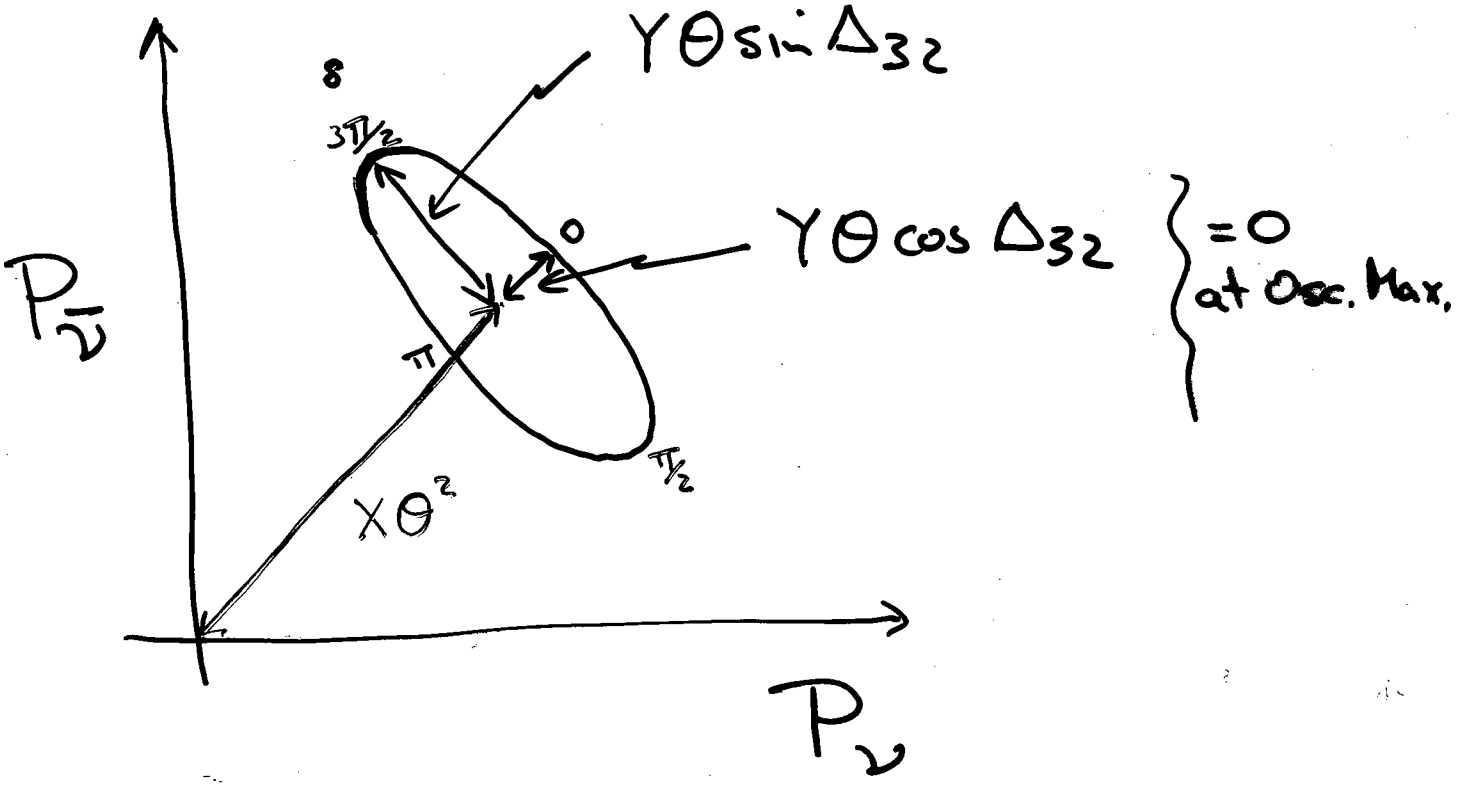
$$\left(\cos\theta_{23}\cos\delta\right)$$

for small θ_{13} (dropping 13 indices)

$$P_{\mu\to e} = X\theta^2 + Y\theta\cos(\Delta_{32} \mp \delta) + P_\odot$$

$$X = 4\sin^2\theta_{23}\sin^2\Delta_{31} : \quad P_\odot = \cos^2\theta_{23}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$

and $Y = 2\sqrt{X P_\odot}$



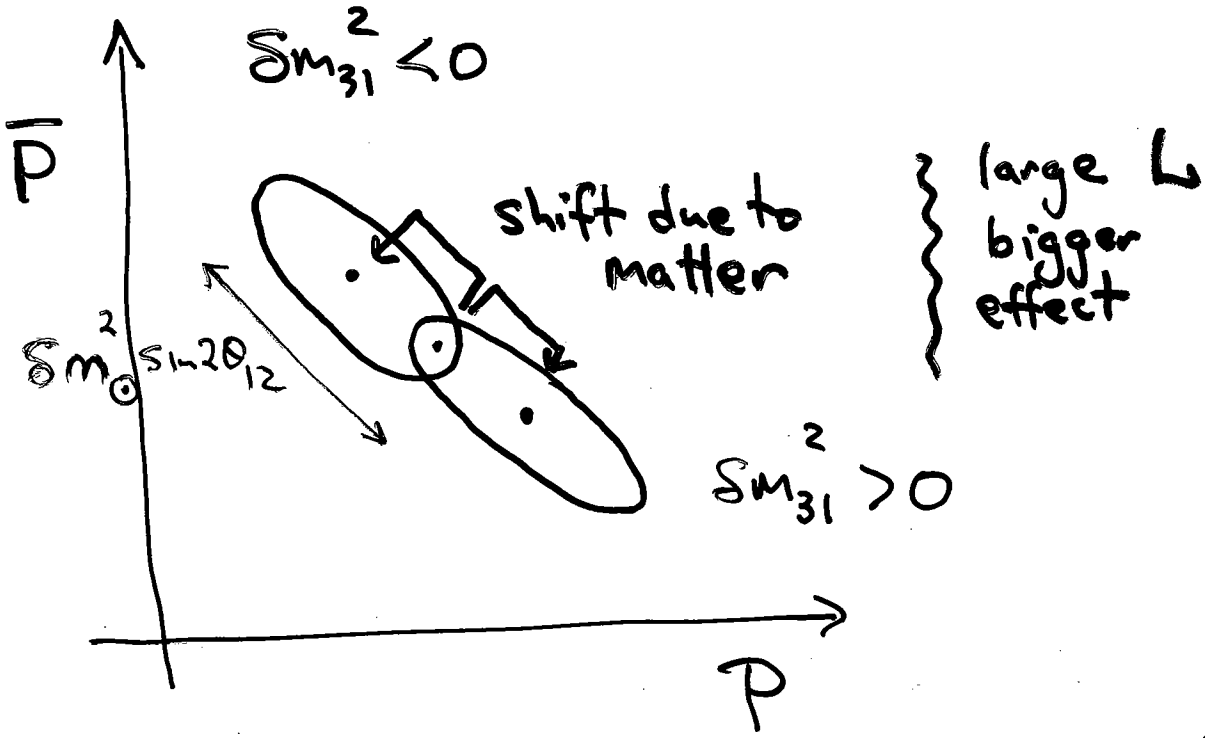
Y depends on solar parameter

$$\delta M^2_{21}\sin 2\theta_{12}$$

known to factor of 2

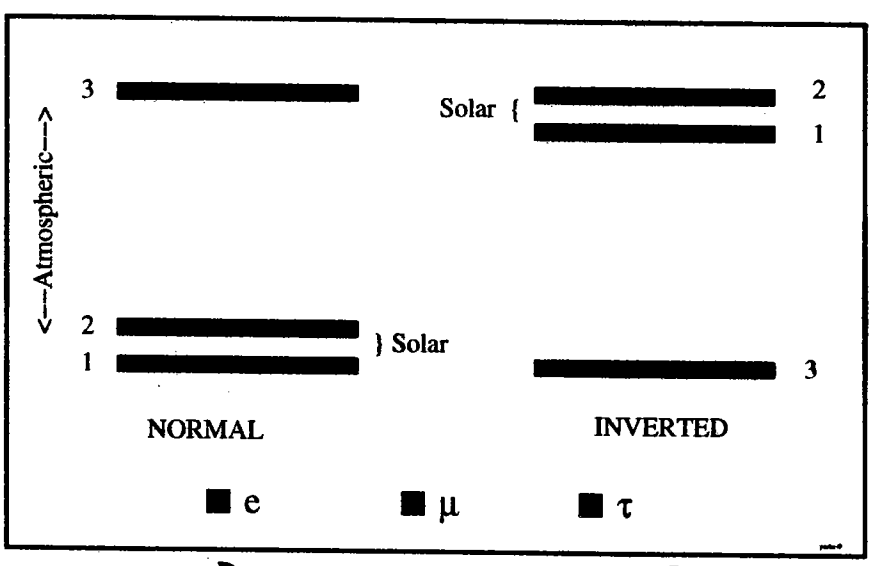
Vacuum:

Matter Effects:



$$P^{center}_{mat} \approx \left(1 \pm \frac{E_\nu}{6\,\mathrm{GeV}}\right) P^{center}_{vac}$$

O.M. for small E_ν

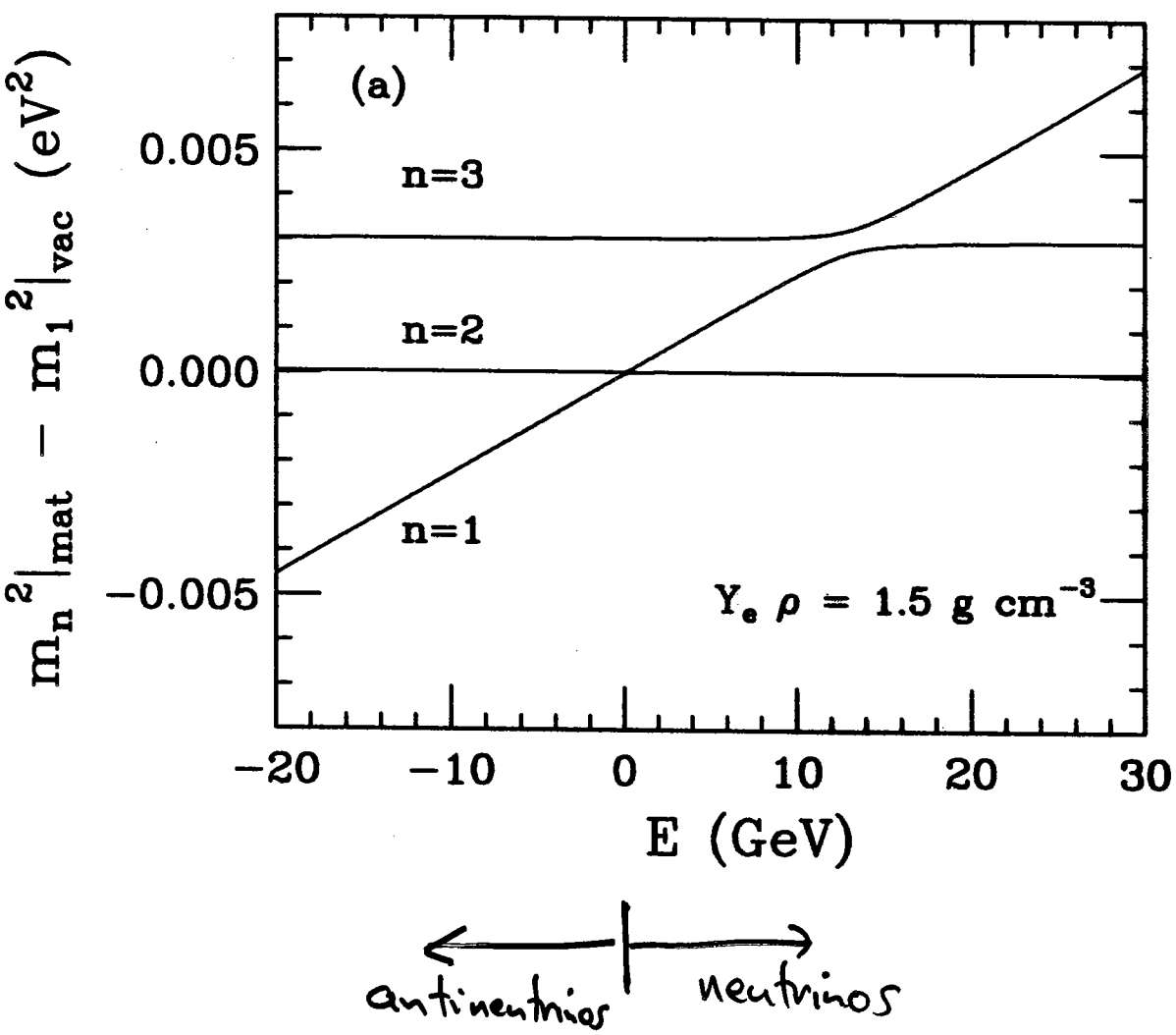


$\delta m^2_{31} > 0$ $\delta m^2_{31} < 0$

$$\left(\sin 2\theta_{13}\, \delta m^2_{31}\right)_{matter} = \left(\sin 2\theta_{13}\, \delta m^2_{31}\right)_{VAC}$$

and

$$\left(\sin 2\theta_{12}\, \delta m^2_{21}\right)_{\cancel{MATTER}} = \left(\sin 2\theta_{12}\, \delta m^2_{21}\right)_{VAC}$$



More Precisely:

$$P_{\mu\to e} = X_\pm \theta^2 + Y_\pm \theta \cos(\pm\Delta_{32} - \delta) + P_\odot$$

where

$$X_\pm = 4 \sin^2\theta_{23} \left(\frac{\Delta_{31}}{\Delta_{31} \mp aL}\right)^2 \sin^2(\Delta_{31} \mp aL)$$

$$P_\odot = \cos^2\theta_{23} \sin^2 2\theta_{12} \left(\frac{\Delta_{21}}{aL}\right)^2 \sin^2(aL)$$

$$Y_\pm = \pm 2\sqrt{X_\pm P_\odot}$$

$$a = G_F N_e / \sqrt{2}$$

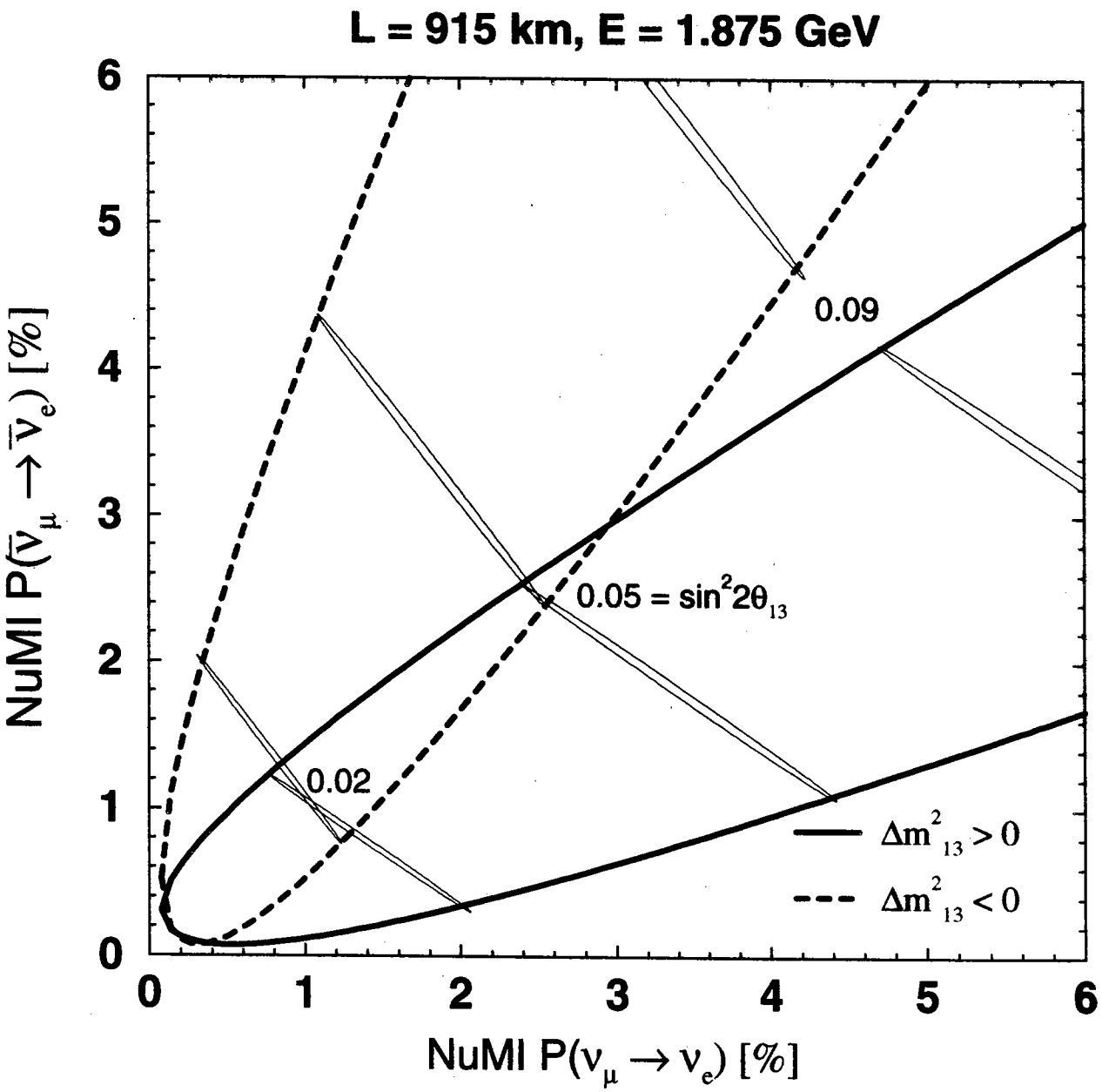
$$\Delta_{ij} = \frac{|\delta m^2_{ij}| L}{4E}$$

Reasonable approximation:

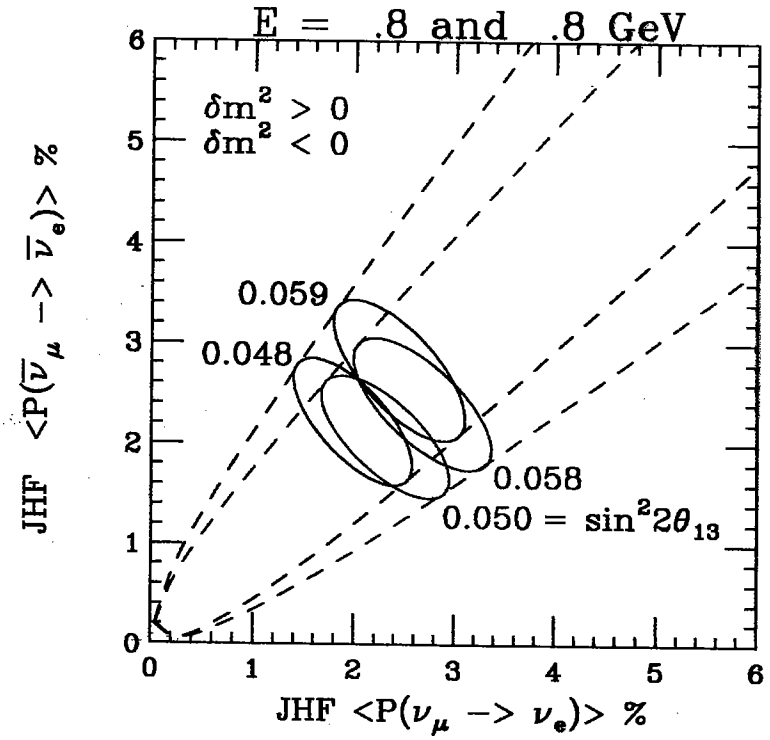
- for small θ_{13}
- energies around 1 GeV
- terrestrial L's

Near Oscillation Maximum:

good separation betwen hierachies
but not perfect.

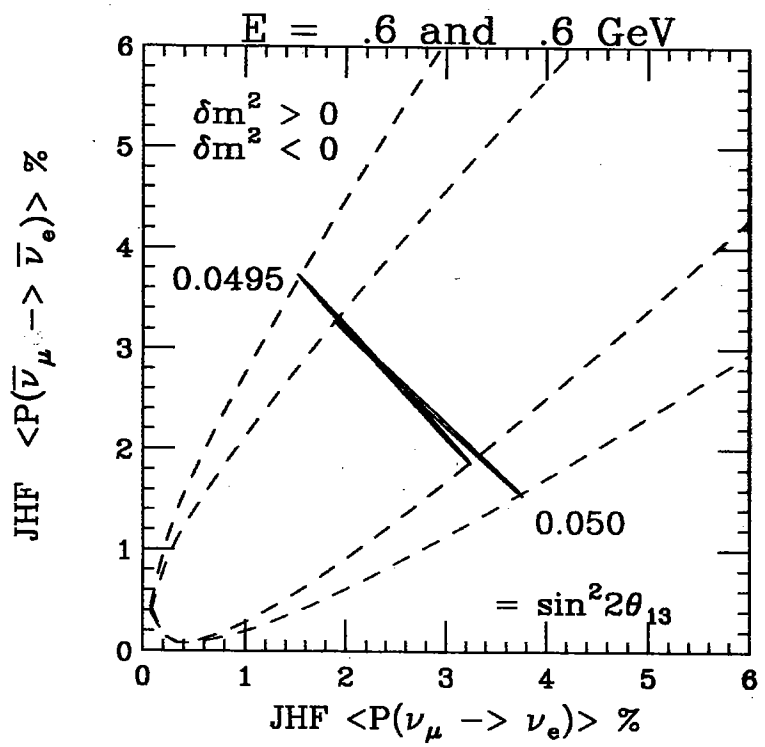


- Two Solutions (θ, δ) for each Hierarchy.



- At Oscillation Maximum (Kajita+Minakata+Nunokawa)

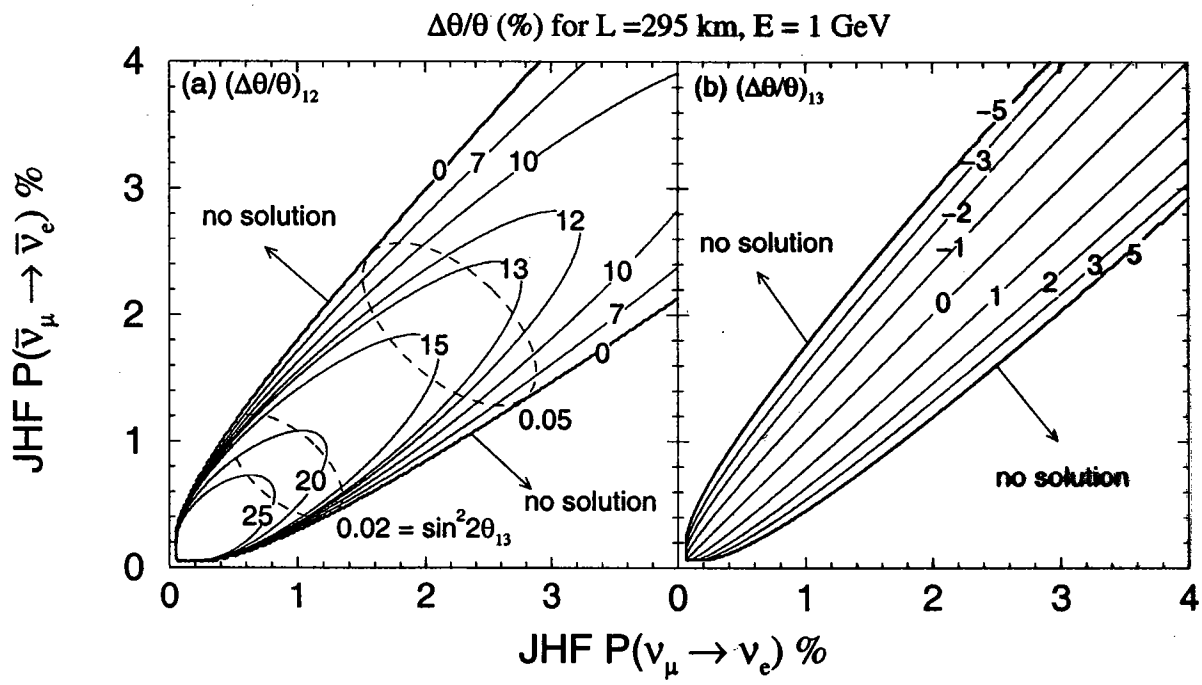
same θ different δ's - δ and $\pi - \delta$



θ measurement insensitive to solar parameters

- The different Hierarchies give almost same θ

$$\frac{\Delta\theta}{\theta} \equiv 2\,\frac{(\theta_i - \theta_j)}{(\theta_i + \theta_j)}$$

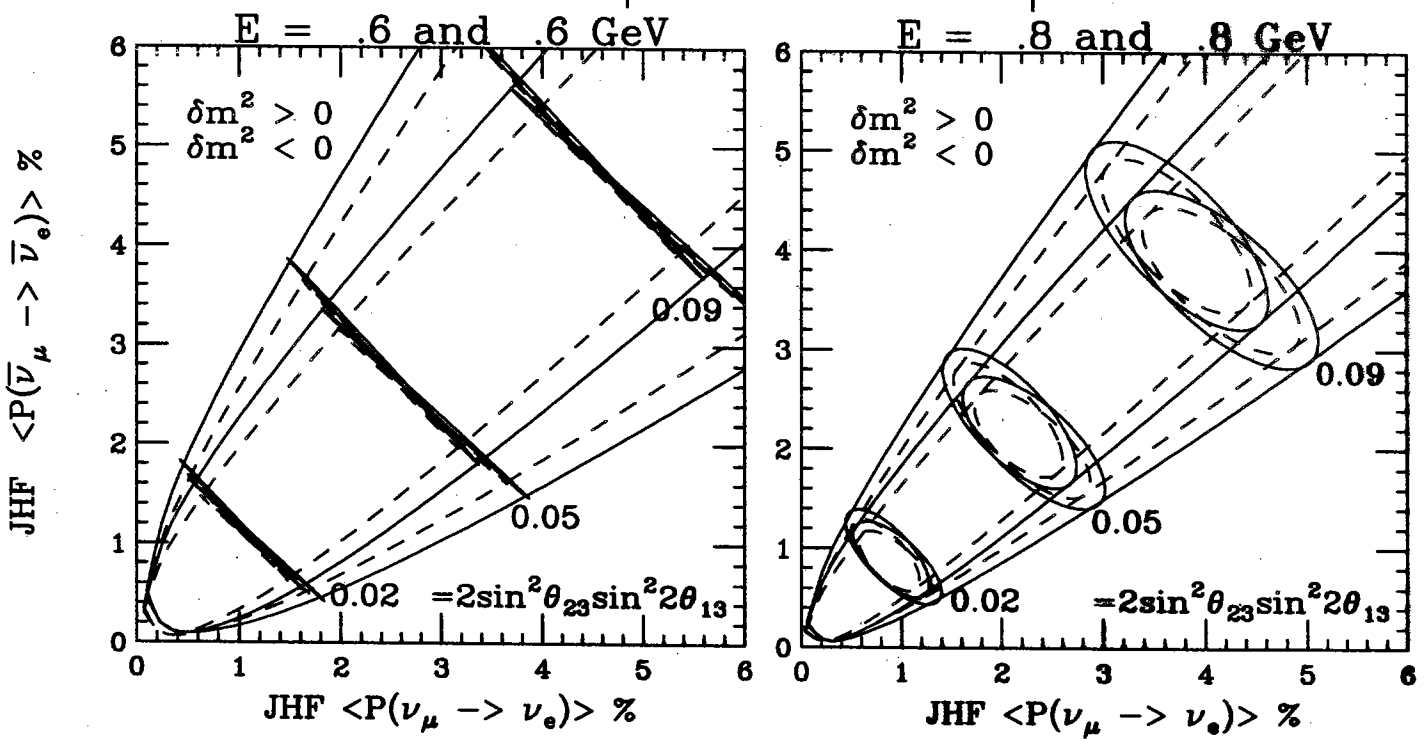


Only relevant for precision (better than 20%) measurements.

$\sin^2 2\theta_{23}$ "Scaling":

- Using $\sin^2 2\theta_{23} = 0.96 = 4 * (0.4) * (0.6)$:

$2\sin^2\theta_{23}\sin^2 2\theta_{13}$	$\sin^2 2\theta_{13}^{(1)}$ $2\sin^2\theta_{23}^{(1)} = 0.8$	$\sin^2 2\theta_{13}^{(2)}$ $2\sin^2\theta_{23}^{(2)} = 1.2$
0.02	0.0250	0.0167
0.05	0.0625	0.0417
0.09	0.1125	0.075
	solid	**dashes**



If $\sin^2 2\theta_{23} \neq 1$ then two solution for θ_{13} are related by $\theta_{13}^{(2)} \approx \frac{\sin\theta_{23}^{(1)}}{\sin\theta_{23}^{(2)}}\,\theta_{13}^{(1)}$

- Exact at Oscillation Maximum:

— small corrects at other energies.

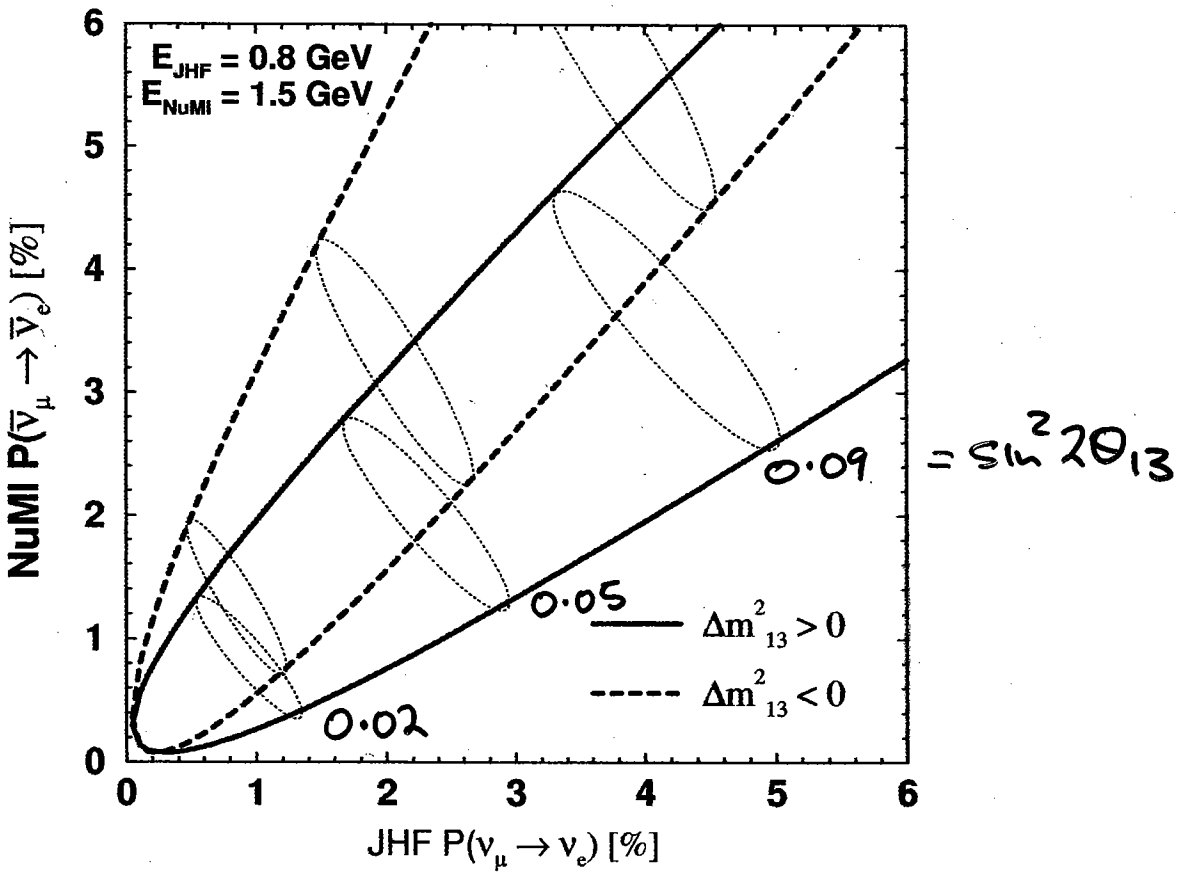
$\nu_\mu \to \nu_e$ JHF/NuMI Complementarity:

Barger+Marfatia+Whisnant hep-ph/0210428

Huber+Lindner+Winter hep-ph/0211300

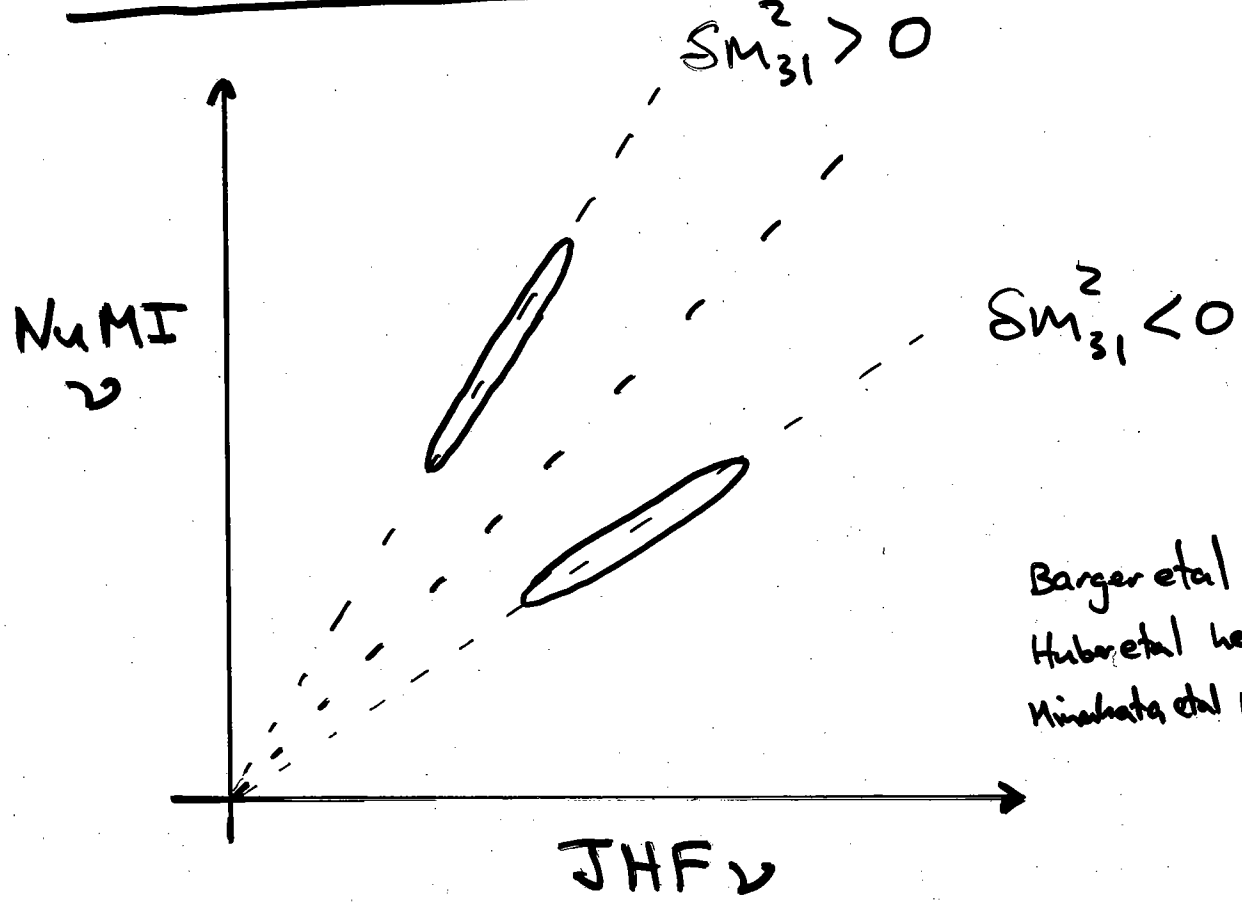
Minakata+Nunokawa+Parke hep-ph/0301210

- JHF Neutrinos - NuMI Anti-Neutrinos



- Similar to JHF (NuMI) ν - JHF (NuMI) $\bar{\nu}$

JHF ν verses NuMI ν



Barger et al hep-ph/0211300
Huber et al hep-ph/0210428
Minakata et al hep-ph/0301210

Ratio of Slopes:

$$\frac{X_+^N}{X_+^J} \Big/ \frac{X_-^N}{X_-^J} \approx 1 + 2G(\Delta^N)(aL)^N - 2G(\Delta^J)(aL)^J$$

$$G(\Delta) = (\Delta^{-1} - \cot\Delta)$$ monot. increasing f^n of Δ $0 \to \pi$

Want larger L^N, smaller E^N
smaller L^J, larger E^J

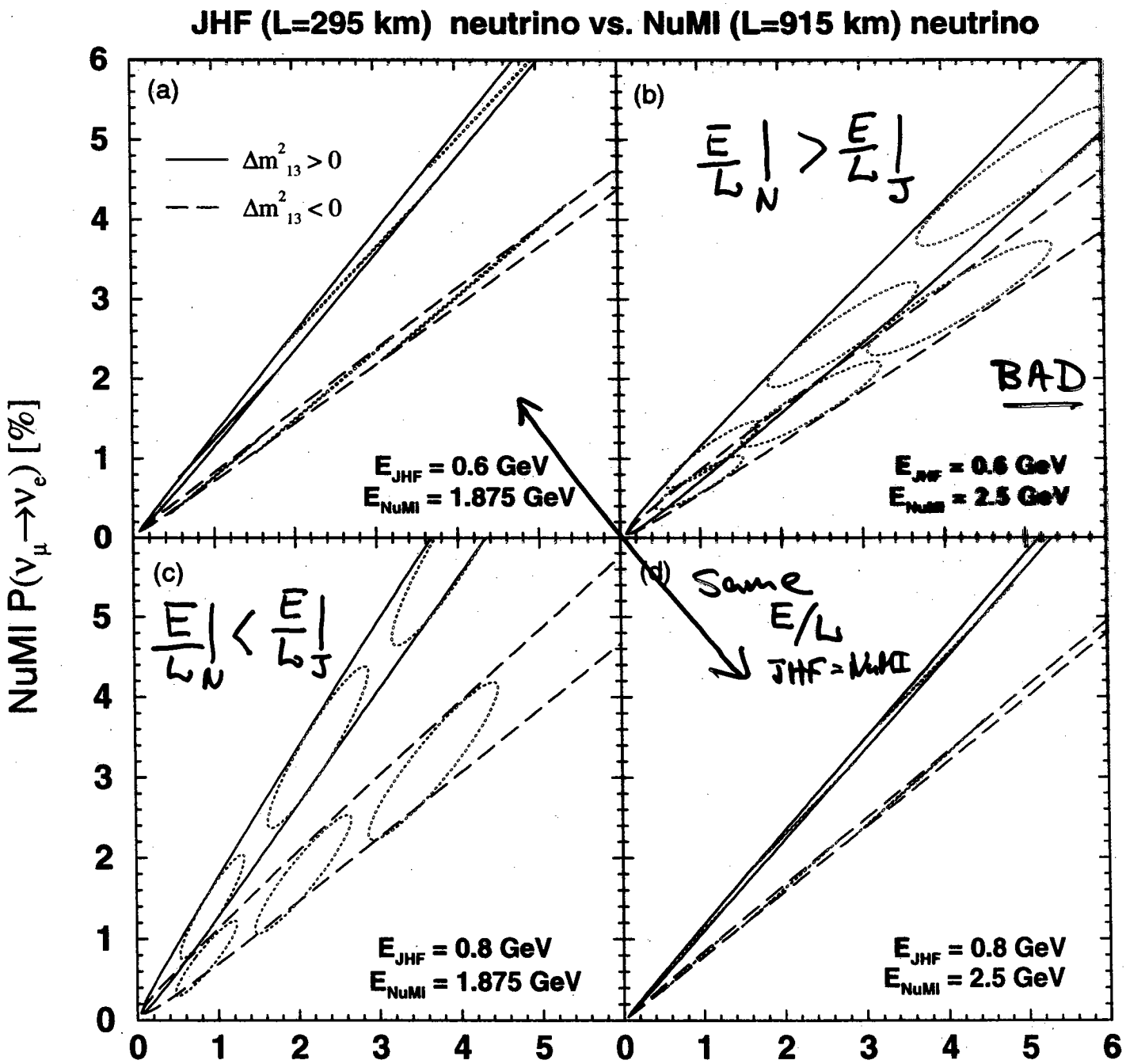
Width of Ellipses: $\left|\frac{Y^N}{X^N} - \frac{Y^J}{X^J}\right| \theta$

Smallest at SAME $\left(\frac{E}{L}\right)$

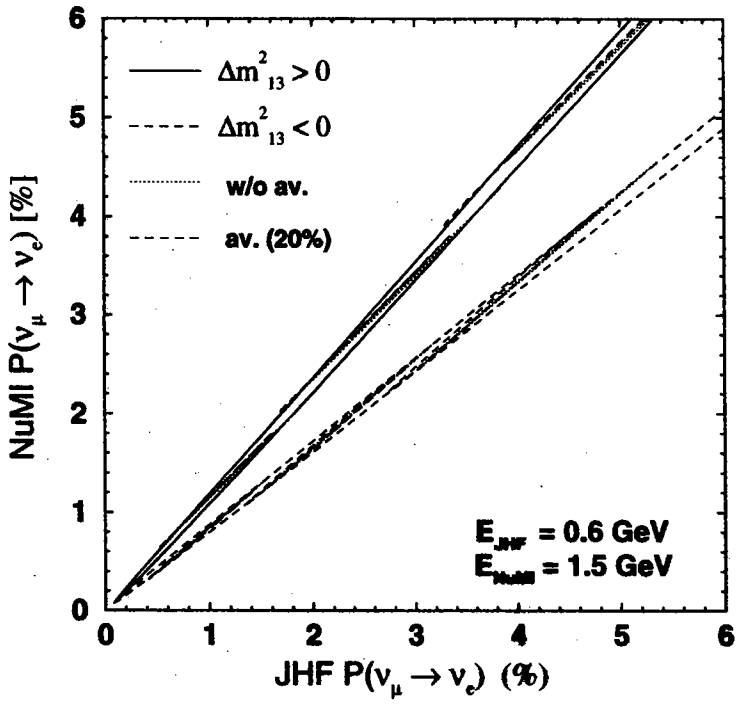
cancellation!
$\frac{Y}{\sqrt{X}}$ the same

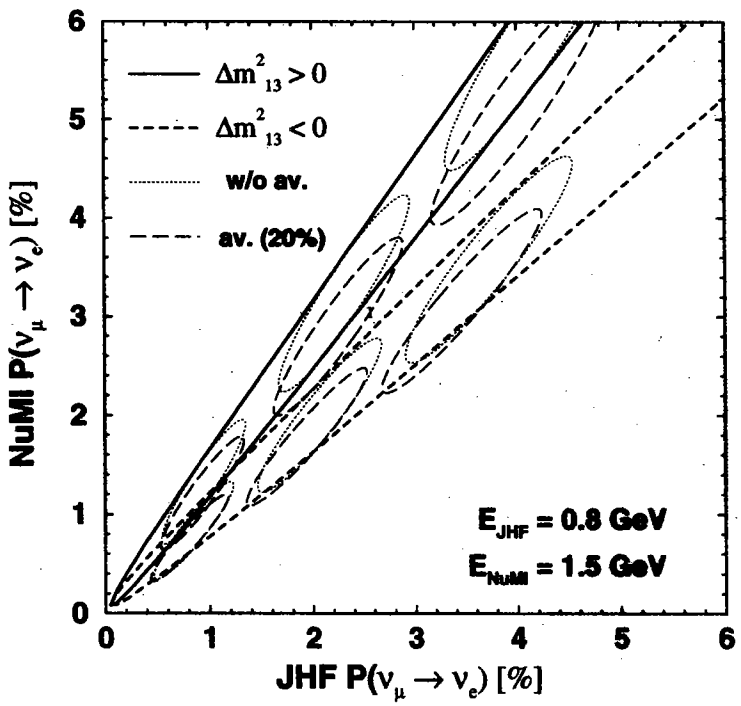
Comparison Between

JHF & NuMI neutrinos



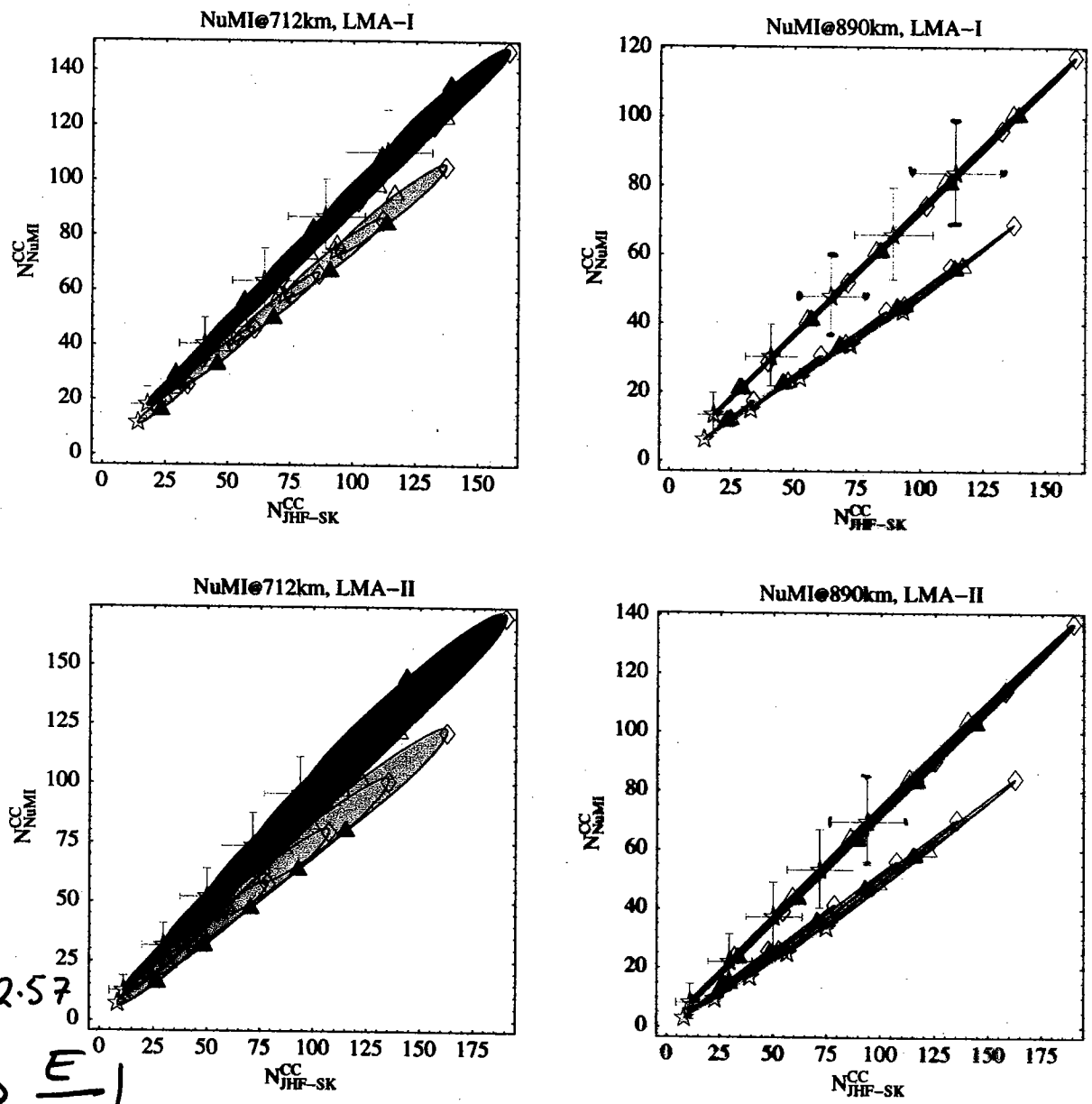
Energy Averaging:





- Energy Averaging does NOT effect separation.

Walter Winter from ~~[illegible]~~ TUM group



$3.11 > 2.57$

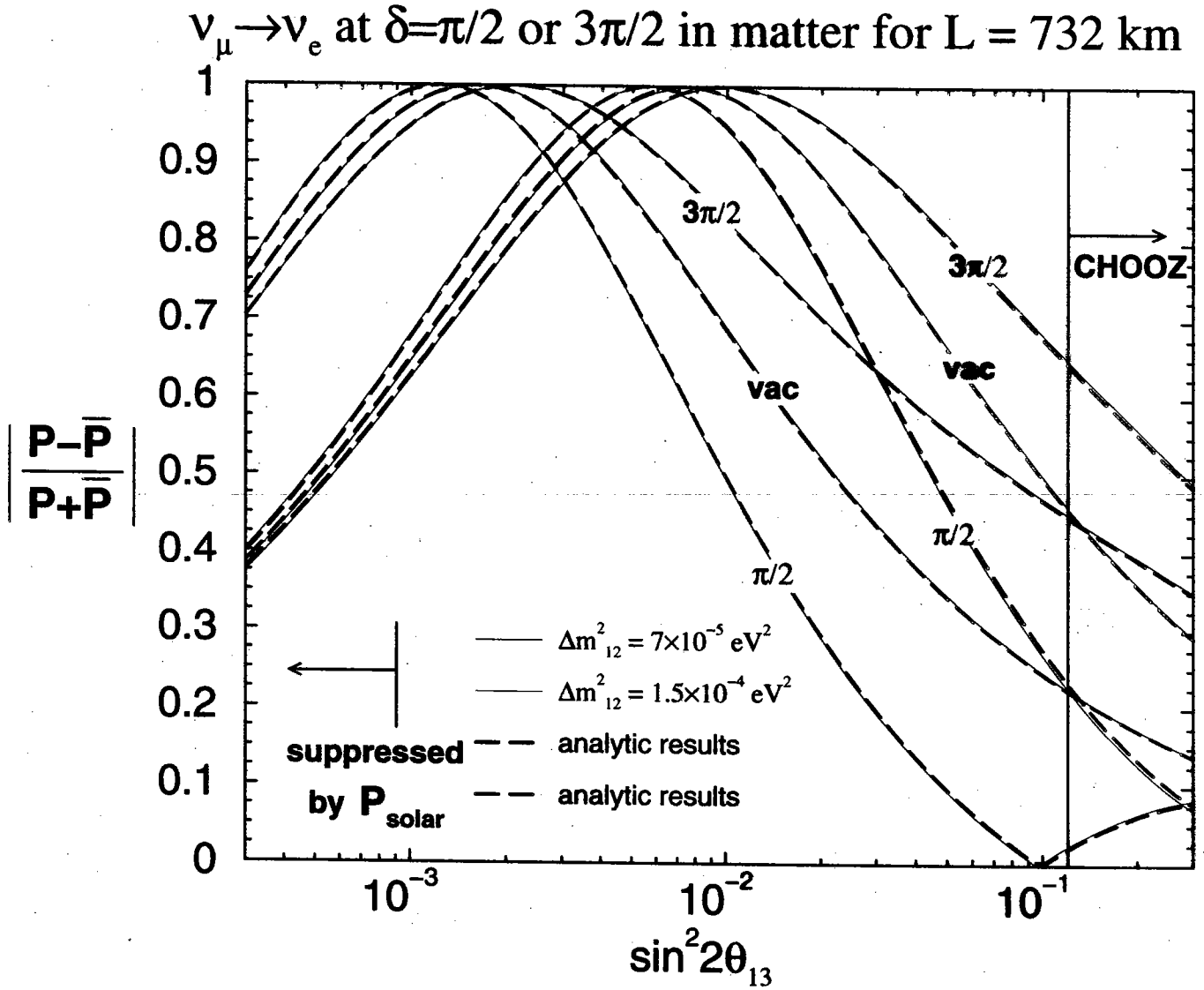
$$\left.\frac{E}{L}\right|_{NuMI} > \left.\frac{E}{L}\right|_{JHF}$$

$$E_{JHF} = 0.76\ \text{GeV}$$

$$E_{NuMI} = 2.22\ \text{GeV}$$

$$\left.\frac{E}{L}\right|_{NuMI} \approx \left.\frac{E}{L}\right|_{JHF}$$

$$(2.49 \approx 2.57) \times 10^{-3}\ \frac{\text{GeV}}{\text{km}}$$


$\nu_\mu \rightarrow \nu_e$ at $\delta=\pi/2$ or $3\pi/2$ in matter for L = 732 km
$\left|\frac{P-\bar{P}}{P+\bar{P}}\right|$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10^{-3}
10^{-2}
10^{-1}
$\sin^2 2\theta_{13}$
3π/2
3π/2
vac
vac
π/2
π/2
CHOOZ
suppressed
by P_{solar}
$\Delta m^2_{12} = 7\times10^{-5}$ eV^2
$\Delta m^2_{12} = 1.5\times10^{-4}$ eV^2
analytic results
analytic results

— Summary —

- $P(\nu_\mu \to \nu_e)$ at the Atmospheric δm^2 is a wonderful opportunity

- The eight fold degeneracies can be solved:
  - ν and $\bar{\nu}$ at O.M. to determine

    $\sin\theta_{23}\sin\theta_{13}$, $\cos\theta_{23}\sin\delta$
  - ν above O.M. to determine

    $\cos\theta_{23}\cos\delta \longrightarrow \cos^2\theta_{23} \lessgtr \frac{1}{2}$ (square + sum)
  - the two ν exp. could also determine the <u>hierarchy</u> if $E/L\,|_{NuMI} \leq E/L\,|_{JHF}$

- The Community needs to exploit this Opportunity Coherently:

- Help Wanted: Reactors, $0\nu\beta\beta$ decay, Supernova, cosmology ....